Chapter 6 Lab

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6.4

$\vec{u} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \vec{w} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}, a_1 \vec{u} + a_2 \vec{v} + a_3 \vec{w} = \vec{b}$ 를 만족하는 a_1, a_2, a_3 ?

(a) $\vec{b} = \begin{bmatrix} 5 \\ 9 \\ 5 \end{bmatrix}$

```
source("./adjoint.R")
(u <- c(2, 1, 4))
```

```
## [1] 2 1 4
```

```
(v <- c(1, -1, 3))
```

```
## [1]  1 -1  3
```

```
(w <- c(3, 2, 5))
```

```
## [1] 3 2 5
```

```
(X <- cbind(u, v, w))
```

```
##      u  v w
## [1,] 2  1 3
## [2,] 1 -1 2
## [3,] 4  3 5
```

```
det(X)
```

```
## [1] 2
```

```
adjoint(X)
```

```
##      [,1] [,2] [,3]
## [1,]  -11    4    5
## [2,]    3   -2   -1
## [3,]    7   -2   -3
```

```
adjoint(X)/det(X)
```

```
##      [,1] [,2] [,3]
## [1,] -5.5    2  2.5
## [2,]  1.5   -1 -0.5
## [3,]  3.5   -1 -1.5
```

```
solve(X)
```

```
##   [,1] [,2] [,3]
## u -5.5    2  2.5
## v  1.5   -1 -0.5
## w  3.5   -1 -1.5
```

```
(b1 <- c(5, 9, 5))
```

```
## [1] 5 9 5
```

```
(adjoint(X)/det(X)) %*% b1
```

```
##      [,1]
## [1,]    3
## [2,]   -4
## [3,]    1
```

```
solve(X, b1)
```

```
##  u  v  w
##  3 -4  1
```

```
solve(X, b1)[1]*u + solve(X, b1)[2]*v + solve(X, b1)[3]*w
```

```
## [1] 5 9 5
```

(b) $\vec{b} = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix}$

```
(b2 <- c(2, 0, 6))
```

```
## [1] 2 0 6
```

```
(adjoint(X)/det(X)) %*% b2
```

```
##               [,1]
## [1,]  4.000000e+00
## [2,] -8.881784e-16
## [3,] -2.000000e+00
```

```
round((adjoint(X)/det(X)) %*% b2, digits = 2)
```

```
##      [,1]
## [1,]    4
## [2,]    0
## [3,]   -2
```

```
solve(X, b2)
```

```
##  u  v  w
##  4  0 -2
```

```
solve(X, b2)[1]*u + solve(X, b2)[2]*v + solve(X, b2)[3]*w
```

```
## [1] 2 0 6
```

(c) $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

```
(b3 <- c(0, 0, 0))
```

```
## [1] 0 0 0
```

```
(adjoint(X)/det(X)) %*% b3
```

```
##      [,1]
## [1,]    0
## [2,]    0
## [3,]    0
```

```
solve(X, b3)
```

```
## u v w
## 0 0 0
```

```
solve(X, b3)[1]*u + solve(X, b3)[2]*v + solve(X, b3)[3]*w
```

```
## [1] 0 0 0
```

6.6 R^3 를 생성할 수 있는 집합은?

(a)

$$\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}.$$

선형독립인 벡터 집합을 찾아내는 문제. 세개의 벡터인 경우는 선형독립 여부만 판단하면 됨. 정방행렬들이므로 행렬식의 존재여부로 판단.

```
(u2 <- c(1, 1, 1))
```

```
## [1] 1 1 1
```

```
(v2 <- c(2, 2, 0))
```

```
## [1] 2 2 0
```

```
(w2 <- c(3, 0, 0))
```

```
## [1] 3 0 0
```

```
(X2 <- cbind(u2, v2, w2))
```

```
##      u2 v2 w2
## [1,]  1  2  3
## [2,]  1  2  0
## [3,]  1  0  0
```

```
det(X2)
```

```
## [1] -6
```

```
(u3 <- c(2, -1, 3))
```

```
## [1]  2 -1  3
```

```
(v3 <- c(4, 1, 2))
```

```
## [1] 4 1 2
```

```
(w3 <- c(8, -1, 8))
```

```
## [1]  8 -1  8
```

```
(X3 <- cbind(u3, v3, w3))
```

```
##      u3 v3 w3
## [1,]  2  4  8
## [2,] -1  1 -1
## [3,]  3  2  8
```

```
det(X3)
```

```
## [1] 0
```

```
2*u3 + v3
```

```
## [1]  8 -1  8
```

4 개의 벡터 중 1, 2개로는 R^3 를 생성할 수 없으므로 3개일 때 집중. 3개씩은 모두 선형독립이므로 R^3 생성 가능. `solve(X, )` 의 결과는 나머지 한 벡터가 다른 세 개의 벡터의 선형결합으로 나타날 때 계수를 찾아준 것임.

```
(u4 <- c(1, 3, 3))
```

```
## [1] 1 3 3
```

```
(v4 <- c(1, 3, 4))
```

```
## [1] 1 3 4
```

```
(w4 <- c(1, 4, 3))
```

```
## [1] 1 4 3
```

```
(t4 <- c(6, 2, 1))
```

```
## [1] 6 2 1
```

```
(X4 <- cbind(u4, v4, w4))
```

```
##      u4 v4 w4
## [1,]  1  1  1
## [2,]  3  3  4
## [3,]  3  4  3
```

```
det(X4)
```

```
## [1] -1
```

```
adjoint(X4)
```

```
##      [,1] [,2] [,3]
## [1,]   -7    1    1
## [2,]    3    0   -1
## [3,]    3   -1    0
```

```
adjoint(X4)/det(X4)
```

```
##      [,1] [,2] [,3]
## [1,]    7   -1   -1
## [2,]   -3    0    1
## [3,]   -3    1    0
```

```
solve(X4)
```

```
##    [,1]          [,2] [,3]
## u4    7 -1.000000e+00   -1
## v4   -3  2.220446e-16    1
## w4   -3  1.000000e+00    0
```

```
round(solve(X4), digits = 2)
```

```
##    [,1] [,2] [,3]
## u4    7   -1   -1
## v4   -3    0    1
## w4   -3    1    0
```

```
(adjoint(X4)/det(X4)) %*% t4
```

```
##      [,1]
## [1,]   39
## [2,]  -17
## [3,]  -16
```

```
solve(X4, t4)
```

```
##  u4  v4  w4
##  39 -17 -16
```

```
adjoint(X4) %*% t4
```

```
##      [,1]
## [1,]  -39
## [2,]   17
## [3,]   16
```

```
solve(X4, t4)[1]*u4 + solve(X4, t4)[2]*v4 + solve(X4, t4)[3]*w4
```

```
## [1] 6 2 1
```

```
(X5 <- cbind(u4, v4, t4))
```

```
##      u4 v4 t4
## [1,]  1  1  6
## [2,]  3  3  2
## [3,]  3  4  1
```

```
det(X5)
```

```
## [1] 16
```

```
adjoint(X5)
```

```
##      [,1] [,2] [,3]
## [1,]   -5   23  -16
## [2,]    3  -17   16
## [3,]    3   -1    0
```

```
adjoint(X5)/det(X5)
```

```
##         [,1]    [,2] [,3]
## [1,] -0.3125  1.4375   -1
## [2,]  0.1875 -1.0625    1
## [3,]  0.1875 -0.0625    0
```

```
solve(X5)
```

```
##       [,1]    [,2] [,3]
## u4 -0.3125  1.4375   -1
## v4  0.1875 -1.0625    1
## t4  0.1875 -0.0625    0
```

```
(adjoint(X5)/det(X5)) %*% w4
```

```
##         [,1]
## [1,]  2.4375
## [2,] -1.0625
## [3,] -0.0625
```

```
solve(X5, w4)
```

```
##      u4      v4      t4
##  2.4375 -1.0625 -0.0625
```

```
adjoint(X5) %*% w4
```

```
##      [,1]
## [1,]   39
## [2,]  -17
## [3,]   -1
```

```
solve(X5, w4)[1]*u4 + solve(X5, w4)[2]*v4 + solve(X5, w4)[3]*t4
```

```
## [1] 1 4 3
```

```
(X6 <- cbind(u4, w4, t4))
```

```
##      u4 w4 t4
## [1,]  1  1  6
## [2,]  3  4  2
## [3,]  3  3  1
```

```
det(X6)
```

```
## [1] -17
```

```
adjoint(X6)
```

```
##      [,1] [,2] [,3]
## [1,]   -2   17  -22
## [2,]    3  -17   16
## [3,]   -3    0    1
```

```
adjoint(X6)/det(X6)
```

```
##            [,1] [,2]        [,3]
## [1,]  0.1176471   -1  1.29411765
## [2,] -0.1764706    1 -0.94117647
## [3,]  0.1764706    0 -0.05882353
```

```
solve(X6)
```

```
##          [,1]          [,2]        [,3]
## u4  0.1176471 -1.000000e+00  1.29411765
## w4 -0.1764706  1.000000e+00 -0.94117647
## t4  0.1764706 -9.796086e-18 -0.05882353
```

```
(adjoint(X6)/det(X6)) %*% v4
```

```
##             [,1]
## [1,]  2.29411765
## [2,] -0.94117647
## [3,] -0.05882353
```

```
solve(X6, v4)
```

```
##          u4          w4          t4
##  2.29411765 -0.94117647 -0.05882353
```

```
adjoint(X6) %*% v4
```

```
##      [,1]
## [1,]  -39
## [2,]   16
## [3,]    1
```

```
solve(X6, v4)[1]*u4 + solve(X6, v4)[2]*w4 + solve(X6, v4)[3]*t4
```

```
## [1] 1 3 4
```

```
(X7 <- cbind(v4, w4, t4))
```

```
##      v4 w4 t4
## [1,]  1  1  6
## [2,]  3  4  2
## [3,]  4  3  1
```

```
det(X7)
```

```
## [1] -39
```

```
adjoint(X7)
```

```
##      [,1] [,2] [,3]
## [1,]   -2   17  -22
## [2,]    5  -23   16
## [3,]   -7    1    1
```

```
adjoint(X7)/det(X7)
```

```
##             [,1]        [,2]        [,3]
## [1,]  0.05128205 -0.43589744  0.56410256
## [2,] -0.12820513  0.58974359 -0.41025641
## [3,]  0.17948718 -0.02564103 -0.02564103
```

```
solve(X7)
```

```
##           [,1]        [,2]        [,3]
## v4  0.05128205 -0.43589744  0.56410256
## w4 -0.12820513  0.58974359 -0.41025641
## t4  0.17948718 -0.02564103 -0.02564103
```

```
(adjoint(X7)/det(X7)) %*% u4
```

```
##            [,1]
## [1,] 0.43589744
## [2,] 0.41025641
## [3,] 0.02564103
```

```
solve(X7, u4)
```

```
##          v4         w4         t4
## 0.43589744 0.41025641 0.02564103
```

```
adjoint(X7) %*% u4
```

```
##      [,1]
## [1,]  -17
## [2,]  -16
## [3,]   -1
```

```
solve(X7, u4)[1]*v4 + solve(X7, u4)[2]*w4 + solve(X7, u4)[3]*t4
```

```
## [1] 1 3 3
```

6.7

$$\overrightarrow{x_1} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \overrightarrow{x_2} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \overrightarrow{x_3} = \begin{bmatrix} -13 \\ -1 \\ 2 \end{bmatrix}, \overrightarrow{x_4} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

(a) $\overrightarrow{x_1}, \overrightarrow{x_2}, \overrightarrow{x_3}$ 는 선형종속

```
(u8 <- c(1, 2, 1))
```

```
## [1] 1 2 1
```

```
(v8 <- c(-1, 3, 2))
```

```
## [1] -1  3  2
```

```
(w8 <- c(-13, -1, 2))
```

```
## [1] -13  -1   2
```

```
(t8 <- c(1, 1, 0))
```

```
## [1] 1 1 0
```

```
X8 <- cbind(u8, v8, w8)
det(X8)
```

```
## [1] 0
```

$a_1\overrightarrow{x_1} + a_2\overrightarrow{x_2} = \overrightarrow{x_3}$ 를 풀어주면 $a_1 = -8, a_2 = 5$ 임을 쉽게 파악.

(b) $\overrightarrow{x_1}, \overrightarrow{x_2}, \overrightarrow{x_4}$ 는 선형독립

```
(X9 <- cbind(u8, v8, t8))
```

```
##      u8 v8 t8
## [1,]  1 -1  1
## [2,]  2  3  1
## [3,]  1  2  0
```

```
det(X9)
```

```
## [1] -2
```

```
adjoint(X9)
```

```
##      [,1] [,2] [,3]
## [1,]   -2    2   -4
## [2,]    1   -1    1
## [3,]    1   -3    5
```

```
adjoint(X9)/det(X9)
```

```
##      [,1] [,2] [,3]
## [1,]  1.0 -1.0  2.0
## [2,] -0.5  0.5 -0.5
## [3,] -0.5  1.5 -2.5
```

$a_1\overrightarrow{x_1} + a_2\overrightarrow{x_2} + a_3\overrightarrow{x_4} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ 를 정리하면

$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ 를 풀어주면,

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 0 \end{bmatrix}^{-1}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = (-1/2)\begin{bmatrix} -2 & 2 & 4 \\ 1 & -1 & 1 \\ 1 & -3 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a - b + 2c \\ -a/2 + b/2 - c/2 \\ -a/2 + 3b/2 - 5c/2 \end{bmatrix}$$

6.10

(a)

```
(X10 <- cbind(c(2, -1, 4), c(3, 6, 2), c(2, 10, -4)))
```

```
##      [,1] [,2] [,3]
## [1,]    2    3    2
## [2,]   -1    6   10
## [3,]    4    2   -4
```

```
det(X10)
```

```
## [1] -32
```

(b)

```
(X11 <- cbind(c(3, 1, 1), c(2, -1, 5), c(4, 0, -3)))
```

```
##      [,1] [,2] [,3]
## [1,]    3    2    4
## [2,]    1   -1    0
## [3,]    1    5   -3
```

```
det(X11)
```

```
## [1] 39
```

(c) $a_1 \begin{bmatrix} 6 \\ 0 \\ -1 \end{bmatrix} + a_2 \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = \vec{0}$ 을 만족시키는 $a_1 = a_2 = 0$ 임을 보인다.

```
(u10 <- c(6, 0, -1))
```

```
## [1]  6  0 -1
```

```
(v10 <- c(1, 1, 4))
```

```
## [1] 1 1 4
```

```
(X12 <- cbind(u10, v10))
```

```
##      u10 v10
## [1,]   6   1
## [2,]   0   1
## [3,]  -1   4
```

```
det(X12[1:2, ])
```

```
## [1] 6
```

```
det(X12[c(1, 3), ])
```

```
## [1] 25
```

```
det(X12[2:3, ])
```

```
## [1] 1
```

(d)

```
(u11 <- c(1, 3, 3))
```

```
## [1] 1 3 3
```

```
(v11 <- c(0, 1, 4))
```

```
## [1] 0 1 4
```

```
(w11 <- c(5, 6, 3))
```

```
## [1] 5 6 3
```

```
(t11 <- c(7, 2, -1))
```

```
## [1]  7  2 -1
```

```
(X13 <- cbind(u11, v11, w11, t11))
```

```
##      u11 v11 w11 t11
## [1,]   1   0   5   7
## [2,]   3   1   6   2
## [3,]   3   4   3  -1
```

```
det(X13[, 1:3])
```

```
## [1] 24
```

```
solve(X13[, 1:3], t11)
```

```
##   u11   v11   w11
## -4.25  1.25  2.25
```

```
solve(X13[, 1:3], t11)*det(X13[, 1:3])
```

```
##  u11  v11  w11
## -102   30   54
```

```
solve(X13[, 1:3], t11)[1]*u11 + solve(X13[, 1:3], t11)[2]*v11 + solve(X13[,
1:3], t11)[3]*w11
```

```
## [1]  7  2 -1
```

```
det(X13[, c(1, 2, 4)])
```

```
## [1] 54
```

```
solve(X13[, c(1, 2, 4)], w11)
```

```
##        u11        v11        t11
##  1.8888889 -0.5555556  0.4444444
```

```
solve(X13[, c(1, 2, 4)], w11)*det(X13[, c(1, 2, 4)])
```

```
## u11 v11 t11
## 102 -30  24
```

```
solve(X13[, c(1, 2, 4)], w11)[1]*u11 + solve(X13[, c(1, 2, 4)], w11)[2]*v11 + s
olve(X13[, c(1, 2, 4)], w11)[3]*t11
```

```
## [1] 5 6 3
```

```
det(X13[, c(1, 3, 4)])
```

```
## [1] -30
```

```
solve(X13[, c(1, 3, 4)], v11)
```

```
##  u11  w11  t11
##  3.4 -1.8  0.8
```

```
solve(X13[, c(1, 3, 4)], v11)*det(X13[, c(1, 3, 4)])
```

```
##  u11  w11  t11
## -102   54  -24
```

```
solve(X13[, c(1, 3, 4)], v11)[1]*u11 + solve(X13[, c(1, 3, 4)], v11)[2]*w11 + s
olve(X13[, c(1, 3, 4)], v11)[3]*t11
```

```
## [1] 8.881784e-16 1.000000e+00 4.000000e+00
```

```
round(solve(X13[, c(1, 3, 4)], v11)[1]*u11 + solve(X13[, c(1, 3, 4)], v11)[2]*w
11 + solve(X13[, c(1, 3, 4)], v11)[3]*t11, digits = 2)
```

```
## [1] 0 1 4
```

```
det(X13[, 2:4])
```

```
## [1] -102
```

```
solve(X13[, 2:4], u11)
```

```
##        v11        w11        t11
##  0.2941176  0.5294118 -0.2352941
```

```
solve(X13[, 2:4], u11)*det(X13[, 2:4])
```

```
## v11 w11 t11
## -30 -54  24
```

```
solve(X13[, 2:4], u11)[1]*v11 + solve(X13[, 2:4], u11)[2]*w11 + solve(X13[,
2:4], u11)[3]*t11
```

```
## [1] 1 3 3
```

6.14

$$\overrightarrow{x_1} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \overrightarrow{x_2} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \overrightarrow{x_3} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \overrightarrow{x_4} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}.$$

(a) $\overrightarrow{x_1}, \overrightarrow{x_2}, \overrightarrow{x_3}$ 는 선형종속

```
(u14 <- c(1, 2, 1))
```

```
## [1] 1 2 1
```

```
(v14 <- c(-1, 3, 2))
```

```
## [1] -1  3  2
```

```
(w14 <- c(3, 1, 0))
```

```
## [1] 3 1 0
```

```
(t14 <- c(3, 1, 1))
```

```
## [1] 3 1 1
```

```
X14 <- cbind(u14, v14, w14)
det(X14)
```

```
## [1] 0
```

$a_1 \overrightarrow{x_1} + a_2 \overrightarrow{x_2} = \overrightarrow{x_3}$ 를 풀어주면 $a_1 = 2, a_2 = -1$ 임을 쉽게 파악.

(b) $\overrightarrow{x_1}, \overrightarrow{x_2}, \overrightarrow{x_4}$ 는 선형독립

```
(X15 <- cbind(u14, v14, t14))
```

```
##      u14 v14 t14
## [1,]   1  -1   3
## [2,]   2   3   1
## [3,]   1   2   1
```

```
det(X15)
```

```
## [1] 5
```

```
adjoint(X15)
```

```
##      [,1] [,2] [,3]
## [1,]    1    7  -10
## [2,]   -1   -2    5
## [3,]    1   -3    5
```

```
adjoint(X15)/det(X15)
```

```
##       [,1] [,2] [,3]
## [1,]  0.2  1.4   -2
## [2,] -0.2 -0.4    1
## [3,]  0.2 -0.6    1
```

$a_1\overrightarrow{x_1} + a_2\overrightarrow{x_2} + a_3\overrightarrow{x_4} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$를 정리하면

$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$를 풀어주면,

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = (1/5)\begin{bmatrix} 1 & 7 & -10 \\ -1 & -2 & 5 \\ 1 & -3 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a/5 + 7b/5 - 2c \\ -a/5 - 2b/5 + c \\ a/5 - 3b/5 + c \end{bmatrix}$$

자료 저장

```
save.image("chapter_6_lab.rda")
```