Development

Contents

Basics and Commonalities

Before stepping into fitting development patterns, its worth reviewing the basics of Estimators. The main modeling API implemented by chainladder follows that of the scikit-learn estimator. An estimator is any object that learns from data.

Scikit-Learn API

The scikit-learn API is a common modeling interface that is used to construct and fit a countless variety of machine learning algorithms. The common interface allows for very quick swapping between models with minimal code changes. The `chainladder` package has adopted the interface to promote a standardized approach to fitting reserving models.

All estimator objects can optionally be configured with parameters to uniquely specify the model being built. This is done ahead of pushing any data through the model.

```
estimator = Estimator(param1=1, param2=2)
```

All estimator objects expose a `fit` method that takes a `Triangle` as input, `X`:

```
estimator.fit(X=data)
```

All estimators include a `sample_weight` option to the `fit` method to specify an exposure basis. If an exposure base is not applicable, then this argument is ignored.

```
estimator.fit(X=data, sample_weight=weight)
```

All estimators either `transform` the input Triangle or `predict` an outcome.

Transformers

All transformers include a `transform` method. The method is used to transform a Triangle and it will always return a Triangle with added features based on the specifics of the transformer.

```
transformed_data = estimator.transform(data)
```

Other than final IBNR models, `chainladder` estimators are transformers. That is, they return your `Triangle` back to you with additional properties.

Transforming can be done at the time of fit.

```
# Fitting and Transforming
estimator.fit(data)
transformed_data = estimator.transform(data)
# One line equivalent
transformed_data = estimator.fit_transform(data)
assert isinstance(transformed_data, cl.Triangle)
```

Predictors

All predictors include a `predict` method.

```
prediction = estimator.predict(new_data)
```

Predictors are intended to create new predictions. It is not uncommon to fit a model on a more aggregate view, say national level, of data and predict on a more granular triangle, state or provincial.

Parameter Types

Estimator parameters: All the parameters of an estimator can be set when it is instantiated or by modifying the corresponding attribute. These parameters define how you'd like to fit an estimator and are chosen before the fitting process. These are often referred to as hyperparameters in the context of Machine Learning, and throughout these documents. Most of the hyperparameters of the `chainladder` package take on sensible defaults.

```
estimator = Estimator(param1=1, param2=2)
assert estimator.param1 == 1
```

Estimated parameters: When data is fitted with an estimator, parameters are estimated from the data at hand. All the estimated parameters are attributes of the estimator object ending by an underscore. The use of the underscore is a key API design style of scikit-learn that allows for the quicker recognition of fitted parameters vs hyperparameters:

```
estimator.estimated_param_
```

In many cases the estimated parameters are themselves Triangles and can be manipulated using the same methods we learned about in the `Triangle` class.

```
import chainladder as cl
import pandas as pd
import matplotlib.pyplot as plt
plt.style.use('ggplot')
%config InlineBackend.figure_format = 'retina'
```

```
dev = cl.Development().fit(cl.load_sample('ukmotor'))
type(dev.cdf_)
```

latest

```
chainladder.core.triangle.Triangle
```

Commonalities

All “Development Estimators” are transformers and reveal common a set of properties when they are fit.

1. `ldf_` represents the fitted age-to-age factors of the model.
2. `cdf_` represents the fitted age-to-ultimate factors of the model.
3. All “Development estimators” implement the `transform` method.

`cdf_` is nothing more than the cumulative representation of the `ldf_` vectors.

```
dev = cl.Development().fit(cl.load_sample('raa'))
dev.ldf_.incr_to_cum() == dev.cdf_
```

```
True
```

Development

`Development` allows for the selection of loss development patterns. Many of the typical averaging techniques are available in this class: `simple`, `volume` and `regression` through the origin. Additionally, `Development` includes patterns to allow for fine-tuned exclusion of link-ratios from the LDF calculation.

```
raa = cl.load_sample('raa')
cl.Development(average='simple')
```

```
▾ Development
Development(average='simple')
```

Alternatively, you can provide a list to parameterize each development peri adjusting individual development periods the list must be the same length a `link_ratio` development axis.

latest

```
len(raa.link_ratio.development)
```

```
9
```

```
cl.Development(average=['volume']+['simple']*8)
```

```
▾ Development
Development(average=['volume', 'simple', 'simple', 'simple', 'simple', 'simple',
                     'simple', 'simple', 'simple'])
```

This approach works for `average`, `n_periods`, `drop_high` and `drop_low`.

Notice where you have not specified a parameter, a sensible default is chosen for you.

Omitting link ratios

There are several arguments for dropping individual cells from the triangle as well as excluding whole valuation periods or highs and lows. Any combination of the 'drop' arguments is permissible.

```
cl.Development(
    drop_high=[True]*5+[False]*4,
    drop_low=[True]*5+[False]*4).fit(raa)
```

```
▾ Development
Development(drop_high=[True, True, True, True, True, False, False, False,
                       False],
            drop_low=[True, True, True, True, True, False, False, False, False])
```

```
cl.Development(drop_valuation='1985').fit(raa)
```

```
▾ Development
Development(drop_valuation='1985')
```

latest

```
cl.Development(drop=[('1985', 12), ('1987', 24)]).fit(raa)
```

```
▾ Development
Development(drop=[('1985', 12), ('1987', 24)])
```

```
cl.Development(drop=('1985', 12), drop_valuation='1988').fit(raa)
```

```
▾ Development
Development(drop=('1985', 12), drop_valuation='1988')
```

When using `drop`, the earliest age of the `link_ratio` should be referenced. For example, use `12` to drop the `12-24` ratio.

Note

`drop_high` and `drop_low` are ignored in cases where the number of link ratios available for a given development period is less than 1.

Extended Link Ratio Family

The `Development` estimator is based on the regression framework known as the Extended Link Ratio Family (ELRF). A nice property of this family is that we not only get estimates for our patterns (`cdf_`, and `ldf_`), but also measures of variability of our estimates (`sigma_`, `std_err_` and `std_residuals_`). These variability properties are used to develop the stochastic features in the `MackChainladder` method, but even for deterministic methods these variability estimates can be used as a diagnostic tool to validate the appropriateness of using multiplicative link ratios.

The `std_residuals_` in particular is described by Barnett and Zehnwirth as a diagnostic that points to the inferiority of the chainladder method relative to the probablistic trend family.

```
raa = cl.load_sample('raa')
model = cl.Development().fit(raa)
model.std_residuals_
```

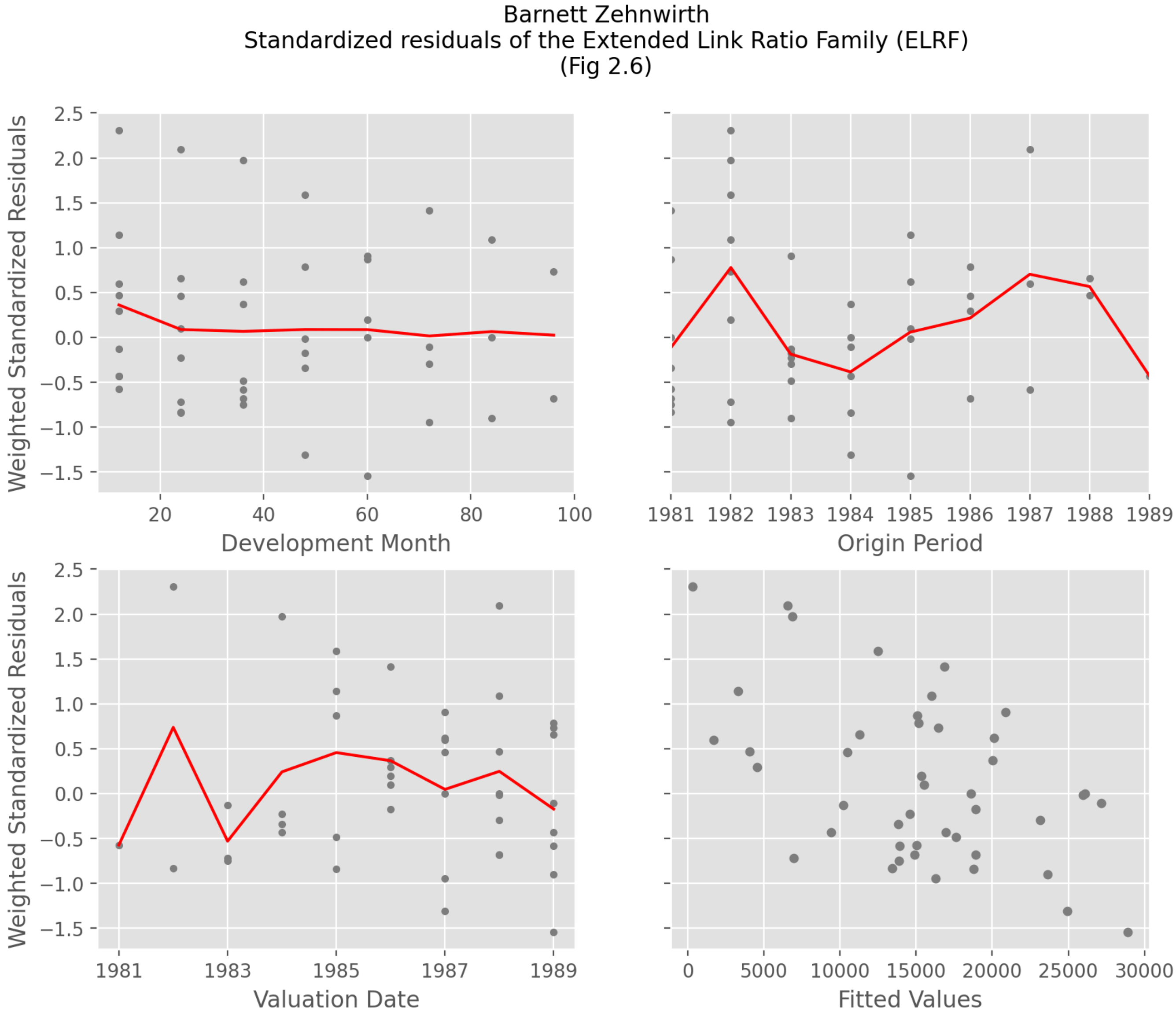
latest

	12	24	36	48	60	72	84	96	108
1981	-0.5722	-0.8317	-0.7489	-0.3442	0.8704	1.4143	-0.0003	-0.6819	
1982	2.3075	-0.7161	1.9716	1.5900	0.1982	-0.9488	1.0919	0.7315	
1983	-0.1267	-0.2299	-0.4811	-0.1780	0.9056	-0.2967	-0.8987		
1984	-0.4305	-0.8365	0.3723	-1.3074	0.0012	-0.1064			
1985	1.1398	0.0943	0.6175	-0.0170	-1.5437				
1986	0.2936	0.4633	-0.6809	0.7825					
1987	0.5961	2.0935	-0.5805						
1988	0.4717	0.6607							
1989	-0.4282								

Replicating **Fig 2.6** from their paper can be accomplished with a bit of manipulation of the residual triangles.

▶ Show code cell source

latest



These residual plots which should should look random are used to highlight whether the chainladder model is violated. Violations generally occur due to trends in the valuation axis which are not accounted for in the basic chainladder method.

[Barnett and Zehnwirth, 2000]

Transforming

When transforming a `Triangle`, you will receive a copy of the original triangle back along with the fitted properties of the `Development` estimator. Where the original Triangle contains all link ratios, the transformed version recognizes any ommissions you specify.

latest

```
triangle = cl.load_sample('raa')
dev = cl.Development(drop=('1982', 12), drop_valuation='1988')
```

```
transformed_triangle = dev.fit_transform(triangle)
transformed_triangle.ldf_
```

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
(All)	2.6625	1.5447	1.2975	1.1719	1.1134	1.0468	1.0294	1.0331	1.0092

```
transformed_triangle.link_ratio.heatmap()
```

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1981	1.6498	1.3190	1.0823	1.1469	1.1951	1.1130	1.0333		1.0092
1982		1.2593	1.9766	1.2921	1.1318	0.9934		1.0331	
1983	2.6370	1.5428	1.1635	1.1607	1.1857		1.0264		
1984	2.0433	1.3644	1.3489	1.1015		1.0377			
1985	8.7592	1.6556	1.3999		1.0087				
1986	4.2597	1.8157		1.2255					
1987	7.2172		1.1250						
1988		1.8874							
1989	1.7220								

By decoupling the `fit` and `transform` methods, we can apply our `Development` estimator to new data. This is a common pattern of the scikit-learn API. In this example we generate development patterns at an industry level and apply those patterns to individual companies.

```
clrd = cl.load_sample('clrd')
clrd = clrd[clrd['LOB']=='wkcomp']['CumPaidLoss']
# Summarize Triangle to industry level to estimate patterns
dev = cl.Development().fit(clrd.sum())
# Apply Industry patterns to individual companies
dev.transform(clrd)
```

Triangle Summary	
Valuation:	1997-12
Grain:	OYDY
Shape:	(132, 1, 10, 10)
Index:	[GRNAME, LOB]
Columns:	[CumPaidLoss]

Groupby

Triangles have a `groupby` method that follows pandas syntax and this allows for aggregating triangle data to a more reasonable level for any particular analysis. However, it is often the desire of an actuary to estimate development factors at a more aggregate grain generally and then apply it to a more detailed triangle.

We can, for example, pick volume-weighted development patterns at a Line of Business level and subsequently apply them to each company within the line of business as follows:

```
clrd = cl.load_sample('clrd')['CumPaidLoss']
clrd = cl.Development(groupby='LOB').fit_transform(clrd)

clrd.shape, clrd.ldf_.shape
```

```
((775, 1, 10, 10), (6, 1, 1, 9))
```

Notice we've retained the grain of the original triangle, but there are six sets of development patterns, one for each line of business. Using this transformed triangle in an IBNR esimtator will result in IBNR at the original grain but using patterns at the Line of Business grain.

It is worth noting that fitting and transforming are entirely decoupled from one another, and we could achieve the same outcome by directly aggregating the Triangle before passing to the fit method.

```
clrd = cl.load_sample('clrd')['CumPaidLoss']
model = cl.Development().fit(clrd.groupby('LOB').sum())
clrd = model.transform(clrd)
```

latest

```
clrd.shape, clrd.ldf_.shape
```

```
((775, 1, 10, 10), (6, 1, 1, 9))
```

This begs the question, why do we need a `groupby` hyperparameter as part of the `Development` estimator when we can aggregate the Triangle before fitting? In more advanced situations, we will be creating compound estimators called `Pipelines` which are very powerful for building custom workflows, but with the limitation that fitting and transforming have to be coupled together. You can explore this in more detail in the Pipeline section.

DevelopmentConstant

The `DevelopmentConstant` estimator simply allows you to hard code development patterns into a Development Estimator. A common example would be to include a set of industry development patterns in your workflow that are not directly estimated from any of your own data.

```
triangle = cl.load_sample('ukmotor')
patterns={12: 2, 24: 1.25, 36: 1.1, 48: 1.08, 60: 1.05, 72: 1.02}
cl.DevelopmentConstant(patterns=patterns, style='ldf').fit(triangle).ldf_
```

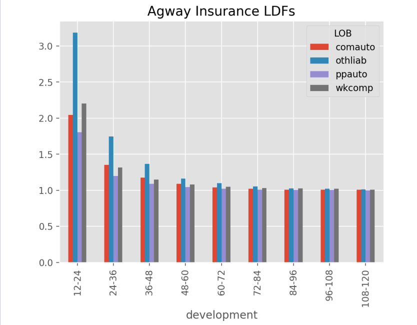
	12-24	24-36	36-48	48-60	60-72	72-84
(All)	2.0000	1.2500	1.1000	1.0800	1.0500	1.0200

By wrapping patterns in the `DevelopmentConstant` estimator, we can integrate into a larger workflow with tail extrapolation and IBNR calculations.

Examples

DevelopmentConstant Callable

latest



hard

IncrementalAdditive

The `IncrementalAdditive` method uses both the triangle of incremental losses and the exposure vector for each accident year as a base. Incremental additive ratios are computed by taking the ratio of incremental loss to the exposure (which has been adjusted for the measurable effect of inflation), for each accident year. This gives the amount of incremental loss in each year and at each age expressed as a percentage of exposure, which we then use to square the incremental triangle.

```
tri = cl.load_sample("ia_sample")
ia = cl.IncrementalAdditive().fit(
    X=tri['loss'],
    sample_weight=tri['exposure'].latest_diagonal)
ia.incremental_.round(0)
```

	12	24	36	48	60	72
2000	1,001.00	854.00	568.00	565.00	347.00	148.00
2001	1,113.00	990.00	671.00	648.00	422.00	164.00
2002	1,265.00	1,168.00	800.00	744.00	482.00	195.00
2003	1,490.00	1,383.00	1,007.00	849.00	543.00	220.00
2004	1,725.00	1,536.00	1,068.00	984.00	629.00	255.00
2005	1,889.00	1,811.00	1,256.00	1,157.00	740.00	300.00

These `incremental_` values are then used to determine an implied set of development patterns. Because incremental additive values are unique for [illegible] `[illegible]`, [illegible] too will be the `ldf_`.

latest

```
ia.ldf_
```

	12-24	24-36	36-48	48-60	60-72
2000	1.8531	1.3062	1.2332	1.1161	1.0444
2001	1.8895	1.3191	1.2336	1.1233	1.0426
2002	1.9233	1.3288	1.2301	1.1212	1.0438
2003	1.9282	1.3505	1.2188	1.1148	1.0418
2004	1.8904	1.3276	1.2274	1.1184	1.0429
2005	1.9586	1.3395	1.2335	1.1210	1.0438

Incremental calculation

The estimation of the incremental triangle can be done with varying hyperparameters of `n_period` and `average` similar to the `Development` estimator. Additionally, a `trend` in the origin period can also be selected.

Suppose there is a vector `zeta_` that represents an estimate of the incremental losses, `X` for a development period as a percentage of some exposure or `sample_weight`. Using a 'volume' weighted estimate for all origin periods, we can manually estimate `zeta_`.

```
zeta_ = tri['loss'].cum_to_incr().sum('origin') / tri['exposure'].sum('origin')
zeta_
```

	12	24	36	48	60	72
2000	0.2432	0.2220	0.1540	0.1419	0.0907	0.0368

The `zeta_` vector along with the `sample_weight` and optionally a `trend` are used to propagate incremental losses to the lower half of the `Triangle`. In the trivial case of no trend, we can estimate the incrementals for age 72.

```
zeta_.loc[..., 72] * tri['exposure'].latest_diagonal
```

latest

	72
2000	148.00
2001	163.85
2002	195.43
2003	220.11
2004	255.15
2005	299.97

These are the same incrementals that the IncrementalAdditive method produces.

```
zeta_.loc[..., 72]*tri['exposure'].latest_diagonal == ia.incremental_.loc[...,
```

```
True
```

Trending

The `IncrementalAdditive` method supports trending through the `trend` and the `future_trend` hyperparameters. The `trend` parameter is used in the fitting of `zeta_` and it trends all inner diagonals of the `Triangle` to its `latest_diagonal` before estimating `zeta_`.

The `future_trend` hyperparameter is used to trend beyond the `latest_diagonal` into the lower half of the `Triangle`. If no future trend is supplied, then the `future_trend` is assumed to be that of the `trend` parameter.

```
cl.IncrementalAdditive(trend=0.02, future_trend=0.05).fit(
    X=tri['loss'],
    sample_weight=tri['exposure'].latest_diagonal
).incremental_.round(0)
```

latest

	12	24	36	48	60	72
2000	1,001.00	854.00	568.00	565.00	347.00	148.00
2001	1,113.00	990.00	671.00	648.00	422.00	172.00
2002	1,265.00	1,168.00	800.00	744.00	511.00	215.00
2003	1,490.00	1,383.00	1,007.00	908.00	604.00	255.00
2004	1,725.00	1,536.00	1,151.00	1,105.00	735.00	310.00
2005	1,889.00	1,967.00	1,420.00	1,364.00	907.00	383.00

> **Note**
>
> These trend assumptions are applied to the incremental Triangle which produces drastically different answers from the same trends applied to a cumulative Triangle.

A nice property of this estimator is that it really only requires incremental amounts so a `Triangle` that has cumulative data censored data in earlier diagonals can leverage this method. Another nice property is that it allows for more explicit recognition of future inflation in your estimate via the `trend` factor.

[Schmidt, 2006]

MunichAdjustment

The `MunichAdjustment` is a bivariate adjustment to loss development factors. There is a fundamental correlation between the paid and the case incurred data of a triangle. The ratio of paid to incurred **(P/I)** has information that can be used to simultaneously adjust the basic development factor selections for the two separate triangles.

Depending on whether the momentary **(P/I)** ratio is below or above average, one should use an above-average or below-average paid development factor and/or a below-average or above-average incurred development factor. In doing so, the model replaces a set of development patterns that would be used for all `origins` with individualized development curves that reflect the unique levels of **(P/I)** per origin period.

BerquistSherman Comparison

This method is similar to the `BerquistSherman` approach in that it tries to adjust for case reserve adequacy. However it is different in two distinct ways.

1. The `BerquistSherman` method is a direct adjustment to the data whereas the MunichAdjustment keeps the `Triangle` intact and adjusts the development patterns.
2. The MunichAdjustment is built in the context of a stochastic framework.

Residuals

The MunichAdjustment uses the correlation between the residuals of the univariate (basic) model and the (P/I) model. These correlations spin off a property `lambda_` which is represented by the line through the origin of the correlation plots.

With the correlations, `lambda_` known, the basic development patterns can be adjusted based on the **(P/I)** ratio at any given cell of the `Triangle`.

Examples

MunichAdjustment Residuals	MunichAdjustment Basics
medium	easy

[Quarg and Mack, 2004]

ClarkLDF

`ClarkLDF` estimates growth curves of the form 'loglogistic' or 'weibull' for the incremental loss development of a `Triangle`. These growth curves are monotonic increasing and are more relevant for paid data. While the model can be used for case incurred data, if there is too much "negative" development, other Estimators should be used.

latest

The `Loglogistic` Growth Function:

$G(x|\omega, \theta) =\frac{x^{\omega }}{x^{\omega } + \theta^{\omega }}$

The `Weibull` Growth Function:

$G(x|\omega, \theta) =1-exp(-\left (\frac{x}{\theta} \right)^\omega)$

Parameterized growth curves can produce patterns for any age and can even be used to estimate a tail beyond the latest age in a Triangle. In general, the loglogistic growth curve produces a larger tail than the weibull growth curve.

LDF and Cape Cod methods

Clark approaches curve fitting with two different methods, an LDF approach and a Cape Cod approach. The LDF approach only requires a loss triangle whereas the Cape Cod approach would also need a premium vector. Choosing between the two methods occurs at the time you fit the estimator. When a premium vector is included, the Cape Cod method is invoked.

A simple example of using `ClarkLDF` LDF Method. Upon fitting the Estimator, we obtain both `omega_` and `theta_`.

```
clrd = cl.load_sample('clrd').groupby('LOB').sum()
dev = cl.ClarkLDF(growth='weibull').fit(clrd['CumPaidLoss'])
dev.omega_
```

	CumPaidLoss
LOB	
comauto	0.928929
medmal	1.569649
othliab	1.330082
ppauto	0.831529
prodliab	1.456171
wkcomp	0.898279

Perhaps more useful than the parameters is the growth curve `G_` function they represent which can be used to deetermine the development factor at any age.

latest

```
1/dev.G_(37.5).to_frame()
```

```
LOB
comauto     1.270899
medmal      1.707238
othliab     1.619168
ppauto      1.118801
prodliab    2.126153
wkcomp      1.311587
dtype: float64
```

Another example showing the usage of the `ClarkLDF` Cape Cod approach. With the Cape Cod, an Expected Loss Ratio is included as an extra feature in the `elr_` property.

```
cl.ClarkLDF().fit(
    X=clrd['CumPaidLoss'],
    sample_weight=clrd['EarnedPremDIR'].latest_diagonal
).elr_
```

	CumPaidLoss
LOB	
comauto	0.680325
medmal	0.701443
othliab	0.623806
ppauto	0.825932
prodliab	0.671024
wkcomp	0.697930

Residuals

Clark's model assumes Incremental losses are independent and identically distributed. To ensure compatibility with this assumption, he suggests reviewing the "Normalized Residuals" of the fitted incremental losses to ensure the assumption is not violated.

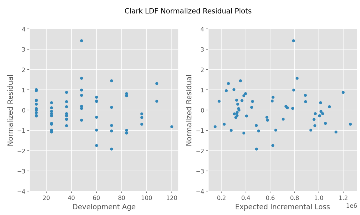
Stochastics

latest

Using MLE to solve for the growth curves, we can produce statistics about t
process uncertainty of our model.

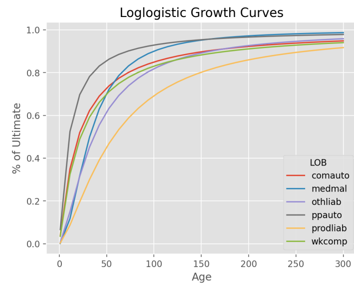
Examples

ClarkLDF Residuals



medium

ClarkLDF Growth Curves



easy

[Clark, 2003]

CaseOutstanding

The `CaseOutstanding` method is a deterministic method that estimates incremental payment patterns from prior lag carried case reserves. Included in this is also patterns for the carried case reserves based on the prior lag carried case reserve.

Like the MunichAdjustment and `BerquistSherman`, this estimator is useful when you want to incorporate information about case reserves into paid ultimates.

To use it, a triangle with both paid and incurred amounts must be available.

```
tri = cl.load_sample('usauto')
model = cl.CaseOutstanding(paid_to_incurred=('paid', 'incurred')).fit(tri)
model.paid_ldf_
```

```
=== in _set_ldf ===
=== self.case ===
                12            24            36            48            60
1998  1.798098e+07  9.602188e+06  5.414189e+06  2.867316e+06  1.405073e+06  722
1999  1.968226e+07  1.051070e+07  5.926455e+06  3.138608e+06  [illegible]
2000  2.181513e+07  1.164970e+07  6.568678e+06  3.478725e+06  [illegible]
2001  1.904741e+07  1.017168e+07  5.735298e+06  3.037373e+06  [illegible]
2002  1.956232e+07  1.044665e+07  5.890341e+06  3.119483e+06  1.528642e+06  785
2003  2.091944e+07  1.117138e+07  6.298978e+06  3.335894e+06  1.634690e+06  839
```

```
2004  2.007969e+07  1.072294e+07  6.046125e+06  3.201985e+06  1.569070e+06  806
2005  2.039616e+07  1.089194e+07  6.141416e+06  3.252450e+06  1.593800e+06  818
2006  2.066376e+07  1.103484e+07  6.221991e+06  3.295122e+06  1.614710e+06  829
2007  2.162359e+07  1.154741e+07  6.511004e+06  3.448181e+06  1.689714e+06  868
=== self.paid ===
              12            24            36            48            60
1998  18539254.0  3.323104e+07  4.006201e+07  4.389204e+07  4.589654e+07  4.676
1999  20410193.0  3.609068e+07  4.325940e+07  4.715924e+07  4.920853e+07  5.016
2000  22120843.0  3.897601e+07  4.638928e+07  5.056238e+07  5.273528e+07  5.374
2001  22992259.0  4.009620e+07  4.776784e+07  5.209392e+07  5.436344e+07  5.537
2002  24092782.0  4.179531e+07  4.990380e+07  5.435288e+07  5.675438e+07  5.780
2003  24084451.0  4.139961e+07  4.907033e+07  5.358420e+07  5.593065e+07  5.697
2004  24369770.0  4.148986e+07  4.923668e+07  5.377467e+07  5.600590e+07  5.700
2005  25100697.0  4.270223e+07  5.064499e+07  5.499527e+07  5.726166e+07  5.827
2006  25608776.0  4.360650e+07  5.144103e+07  5.584838e+07  5.814450e+07  5.917
2007  27229969.0  4.545463e+07  5.365308e+07  5.826515e+07  6.066793e+07  6.174
=== set LDF return ===
             Triangle Summary
Valuation:           2261-12
Grain:                  OYDY
Shape:         (1, 2, 10, 9)
Index:               [Total]
Columns:    [incurred, paid]
```

	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132
(All)	0.8428	0.7100	0.7084	0.6968	0.6376	0.6220	0.5534	0.4374	0.5243

In the example above, the incremental paid losses during the period 12-24 is expected to be 84.28% of the outstanding case reserve at lag 12. The set of patterns produced by `CaseOutstanding` don't follow the multiplicative approach commonly used in the various IBNR methods making them not directly usable. Because of this, the estimator determines the 'implied' multiplicative pattern so that a broader set of IBNR methods can be used. Due to the origin period specifics on case reserves, each origin gets its own set of multiplicative `ldf_` patterns.

```
model.ldf_['paid']
```

latest

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1998	1.7925	1.2056	1.0956	1.0457	1.0189	1.0097	1.0048	1.0023	1.0019
1999	1.7683	1.1986	1.0902	1.0435	1.0194	1.0092	1.0050	1.0024	1.0019
2000	1.7620	1.1902	1.0900	1.0430	1.0191	1.0101	1.0046	1.0024	1.0020
2001	1.7439	1.1913	1.0906	1.0436	1.0187	1.0090	1.0042	1.0021	1.0017
2002	1.7348	1.1940	1.0892	1.0442	1.0186	1.0085	1.0041	1.0020	1.0016
2003	1.7189	1.1853	1.0920	1.0438	1.0186	1.0092	1.0045	1.0022	1.0018
2004	1.7025	1.1867	1.0922	1.0415	1.0179	1.0088	1.0043	1.0021	1.0017
2005	1.7012	1.1860	1.0859	1.0412	1.0177	1.0087	1.0042	1.0021	1.0017
2006	1.7028	1.1797	1.0857	1.0411	1.0177	1.0087	1.0042	1.0021	1.0017
2007	1.6693	1.1804	1.0860	1.0412	1.0178	1.0087	1.0042	1.0021	1.0017

Incremental patterns

The incremental patterns of the `CaseOutstanding` method are avilable as additional properties for review. They are the `paid_to_prior_case_` and the `case_to_prior_case_`. These are useful to review when deciding on the appropriate hyperparameters for `paid_n_periods` and `case_n_periods`. Once you are satisfied with your hyperparameter tuning, you can see the fitted selections in the `paid_ldf_` and `case_ldf_` incremental patterns.

```
model.case_to_prior_case_
```

latest

	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132
1998	0.5378	0.5541	0.5253	0.4981	0.5329	0.5380	0.5877	0.6970	0.5798
1999	0.5368	0.5649	0.5442	0.4969	0.5029	0.5800	0.6420	0.6506	
2000	0.5461	0.5742	0.5391	0.4872	0.5376	0.5432	0.6655		
2001	0.5406	0.5660	0.5148	0.5013	0.5077	0.5414			
2002	0.5409	0.5546	0.5406	0.4802	0.4881				
2003	0.5265	0.5765	0.5363	0.4764					
2004	0.5298	0.5665	0.5069						
2005	0.5215	0.5539							
2006	0.5261								
2007									

```
model.case_ldf_
```

	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132
(All)	0.5340	0.5638	0.5296	0.4900	0.5139	0.5506	0.6317	0.6738	0.5798

[Friedland, 2010]

TweedieGLM

The `TweedieGLM` implements the GLM reserving structure discussed by Taylor and McGuire. A nice property of the GLM framework is that it is highly flexible in terms of including covariates that may be predictive of loss reserves while maintaining a close relationship to traditional methods. Additionally, the framework can be extended in a straightforward way to incorporate various approaches to measuring prediction errors. Behind the scenes, `TweedieGLM` is using scikit-learn's `TweedieRegressor` estimator.

Long Format

GLMs are fit to triangles in "Long Format". That is, they are converted to pa behind the scenes. Each axis of the `Triangle` is included in the dataframe. The `origin` and

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`development` axes are in columns of the same name. You can inspect what your `Triangle` looks like in long format by calling `to_frame` with `keepdims=True`

```
cl.load_sample('clrd').to_frame(keepdims=True).reset_index().head()
```

	GRNAME	LOB	origin	development	IncurLoss	CumPaidLoss	BulkLoss	Ea
0	Adriatic Ins Co	othliab	1995-01-01	12	8.0	NaN	8.0	
1	Adriatic Ins Co	othliab	1995-01-01	24	11.0	NaN	4.0	
2	Adriatic Ins Co	othliab	1995-01-01	36	7.0	3.0	4.0	
3	Adriatic Ins Co	othliab	1996-01-01	12	40.0	NaN	40.0	
4	Adriatic Ins Co	othliab	1996-01-01	24	40.0	NaN	40.0	

Warning

'origin', 'development', and 'valuation' are reserved keywords for the dataframe. Declaring columns with these names separately will result in error.

While you can inspect the `Triangle` in long format, you will not directly convert to long format yourself. The `TweedieGLM` does this for you. Additionally, the `origin` of the design matrix is restated in years from the earliest origin period. That is, is if the earliest origin is '1995-01-01' then it gets replaced with 0. Consequently, '1996-04-01' would be replaced with 1.25. This is done because datetimes have limited support in scikit-learn. Finally, the `TweedieGLM` will automatically convert the response to an incremental basis.

R-style formulas

We use the `patsy` library to allow formulation of the the feature set `X` of the GLM. Because `X` is a parameter that used extensively throughout `chainladder`, the `Tweed:` ... the `design_matrix`. Those familiar with the R programming language will

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notation used by `patsy`. For example, we can include both `origin` and `development` as terms in a model.

```
genins = cl.load_sample('genins')
glm = cl.TweedieGLM(design_matrix='development + origin').fit(genins)
glm.coef_
```

	coef_
Intercept	13.516322
development	-0.006251
origin	0.033863

ODP Chainladder

Replicating the results of the volume weighted chainladder development patterns can be done by fitting a Poisson-log GLM to incremental paids. To do this, we can specify the `power` and `link` of the estimator as well as the `design_matrix`. The volume-weighted chainladder method can be replicated by including both `origin` and `development` as categorical features.

```
dev = cl.TweedieGLM(
    design_matrix='C(development) + C(origin)',
    power=1, link='log').fit(genins)
```

A trivial comparison against the traditional `Development` estimator shows a comparable set of `ldf_` patterns.

Parsimonious modeling

Having full access to all axes of the `Triangle` along with the powerful formulation of `patsy` allows for substantial customization of the model fit. For example, we can include 'LOB' interactions with piecewise linear coefficients to reduce model complexity.

```
clrd = cl.load_sample('clrd')['CumPaidLoss'].groupby('LOB').sum()
clrd=clrd[clrd['LOB'].isin(['ppauto', 'comauto'])]
dev = cl.TweedieGLM(
    design_matrix='LOB+LOB:C(np.minimum(development, 36))+LOB:development+LOB:o
```

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```
    max_iter=1000).fit(clrd)
dev.coef_
```

	coef_
Intercept	12.550094
LOB[T.ppauto]	3.202525
LOB[comauto]:C(np.minimum(development, 36))[T.24]	0.578636
LOB[ppauto]:C(np.minimum(development, 36))[T.24]	0.449818
LOB[comauto]:C(np.minimum(development, 36))[T.36]	0.790583
LOB[ppauto]:C(np.minimum(development, 36))[T.36]	0.321155
LOB[comauto]:development	-0.044631
LOB[ppauto]:development	-0.054813
LOB[comauto]:origin	0.054570
LOB[ppauto]:origin	0.057791

This model is limited to 10 coefficients across two lines of business. The basic chainladder model is known to be overparameterized with at least 18 parameters requiring estimation. Despite drastically simplifying the model, the `cdf_` patterns of the GLM are within 1% of the traditional chainladder for every lag and for both lines of business:

```
((dev.cdf_.iloc[..., 0, :] /
  cl.Development().fit(clrd).cdf_) - 1
).to_frame().round(3)
```

development	12-Ult	24-Ult	36-Ult	48-Ult	60-Ult	72-Ult	84-Ult	96-Ult	108-Ult
LOB									
comauto	0.002	0.003	-0.01	0.003	0.011	0.008	0.005	-0.000	-0.002
ppauto	0.006	0.003	-0.00	0.001	0.002	0.001	0.001	0.001	0.001

Like every other Development estimator, the `TweedieGLM` produces a set of `ldf_` patterns and can be used in a larger workflow with tail extrapolation and reserve estimation

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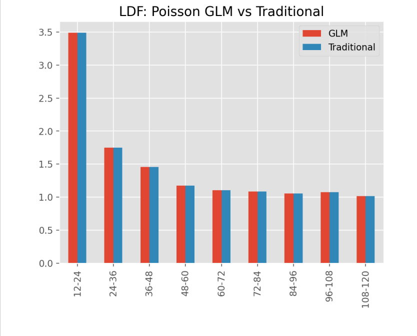
TweedieGLM Basics



easy

[Taylor and G., 2016]

DevelopmentML

`DevelopmentML` is a general development estimator that works as an interface to scikit-learn compliant machine learning (ML) estimators. The `TweedieGLM` is a special case of `DevelopmentML` with the ML algorithm limited to scikit-learn's `TweedieRegressor` estimator.

The Interface

ML algorithms are designed to be fit against tabular data like a pandas DataFrame or a 2D numpy array. A `Triangle` does not meet the definition and so `DevelopmentML` is provided to incorporate ML into a broader reserving workflow. This includes:

1. Automatic conversion of Triangle to a dataframe for fitting
2. Flexibility in expressing any preprocessing as part of a scikit-learn `Pipeline`
3. Predictions through the terminal development age of a `Triangle` to fill in the lower half
4. Predictions converted to `ldf_` patterns so that the results of the estimator are compliant with the rest of `chainladder`, like tail selection and IBNR modeling.

Features

Data from any axis of a `Triangle` is available to be used in the `DevelopmentML` estimator. For example, we can use many of the scikit-learn components to generate deve latest from both the time axes as well as the `index` of the `Triangle`.

```
from sklearn.ensemble import RandomForestRegressor
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import OneHotEncoder
from sklearn.compose import ColumnTransformer

clrd = cl.load_sample('clrd').groupby('LOB').sum()['CumPaidLoss']

# Decide how to preprocess the X (ML) dataset using sklearn
design_matrix = ColumnTransformer(transformers=[
    ('dummy', OneHotEncoder(drop='first'), ['LOB', 'development']),
    ('passthrough', 'passthrough', ['origin'])
])

# Wrap preprocessing and model in a larger sklearn Pipeline
estimator_ml = Pipeline(steps=[
    ('design_matrix', design_matrix),
    ('model', RandomForestRegressor())
])

# Fitting DevelopmentML fits the underlying ML model and gives access to ldf_
cl.DevelopmentML(estimator_ml=estimator_ml, y_ml='CumPaidLoss').fit(clrd).ldf_
```

	Triangle Summary
Valuation:	2261-12
Grain:	OYDY
Shape:	(6, 1, 10, 9)
Index:	[LOB]
Columns:	[CumPaidLoss]

Autoregressive

The time-series nature of loss development naturally lends to an urge for autoregressive features. That is, features that are based on predictions, albeit on a lagged basis. `DevelopmentML` includes an `autoregressive` parameter that can be used to express the response as a lagged feature as well.

Note

When using `autoregressive` features, you must also declare it as a `estimator_ml` Pipeline.

PatsyFormula

While the sklearn preprocessing API is powerful, it can be tedious work with in some instances. In particular, modeling complex interactions is much easier to do with Patsy. The `chainladder` package includes a custom sklearn estimator to gain access to the patsy API. This is done through the `PatsyFormula` estimator.

```
estimator_ml = Pipeline(steps=[
    ('design_matrix', cl.PatsyFormula('LOB:C(origin)+LOB:C(development)+develop
    ('model', RandomForestRegressor())
])
cl.DevelopmentML(
    estimator_ml=estimator_ml,
    y_ml='CumPaidLoss').fit(clrd).ldf_.iloc[0, 0, 0].round(2)
```

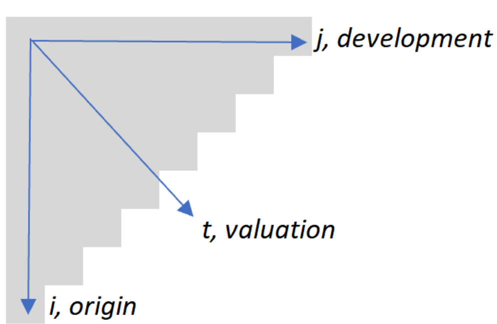
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
(All)	2.6100	1.4100	1.1900	1.1000	1.0400	1.0200	1.0100	1.0100	1.0100

Note

`PatsyFormula` is not an estimator designed to work with triangles. It is an sklearn transformer designed to work with pandas DataFrames allowing it to work directly in an sklearn Pipeline.

BarnettZehnwirth

The `BarnettZehnwirth` estimator solves for development patterns using the Probabilistic Trend Family (PTF) regression framework. Unlike the ELRF framework, which assumes no `valuation` covariate, the PTF framework allows for this.



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Structurally, the PTF regression is different from the `ELRF` (ELRF) regression framework in two distinct ways:

1. Where the ELRF fits independent regressions to each adjacent development lag, the PTF regression is fit to the entire triangle
2. Where the ELRF is fit to cumulative amounts, the PTF is fit to the log of the incremental amounts of the `Triangle`.

Formulation

The PTF framework is an ordinary least squares (OLS) model with the response, `y` being the log of the incremental amounts of a Triangle. These are assumed to be normally distributed which implies the incrementals themselves are log-normal distributed.

The framework includes coefficients for origin periods (alpha), development periods (gamma) and calendar period (iota).

$y(i, j) = \alpha *{i}* + \sum{k=1}^{j}\gamma *{k}*+ \sum{\iota =1}^{i+j}\gamma _{\iota}+ \varepsilon _{i,j}$

These coefficients can be categorical or continuous, and to support a wide range of model forms, patsy formulas are used.

```
abc = cl.load_sample('abc')

# Discrete origin, development, valuation
cl.BarnettZehnwirth(formula='C(origin)+C(development)').fit(abc).coef_.T
```

	Intercept	C(origin) [T.1.0]	C(origin) [T.2.0]	C(origin) [T.3.0]	C(origin) [T.4.0]	C(origin) [T.5.0]	C(origin) [T.6.0]	C(or [T
coef_	11.836863	0.178824	0.345112	0.378133	0.405093	0.427041	0.431076	0.66

1 rows × 21 columns

```
# Linear coefficients for origin, development, and valuation
cl.BarnettZehnwirth(formula='origin+development+valuation').f
```

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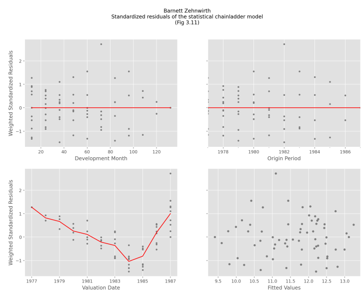
	Intercept	origin	development	valuation
coef_	8.359157	4.215981	0.319288	-4.116569

The PTF framework is particularly useful when there is calendar period inflation influences on loss development.

PTF Residuals



medium

[Barnett and Zehnwirth, 2000]

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Triangle

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Tail Estimators

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