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Interactions Between 3D Surface Shape and Material Perception

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Abstract

Our visual systems are remarkably adept at deriving the shape and material properties of surfaces even when only one image of a surface is available. This ability implies that a single image of a surface contains potent information about both surface shape and material. However, from a computational perspective, the problem of deriving surface shape and material is formally ill posed. Any given image could be due to many combinations of shape, material, and illumination. Early computational models required prior knowledge about two of the three scene variables to derive the third. However, such models are biologically implausible because our visual systems are tasked with extracting all relevant scene variables from images simultaneously. This review describes recent progress in understanding how the visual system solves this problem by identifying complex forms of image structure that support its ability to simultaneously derive the shape and material properties of surfaces from images.

INTRODUCTION

Our visual systems exploit the structure in light to derive properties of our environment. This structure is created by the way that light interacts with the shape and material properties of surfaces, substances, and objects in the world. The 3D shape of surfaces and the position of light sources determine how much illumination reaches the surface. The material properties of surfaces determine what occurs after light strikes a surface, which could include a variety of possible optical behaviors. For example, glossy materials reflect some proportion of light at the interface between their surfaces and the surrounding medium. Opaque, matte materials allow light to penetrate the superficial layers of a surface, where it is internally scattered and re-emitted approximately equally in all directions. Translucent materials allow light to penetrate more deeply into a material, where it is internally scattered before some light re-emerges toward the viewer—often re-emerging from a different location than where the light entered. Somehow, the brain has to exploit these different forms of optical interactions to derive the material properties of surfaces.

From a computational point of view, the problem of deriving both shape and material from an image of a surface should be impossible. Mathematically, it is an ill-posed problem: Any given image is a product of three scene variables—shape, material, and illumination. It is not possible to invert the image formation process and recover the true scene variables because there are a potentially infinite number of different combinations of these variables that could generate any given image. Yet despite the computational impossibility, our brains somehow manage to generate a percept of 3D surface shape and material, even when we view a single, static image. This review focuses on describing what is currently known about how our visual systems accomplish this seemingly impossible computational feat.

One of the central themes that emerges in this review is that progress on this problem has been made by searching for increasingly complex and informative forms of image structure. Early computational models required prior knowledge about two of the three variables to derive the third (unknown) variable. For example, models of shape from shading required knowledge of both surface reflectance (i.e., that the surface is opaque and matte) and illumination direction to derive estimates of 3D shape (Horn 1975, Ikeuchi & Horn 1981). In contrast, more recent models of shape from shading have attempted to eliminate the reliance on knowledge of illumination by identifying forms of image structure that remain invariant to changes in illumination (Kunsberg & Zucker 2013, 2018; Xiong et al. 2013). In a related vein, some recent work has also attempted to eliminate the reliance on knowledge of material by identifying forms of image structure that can simultaneously provide information about the shape and the material properties of surfaces (Marlow & Anderson 2021; Marlow et al. 2019, 2023). A key insight from this work is that information about shape and material is conveyed by particular relationships that arise among different kinds of image structure (such as contours, shading, and specular reflections)—not by any one particular form of structure alone. Whereas most work has attempted to understand the problem of shape and that of material perception as separate problems, the work aiming to derive both shape and material is based on the premise that both problems are coupled; one cannot be solved without also solving the other.

The sections that follow consider how the problems of shape and material are related for three different classes of materials: matte and opaque, glossy, and translucent. These sections show how different materials generate particular signatures of their shape and material properties in images and how these variables appear to be exploited by the visual system.

SURFACE SHADING: COMPUTATIONAL WORK FOCUSED ON ORIENTATION

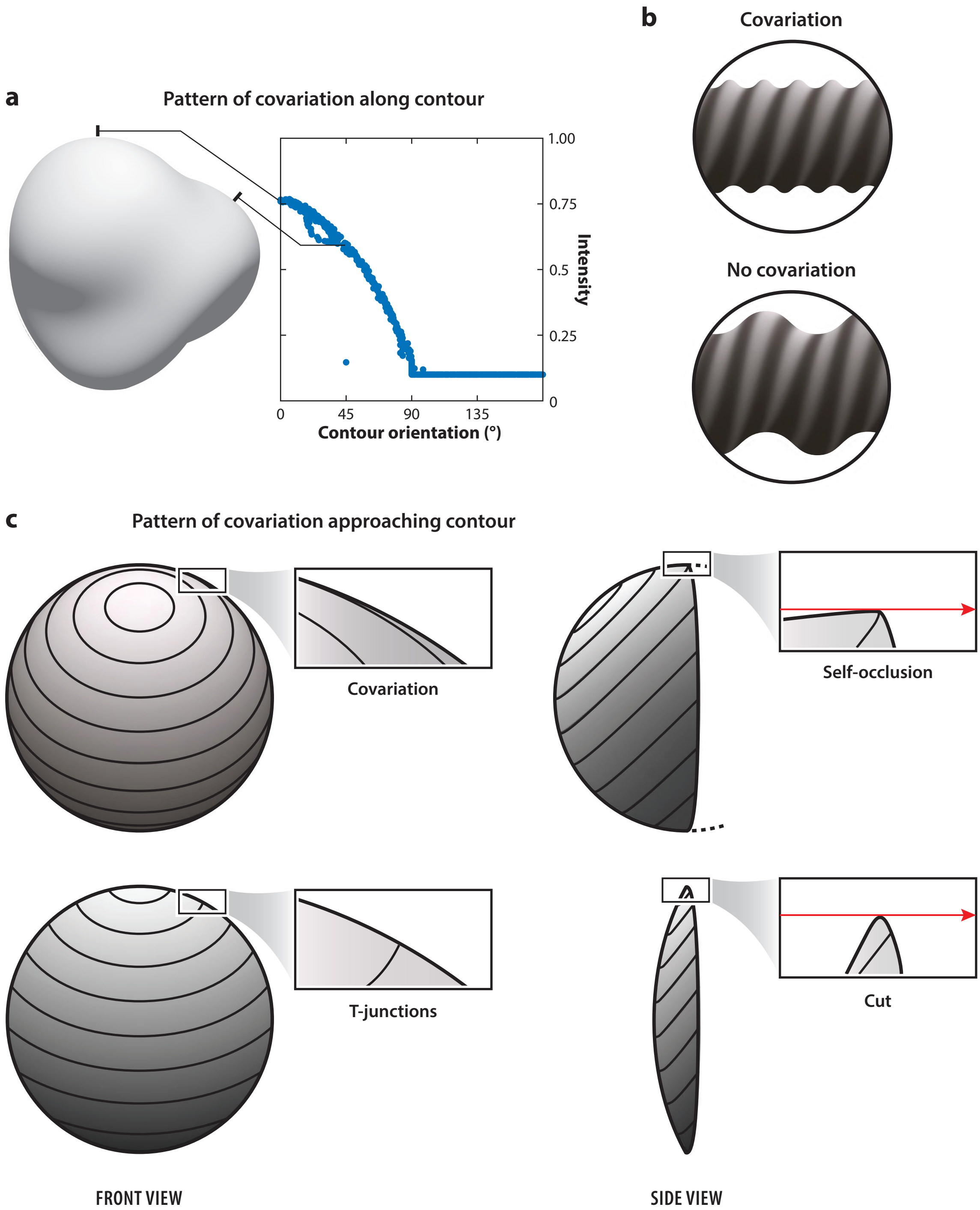
A single, stationary image of a shaded surface can elicit compelling experiences of 3D shape, material, and illumination. **Figure 1*a*** depicts an example in which 3D shape resembles a bumpy potato, composed of a material that appears opaque and matte (i.e., dull, not shiny) and illuminated by a light source approximately 45° above the viewing direction. One of the most puzzling problems in vision science is understanding what information exists in such images that enables the visual system to construct our experience of all three scene variables from single images such as this. Early computational models compute shape from shading by assuming that material and illumination direction are somehow both known (e.g., Horn 1975, Ikeuchi & Horn 1981); however, the visual system never has any of this information. Our brains somehow derive all three scene variables without knowing any.

The amount of light that a shaded surface projects toward the viewer depends on two scene variables: the proportion of light reflected by the surface and the amount of light falling on the surface, the latter of which depends on the surface geometry and illumination. The proportion of reflected light is called albedo or diffuse reflectance and is associated with our experience of lightness; black surfaces reflect approximately 5% of the light falling on them, and white surfaces can reflect roughly 90% (Murray 2021). The amount of light reflected from a diffuse surface does not depend on the observer's viewing direction; surfaces with a diffuse (or Lambertian) reflectance function reflect equal amounts of light to all possible viewing directions. Lambertian surfaces appear matte; the amount of light that they project to an observer is simply the product of the albedo and the amount of light falling onto the surface.

The amount of light falling onto a surface mainly depends on the surface's (3D) orientation and the illumination direction. A shaded surface is brightest when it faces the illumination, i.e., when the direction perpendicular to the surface (the surface normal) points at the illuminant. A shaded surface is increasingly dark where it is oriented more side-on to the illuminant. There are contexts where the amount of light falling on the surface does not depend only on orientation: One surface can cast shadows onto another, light from one surface can illuminate another through interreflections, and the amount of light can vary as a function of both surface orientation and distance for illuminants located very near a surface. However, nearly all computational work begins with the assumption that variations in surface orientation are the sole cause of the variations in image intensity (see Todd et al. 2014; e.g., Horn 1975, Ikeuchi & Horn 1981). Below, we use the term shading to refer to variations in image intensity caused by variations in surface orientation.

Early models focused on computing a surface's (3D) orientation from shading intensity, but ambiguities about orientation remain even when it is the only unknown variable (Horn 1975). The problem is visualized in **Figure 1*c***, which depicts a shaded sphere illuminated from a direction that is 45° above the viewing direction. Every position on the sphere has a unique 3D orientation, but many positions and orientations have identical shading intensities. Contours called isophotes, drawn in **Figure 1*c***, connect locations on the surface that have the same image intensity but different 3D orientations. This means that the intensity of shading does not fully specify orientation; there are many 3D surface orientations that have the same luminance.

To understand why this many-to-one mapping occurs, it is useful to describe orientation using spherical coordinates—elevation and azimuth. Elevation is the surface's slant relative to some reference axis on the sphere that serves as an origin, which is typically aligned with the illuminant. The amount of reflected light varies as a cosine function of the elevation only (also referred to as slant); the intensity of reflected light is independent of the azimuth (also referred to as tilt). Intuitively, this just means that shading intensity only depends on how much a surface is slanted



(*Caption appears on following page*)

Figure 1 (*Figure appears on preceding page*)

Shape affects both shading and the bounding contour of a surface, so both kinds of structure covary. (*a*) A matte (Lambertian) material illuminated from 45° above the viewing direction. The graph plots the intensity of the diffuse shading, sampled a few pixels inside the bounding contour, as a function of the orientation of the bounding contour. Note that intensity covaries with the orientation of the bounding contour and exhibits a cosine fall-off. (*b*) A demonstration that the visual system uses this covariation to determine whether image gradients are due to shading. The upper 3D screw shape is a Lambertian material and exhibits a strong covariation between intensity and contour orientation. The lower surface was created by projecting the same image gradients as the surface on the upper image onto a different screw, which has differently shaped bounding contours; this eliminates the covariation between intensity and contour orientation, and the image gradients fail to appear due to shading (they appear as zebra-stripe changes in reflectance). (*c*) Diffuse shading approaches self-occluding contours and cuts differently. The isophotes bend when approaching self-occluding contours; this covariation does not arise when the bounding contour is due to a smooth surface having been cut. Panel *b* adapted from Marlow et al. (2019) with permission from Elsevier.

relative to the illuminant, not in which direction it is slanted. Thus, shading intensity can specify the elevation but not the azimuth of a surface's orientation relative to a known illumination direction. Although Koenderink & van Doorn (1980) used a sphere as an example, the conclusions generalize irrespective of shape and illumination direction.

To overcome this ambiguity, early shape-from-shading algorithms operated by iteratively improving estimates about the distribution of surface orientations that can reproduce a given pattern of shading (Ikeuchi & Horn 1981). The idea of recovering surface orientation is motivated by the observation that intensity specifies the elevation component of the orientation relative to a known illuminant. The idea of iteratively improving guesses is motivated by the observation that the azimuth component of the orientation cannot be derived even when illumination is known. This approach aims to derive only surface orientation and relies on many strong assumptions: The material is opaque and matte, color and lightness are fixed, illumination is known, and interreflections and shadows are absent.

More recent versions of this approach aim to derive illumination direction and surface color and lightness in addition to surface orientation (Barron & Malik 2011, 2012, 2015). The main idea remains the same: Surface orientation is a free parameter that is iteratively adjusted to try to reproduce the input image; the illumination and distribution of surface albedos are additional free parameters. Both early and recent versions of this approach rely on prior knowledge about what shapes, materials, and illuminations likely generated an image because there are infinitely many scenes that could have generated an image. The main difference between early and recent versions is that early versions assumed that illumination direction and albedo are known, and recent versions impose weaker, more generally applicable priors: Albedo rarely changes, but where it does, it changes rapidly; 3D shape tends to curve smoothly; and illumination can be approximated quite well using only a few spherical harmonics. Overall, this approach is quite computationally time consuming because the surface orientations (plus albedos and illumination) that the algorithms converge toward are the result of iteratively improving guesses, which in turn require both shading and surface curvatures to be recalculated each iteration (to assess shape's smoothness and ability to reproduce the input image).

SURFACE SHADING: COMPUTATIONAL WORK FOCUSED ON CURVATURE

Computational work initially focused on recovering surface orientation (e.g., Horn 1975) but has increasingly been focusing on recovering changes in surface orientation, i.e., surface curvatures (e.g., Kunsberg & Zucker 2013, Xiong et al. 2013). One reason for this is that curvatures control relationships between neighboring shading intensities, which must contain at least as much information as the intensities considered in isolation of their neighbors. Another reason is that shape itself is strongly constrained when it is smoothly curved. Many rules about the behavior of smoothly curved surfaces are being borrowed from differential geometry to help constrain thinking about shape from shading.

Curvature-based approaches to shape from shading (Han & Zickler 2023, Kunsberg & Zucker 2013, Xiong et al. 2013) differ in three respects compared to orientation-based approaches (e.g., Ikeuchi & Horn 1981). First, these different approaches aim to recover shape using different image properties. Orientation-based approaches derive 3D shape from local shading intensity, whereas curvature-based approaches focus on more complex forms of spatial structure characterized by variations in shading intensity. Second, models attempting to derive surface orientation require the illumination direction (or an estimate of it), whereas curvature-based approaches do not; they recognize that the human visual system knows neither the 3D shape nor the illumination and seek constraints on surface curvature that arise irrespective of where the illuminant happens to be located. Third, orientation-based models require many iterations to improve their guesses, whereas curvature-based approaches have two stages: The first stage identifies local 3D shape possibilities within a region of interest or a patch of shading, and the second stage searches for a global 3D shape that is consistent with finite local 3D shape possibilities at multiple image locations determined by the first stage (Han & Zickler 2023, Kunsberg & Zucker 2013, Xiong et al. 2013).

The general insight motivating the first stage of recent approaches is that the spatial structure of shading provides information about local 3D shape because only some local 3D shapes can generate particular patterns of shading. The basic intuition can be visualized by considering the sphere's shading depicted in **Figure 1*c***. There is no illumination direction that can cause a cylinder to project curved isophotes like the sphere does. A cylinder only curves in one direction, so its shading can only vary in that direction irrespective of the illumination direction. The presence of curved isophotes implies that the local 3D shape has two nonzero directions of curvature (i.e., like a sphere), and it rules out local 3D shapes that only curve in one direction (i.e., like a cylinder). Recent approaches generalize this intuition by demonstrating (using differential geometry) that the spatial structure of shading is constrained by the principal curvatures: the directions and rates of the most rapid and least rapid surface curvature (Han & Zickler 2023, Kunsberg & Zucker 2013, Xiong et al. 2013). The constraints that they have found limit possible interpretations to convexities (i.e., a bump), concavities (i.e., a dent), and saddle shapes that curve convexly in one direction and concavely in another direction. Each has restricted directions and rates of curvatures and implies a different direction of illumination.

The second stage of recent shape-from-shading approaches attempts to combine the finite local 3D shape possibilities identified in the first stage to reconstruct a globally coherent 3D shape. Multiple shape possibilities exist at multiple image locations, but many of these local ambiguities can be eliminated if the surface curves smoothly (Han & Zickler 2023, Xiong et al. 2013). For example, convexities cannot abut concavities because convexities and concavities must each be surrounded by saddles if the surface is smoothly curved (e.g., Koenderink & van Doorn 1980). This constraint on shape itself further limits possible distributions of the possible local 3D shapes.

More information about local 3D shapes can be derived from critical contours characterized by bright-dark-bright or dark-bright-dark reversals in shading intensity. Kunsberg & Zucker (2018, 2021) argue that the image of a shaded surface can be subdivided into regions that are convexities, concavities, or saddles. The logic suggesting that this is possible is that surface orientation, and thus shading intensity, has to reverse across the boundaries of these local 3D shapes. The boundaries of convexities, concavities, and saddles are inflection points of surface curvature; they curve convexly on one side of the inflection point and curve concavely on the other side (e.g., Koenderink & van Doorn 1980). The sign of curvature reverses across the inflection point, causing a repetition of the same surface orientations—and thus the same shading intensities—on either side of the inflection point. In other words, shading varies bright-dark-bright (or dark-bright-dark) where the angle of the surface relative to the illuminant is varying low-high-low (or high-low-high) across the boundaries of convexities, concavities, or saddles.

Overall, the computational work is increasingly focusing on recovering qualitative shape instead of metric shape properties like surface orientation. Although this work has tested its hypotheses relative to the ground-truth 3D shape of a shaded surface, experiments measuring perceived 3D shape are also needed to test the applicability of the ideas to human perception of shape from shading (for an example of such experiments, see Kunsberg et al. 2018).

SHAPE FROM SHADING: PERCEPTION RESEARCH

Perception research has also attempted to determine what shape properties human observers experience when viewing surface shading (e.g., Koenderink et al. 2001, Todd 2004, Todd & Norman 1995). Shape can be approximated using many primitives, such as the triangles common in computer graphics, Monge patches, and generalized cones, to name a few (e.g., Marr 1977, Todd & Norman 2003). There are also infinitely many derivatives of surface depth that one could attempt to derive from shading: Orientation is the first derivative, curvatures are the second derivative, and higher-order derivatives are possible and increasingly complex (Kunsberg & Zucker 2013). What can psychophysical experiments reveal about which aspects of 3D shape the human visual system perceives, particularly when viewing shading?

A recurring theme of studies measuring the apparent 3D shape of shaded surfaces has been that the results can differ significantly across tasks, observers, and illumination contexts (Egan & Todd 2015; Erens et al. 1993; Koenderink et al. 1996a,b, 2001; Todd et al. 1996; Wilder et al. 2019). However, many of these apparent differences are eliminated, or at least greatly reduced, by affine transformations of observers' depth settings (or the depths implied by their judgments of apparent surface orientation; Koenderink et al. 2001). Below, we describe what an affine transformation is, how such transformations have helped understand many results, and what they imply about how and how well the visual system can perceive the 3D shape of shaded surfaces.

Affine transformations of depth stretch, squash, and shear a surface in ways that preserve the locations of bumps, dents, and saddles but alter their heights, depths, and average orientations. Affine transformations can cause a surface to be more slanted in some (X,Y) direction (e.g., a frontoparallel plane slanted back in depth). Specifically, an affine transformation of depth is a new set of depth values, Z_2, that is a weighted sum of the original depth values, Z_1, and the image coordinates, X and Y, such that $Z_2 = AX + BY + CZ_1$. (The only constraint on the weights is that the depth scaling, C, cannot be 0; e.g., Belheumer et al. 1999). Affine transformations of depth preserve the locations of regions that are convex, concave, or saddle-shaped. In other words, these transformations preserve the same features that the most recent computational work has focused on deriving from shading (e.g., Kunsberg & Zucker 2018).

Affine transformations set an upper bound on the accuracy of 3D shape from shading because they capture some inherent ambiguities about depth and local 3D surface orientation. Nearly identical patterns of shading occur for all affine transformations of the depth of a shaded surface if the illumination direction is also appropriately transformed. For example, nearly identical images can arise when short bumps (and/or shallow dents) are illuminated from a more grazing direction than taller bumps (and/or deeper dents). Indeed, identical images arise if the albedo of the surface is also appropriately adjusted, which is known as the generalized bas-relief ambiguity (for orthogonal projection; Belheumer et al. 1999). One implication of the bas-relief ambiguity is that there is no way an observer could use shading alone to distinguish surface geometries that are affine transformations of each other. Another implication is that all affine transformations of the ground-truth surface depths are equivalently accurate shape-from-shading percepts (Koenderink et al. 2001). In other words, the geometry and illumination that the experimenter used to render an image are arbitrary; all of the affine transformations need to be considered when reaching a conclusion about whether perceived 3D shape is accurate in the sense of being one of many physically correct shape-from-shading solutions.

Affine transformations have helped us understand why observers' shape judgments can differ among tasks, observers, and illuminations (Koenderink et al. 2001, Nefs et al. 2005, Todd et al. 1996). One common task requires observers to match the depth interval between two locations on a surface marked by dots. Another requires observers to match local 3D surface orientation by making the circular base of a thumbtack superimposed on the surface appear parallel to the tangent plane to a surface (gauge probe figures; Koenderink et al. 1992). The results of both tasks can be directly compared by fitting a surface to many such judgements made at many image locations. Koenderink et al. (2001) found that the results of these and other methods of measuring apparent 3D shape are poorly correlated if the raw response surfaces are compared but highly correlated if affine transformations of the response surfaces are compared. Several studies have found that affine transformations also reduce individual differences among the results of observers performing the same task and help us understand the sense in which perceived 3D shape tends to remain constant as illumination direction varies (Koenderink et al. 1996a, 2001; Nefs et al. 2005, 2006; Todd et al. 1996).

Overall, research in the perception of shading suggests that observers are good at perceiving the same aspects of 3D shape that the most recent computational work focuses on recovering from images of shaded surfaces. Both computational studies and perception research initially focused on recovering local surface orientation and/or relative depth of points along a surface. However, these shape properties are inherently ambiguous; they can be altered by affine transformations that are perceptually invisible because they generate identical patterns of shading. More recent computation work and perception research has focused on more complex shape properties that are less ambiguous and seem to be perceived more robustly across different tasks, observers, and illumination contexts. These more complex shape properties include the boundaries of convexities, concavities, and saddles and the rates of surface curvature across these qualitative local 3D shapes. The ability to perceptually distinguish surface geometries that are relatable through affine transformations seems to require other sources of information in addition to shading, such as occluding contours.

INTERACTIONS BETWEEN SHADING AND CONTOURS

Patterns of surface shading are often bounded by self-occluding contours formed by a surface curving behind itself from the observer's viewpoint. One remarkable fact about such contours is that 3D surface orientation can be derived precisely from 2D image orientations (Barrow & Tenenbaum 1978). This is particularly remarkable when compared to shading, which only specified one out of two of the degrees of freedom that surface orientation has. In the case of self-occluding contours, both degrees of freedom—slant and tilt—can be derived, and they are relative to a known, instead of an unknown, reference direction—the observer's viewing direction, instead of the illumination direction. Specifically, the slant of the surface relative to the viewing direction is known because it is a fixed value (90°) along self-occluding contours; the other degree of freedom—surface tilt—determines the projected 2D orientation of the contour and thus can also be derived. This insight about the ease of deriving surface orientation along self-occluding contours was rapidly incorporated by research into shape from shading, which used the contours as boundary conditions for guessing surface orientation of the shading inside the contours (e.g., Barron & Malik 2011, 2012, 2015; Ikeuchi & Horn 1981; Oxholm & Nishino 2015). Perceived 3D shape from shading is particularly accurate near self-occlusions, suggesting that the visual system has also learned that such contours contribute potent information about 3D shape (Mamassian & Kersten 1996, Todd & Reichel 1989; see also Egan & Todd 2015).

Two fundamental problems remain despite these insights about how to derive shape from shading, contours, or both. First, how does the visual system know whether contours are self-occlusions and not from some other source, such as cuts created by the surface abruptly terminating where it was cut in some arbitrary way? The capacity to infer surface orientation from self-occlusions presumably depends on the visual system somehow solving this more fundamental problem (Huggins & Zucker 2001a,b; Huggins et al. 2001). Second, how does the visual system know whether it is or is not viewing a shaded surface? Smooth variations in image intensity can arise from many other sources in addition to shading, such as shadows, pigment variations, and optical defocus (Marlow et al. 2019, Mooney et al. 2019). The capacity of shading to specify 3D shape, even qualitatively, is presumably only possible after the visual system has somehow determined that it is in fact viewing a shaded surface. Assuming that shading and self-occlusions generated the gradients and contours would lead to inaccurate percepts of 3D shape if either of these two core assumptions were violated. Indeed, wrong assumptions that the luminance gradients are diffuse shading caused by a matte material could lead to gross misperceptions of both surface shape and material.

These two problems seem computationally coupled because contours have been observed to affect the apparent cause(s) of gradients, and conversely, gradients affect whether contours appear as self-occlusions or cuts (e.g., Ramachandran 1988). Manipulating the 2D shape of the bounding contours that encircle a luminance gradient can alter many aspects of the gradient's appearance. These include the apparent 3D shape; the apparent illumination context; and the material appearance, for example, whether the gradient appears to be due to a shaded or matte surface (e.g., **Figure 1*b***) (Knill & Kersten 1991; Marlow et al. 2015, 2019, 2023; Ramachandran 1988; Todorović 2014). The gradients inside a contour can likewise affect whether the contour appears to be due to a self-occlusion or a cut. For example, **Figure 1*c*** depicts two identical circular bounding contours, but one is interpreted as being due to a self-occlusion and the other as being due to a cut. The only difference is the luminance gradients within the identical contours (and their isophotes superimposed on the gradients), which implies that the luminance gradients disambiguate the distal cause of the contours despite the distal meaning of the gradients also being ambiguous. In other words, determining what in the world caused the gradients and determining what caused the contours are computationally coupled problems.

An approach offering simultaneous solutions to both problems has focused on understanding how different types of image structure covary in ways that cospecify surface shape and material (Marlow & Anderson 2021; Marlow et al. 2019, 2023). The visual system never has access to either the shape or material properties of the surfaces that caused a particular image; there must be something in images that allows the visual system to simultaneously infer the shape and material properties of the surface being viewed. In the case of shading and contours, it involves determining both whether the gradients are due to a matte, shaded material and whether the contours are due to self-occlusion. Patterns of covariation between the two types of image structure provide the means to that end. In this example, the gradients and the contours covary in ways that are diagnostic about whether shading and self-occlusions are being viewed. This approach seems fruitful because it promises to obtain multiple forms of information about the scene directly from images without needing assumptions about shape, material, or illumination.

The reason that shading and self-occlusions covary in meaningful ways is that these two types of image structure have overlapping sets of distal causes. Self-occlusions depend on surface orientation and the viewing direction, and shading depends on surface orientation and the illumination direction. Surface orientation, or shape more generally, is thus a shared distal cause that ensures that patterns of covariation will emerge between patterns of shading and self-occluding contours.

Research into shading and self-occluding contours has identified two specific forms of covariation: One occurs along the contour and is diagnostic about whether material is matte (i.e., whether a shaded surface exists; Marlow et al. 2019), and the other occurs approaching the contour and may be diagnostic about whether the contour is due to either a self-occlusion or a cut (Huggins & Zucker 2001a,b). **Figure 1*a*** plots the intensity of the shading abutting the contour as a function of the contour's orientation, which in this example is due to a self-occlusion. More precisely, the graph plots image intensity next to the contour as a function of the angular difference of the contour's orientation relative to the brightest point along the contour. This covariation exhibits the cosine nonlinearity characteristic of Lambertian shaded surfaces. Marlow et al. (2019) assessed the robustness of this covariation for thousands of images of opaque, matte materials varying in 3D shape, illumination, and whether the bounding contour of the surface was due to either a self-occlusion or a cut. With the exception of a few degenerate cases (i.e., frontal illumination or cuts aligned with the viewing direction), the same qualitative pattern of covariation emerges for both self-occluding contours and cuts. Marlow et al. tested whether the visual system exploits this information by parametrically manipulating this form of covariation occurring along a contour and measuring both perceived 3D shape and how strongly the gradients appeared due to shading. Vivid percepts of shading and 3D shape arose whenever the gradient's intensity covaried with the contour's orientation, but this perception was degraded or eliminated when this covariation was absent (**Figure 1*b***).

Another form of covariation between gradients and contours could in theory signify that a self-occlusion is the distal cause of a contour (Huggins & Zucker 2001a,b; Huggins et al. 2001). This form of covariation involves assessing how the isophotes—contours connecting points of equal intensity—change as they approach the bounding contour (**Figure 1*c***). The isophotes that diffuse shading become increasingly parallel to the orientation of the self-occluding contour as they approach it, which creates a pattern of orientation–orientation covariation between the gradients and the contour that is diagnostic of a shaded surface smoothly curving out of sight. This covariation does not arise when the contour is due to a cut across the surface; in the case of a cut, the bounding contour of the surface forms abrupt T-junctions with the isophotes. However, at this juncture, there are no compelling data to determine whether the visual system exploits this information. Experiments are needed to assess whether these are the forms of image structure that the visual system uses to distinguish self-occlusions from abrupt surface terminations.

SHAPE AND MATERIAL OF SPECULAR SURFACES

The preceding section focuses on diffuse reflectance, which arises from light penetrating the superficial layers of a material. Light can also reflect off the surface, which is dubbed specular reflectance and is associated with our experience of gloss (Cook & Torrance 1982, Hunter & Harold 1987).

The spatial structure of the reflections strongly depends on the 3D shape of the specular surface. Regions of high surface curvature are exposed to a large region of the light field, and therefore compress a large portion of the visual field into a small image region, whereas regions of low curvature compress a smaller portion of the scene, causing less image compression. This differential compression generates systematic distortions of the visual field: The reflected scene appears elongated along directions of low surface curvature and compressed in directions of high curvature

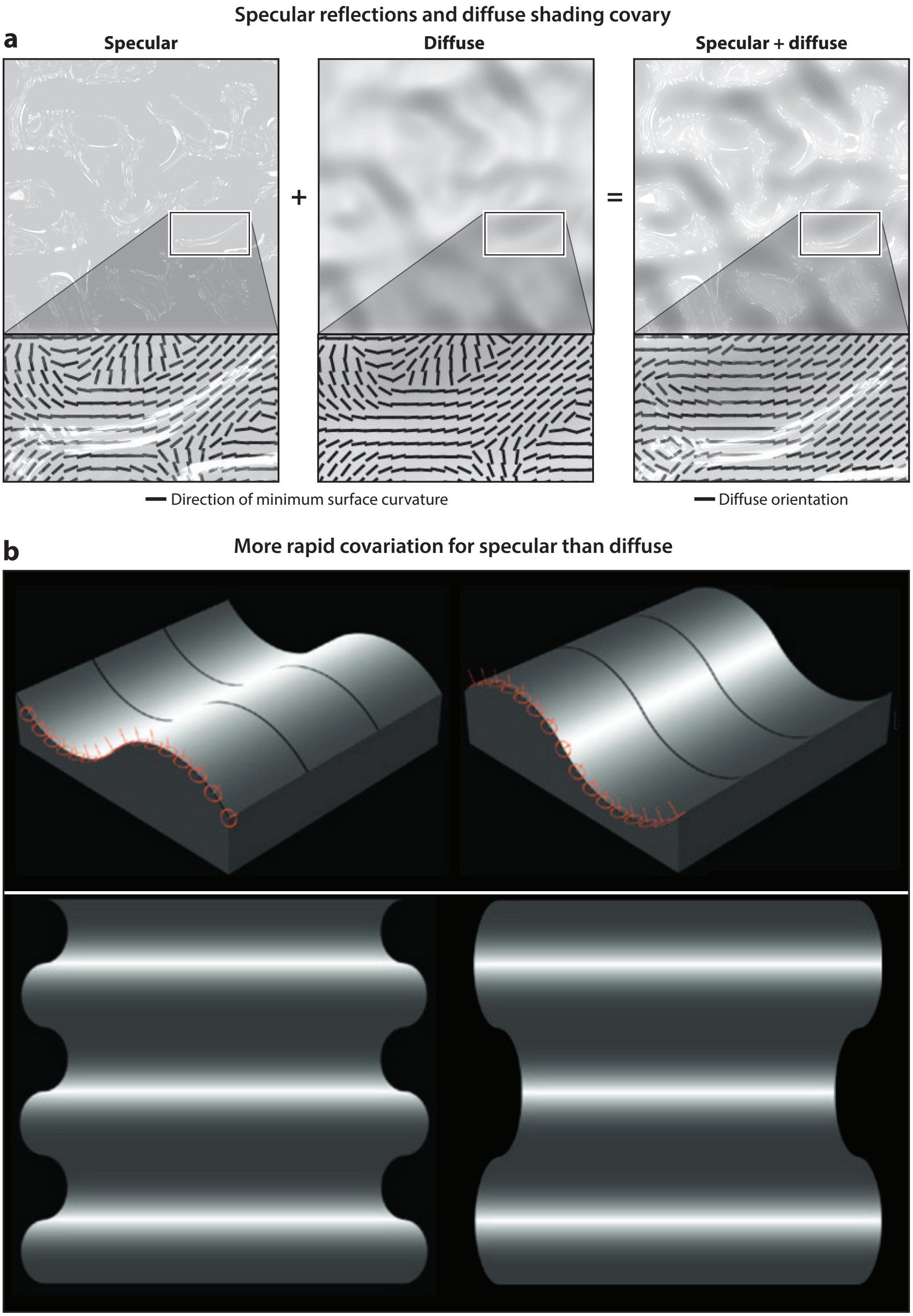
(**Figure 2*a***). For purely specular surfaces (mirrors), these distortions generate patterns of oriented image structure dubbed orientation fields that carry information related to the directions of the principal curvatures (i.e., the second derivative of the depth map of the surface; Fleming et al. 2004, 2009). The stability of the orientation fields across different illumination environments suggests that they play an important role in explaining shape constancy for surfaces that reflect light like a mirror (Fleming et al. 2004, 2009; Shimokawa et al. 2019). However, this shape constancy is not perfect. It has been shown that gross errors in shape constancy can arise, particularly when a specular surface is rough at the microscale, which blurs the reflected scene (Mooney & Anderson 2014). The image gradients projected to the viewer depend on both the rates of surface curvature and the microroughness of the specular material, and the visual system cannot fully disentangle the contributions of these two causes of the image gradients.

Purely specular surfaces are, however, exceedingly rare in natural scenes. Most glossy surfaces typically also reflect some light diffusely; such materials are reasonably well modeled by a dichromatic reflectance function, where the light reflected from a surface is a weighted combination of diffuse and specular reflectance (Larson & Shakespeare 1998, Nicodemus 1965). The perception of gloss of such surfaces exhibits striking failures of constancy; surfaces with physically identical levels of gloss can appear very different (Fleming et al. 2003; Ho et al. 2008; Klinker et al. 1988; Nishida & Shinya 1998; Olkkonen & Brainard 2010, 2011; Pont & te Pas 2006; Storrs et al. 2021; Vangorp et al. 2007). Light surfaces appear less glossy than dark surfaces because the specular reflections have lower contrast when overlaid on a light surface (Ferwerda et al. 2001). The perception of gloss is also strongly affected by variations in the 3D surface geometry and distribution of light sources, which affect the proportion of the surface that is visibly covered in specular reflections and the sharpness of these reflections (Marlow & Anderson 2013, Marlow et al. 2012). The perception and misperception of gloss are well predicted by a set of image properties—the contrast, sharpness, and coverage of specular reflections—that vary as a function of surface geometry and illumination (Marlow & Anderson 2013, Marlow et al. 2012). The dependence of perceived gloss on 3D shape represents a strong failure of gloss constancy, which can provide insight into both what the visual system uses to generate our experience of gloss and how sensitivity to this input could emerge (Storrs et al. 2021).

Dichromatic surfaces also generate systematic forms of covariation that the visual system exploits to derive shape and material. Both diffuse and specular reflectance are constrained by 3D shape, which causes their image structure to covary in specific ways. The geometry of specular reflections and the gradients of diffuse shading both depend on surface curvature, which causes their orientations to run parallel to each other (**Figure 2*a***). The perception of gloss is lost if the images are manipulated to destroy this orientation congruence, for example, by rotating the specular image structure to make it misaligned with the orientations of the diffuse shading gradients (Anderson & Kim 2009, Beck & Prazdny 1981, Kim et al. 2011, Marlow et al. 2011, Todd et al. 2004).

Specular reflections also depend on viewpoint; they occur at surface normals that bisect the viewing and light source directions. The brightest points of surface shading and specular reflections are only aligned when the viewing and illumination axis coincide; for all other contexts, they will be close, particularly in regions of high surface curvature (which contain a broad range of surface normals). This induces a second, weaker form of covariation between shading and specular reflections—position congruence—that the visual system exploits to identify specular reflections. The perception of gloss is also lost when this form of congruence is destroyed (Kim et al. 2011, Marlow et al. 2011).

The structure of specular reflections also depends on the microscopic roughness of the surface, which determines how much the reflected scene is blurred (Fleming et al. 2003). In the limit,



(Caption appears on following page)

Figure 2 (*Figure appears on preceding page*)

Specular reflections covary with other kinds of image structure that also depend on 3D shape, and elicit more compelling percepts of surface gloss when patterns of covariation are present. (*a*) The image on the left shows only specular reflections of a bumpy plane illuminated 45° above the viewing direction. The enlarged region of interest depicts the direction of minimum surface curvature; note that specular reflections are stretched in this direction. The middle image shows only the diffuse shading generated by the same surface geometry and illumination, which also tends to be stretched in the direction of minimum surface curvature. The right image shows the specular reflections overlaying the diffuse shading, and both of their 2D orientations covary because both depend on surface curvature. Note that the perception of gloss is stronger when both the specular reflections and the diffuse shading are present and have covarying orientations. (*b*) Different rates of covariation between image intensity and contour orientation can cause identical image gradients to appear due to either a specular or a matte material. The gradients in the left column are identical to the image gradients in the right column; only the shape of the bounding contours differs, but the material appears shinier on the right than on the left. Panel *b* adapted from Marlow et al. (2015) with permission from Elsevier.

surface roughness can transform a perfectly smooth mirror into a diffuse reflector. Consequently, identical image gradients could be generated by a highly curved matte surface and by a specular surface with lower surface curvature. As noted above, the covariation between image intensity and contour orientation exhibits a strong cosine shape when reflectance is Lambertian. The quantitative form of this covariation provides information that the image gradients are caused by diffuse shading, which elicits percepts of 3D shape and a (matte) material (Marlow et al. 2019). A similar qualitative form of contour–intensity covariation will also be exhibited by rough specular surfaces, but the quantitative form of this covariation will be different (i.e., it will be much steeper than cosine). If the visual system uses rates of covariation to distinguish different forms of surface reflectance, then it should be possible to cause gradients to appear as either diffuse or specular reflectance by simply changing the rate at which intensity covaries with the orientation of image contours. We present evidence in support of this prediction in **Figure 2*b*** (Marlow & Anderson 2015, 2016; Marlow et al. 2015). The intensity gradients in the two images are identical; the only difference is the 2D shape of their bounding contours. In the image on the right, intensity varies rapidly as a function of bounding contour orientation, which is consistent with surface reflectance having a narrow scattering function. In the image on the left, intensity varies slowly relative to the orientation of the bounding contour, which is consistent with the surface scattering light more broadly. The visual system is clearly sensitive to this information: The image on the right appears metallic, like brushed metal, whereas the image on the left appears much more matte.

SHAPE AND MATERIAL OF SUBSURFACE SCATTERING

Diffuse reflectance is a special form of subsurface scattering, which occurs whenever light penetrates a material before re-emerging. For diffuse reflectance, light exits close to where it penetrates, which allows it to be modeled as bouncing off of a point on the surface. For more light-permeable (translucent) materials, light penetrates at one location on a surface, scatters within the material's volume, and re-emerges at many locations remote from where it entered. This means that the amount of light returned from a surface is not simply a function of the amount of incident light; it depends on the complex light-transport processes that occur within the body of a material (Jensen 2001, Koenderink & van Doorn 2001).

The physical complexity of subsurface scattering makes it unlikely that the visual system could be sensitive to all of the physical variables that affect the amount of visible light that emerges after the subsurface light transport (Fleming & Bülthoff 2005). Indeed, work has shown that identical images can be generated by different combinations of parameters that affect the light-transport processes that occur in translucent materials (Gkioulekas et al. 2013). These translucency metamers occur even when 3D shape and illumination are held fixed, which implies that images of translucent materials do not contain sufficient information to distinguish the true causes of a given image.

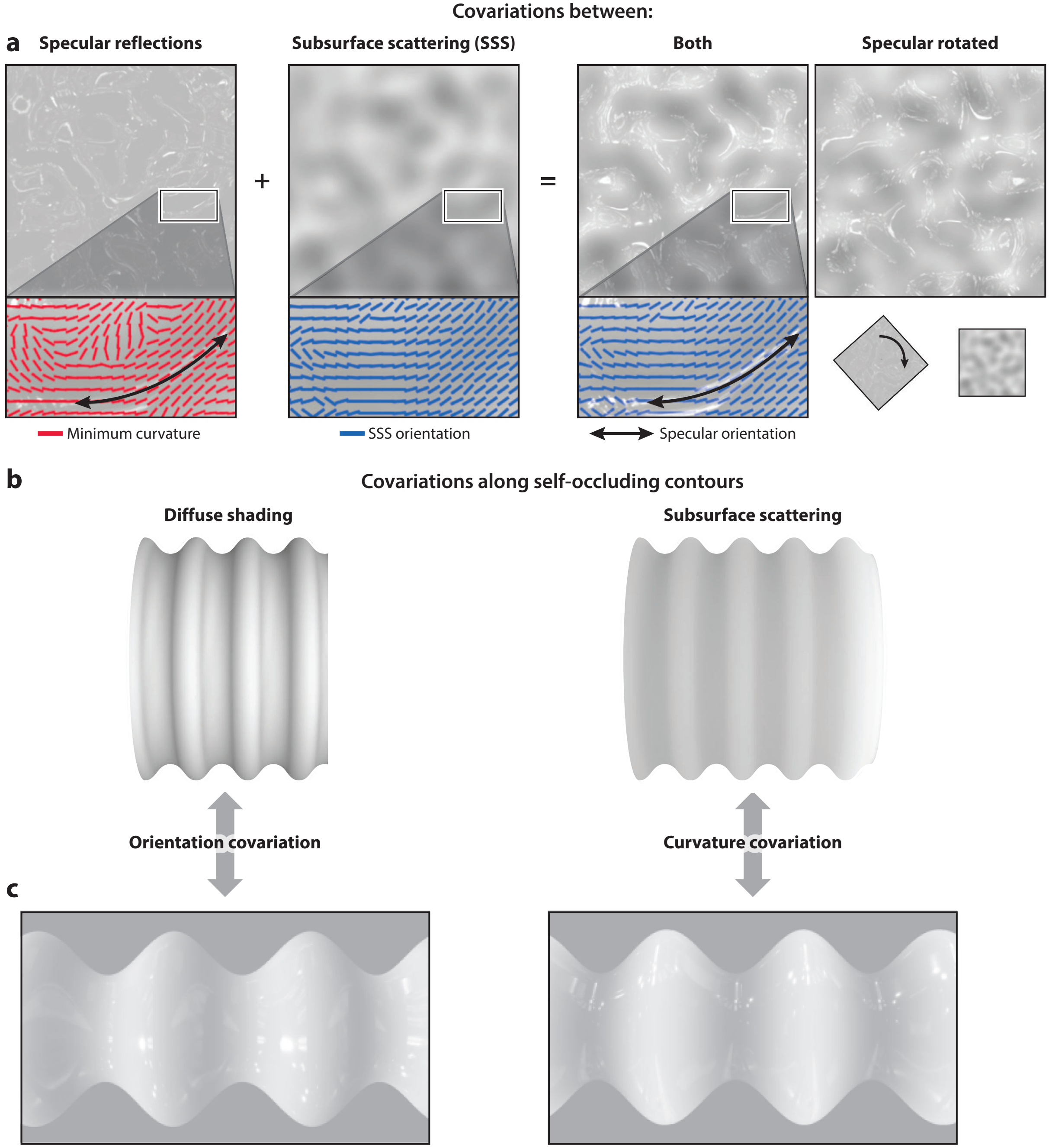
The perception of translucency is further complicated by the fact that the amount of light returned can depend on the volumetric properties of a material, not just the shape of its surface. The light reflected from translucent materials can also be affected by their thickness; light illuminating the back of a material can bleed though to the front. Psychophysical experiments have shown substantial failures of perceptual constancy for the material properties of translucent surfaces: They appear more dense when illuminated from the front than when illuminated from behind (Fleming & Bülthoff 2005, Xiao et al. 2014), and materials have to be physically more transmissive when illuminated from the front to appear equally translucent as when illuminated from the back.

How does the visual system distinguish opaque and light-permeable materials? One characteristic difference between opaque and translucent materials is that the light bleeding through translucent materials lightens regions in shadows, which reduces the contrast of attached shadows. Subsurface light transport also blurs the gradients across the surface and boundaries of attached shadows. Early work suggested that these contrast and spatial frequency differences were the primary forms of information that the visual system uses to perceive translucency (Fleming & Bülthoff 2005, Motoyoshi 2010, Nagai et al. 2013).

There are, however, also strong differences between opaque and translucent materials in terms of how 3D shape constrains the distribution of intensities (Koenderink & van Doorn 2001, Marlow et al. 2017). Diffuse reflectance primarily depends on 3D surface orientation and illumination direction, which causes intensity to covary with surface orientation. The same is not true for surfaces that involve much deeper forms of subsurface scattering; light can emerge from locations remote from where it entered the material, which weakens the covariation between intensity and surface orientation. Stereopsis and structure from motion have been used to create or destroy this covariation between intensity and orientation, which is strong for opaque materials but weaker for translucent materials (Marlow et al. 2017). The results show that monocularly identical images appear as opaque surfaces when the covariation is present and as translucent materials when it is absent—irrespective of whether an opaque or translucent material was used to render the images. Kim et al. (2006) also manipulated apparent 3D shape but argued that the percepts of glow and translucency they found were due to bright concavities.

The complexity of subsurface scattering has suggested to some that there may be no clear signature, like the intensity–orientation covariation observed with opaque materials, that can be used to identify light-permeable materials (Koenderink & van Doorn 2001). However, our visual systems typically do distinguish these different light–material interactions, which implies that there must be something in images that supports these perceptual abilities. What image signatures could be used to identify translucent materials? How might such signatures be discovered?

One answer to these questions emerged through the recognition that observers only receive light re-emerging from the front of translucent surfaces, so the strongest forms of covariation likely arise when surfaces at the front—not the back—are being illuminated. Marlow & Anderson (2021) constructed smooth, bumpy planes that were sufficiently thick that no light from behind bled through the surfaces at the front. They varied parameters affecting the light-transport processes within the material and the dominant direction of illumination (over the front hemisphere). They found that, as the depth of subsurface scattering increases, the intensity of reflected light shifted from covarying with 3D surface orientation to covarying with the sign of surface curvature: Convexities tend to be bright, and concavities tend to be dark (**Figure 3*b***). This form of covariation is extremely robust to changes in illumination direction; indeed, the intensity shifts that occur when the illumination direction changes are smaller for translucent surfaces than for opaque surfaces with the same surface geometry. Thus, there are clear differences in the forms of covariation exhibited by surfaces that vary in opacity, which suggests that the visual system might use these different forms of covariation to distinguish opaque and translucent materials. However, this cannot be done simply by considering the image gradients generated by either shading or subsurface scattering. Local intensity gradients are inherently ambiguous; their environmental



(Caption appears on following page)

Figure 3 (*Figure appears on preceding page*)

Subsurface scattering depends on 3D surface curvatures, causing it to covary with other kinds of image structure that also depend on curvature. (*a*) The left image depicts only the specular reflections generated by a bumpy plane illuminated 45° above the viewing direction, which are stretched in the direction of minimum surface curvature. The image next to it only depicts patterns of subsurface scattering caused by light that has penetrated beneath the same surface, scattered inside the material, and then re-emerged toward the viewer (i.e., subsurface scattering). Note that the image gradients generated by either are stretched along directions of minimum surface curvature. Superimposing the specular reflections and subsurface scattering elicits a compelling percept of 3D shape, gloss, and translucency but only if the specular reflections are in the geometrically correct place and, thus, covary with the subsurface scattering. (*b*) Two cylinders with corrugated sides rendered with diffuse reflectance or subsurface scattering. Both are illuminated from the right, but their intensities depend differently on their 3D shapes. The opaque surface is brighter where it faces the light source, whereas the translucent surface is brighter where its curvature is convex. (*c*) Stimuli testing whether two different patterns of covariation elicit percepts of an opaque or translucent material. The spatial frequency and contrast of the gradients are identical, and specular reflections appropriate for each 3D shape percept have been superimposed. Panel *c* adapted from Marlow et al. (2023) with permission from Elsevier.

cause(s) can only be determined by considering how intensity covaries with some other forms of image structure (i.e., by considering the surrounding context).

There are two forms of image structure that have been shown to influence the perception of the shape and material properties of light-permeable surfaces: specular reflections and self-occluding contours. We consider each in turn.

Early work on the perception of translucency showed that perceived translucency is much more compelling for surfaces that generate specular reflections than for those that do not (Fleming & Bülthoff 2005, Motoyoshi 2010). The presence of specular reflections provides some information about the strength and directionality of the light sources in a scene, whereas the strength of gradients generated by subsurface scattering provides information about how light permeable a material is. These clear image differences suggested that the perception of translucency depends primarily on the contrast relationships and spatial frequency content of the specular reflections relative to gradients of subsurface scattering. The images in these studies were of complex objects that generated specular reflections, self-occluding contours, and patterns of subsurface scattering, which elicited a vivid and stable perception of 3D shape. Thus, it remained unclear whether (or how) the perception of translucency depends on the perception of 3D shape, or how the perception of 3D shape could be derived from the different forms of covariation exhibited in these images.

More recent work showed that the covariation that arises between gradients of subsurface scattering and specular reflections can provide vivid information about both the shape and material properties of translucent materials (Marlow & Anderson 2021, Marlow et al. 2023). The structure of specular reflections and gradients of subsurface scattering covary because they are both constrained by surface curvature (**Figure 3*a***). The image structure that specular reflections generate is elongated along the direction of minimal surface curvature. The same is true for gradients of subsurface scattering, where intensity varies slowest in directions where the curvature is changing the least. Thus, both forms of image structure generate local orientation fields that are highly congruent: Specular reflections tend to be oriented parallel to the orientation structure generated by gradients of subsurface scattering. The covariation exhibited by these two forms of image structure may therefore provide information that cospecifies the shape and material properties of translucent surfaces.

Marlow & Anderson (2021) tested this possibility by constructing images formed by glossy, translucent, bumpy planes, which generated only two forms of image structure: specular reflections and subsurface scattering. When the gradients of subsurface scattering were shown in the absence of specular reflections, or the specular reflections were shown alone, the perception of shape and material was extremely weak. However, when both sources of information were present, their 3D shape and material properties were perceptually vivid (**Figure 3*a***). These results suggest that the information about shape and material is provided by the covariation generated by specular reflections and subsurface scattering. Moving the specular reflections away from their geometrically correct locations destroys the perception of gloss and significantly weakens both percepts of 3D shape and those of translucency (**Figure 3*a***).

Subsurface scattering also generates unique patterns of covariation along the self-occluding contours of translucent materials (Marlow & Anderson 2021, Marlow et al. 2023) (**Figure 3*b***). Koenderink (1984) showed that (2D) convexities along self-occluding contours are generated by convexities of surface curvature, and that (2D) concavities along self-occluding contours are caused by surface saddles, which are concave along the contour and convex in the direction that grazes the surface along the viewing direction. Marlow & Anderson (2021) showed that the sign of the curvature also affects the intensity of subsurface scattering; convexities are bright, concavities are dark, and saddles have an intermediate brightness. Thus, two kinds of image structure—occluding contours and subsurface scattering—depend on the same thing in the world (the sign of curvature); thus, they covary. The generality of the covariation was tested using simulations and found to increase in strength as material becomes translucent and arise for many illumination directions (Marlow et al. 2023). This implies that there are unique forms of covariation along self-occluding contours that the visual system could use to distinguish opaque and translucent surfaces: Intensity covaries with orientation and curvature of self-occluding contours for opaque and translucent surfaces, respectively. **Figure 3*c*** shows one of Marlow et al.'s (2023) experiments, which provides evidence that the visual system exploits this information: When intensity covaried with contour orientation, the surface appeared opaque; when intensity covaried with curvature, the surface appeared more translucent.

CONCLUSION

Research has historically taken a divide-and-conquer approach toward understanding the perception of shape and material. The perception of shape was studied for particular reflectance functions (most commonly, diffuse reflectance), and the perception of material was typically studied for fixed shapes. The presumption was that this somehow simplified the problem, at least for the scientist performing the study. However, these studies typically offered no insight into how the visual system knows the values of the scene variables that were not varied. An alternative approach suggests that it is more fruitful to pursue a unite-and-conquer methodology. There are two intuitions shaping this approach. The first is that our brains do not have a priori knowledge of any of the scene variables that generated an image; all relevant scene variables—shape, material, and illumination—must somehow be derived simultaneously. The second is that there must be something in images that allows our brains to derive shape and material properties simultaneously. The work reviewed above provides evidence that this information is conveyed through the relationships that arise among the kinds of image structure that traditional approaches studied in isolation.

Finally, it should be noted that the idea of complex patterns of covariation is not simply a form of cue combination, which has been extensively studied (e.g., Landy et al. 1995). This is because the cues themselves are only defined as relationships among different forms of image structure; the relevant cues are emergent properties of the relationships among different kinds of image structure. These emergent properties are strongly reminiscent of Gibson's (1979) idea of higher-order invariants. Gibson argued that vision was not a form of inference based on ambiguous cues that needed to be supplemented with prior knowledge about the world. Instead, he argued that vision was a process of detecting higher-order invariants in the flux of light that projects to our eyes that provide information about the properties in the world that we can sense. This idea has languished for decades because he offered no method by which these complex image properties could be discovered. Recent work offers a method for discovering the existence of such complex forms of covariation and, by exploiting modern computer graphics to explicitly manipulate the covariations, to test whether they are used by our visual systems (Marlow & Anderson 2021; Marlow et al. 2019, 2023; Mooney et al. 2019).

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