Chapter 2
Halley's life table (1693)

Edmond Halley was born near London in 1656. His father was a rich soap maker. Edmond became interested in astronomy at a young age. He started studying at Queen's College of Oxford University. When the Greenwich Observatory was inaugurated in 1675, Halley could already visit Flamsteed, the Astronomer Royal. He interrupted his studies from 1676 to 1678 to go to the island of Saint Helena and establish a catalog of the stars that can be seen from the southern hemisphere. At his return to England he became a fellow of the Royal Society. He published also the observations he had made on the circulation of winds during his journey to Saint Helena. In 1684 he visited Newton in Cambridge to discuss the link between Kepler's laws of planetary motion and the force of attraction exerted by the Sun. He encouraged Newton to write the famous *Mathematical Principles of Natural Philosophy*, a book which he finally published at his own expense. He was then working as clerk of the Royal Society. In 1689 he designed a bell for underwater diving, which he tested himself.

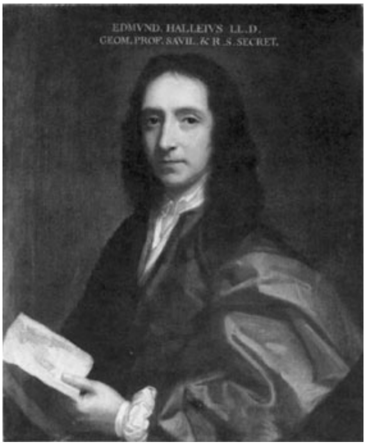

Fig. 2.1 Edmond Halley (1656–1742)

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At about the same time, Caspar Neumann, a theologian living in Breslau, was collecting data about the number of births and deaths in his city. Breslau belonged to the Habsburg empire (it is now in Poland and called Wrocław). The data included the age at which people had died. So it could be used to construct a life table showing the probability of surviving until any given age.

The first life table had been published in London in 1662 in a book entitled *Natural and Political Observations Made upon the Bills of Mortality*. This book is usually considered as the founding text of both statistics and demography and has a strange particularity: people still wonder nowadays if it was written by John Graunt, a London merchant and author indicated on the book cover, or by his friend William Petty, one of the founders of the Royal Society[1]. In any case the life table contained in the book tried to take advantage of the bulletins that had been regularly reporting the burials and baptisms in London since the beginning of the seventeenth century. These bulletins were mainly used to inform people on the recurrent epidemics of plague. This is the reason why they indicated the cause of death and not the age at which people died. To obtain a life table giving the chance of survival as a function of age, Graunt or Petty had to guess how different causes of death were related to age groups. So their life table could be subject to large errors. The book was nevertheless very successful, with five editions between 1662 and 1676. Several cities in Europe had started to publish bulletins similar to that of London.

So it was nearly thirty years after this first life table that, following the suggestion of Leibniz, Neumann sent to Henry Justel, the secretary of the Royal Society, his demographic data from the city of Breslau for the years 1687–1691. Justel died shortly after, and Halley got hold of the data, analyzed them and in 1693 published his conclusions in the *Philosophical Transactions of the Royal Society*. His article is called "An estimate of the degrees of the mortality of mankind, drawn from curious tables of the births and funerals at the city of Breslaw, with an attempt to ascertain the price of annuities upon lives".

For the period of five years under study, Halley noticed that the number of births in Breslau was more or less equal to the number of deaths, so that the total population was almost constant. To simplify the analysis, he assumed that the population was exactly at steady state: the annual number of births (call it P_0), the total population, the population aged k (P_k) and the annual number of deaths at age k (D_k) are all constant as time goes by. This emphasizes an additional interesting property of the data from Breslau, because such a simplification would not have been possible for a fast growing city such as London, where the statistics were also biased by the flow of population coming from the countryside.

The data from Breslau had a mean of $1,238$ births per year: this is the value that Halley took for P_0. In principle he could also compute from the data the annual mean D_k of the number of deaths among people aged k for all $k \geq 0$. Using the formula

$$P_{k+1} = P_k - D_k\,, \tag{2.1}$$

[1] For a detailed discussion, see the book by Hervé Le Bras in the references.

Table 2.1 Halley's life table showing the population P_k aged k.

Age	Number	Age	Number	Age	Number
1	1000	29	539	57	272
2	855	30	531	58	262
3	798	31	523	59	252
4	760	32	515	60	242
5	732	33	507	61	232
6	710	34	499	62	222
7	692	35	490	63	212
8	680	36	481	64	202
9	670	37	472	65	192
10	661	38	463	66	182
11	653	39	454	67	172
12	646	40	445	68	162
13	640	41	436	69	152
14	634	42	427	70	142
15	628	43	417	71	131
16	622	44	407	72	120
17	616	45	397	73	109
18	610	46	387	74	98
19	604	47	377	75	88
20	598	48	367	76	78
21	592	49	357	77	68
22	586	50	346	78	58
23	579	51	335	79	50
24	573	52	324	80	41
25	567	53	313	81	34
26	560	54	302	82	28
27	553	55	292	83	23
28	546	56	282	84	20

he could construct Table 2.1 giving P_k. Conversely, one can find the values of D_k that he used from the formula $D_k = P_k - P_{k+1}$: $D_0 = 238$, $D_1 = 145$, $D_2 = 57$, $D_3 = 38$ and so on. In fact, Halley rearranged his results a little, either to get round numbers (this is the case of D_1, which has been slightly changed so that $P_1 = 1{,}000$) or to smooth certain irregularities due to the small numbers of deaths at old ages in a five-year study. Taking the sum of all the numbers P_k in the table, Halley obtained an estimate of the total population of Breslau close[2] to 34,000. In summary this method had the great advantage of not requiring a general census but only knowledge of the number of births and deaths and of the age at which people died during a few years.

Halley's life table served as a reference for various works in the eighteenth century (see Chapter 4). Indeed, although the values of P_k were specific to the city of Breslau, one could consider that the ratio P_{k+1}/P_k was the probability of surviving until age $k+1$ knowing that one had already reached age k. This probability could reasonably be used for the populations of other European cities

[2] For people aged over 84 years, Halley just mentioned that their number was 107.

of the time. For example, one might expect a one-year-old child to have 661 chances out of 1,000 of reaching age 10 or 598 chances out of 1,000 of reaching age 20.

Halley also used his life table to compute the price of annuities upon lives. During the sixteenth and seventeenth centuries, several cities and states had sold such annuities to their citizens to raise money. The buyers received each year until their death a fixed amount of money, which was equal to a certain percentage of the sum initially paid, often twice the interest rate of the time, but independently of the age of the buyer. Of course the institution was risking bankruptcy if too many people with a very long life expectancy bought these annuities. The problem could not be correctly addressed without a reliable life table.

In 1671 Johan De Witt, prime minister of Holland, and Johannes Hudde, one of the mayors of the city of Amsterdam, had already thought about the problem of computing the price of life annuities. Fearing an invasion of French troops, they wanted to raise money to strengthen the army. They had data concerning people who had bought annuities upon lives several decades earlier, in particular the age at which the annuities had been bought and the age at which people had died. They had managed to compute the price of annuities more or less correctly, but their method was later forgotten. Holland was invaded the following year and De Witt was lynched by the crowd.

Halley considered the problem anew in 1693 with the life table from Breslau and assuming an interest rate of 6%. The method of computation is simple. Let i be the interest rate. Let R_k be the price at which a person aged k can buy an annuity of, say, one pound per year. This person has a probability P_{k+n}/P_k of being still alive at age $k+n$. The pound that the State promises to pay if he reaches this age can be obtained by placing $1/(1+i)^n$ pounds of the initial sum at the interest rate i. So if one makes the simplifying assumption that the initial sum is used only to pay the annuities, then the price should be

$$R_k = \frac{1}{P_k}\left(\frac{P_{k+1}}{1+i} + \frac{P_{k+2}}{(1+i)^2} + \frac{P_{k+3}}{(1+i)^3} + \cdots\right). \tag{2.2}$$

Halley obtained in this way Table 2.2, which shows the factor R_k by which the desired annuity has to be multiplied to get the necessary initial sum. A man aged 20 would hence get each year $1/12.78 \simeq 7.8\%$ of the initial sum. But a man aged 50 would get $1/9.21 \simeq 10.9\%$, because he would have fewer years to live. Notice that twice the interest rate would correspond to an annuity equal to 12% of the initial sum, or equivalently to a price equal to 8.33 times the annuity.

The computations are of course quite tedious. Halley could nevertheless use tables of logarithms to obtain the general term $P_{k+n}/(1+i)^n$ more quickly. Since he did not show values for P_k above 84 years, it is not possible to check his calculations exactly. Finally, Halley's work did not have any immediate impact: for several decades, annuities upon lives in England and elsewhere continued to be sold at a price independent of the age of the buyer and at a price that was much lower than it could be, for example 7 times the annuity.

Table 2.2 Multiplying factor giving the price of annuities upon lives.

Age k	Price R_k	Age k	Price R_k	Age k	Price R_k
1	10.28	25	12.27	50	9.21
5	13.40	30	11.72	55	8.51
10	13.44	35	11.12	60	7.60
15	13.33	40	10.57	65	6.54
20	12.78	45	9.91	70	5.32

The questions derived from life tables interested many scientists during Halley's time. The Dutch Christiaan Huygens, author in 1657 of the first booklet dedicated to probability theory, discussed in 1669 in his correspondence with his brother Graunt's life table and the calculation of life expectancy[3]. A few years before bringing Neumann into contact with the Royal Society, Leibniz also wrote about the calculation of life expectancy in an essay which remained unpublished. In 1709 it was the turn of Nikolaus I Bernoulli. In 1725 Abraham de Moivre published an entire *Treatise on Annuities*. He noticed in particular that the price R_k could be easily computed for old ages since formula (2.2) contained just a few terms. One could then use the backward recurrence formula

$$R_k = \frac{P_{k+1}}{P_k}\,\frac{1+R_{k+1}}{1+i}\,,$$

which is easily proved starting from (2.2). Using the value that Halley gives for the price at age 70, one can hence check[4] the other values of Table 2.2.

After this break focusing on demography Halley returned to his main research subjects. Between 1698 and 1700 he sailed around the Atlantic Ocean to draw a map of the Earth's magnetic field. In 1704 he became professor at Oxford University. The following year he published a book on comets and predicted that the comet of 1682, which Kepler had observed in 1607, would come back in 1758: it became known as "Halley's comet". He also published a translation of the book by Apollonius of Perga on conics. In 1720 he replaced Flamsteed as Astronomer Royal. He tried to solve the problem of determining longitude at sea precisely from observation of the Moon, a problem of great practical importance for navigation. He died in Greenwich in 1742 at age 86.

Further reading

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[3] The life expectancy at age k is given by formula (2.2) with $i = 0$.

[4] It seems that there are a few errors in the table, in particular for the ages 5 and 15.

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