

# Graunt and US 1993 Life Table

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## Source of Data

```
knitr::include_graphics("../pics/graunt_table.png")
```

Age	Graunt	1993
0	100	100
6	64	99
16	40	99
26	25	98
36	16	97
46	10	95
56	6	92
66	3	84
76	1	70

## Data Input

- Graunt's Life Table

```
graunt <- data.frame(x = c(0, seq(6, 76, by = 10)),  
                    xPo_g = c(100, 64, 40, 25, 16, 10, 6, 3, 1))
```

## More data

- US 1993 life table for the same age group

```
us93 <- data.frame(x = graunt$x,  
                  xPo_us = c(100, 99, 99, 98, 97, 95, 92, 84, 70))
```

## Data Extraction

There are many ways to extract part of `us93` data frame.

```
us93["xPo_us"]
```

```
##    xPo_us  
## 1     100  
## 2      99  
## 3      99  
## 4      98  
## 5      97
```

```
## 6      95
## 7      92
## 8      84
## 9      70
```

```
us93["xPo_us"][[1]]
```

```
## [1] 100 99 99 98 97 95 92 84 70
```

```
us93["xPo_us"]$xPo_us
```

```
## [1] 100 99 99 98 97 95 92 84 70
```

```
us93["xPo_us"]$xPo
```

```
## [1] 100 99 99 98 97 95 92 84 70
```

```
us93[2]
```

```
##      xPo_us
## 1      100
## 2       99
## 3       99
## 4       98
## 5       97
## 6       95
## 7       92
## 8       84
## 9       70
```

```
us93[2] [[1]]
```

```
## [1] 100 99 99 98 97 95 92 84 70
```

```
us93[2]$xPo_us
```

```
## [1] 100 99 99 98 97 95 92 84 70
```

```
us93[ , "xPo_us"]
```

```
## [1] 100 99 99 98 97 95 92 84 70
```

```
us93[ , 2]
```

```
## [1] 100 99 99 98 97 95 92 84 70
```

```
us93$xPo_us
```

```
## [1] 100 99 99 98 97 95 92 84 70
```

```
us93$xPo
```

```
## [1] 100 99 99 98 97 95 92 84 70
```

## Into one single data frame

Combine two data frames into one single data frame, compare the results.

```
(graunt_us <- data.frame(graunt, xPo_us = us93$xPo))
```

```
##      x xPo_g xPo_us
## 1  0    100    100
```

```
## 2 6 64 99
## 3 16 40 99
## 4 26 25 98
## 5 36 16 97
## 6 46 10 95
## 7 56 6 92
## 8 66 3 84
## 9 76 1 70
```

```
(graunt_us_2 <- data.frame(graunt, us93[2]))
```

```
##      x xPo_g xPo_us
## 1  0  100  100
## 2  6  64  99
## 3 16  40  99
## 4 26  25  98
## 5 36  16  97
## 6 46  10  95
## 7 56   6  92
## 8 66   3  84
## 9 76   1  70
```

```
(graunt_us_3 <- data.frame(graunt, us93[, 2]))
```

```
##      x xPo_g us93...2.
## 1  0  100  100
## 2  6  64  99
## 3 16  40  99
## 4 26  25  98
## 5 36  16  97
## 6 46  10  95
## 7 56   6  92
## 8 66   3  84
## 9 76   1  70
```

## Life Expectancy

The basic principle is that the area under the survival function is the life expectancy.

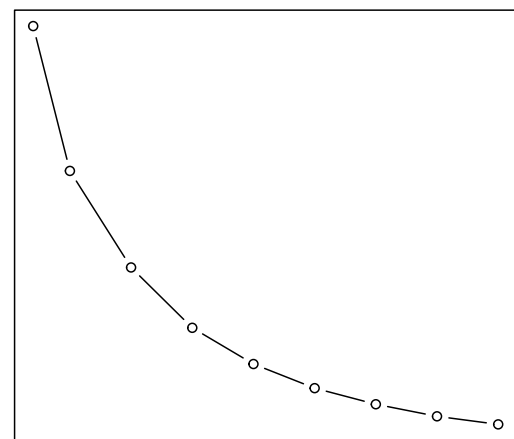
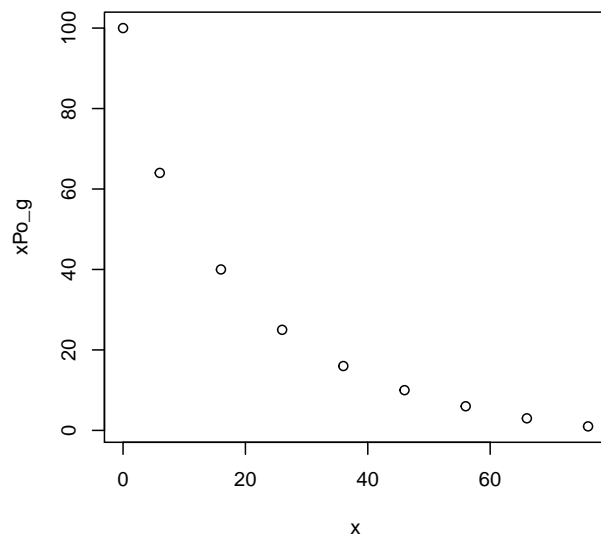
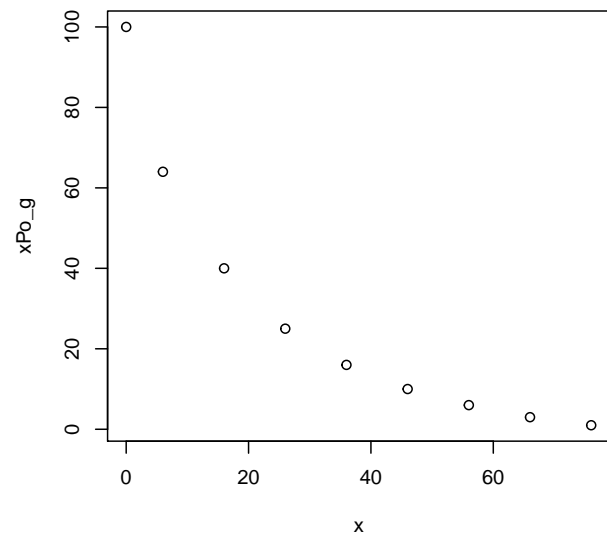
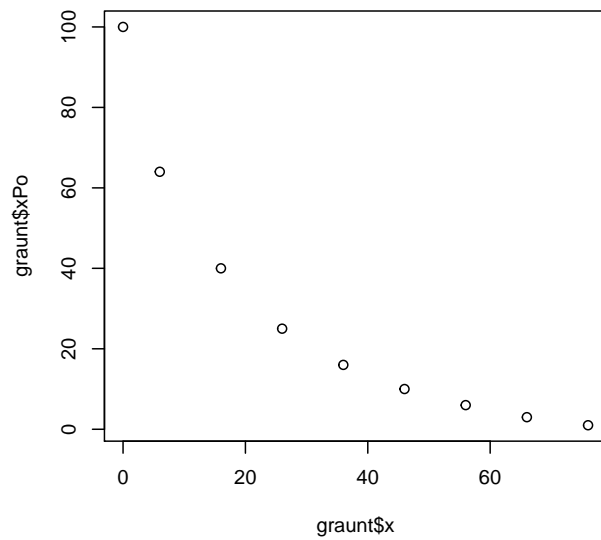
$X \geq 0$ ,  $X \sim F(x) \Rightarrow X \equiv F^{-1}(U)$ ,  $U \sim U(0, 1)$ , therefore,

$$E(X) = E\{F^{-1}(U)\} = \int_0^1 F^{-1}(u) du = \int_0^\infty 1 - F(x) dx = \int_0^\infty S(x) dx$$

## Step by step approach to draw survival function plot

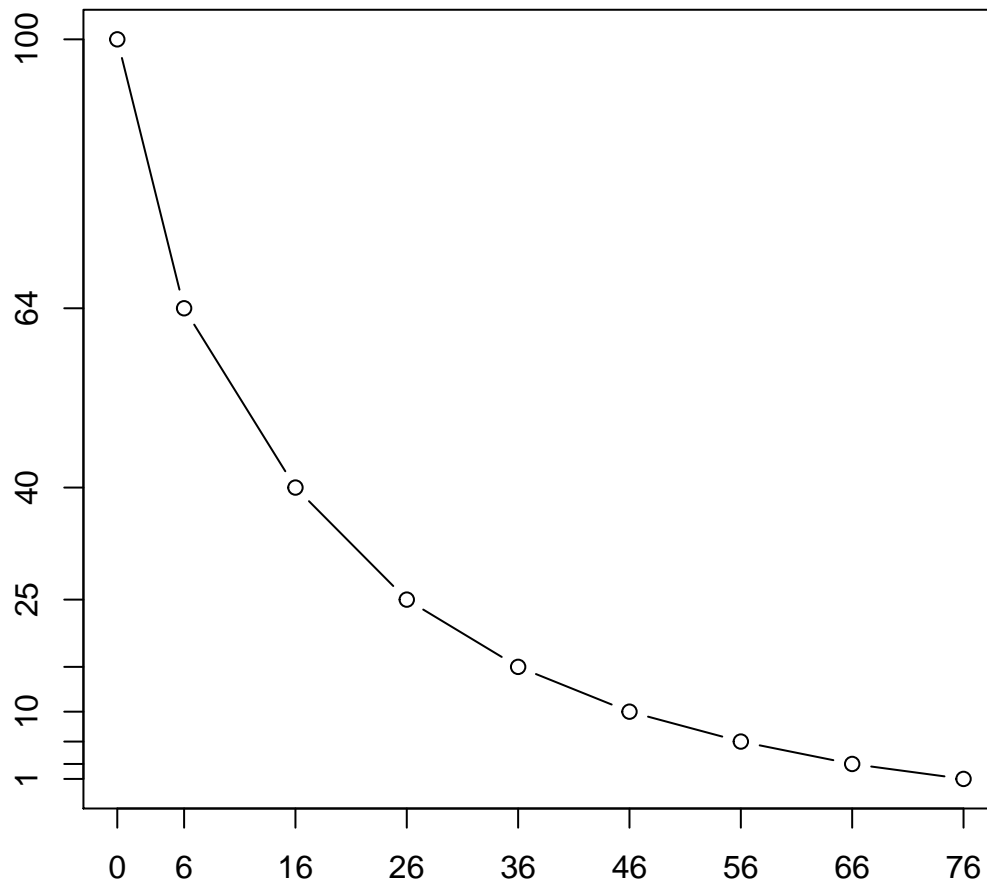
1. Basic plot with points and lines, compare the following threes methods

```
par(mfrow = c(2, 2))
plot(x = graunt$x, y = graunt$xPo)
plot(xPo_g ~ x, data = graunt)
plot(graunt)
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
```



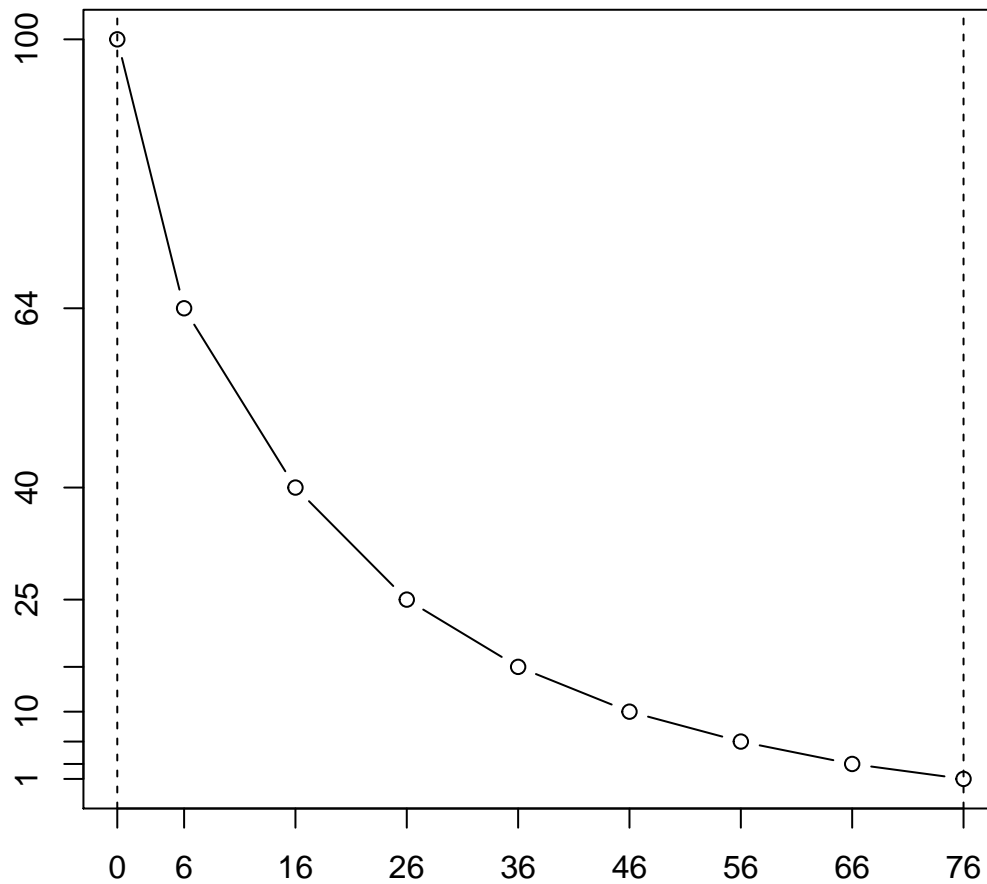
2. Denote the ages and observed survival rates on the axes

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo_g, labels = graunt$xPo_g)
```



3. Denote the age 0 and 76 by dotted lines

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo_g, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
```



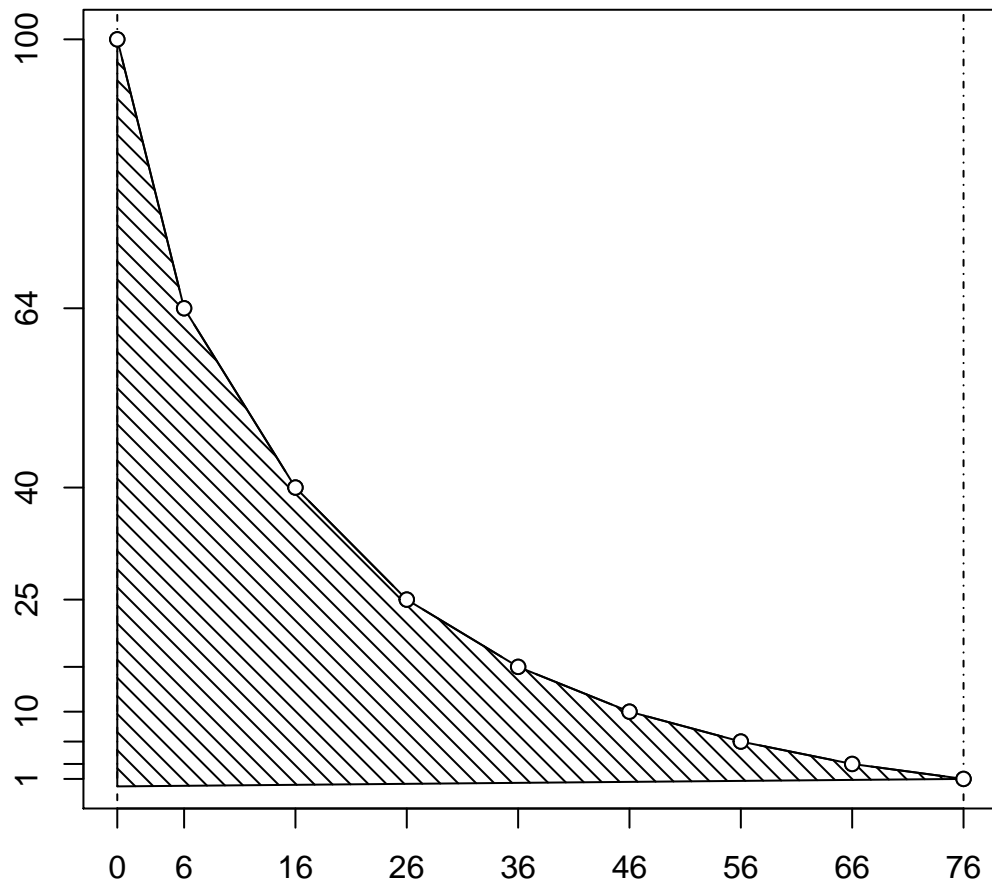
Setting up coordinates for polygon() (Clockwise)

```
graunt_x <- c(graunt$x, 0)
graunt_y <- c(graunt$xPo_g, 0)
graunt_poly <- data.frame(x = graunt_x, y = graunt_y)
```

#### 4. Shading

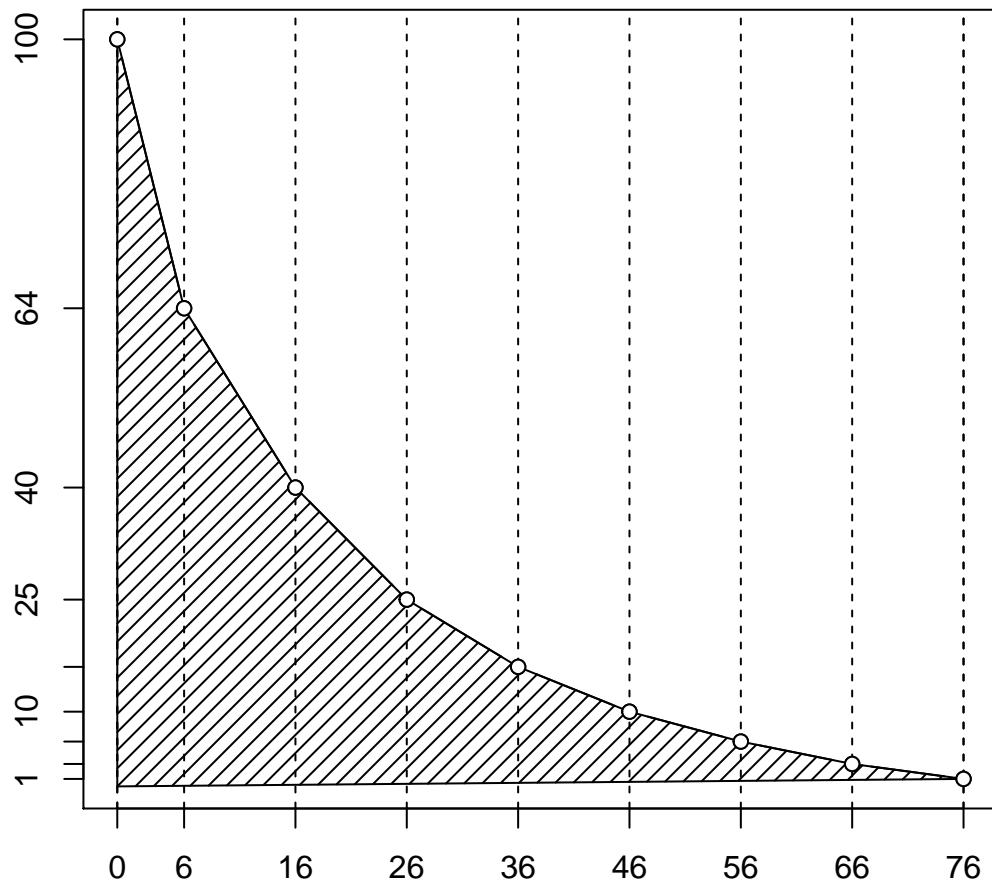
Note the effect of the last line of code.

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo_g, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 4)
polygon(graunt_poly, density = 15, angle = 135)
points(graunt, pch = 21, col = "black", bg = "white")
```



5. Grids

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo_g, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
polygon(graunt_poly, density = 15)
abline(v = graunt$x, lty = 2)
points(graunt, pch = 21, col = "black", bg = "white")
```

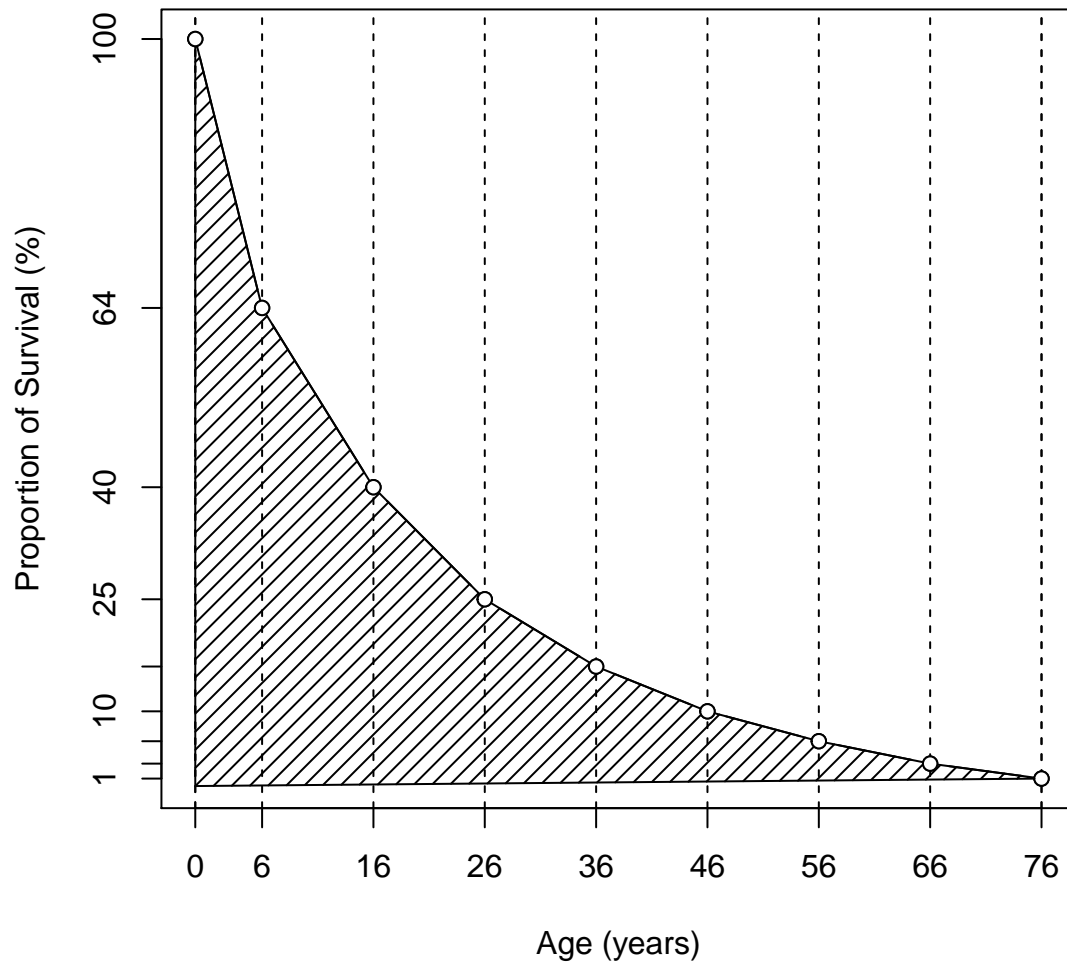


6. Title, x-axis label, and y-axis label

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo_g, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
polygon(graunt_poly, density = 15)
abline(v = graunt$x, lty = 2)
points(graunt, pch = 21, col = "black", bg = "white")
main_title <- "Graunt's Survival Function"
x_lab <- "Age (years)"
y_lab <- "Proportion of Survival (%)"
title(main = main_title, xlab = x_lab, ylab = y_lab)
```



## Graunt's Survival Function



### Area under the curve

The area under the curve can be approximated by the sum of the areas of trapezoids, therefore the area is  $\sum_{i=1}^{n-1} (x_{i+1} - x_i) \times \frac{1}{2}(y_i + y_{i+1})$ .

- `diff()`, `head()`, and `tail()` can be used to write a function to compute the area easily.

```
area.R <- function(x, y) {
  sum(diff(x) * (head(y, -1) + tail(y, -1))/2)
}
area.R(graunt$x, graunt$xPo_g)/100
```

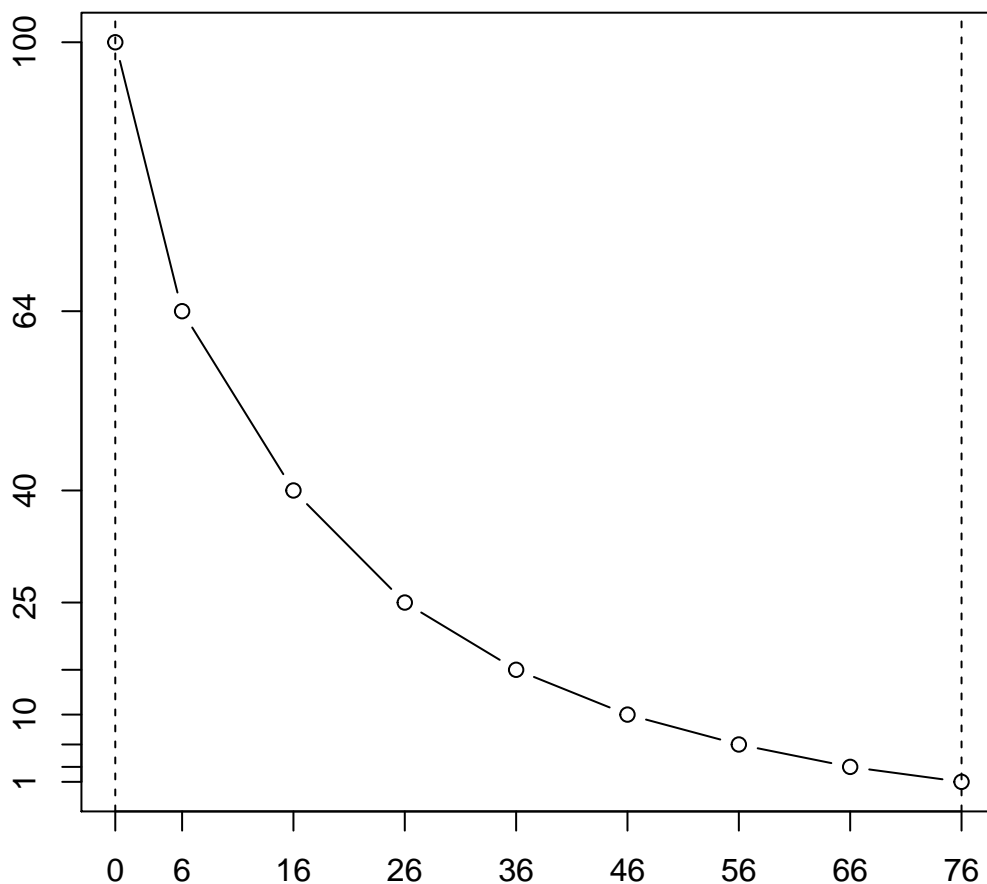
```
## [1] 18.17
```

### Comparison with US 1993 life table

The shaded area between the survival function of Graunt and that of US 1993 represents the difference of life expectancies.

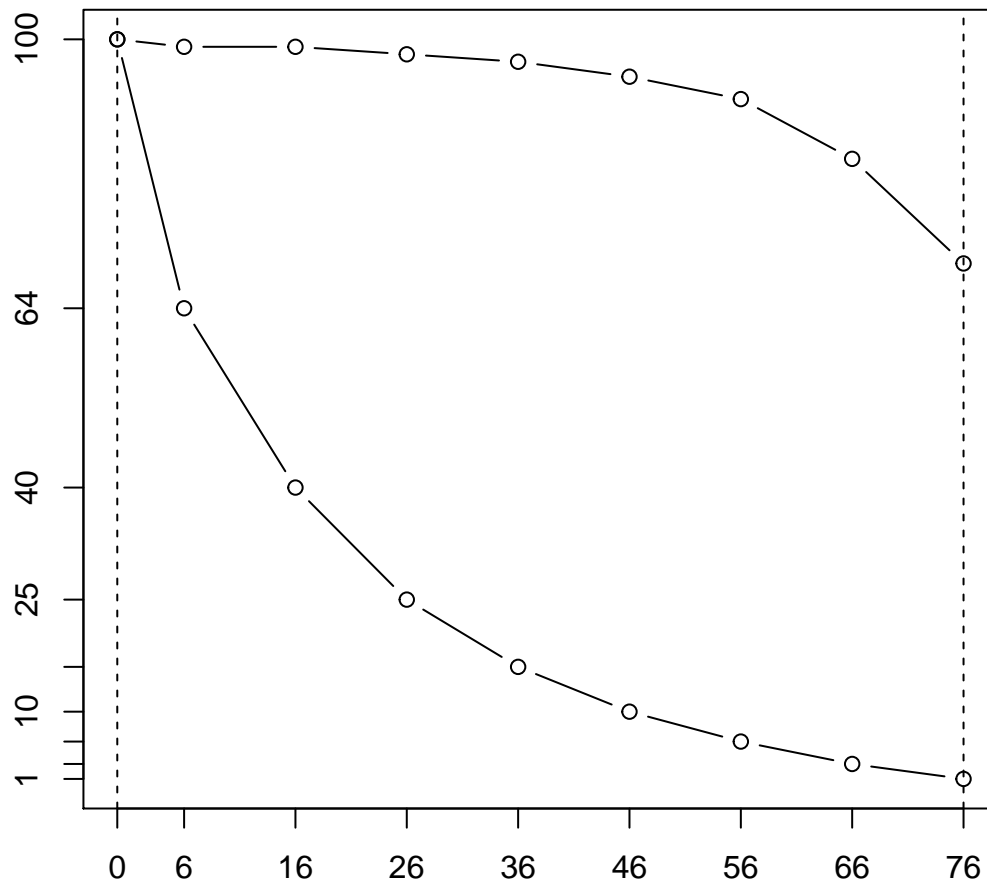
1. Draw Graunt's first with axes, lower and upper limits. Check what happens if you place `abline(...)` right after `plot(...)`.

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
```



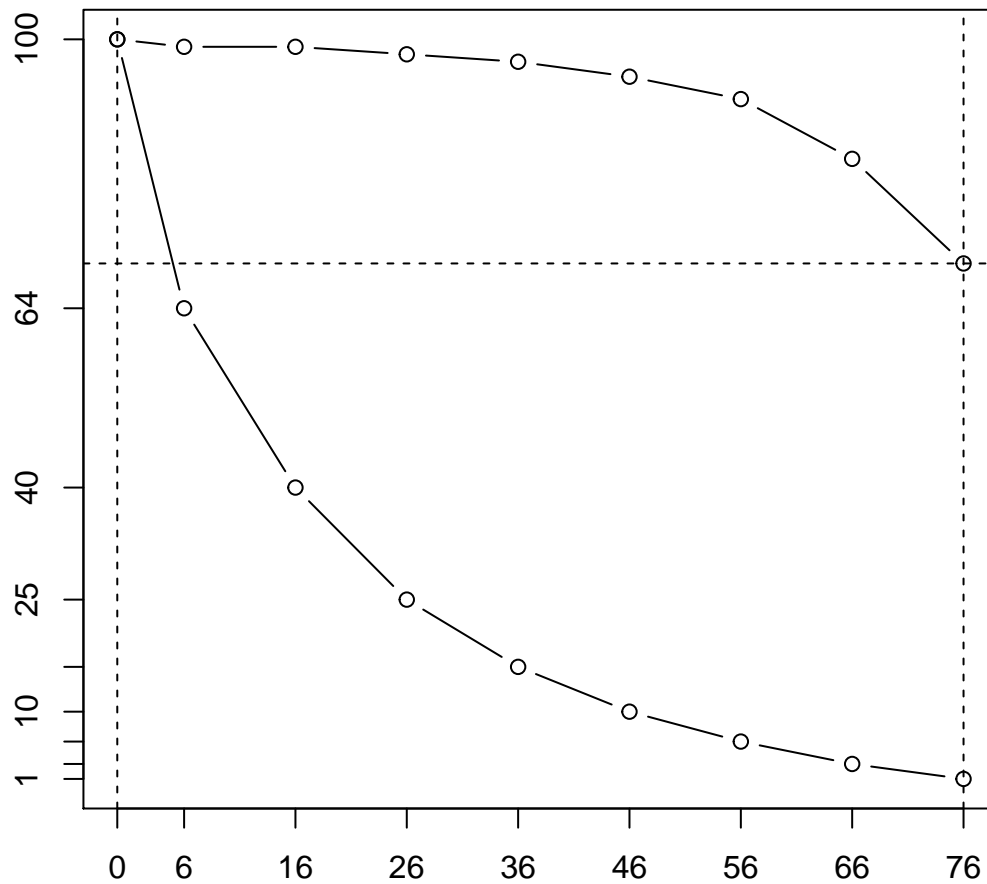
2. Add US 1993 survival function

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
lines(us93, type = "b")
```



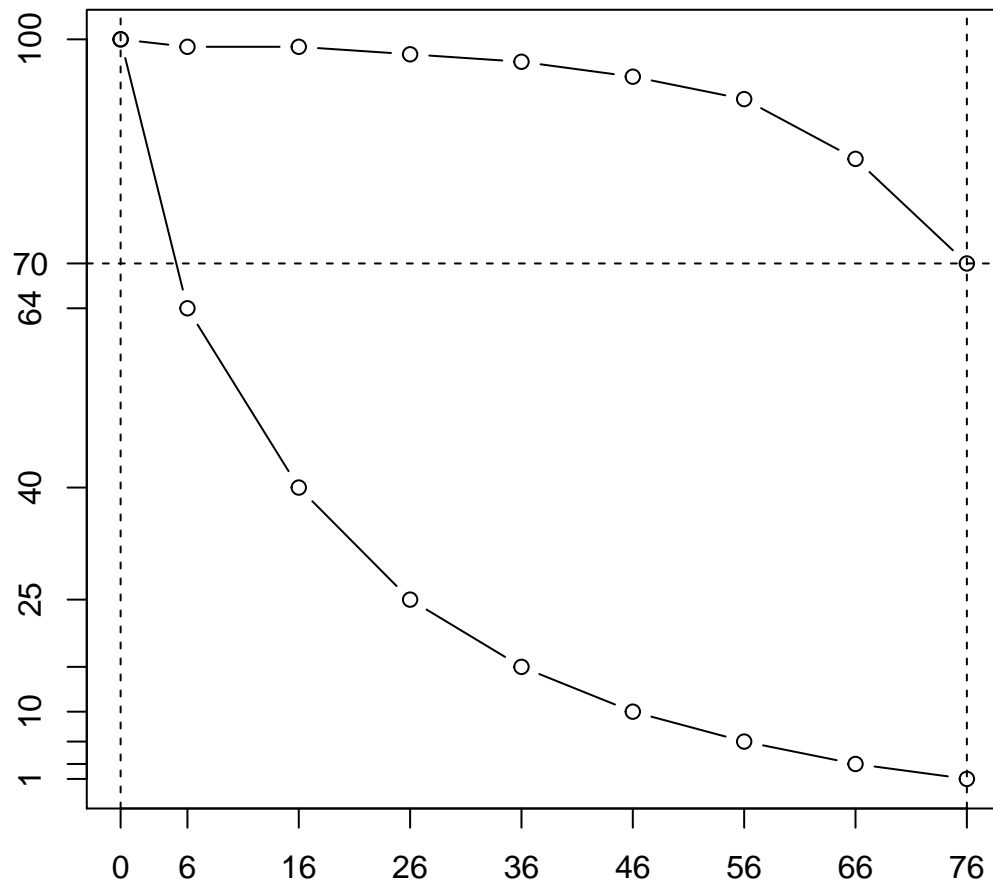
3. Actually, US 1993 life table is truncated at the age 76. Specify that point.

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
lines(us93, type = "b")
abline(h = 70, lty = 2)
```



4. Using `las = 1` to specify 70%.

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
lines(us93, type = "b")
abline(h = 70, lty = 2)
axis(side = 2, at = 70, labels = 70, las = 1)
```



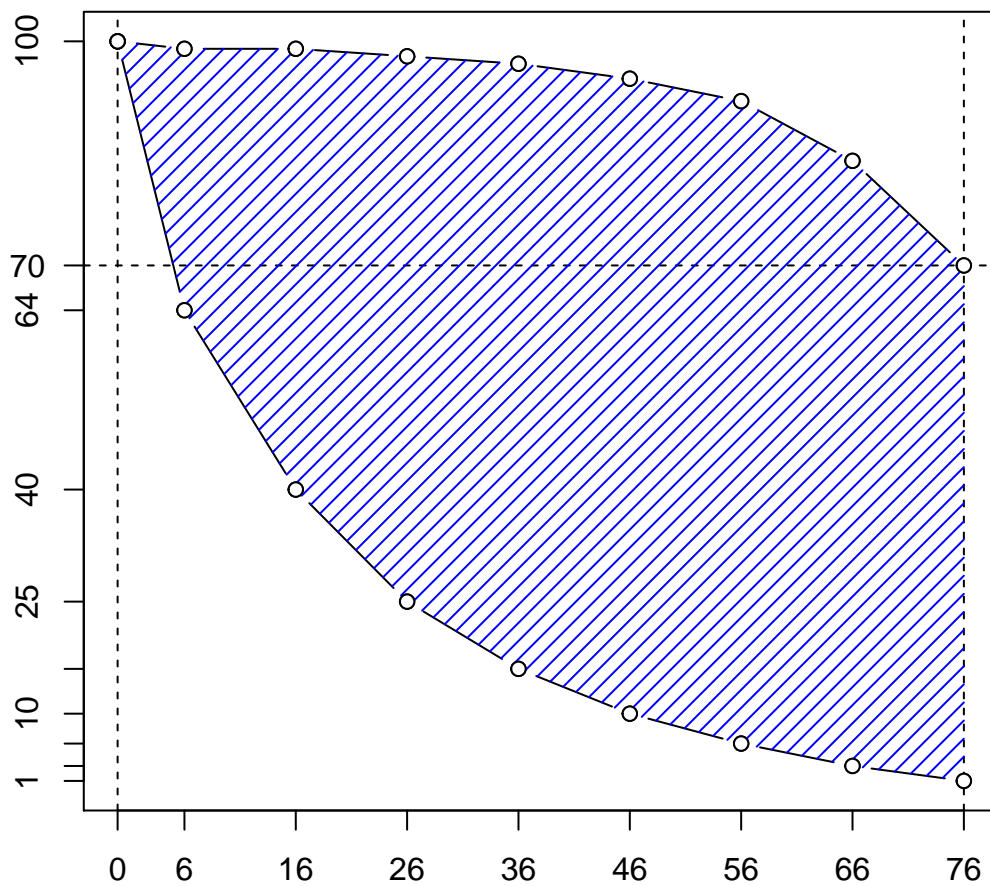
Setting coordinates for polygon()

```
us_graunt_x <- c(us93$x, rev(graunt$x))
us_graunt_y <- c(us93$xPo_us, rev(graunt$xPo_g))
us_graunt <- data.frame(x = us_graunt_x, y = us_graunt_y)
```

#### 5. Shading

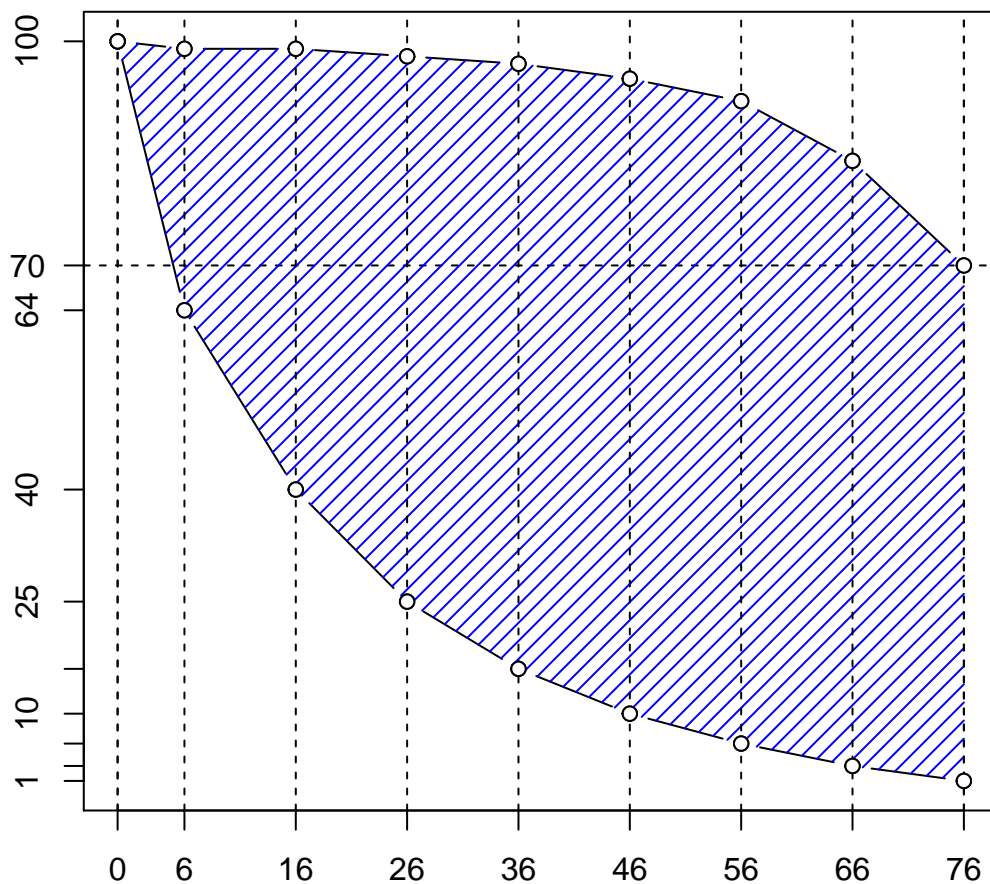
What is the effect of `border = NA`, the last line of code?

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
lines(us93, type = "b")
abline(h = 70, lty = 2)
axis(side = 2, at = 70, labels = 70, las = 1)
polygon(us_graunt, density = 15, col = "blue", border = NA)
points(us_graunt, pch = 21, col = "black", bg = "white")
```



6. Grids

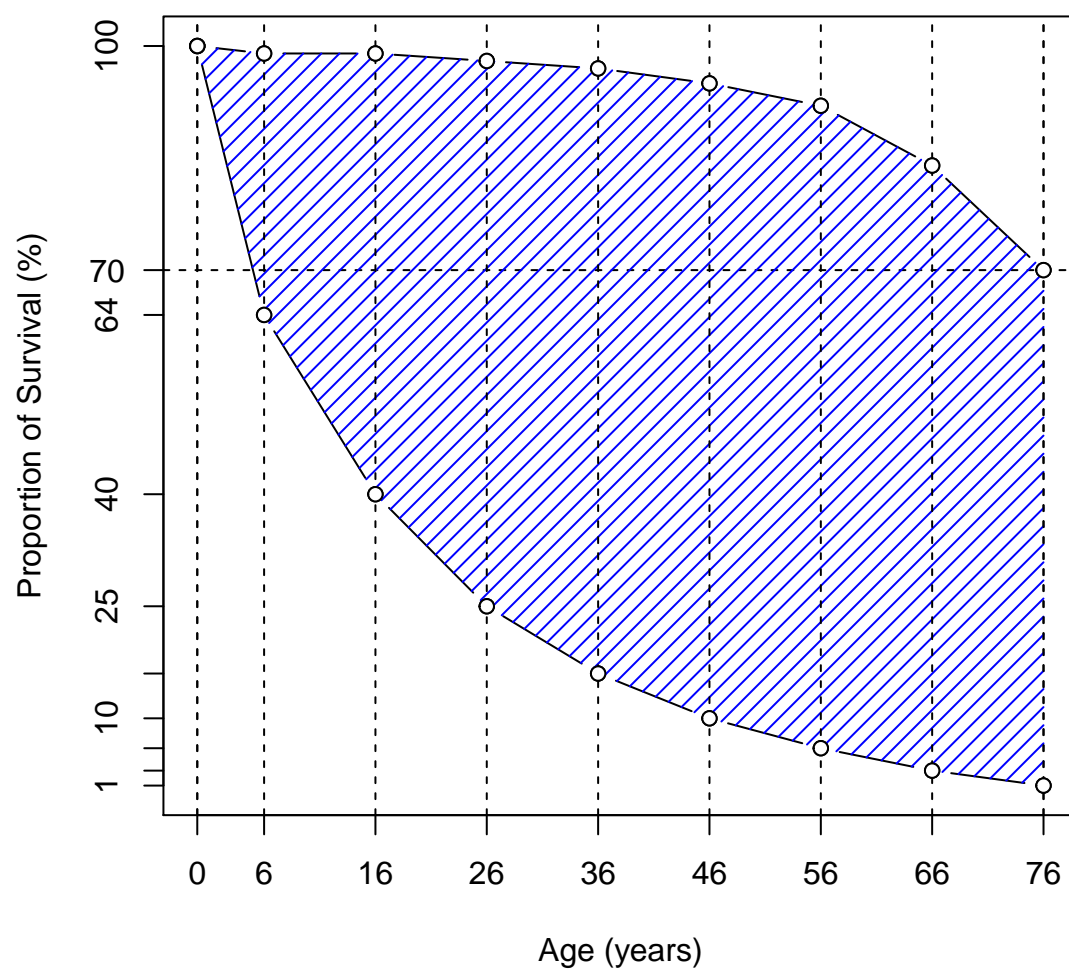
```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
lines(us93, type = "b")
abline(h = 70, lty = 2)
axis(side = 2, at = 70, labels = 70, las = 1)
polygon(us_graunt, density = 15, col = "blue", border = NA)
abline(v = graunt$x, lty = 2)
points(us_graunt, pch = 21, col = "black", bg = "white")
```



7. Title, x-axis and y-axis labels

```
plot(graunt, ann = FALSE, xaxt = "n", yaxt = "n", type = "b")
axis(side = 1, at = graunt$x, labels = graunt$x)
axis(side = 2, at = graunt$xPo, labels = graunt$xPo_g)
abline(v = c(0, 76), lty = 2)
lines(us93, type = "b")
abline(h = 70, lty = 2)
axis(side = 2, at = 70, labels = 70, las = 1)
polygon(us_graunt, density = 15, col = "blue", border = NA)
abline(v = graunt$x, lty = 2)
points(us_graunt, pch = 21, col = "black", bg = "white")
main_title_g_us <- "Survival Function of Graunt and US 1993"
title(main = main_title_g_us, xlab = x_lab, ylab = y_lab)
```

## Survival Function of Graunt and US 1993



```
dev.copy(device = png, file = "../pics/graunt_us93.png")
```

```
## png
## 3
dev.off()
```

```
## pdf
## 2
```

### Life expectancy

The area under the US 1993 survival function is

```
area.R(us93$x, us93$xPo_us)/100
```

```
## [1] 70.92
```

The area of shaded region is

```
area.R(us93$x, us93$xPo_us)/100 - area.R(graunt$x, graunt$xPo_g)/100
```

```
## [1] 52.75
```



## ggplot

```
library(ggplot2)
```

### Data Reshape

Attach `reshape2` package to change wide format to long format

```
library(reshape2)
```

How `melt()` works

```
graunt_us_melt <- melt(graunt_us,
                      id.vars = "x",
                      measure.vars = c("xPo_g", "xPo_us"),
                      value.name = "xPo",
                      variable.name = "times")
```

```
graunt_us_melt
```

```
##      x  times xPo
## 1    0 xPo_g 100
## 2    6 xPo_g  64
## 3   16 xPo_g  40
## 4   26 xPo_g  25
## 5   36 xPo_g  16
## 6   46 xPo_g  10
## 7   56 xPo_g   6
## 8   66 xPo_g   3
## 9   76 xPo_g   1
## 10  0 xPo_us 100
## 11  6 xPo_us  99
## 12 16 xPo_us  99
## 13 26 xPo_us  98
## 14 36 xPo_us  97
## 15 46 xPo_us  95
## 16 56 xPo_us  92
## 17 66 xPo_us  84
## 18 76 xPo_us  70
```

```
str(graunt_us_melt)
```

```
## 'data.frame':   18 obs. of  3 variables:
## $ x      : num  0 6 16 26 36 46 56 66 76 0 ...
## $ times: Factor w/ 2 levels "xPo_g","xPo_us": 1 1 1 1 1 1 1 1 1 2 ...
## $ xPo   : num  100 64 40 25 16 10 6 3 1 100 ...
```

- Change factor levels of times

```
levels(graunt_us_melt$times) <- c("Graunt", "US1993")
graunt_us_melt
```

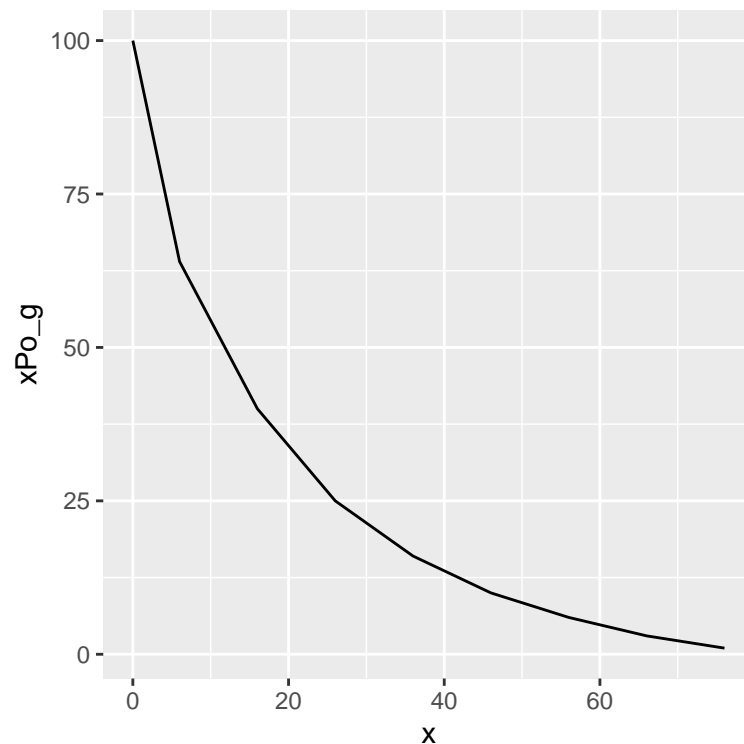
```
##      x  times xPo
## 1    0 Graunt 100
## 2    6 Graunt  64
## 3   16 Graunt  40
## 4   26 Graunt  25
```

```
## 5 36 Graunt 16
## 6 46 Graunt 10
## 7 56 Graunt 6
## 8 66 Graunt 3
## 9 76 Graunt 1
## 10 0 US1993 100
## 11 6 US1993 99
## 12 16 US1993 99
## 13 26 US1993 98
## 14 36 US1993 97
## 15 46 US1993 95
## 16 56 US1993 92
## 17 66 US1993 84
## 18 76 US1993 70
```

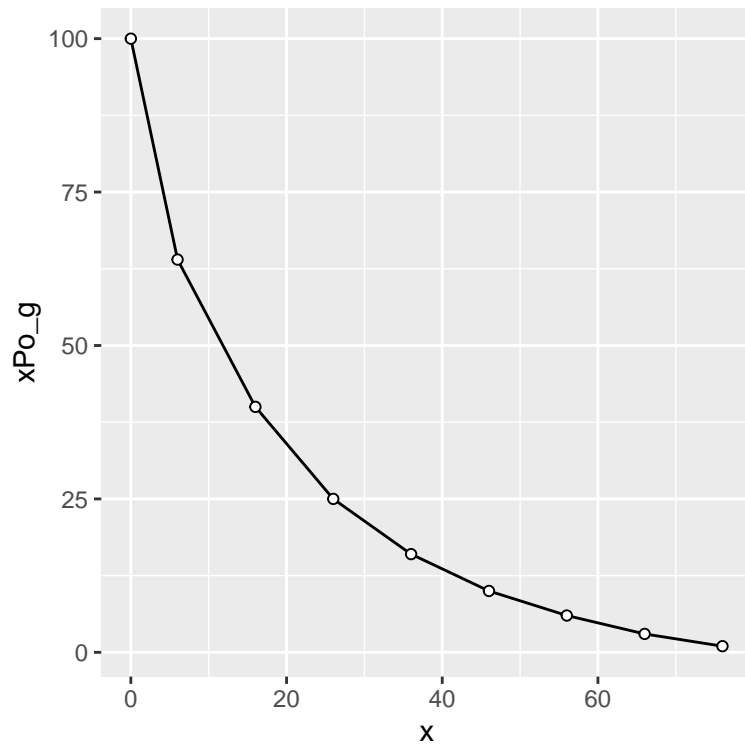
## Graunt

### Structure of ggplot

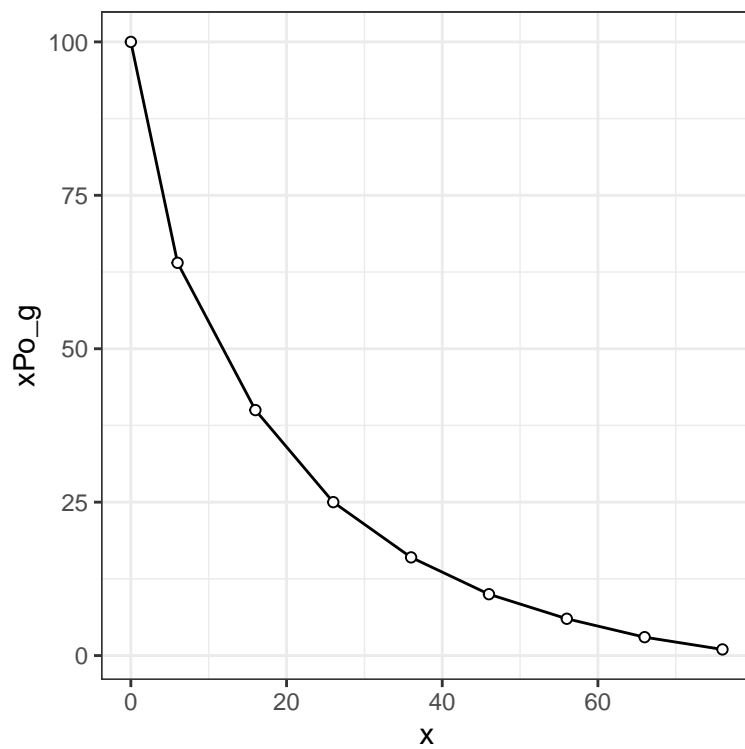
```
(g1 <- ggplot(data = graunt,
              mapping = aes(x = x, y = xPo_g)) +
  geom_line())
```



```
(g2 <- g1 +
  geom_point(shape = 21, fill = "white"))
```



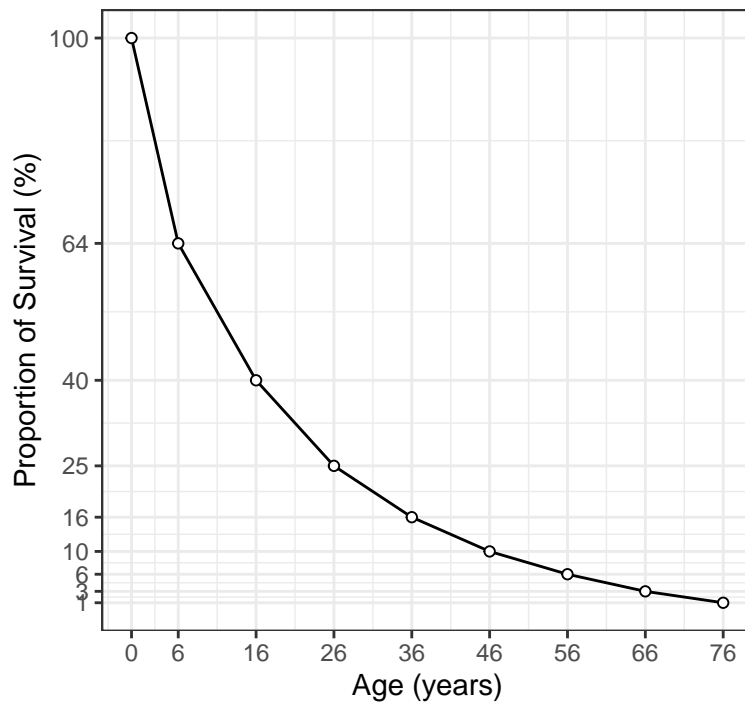
```
(g3 <- g2 +  
  theme_bw())
```



```
(g4 <- g3 +  
  xlab(x_lab) +  
  ylab(y_lab) +  
  ggtitle(main_title) +
```

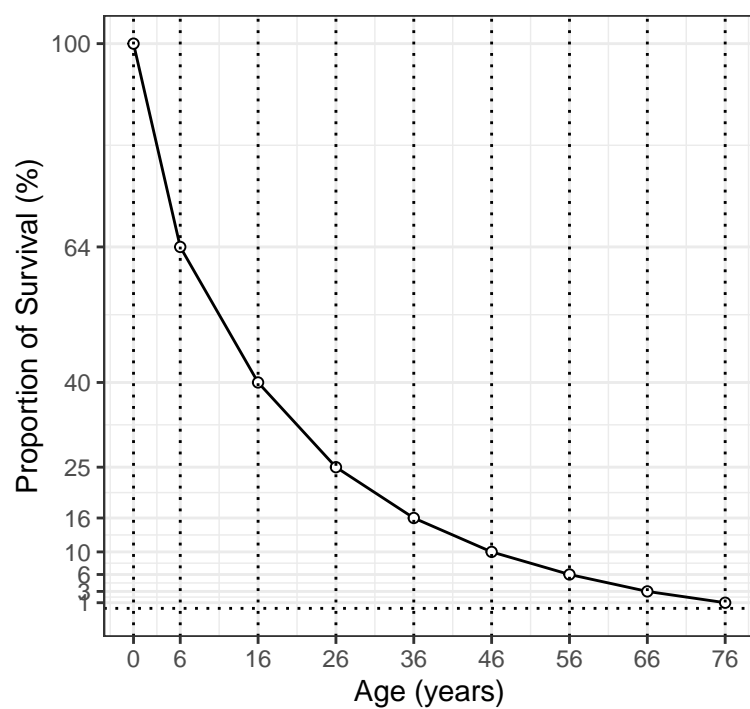
```
scale_x_continuous(breaks = graunt$x) +
scale_y_continuous(breaks = graunt$xPo_g))
```

Graunt's Survival Function



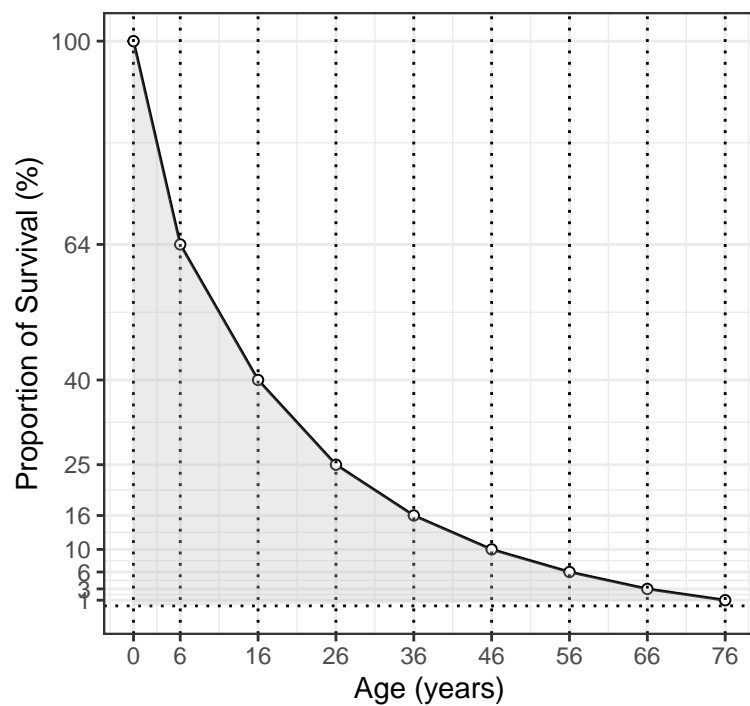
```
(g5 <- g4 +
  geom_vline(xintercept = graunt$x, linetype = "dotted") +
  geom_hline(yintercept = 0, linetype = "dotted"))
```

### Graunt's Survival Function



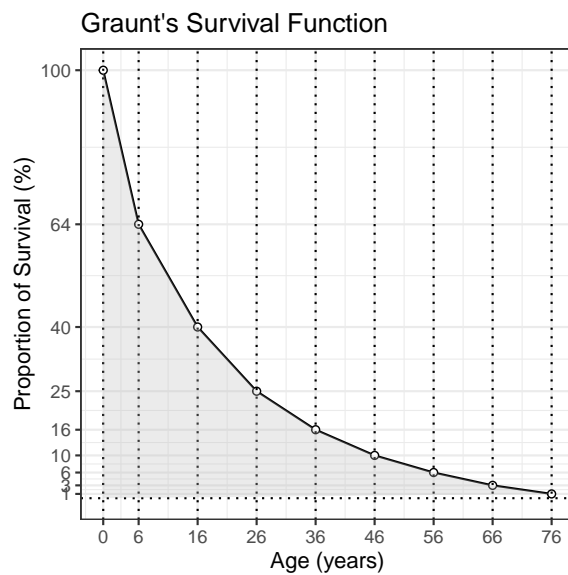
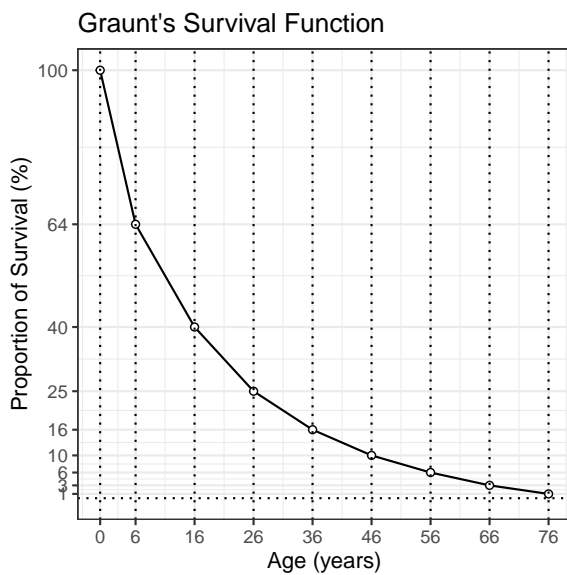
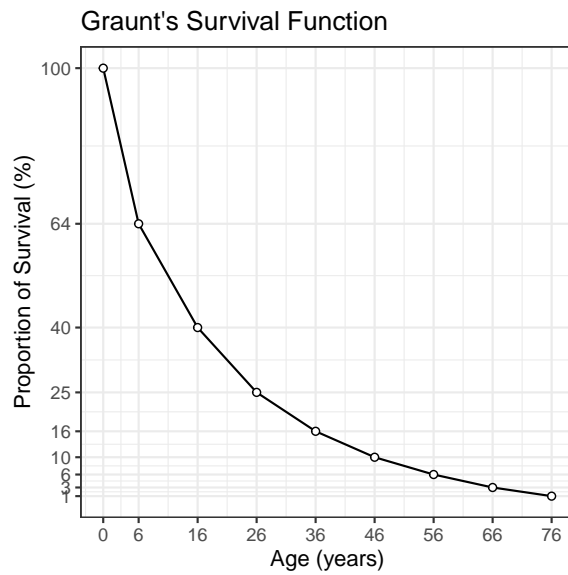
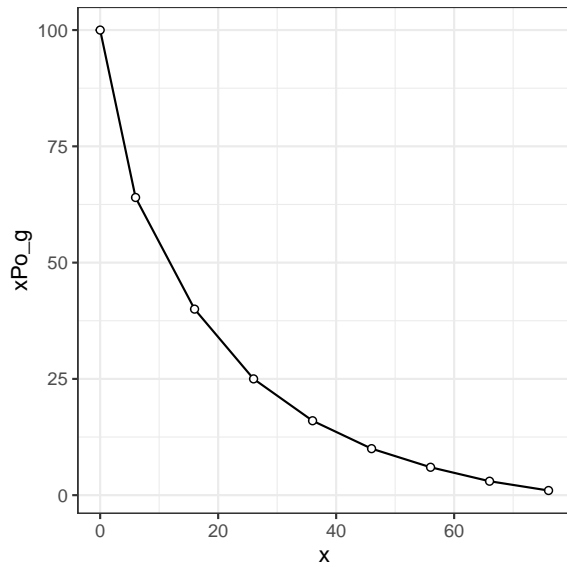
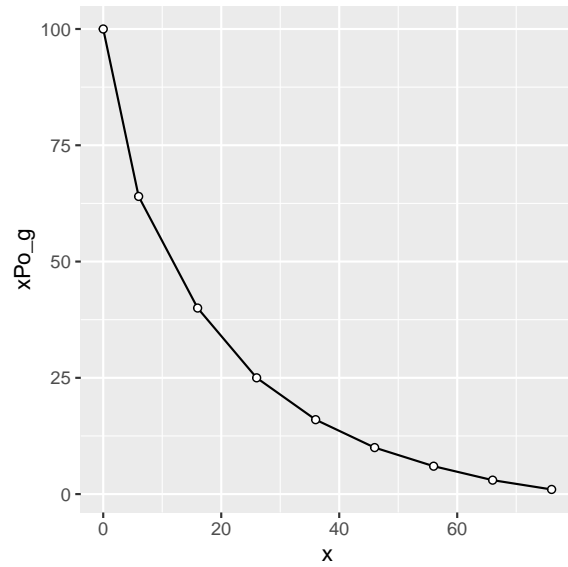
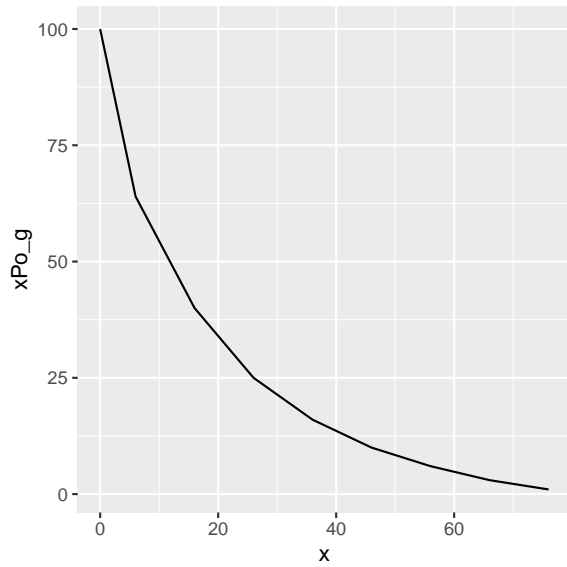
```
(pg5 <- g5 +
  geom_polygon(data = graunt_poly,
    mapping = aes(x = x, y = y),
    alpha = 0.3, fill = "grey"))
```

### Graunt's Survival Function



```
# ggsave("../pics/graunt_poly_ggplot.png", pg5)

library(gridExtra)
g_graunt <- grid.arrange(g1, g2, g3, g4, g5, pg5, nrow = 3)
```



```
# ggsave(g_graunt, file = "../pics/graunt_ggplots.png", width = 8, height = 12)
```

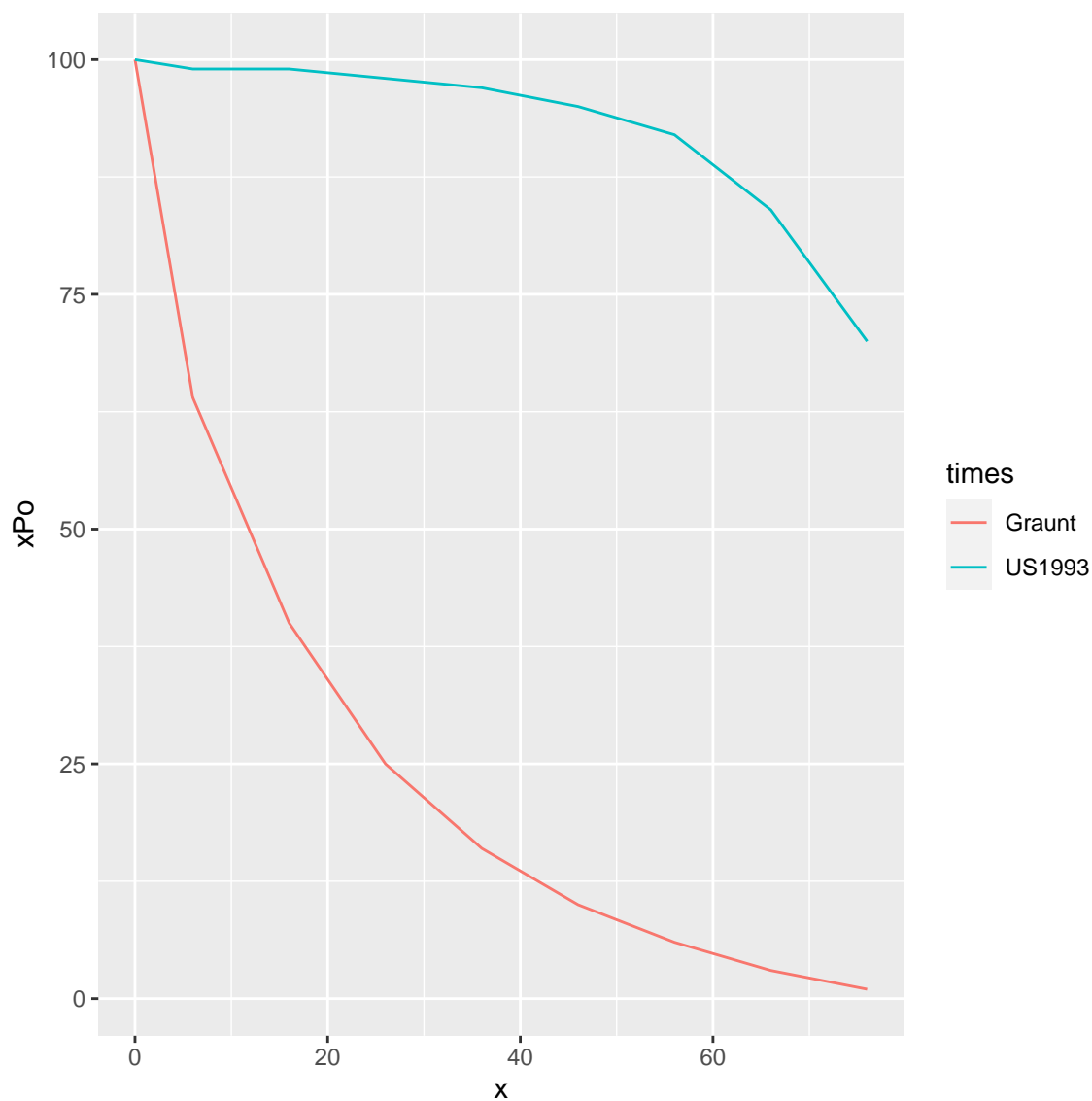
## Graunt and US 1993

### Points and Lines

Step by step approach to understand the grammar of ggplot

- We set `ggplot()` to accept varying `data.frame()` and `aes()` in `geom_polygon`

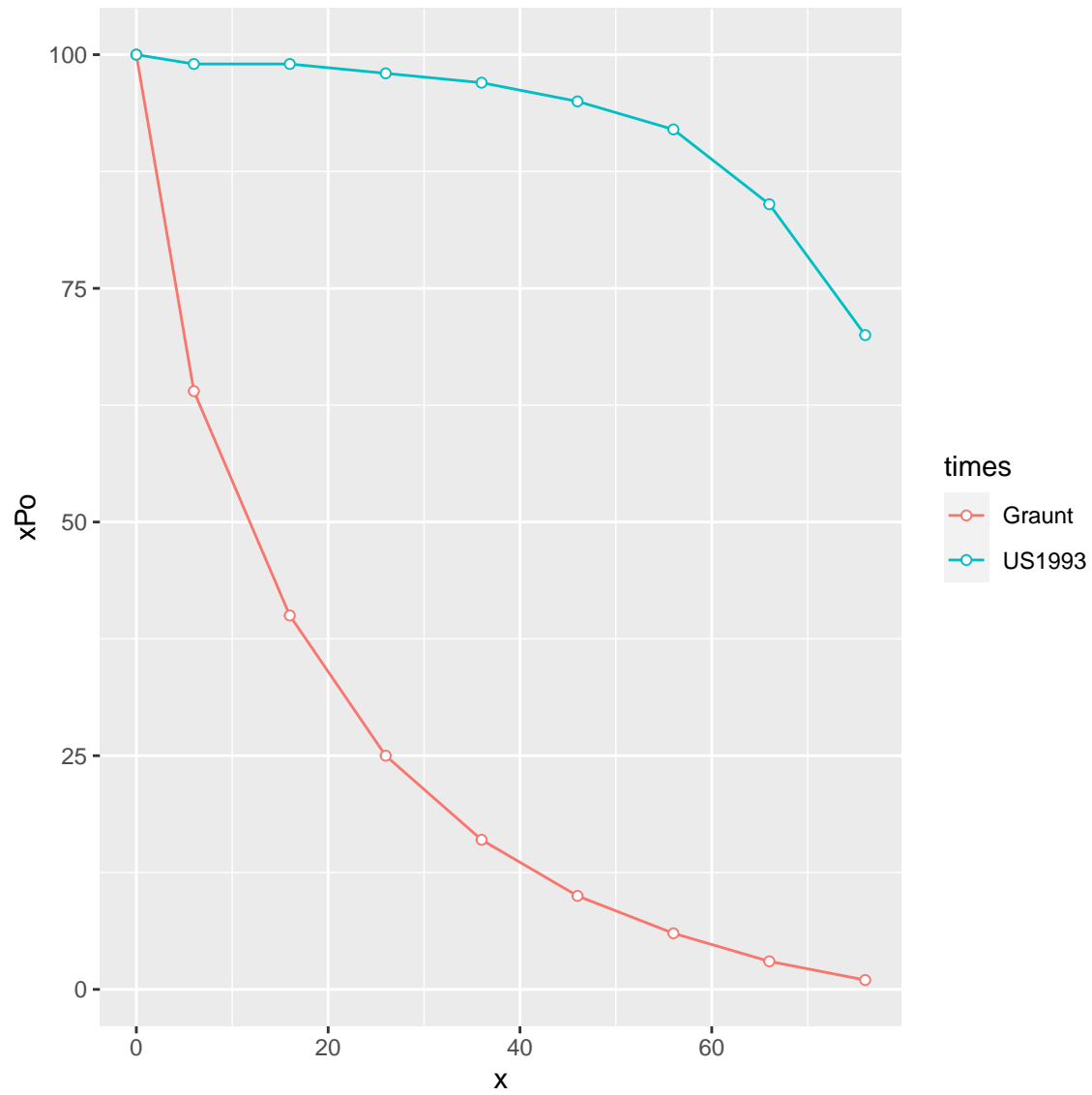
```
(gu1 <- ggplot() +  
  geom_line(data = graunt_us_melt,  
            mapping = aes(x = x, y = xPo, colour = times)))
```



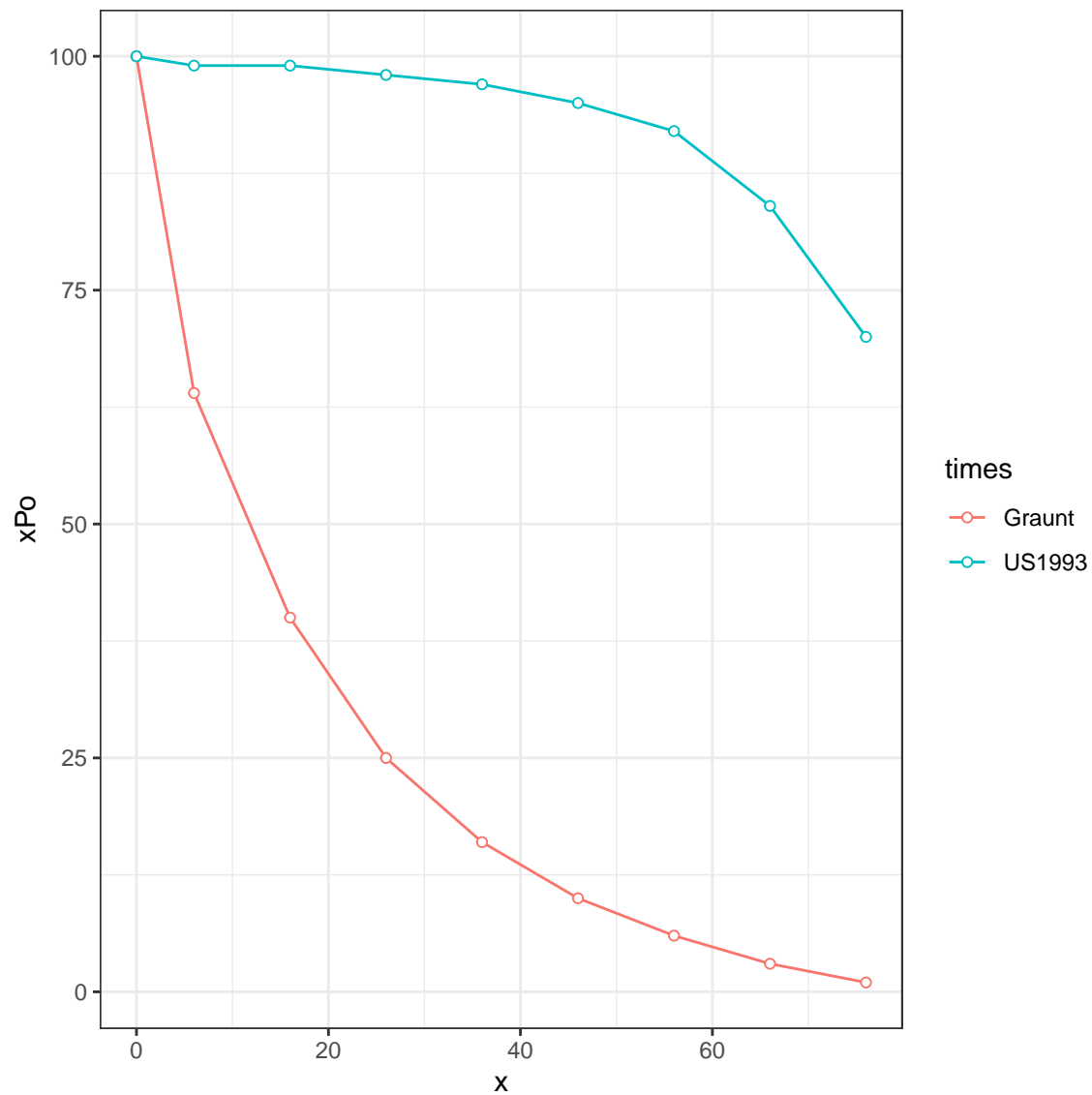
```
(gu2 <- gu1 +  
  geom_point(data = graunt_us_melt,
```



```
mapping = aes(x = x, y = xPo, colour = times),
shape = 21, fill = "white"))
```



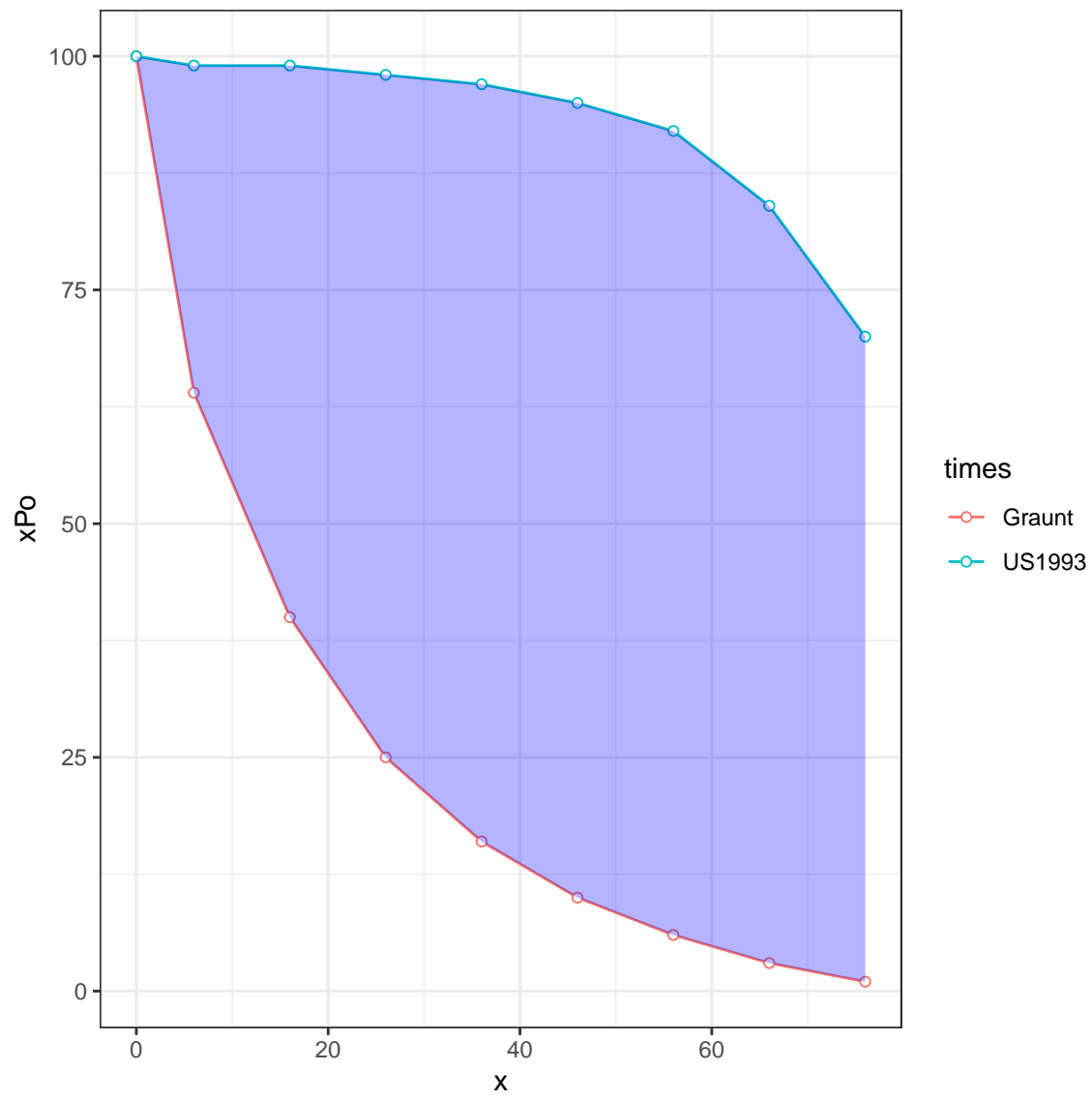
```
(gu3 <- gu2 +
  theme_bw())
```



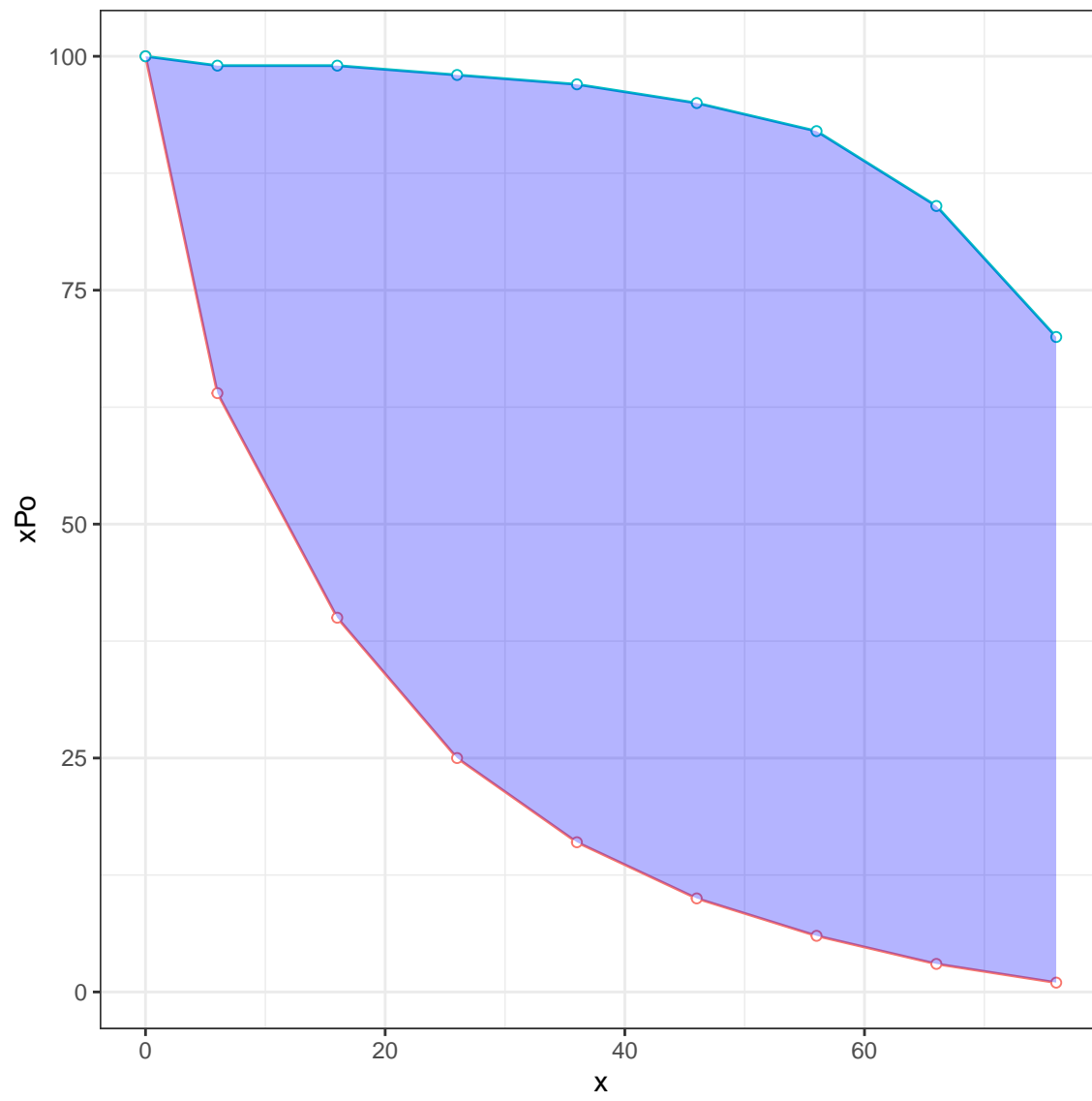
## Polygon

Reuse `us_graunt` which contains `x = us_graunt_x` and `y = us_graunt_y` for `polygon()`. Note that we start with `gu3`, and also note how to remove default legends.

```
(gup3 <- gu3 +  
  geom_polygon(data = us_graunt,  
    mapping = aes(x = x, y = y),  
    alpha = 0.3, fill = "blue"))
```



```
(gup4 <- gup3 +  
  guides(colour = "none"))
```

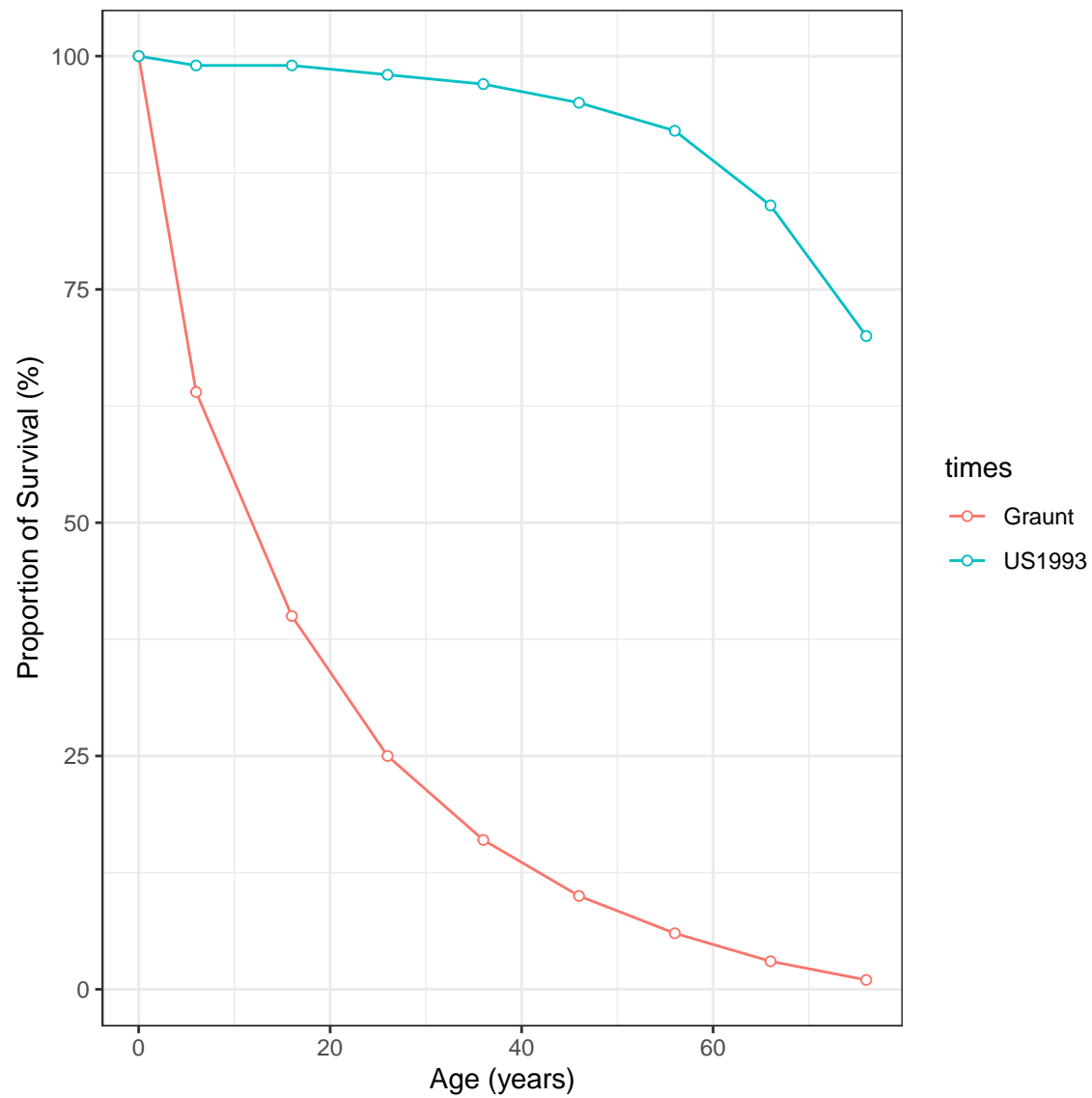


## Change default annotations

### Points and Lines

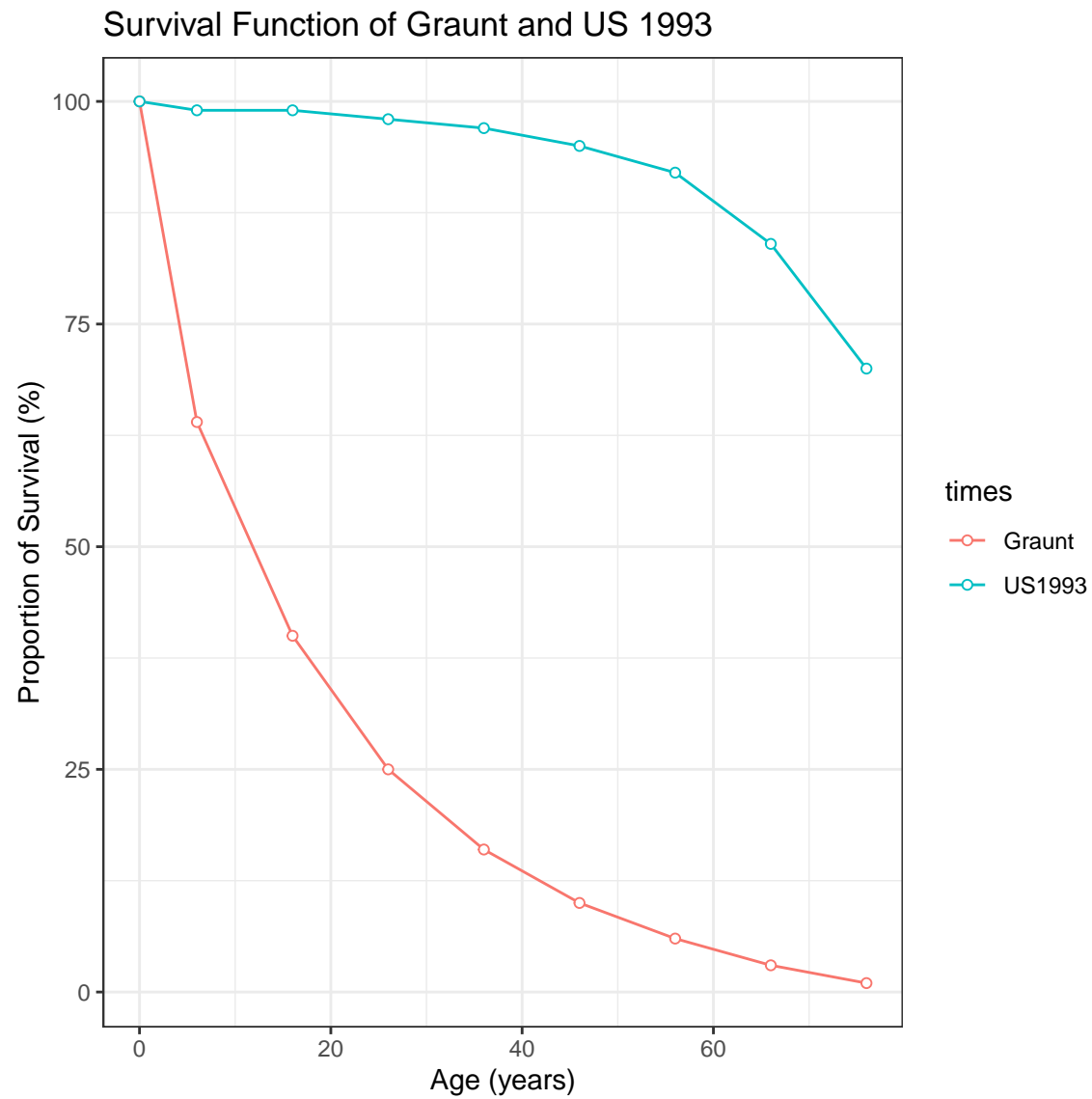
1. Change the x-axis and y-axis labels

```
(gu4 <- gu3 +  
  xlab(x_lab) +  
  ylab(y_lab))
```



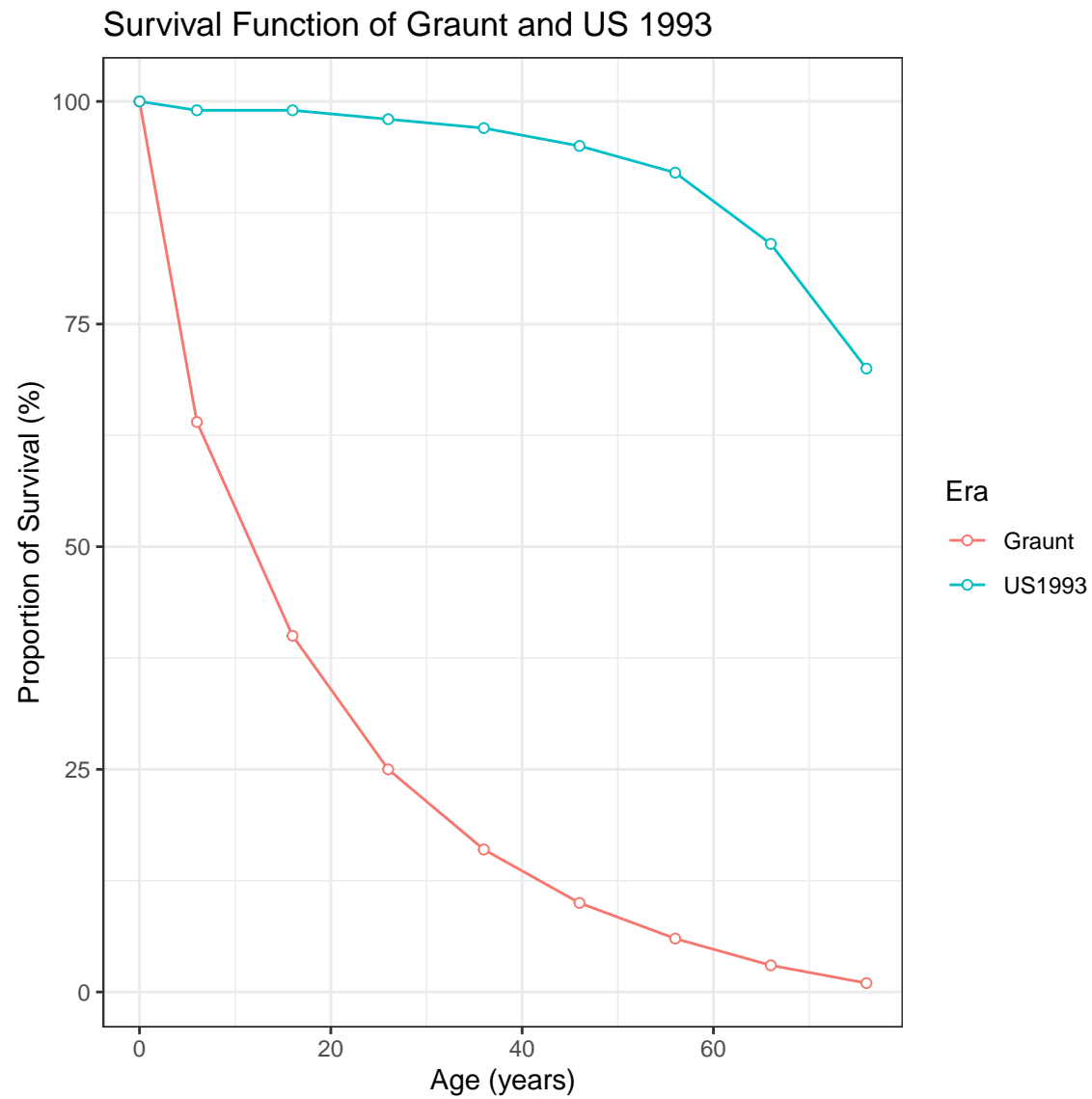
2. Add main title

```
(gu4 <- gu3 +  
  xlab(x_lab) +  
  ylab(y_lab) +  
  ggtitle(main_title_g_us))
```



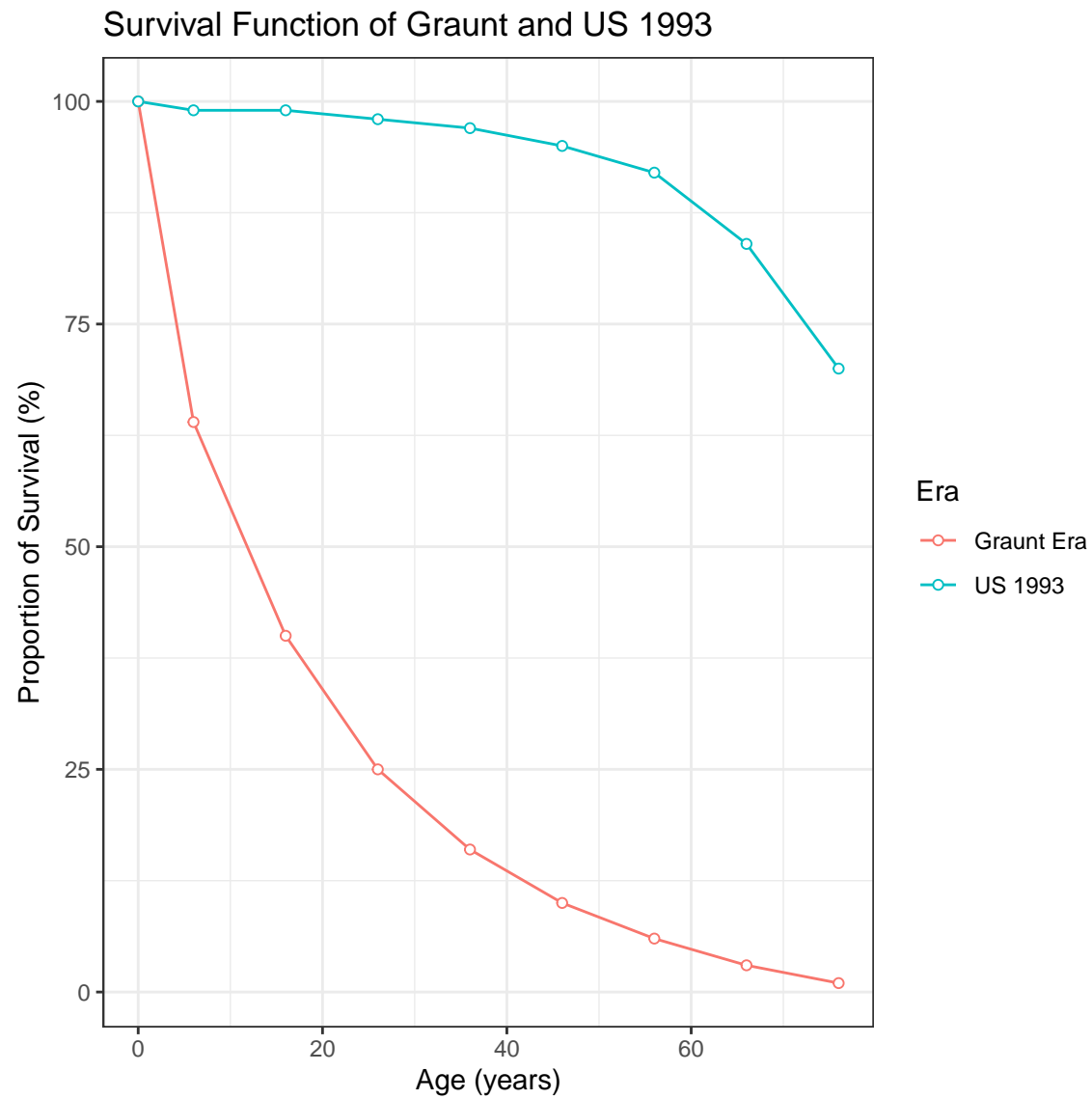
3. Change legend title

```
(gu4 <- gu3 +  
  xlab(x_lab) +  
  ylab(y_lab) +  
  ggtitle(main_title_g_us) +  
  labs(colour = "Era"))
```



4. Change legends.

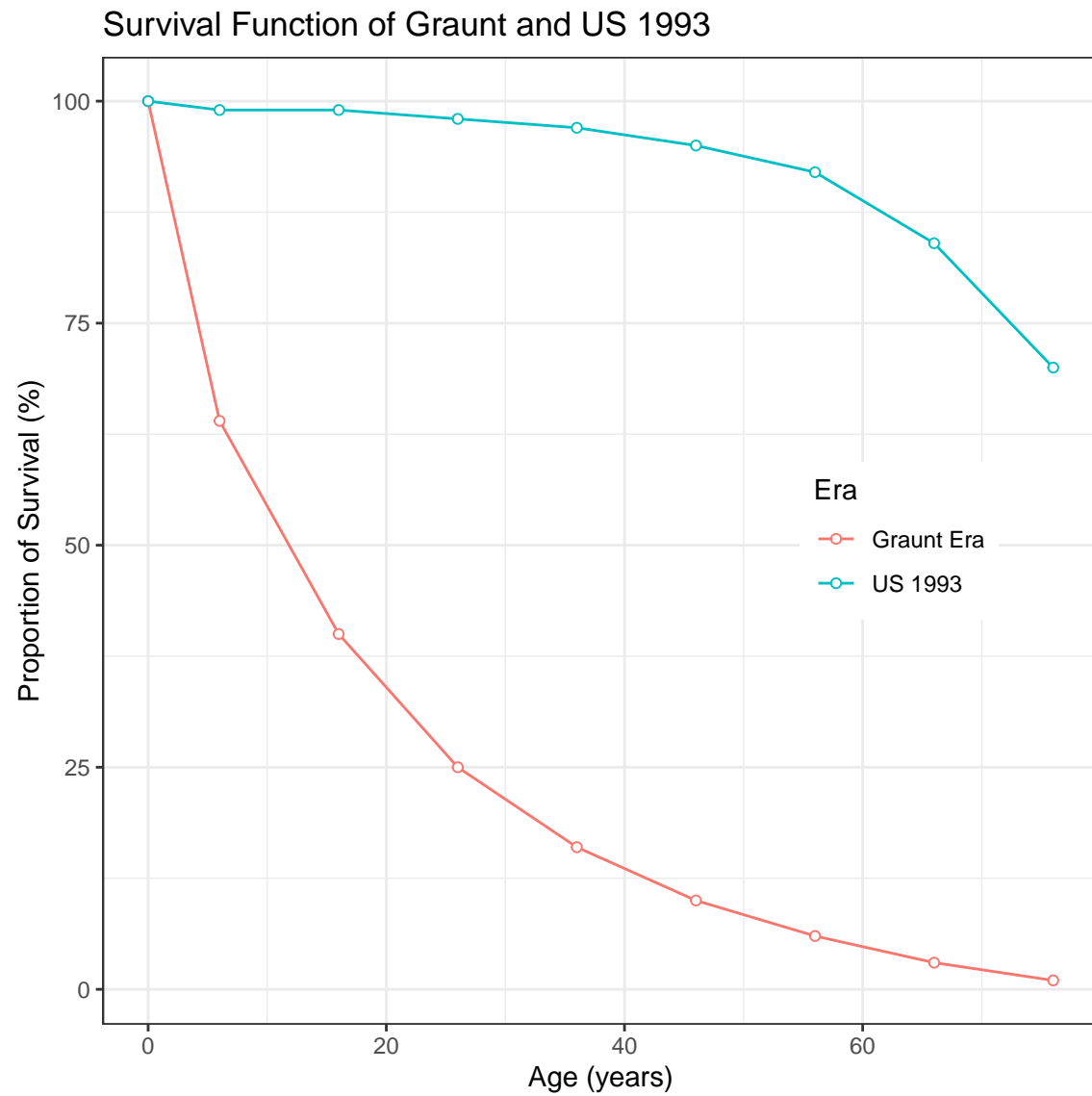
```
(gu4 <- gu3 +  
  xlab(x_lab) +  
  ylab(y_lab) +  
  ggtitle(main_title_g_us) +  
  labs(colour = "Era") +  
  scale_colour_discrete(labels = c("Graunt Era", "US 1993")))
```



5. Place legends inside the plot

```
(gu5 <- gu4 +  
  theme(legend.position = c(0.8, 0.5)))
```

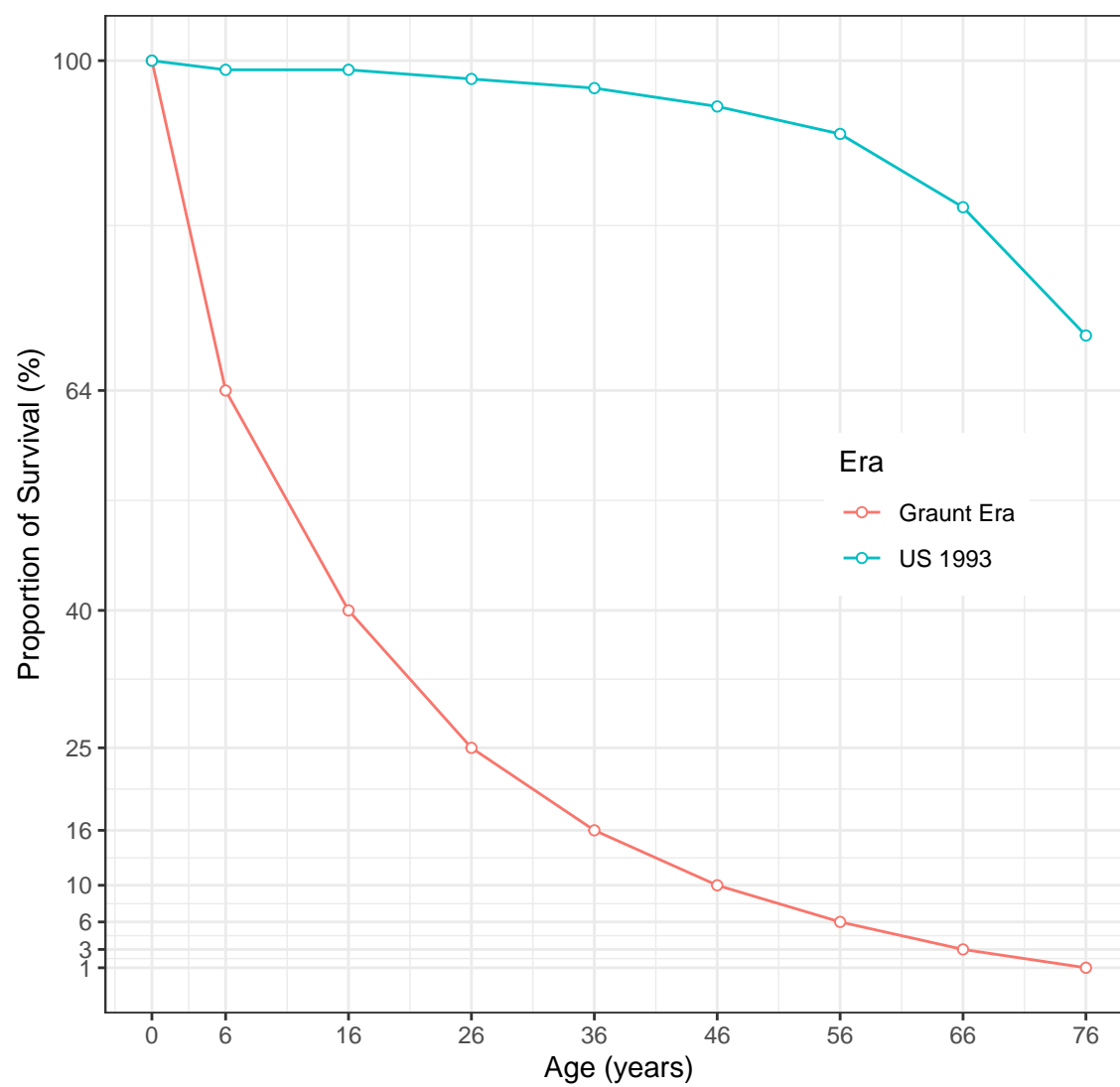




6. Change x-axis and y-axis tick marks

```
(gu6 <- gu5 +  
  scale_x_continuous(breaks = graunt$x) +  
  scale_y_continuous(breaks = graunt$xPo_g))
```

## Survival Function of Graunt and US 1993



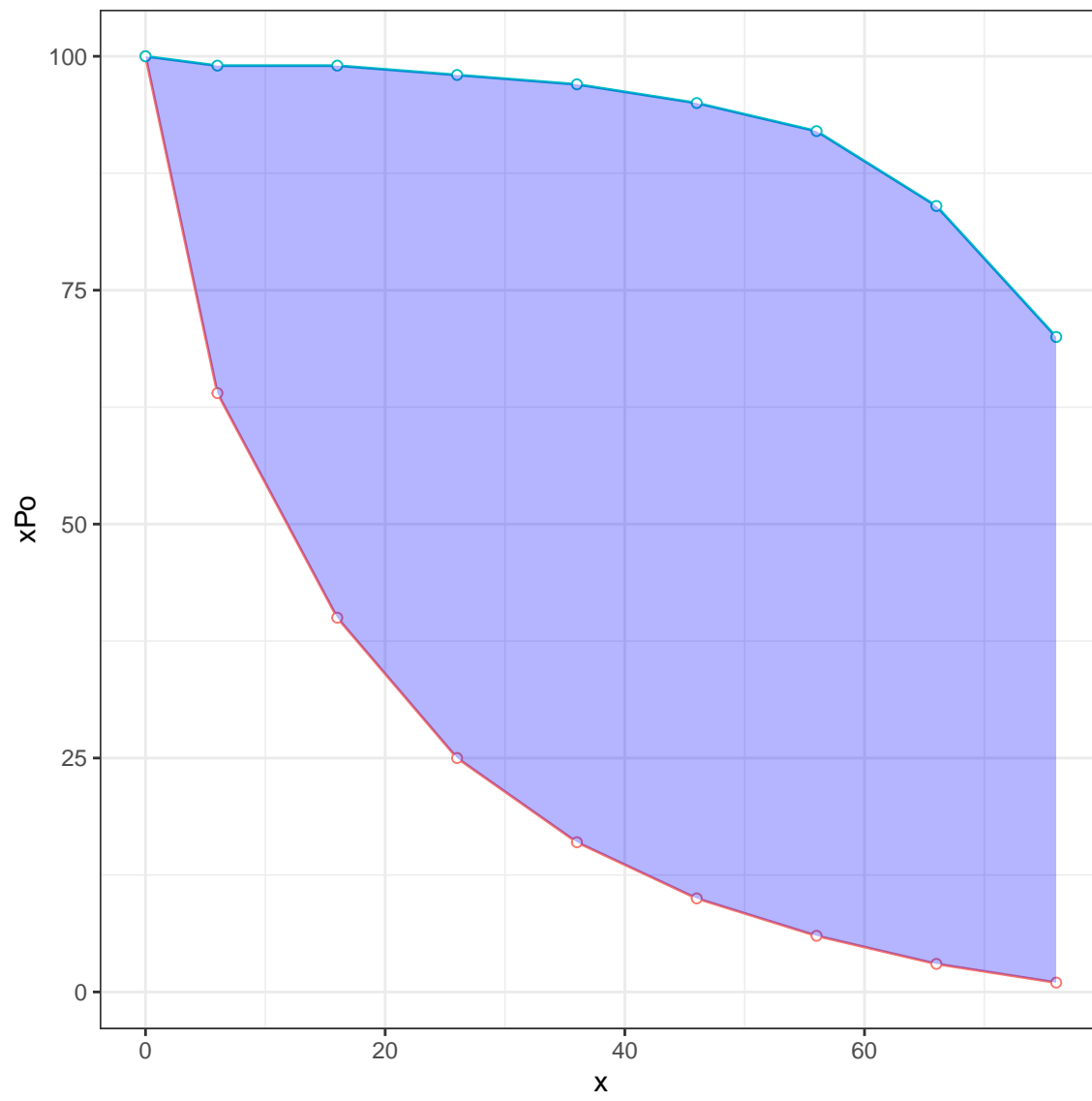
```
# ggsave("../pics/graunt_us_ggplot.png", gu6)
```

## Polygon

Add information to the plot drawn with `polygon()`

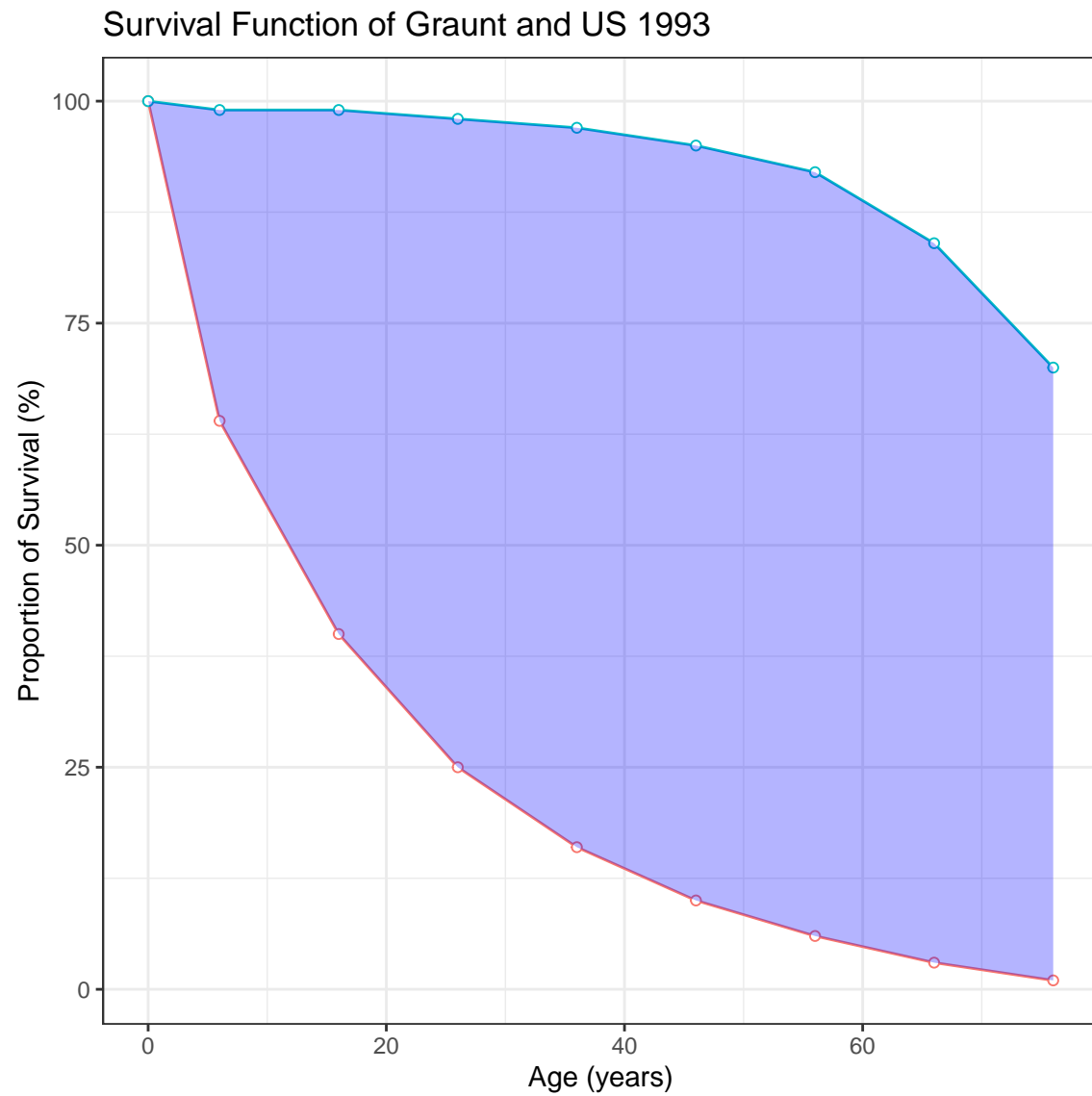
1. Start with `gup4`

```
gup4
```



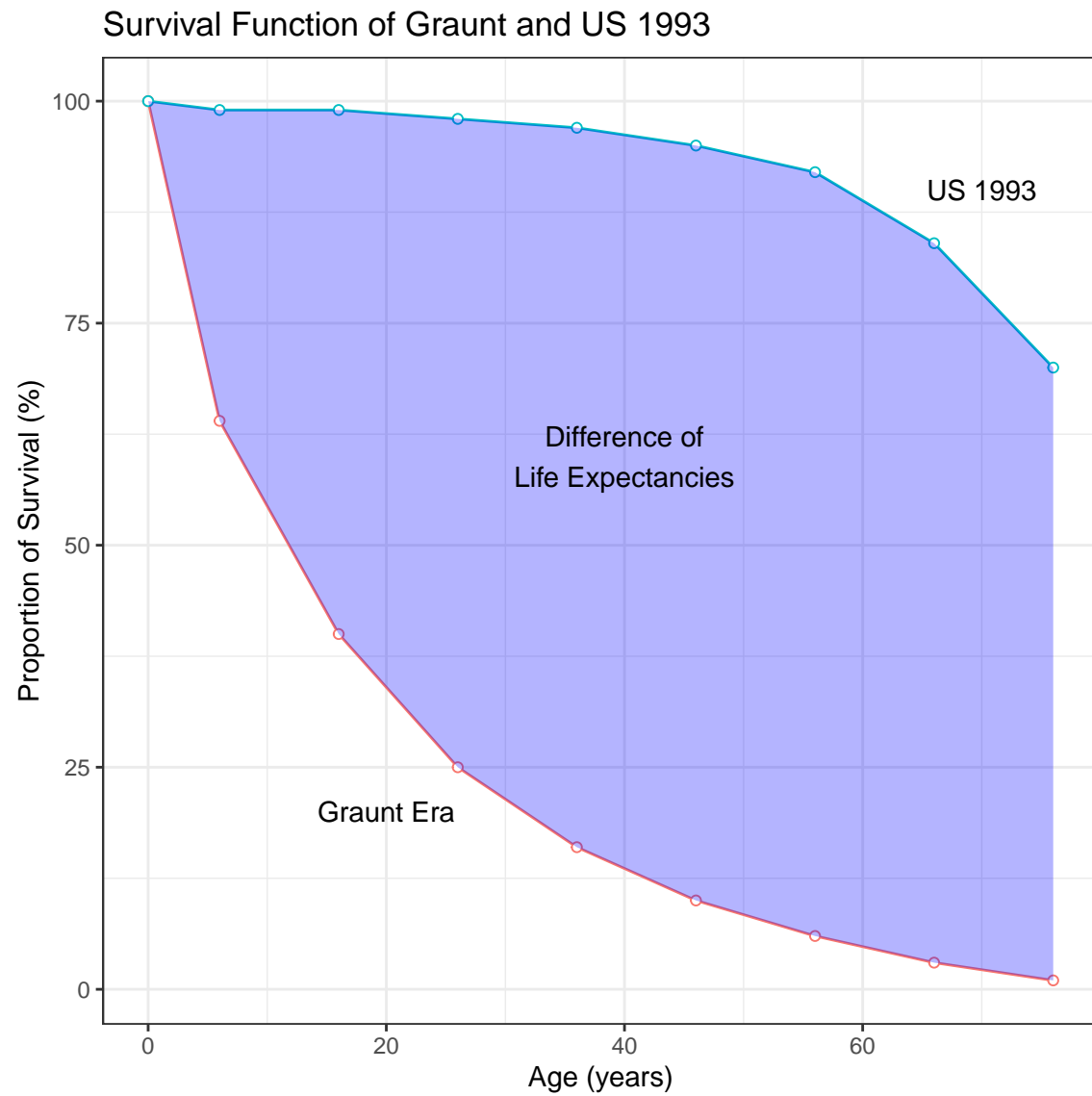
2. Main title, x-axis and y-axis labels

```
(gup5 <- gup4 +
  xlab(x_lab) +
  ylab(y_lab) +
  ggtitle(main_title_g_us))
```



3. "Graunt Era", "US 1993", "Difference of Life Expectancies" at proper positions

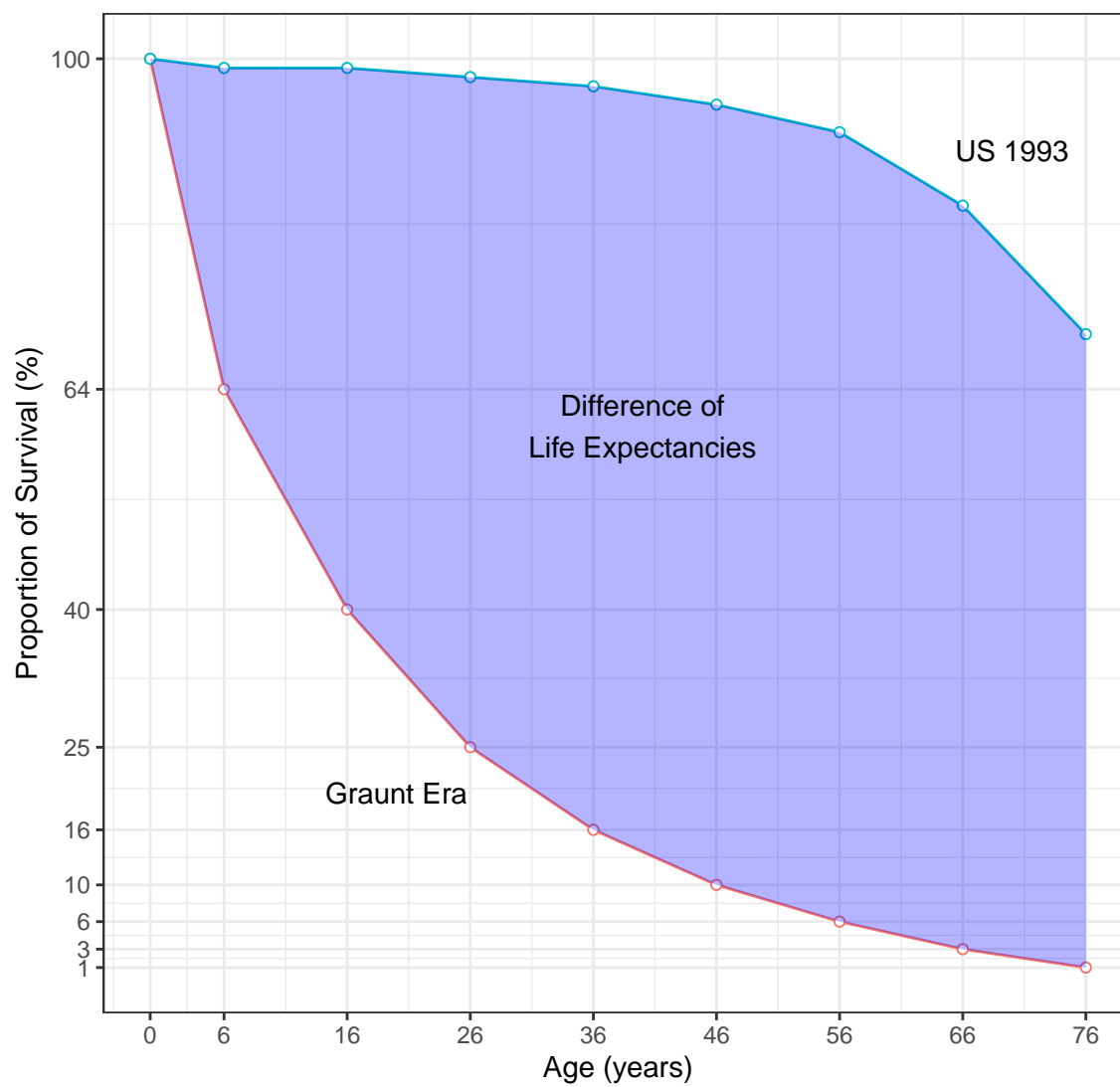
```
(gup6 <- gup5 +
  annotate("text",
    x = c(20, 40, 70), y = c(20, 60, 90),
    label = c("Graunt Era", "Difference of\nLife Expectancies", "US 1993"),
    family = ""))
```



4. x-axis and y-axis tick marks

```
(gup7 <- gup6 +
  scale_x_continuous(breaks = graunt$x) +
  scale_y_continuous(breaks = graunt$xPo_g))
```

## Survival Function of Graunt and US 1993



```
# ggsave("../pics/graunt_us_poly.png", gup7)
```

`dump()` and `source()`

- Check out how to save and retrieve. Use `source()` and `load()` for retrieval.

```
dump("area.R", file = "area.R")
save.image("../graunt_halley.RData")
```

Comments