

1.

(i) String of n bits sent across lossy link. In how many ways can 2 bit errors occur? If $n=3$, list set of possible patterns.

N choose 2 ways.

When $N=3$: {Error, Error, Correct}, {Error, Correct, Error}, {Correct, Error, Error}

(ii) In how many ways can letters MISSISSIPPI be rearranged?

1 M, 4 S, 2 P, 4 I

$$11 C 4 * 7 C 4 * 3 C 2 * 1 = 34650$$

(iii) How many straight flushes are possible in poker?

$$10 * 4 \text{ suits} = 40$$

2.

(i) Six people get in elevator at ground floor, with 10 upper floors. Assuming each person gets off uniformly at random what is the probability no two people get off at the same floor?

$$10 * 9 * 8 * 7 * 6 * 5 / 10^6$$

3.

(i) A random variable X has $P(X=x)=x/15$ for $x=1, 2, 3, 4, 5$, and $P(X=x)$ is 0 for all other values. What is the mean and variance of X ?

$$1/15 + 4/15 + 9/15 + 16/15 + 25/15 = 55/15$$

$$E[X^2] = 1/15 + 16/15 + 81/15 + 256/15 + 525/15 = 879/15$$

$$\text{Var} = (55/15)^2 - 879/15$$

(ii) You perform the following experiment: you take a six-sided die, and roll it. If the number that comes up is six, you stop; otherwise you repeat.

(a) What is the distribution of the number of times you roll the die?

$$\left(\frac{5}{6}\right)^{n-1} \left(\frac{1}{6}\right)$$

(b) What is the expected number of rolls?

$$\frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)(2) + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)(3) + \dots$$