Chapter 1 class

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1.1 합과 곱

합의 기호 $\sum$

- a_1 , a_2 , a_3 , a_4 , a_5 에 대하여 더하기 연산
- $a_1 = 80, a_2 = 75, a_3 = 90, a_4 = 83, a_5 = 70$

```
a <- c(80, 75, 90, 83, 70)
a
```

```
## [1] 80 75 90 83 70
```

```
a[1]
```

```
## [1] 80
```

```
a[c(2, 5)]
```

```
## [1] 75 70
```

```
a[-3]
```

```
## [1] 80 75 83 70
```

```
length(a)
```

```
## [1] 5
```

```
a[length(a)]
```

```
## [1] 70
```

```
l.a <- length(a)
a[l.a]
```

```
## [1] 70
```

- 합의 계산

$\sum_{i=1}^{5} a_i$ 와 $\sum_{i=3}^{10} c_i$

```
a[1] + a[2] + a[3] + a[4] + a[5]
```

```
## [1] 398
```

```
sum(a)
```

```
## [1] 398
```

```
(b <- rev(a))
```

```
## [1] 70 83 90 75 80
```

```
# b
(c <- c(a, b))
```

```
##  [1] 80 75 90 83 70 70 83 90 75 80
```

```
# c
c[3:10]
```

```
## [1] 90 83 70 70 83 90 75 80
```

```
sum(c[3:10])
```

```
## [1] 641
```

a_i의 함수들의 합

- $\sum_{i=1}^{5} a_i^3$, $\sum_{i=1}^{5} (a_i + b_i)$

```
a^3
```

```
## [1] 512000 421875 729000 571787 343000
```

```
sum(a^3)
```

```
## [1] 2577662
```

```
a + b
```

```
## [1] 150 158 180 158 150
```

```
sum(a + b)
```

```
## [1] 796
```

```
sum(a) + sum(b)
```

```
## [1] 796
```

행렬

- a_{ij} 는 행렬의 i번째 행, j번째 열에 있는 원소를 지칭.

```
(A <- rbind(a, b))
```

```
##   [,1] [,2] [,3] [,4] [,5]
## a   80   75   90   83   70
## b   70   83   90   75   80
```

```
# A
str(A)
```

```
##  num [1:2, 1:5] 80 70 75 83 90 90 83 75 70 80
##  - attr(*, "dimnames")=List of 2
##   ..$ : chr [1:2] "a" "b"
##   ..$ : NULL
```

```
paste("r", 1:2)
```

```
## [1] "r 1" "r 2"
```

```
paste("r", 1:2, sep = "")
```

```
## [1] "r1" "r2"
```

```
paste("c", 1:5, sep = "")
```

```
## [1] "c1" "c2" "c3" "c4" "c5"
```

```
(dimnames(A) <- list("Row" = paste("r", 1:2, sep = ""), "Col" = paste("c", 1:5, sep =
 "")))
```

```
## $Row
## [1] "r1" "r2"
##
## $Col
## [1] "c1" "c2" "c3" "c4" "c5"
```

```
A
```

```
##     Col
## Row  c1 c2 c3 c4 c5
##   r1 80 75 90 83 70
##   r2 70 83 90 75 80
```

```
A[1, 1]
```

```
## [1] 80
```

```
A[2, 1]
```

```
## [1] 70
```

```
A[1, c(1, 3)]
```

```
## c1 c3
## 80 90
```

```
A[-1, ]
```

```
## c1 c2 c3 c4 c5
## 70 83 90 75 80
```

```
str(A[2, ])
```

```
##  Named num [1:5] 70 83 90 75 80
##  - attr(*, "names")= chr [1:5] "c1" "c2" "c3" "c4" ...
```

```
A[, 1]
```

```
## r1 r2
## 80 70
```

```
str(A[, 1])
```

```
##  Named num [1:2] 80 70
##  - attr(*, "names")= chr [1:2] "r1" "r2"
```

```
A[, 1, drop = FALSE]
```

```
##     Col
## Row  c1
##   r1 80
##   r2 70
```

```
str(A[, 1, drop = FALSE])
```

```
##  num [1:2, 1] 80 70
##  - attr(*, "dimnames")=List of 2
##   ..$ Row: chr [1:2] "r1" "r2"
##   ..$ Col: chr "c1"
```

```
B <- A
B
```

```
##     Col
## Row  c1 c2 c3 c4 c5
##   r1 80 75 90 83 70
##   r2 70 83 90 75 80
```

```
dimnames(B) <- NULL
B
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   80   75   90   83   70
## [2,]   70   83   90   75   80
```

열의 합과 행의 합

- $i = 1, 2$ 행에 대하여 $j = 1, 2, 3$ 열에 속한 원소들의 합을 구하면

```
A[1, 1:3]
```

```
## c1 c2 c3
## 80 75 90
```

```
sum(A[1, 1:3])
```

```
## [1] 245
```

```
A[2, 1:3]
```

```
## c1 c2 c3
## 70 83 90
```

```
sum(A[2, 1:3])
```

```
## [1] 243
```

```
apply(A[, 1:3], 1, sum)
```

```
##  r1  r2
## 245 243
```

```
sum(A[1, 1:3]) + sum(A[2, 1:3])
```

```
## [1] 488
```

```
sum(apply(A[, 1:3], 1, sum))
```

```
## [1] 488
```

```
sum(A[, 1:3])
```

```
## [1] 488
```

- $j = 1, 2, 3$ 열에 대하여 $i = 1, 2$ 행에 속한 원소들의 합을 구하면

```
A[, 1]
```

```
## r1 r2
## 80 70
```

```
sum(A[, 1])
```

```
## [1] 150
```

```
A[, 2]
```

```
## r1 r2
## 75 83
```

```
sum(A[, 2])
```

```
## [1] 158
```

```
A[, 1]
```

```
## r1 r2
## 80 70
```

```
sum(A[, 2])
```

```
## [1] 158
```

```
apply(A[, 1:3], 2, mean)
```

```
## c1 c2 c3
## 75 79 90
```

```
sum(A[, 1]) + sum(A[, 2]) + sum(A[, 3])
```

```
## [1] 488
```

```
apply(A[, 1:3], 2, sum)
```

```
##  c1  c2  c3
## 150 158 180
```

```
sum(apply(A[, 1:3], 2, sum))
```

```
## [1] 488
```

```
sum(A[, 1:3])
```

```
## [1] 488
```

```
apply(A, 1, sum)
```

```
##  r1  r2
## 398 398
```

```
apply(A, 2, sum)
```

```
##  c1  c2  c3  c4  c5
## 150 158 180 158 150
```

```
rowSums(A)
```

```
##  r1  r2
## 398 398
```

```
colSums(A)
```

```
##  c1  c2  c3  c4  c5
## 150 158 180 158 150
```

곱의 기호 $\prod$

```
a[1]*a[2]*a[3]*a[4]*a[5]
```

```
## [1] 3137400000
```

```
prod(a)
```

```
## [1] 3137400000
```

```
a + b
```

```
## [1] 150 158 180 158 150
```

```
prod(a + b)
```

```
## [1] 101104200000
```

- # 기호의 활용
  - 열의 합이나 행의 합을 간단히 표현
  - $\sum_{i=1}^{n} a_{ij} = a_{\cdot j}$, $\sum_{j=1}^{m} a_{ij} = a_{i\cdot}$

```
A
```

```
##     Col
## Row  c1 c2 c3 c4 c5
##   r1 80 75 90 83 70
##   r2 70 83 90 75 80
```

```
sum(A[, 1])
```

```
## [1] 150
```

```
sum(A[, 2])
```

```
## [1] 158
```

```
sum(A[, 3])
```

```
## [1] 180
```

```
sum(A[, 4])
```

```
## [1] 158
```

```
sum(A[, 5])
```

```
## [1] 150
```

```
apply(A, 2, sum)
```

```
##  c1  c2  c3  c4  c5
## 150 158 180 158 150
```

```
colSums(A)
```

```
##  c1  c2  c3  c4  c5
## 150 158 180 158 150
```

```
sum(A[1, ])
```

```
## [1] 398
```

```
sum(A[2, ])
```

```
## [1] 398
```

```
apply(A, 1, sum)
```

```
##  r1  r2
## 398 398
```

```
rowSums(A)
```

```
##  r1  r2
## 398 398
```

- $a_{..} = \sum_{i=1}^{n} a_{i.} = \sum_{j=1}^{m} a_{.j} = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij}$

```
rowSums(A)
```

```
##  r1  r2
## 398 398
```

```
sum(rowSums(A))
```

```
## [1] 796
```

```
colSums(A)
```

```
##  c1  c2  c3  c4  c5
## 150 158 180 158 150
```

```
sum(colSums(A))
```

```
## [1] 796
```

- $a_{\cdot j}^2 = (\sum_{i=1}^n a_{ij})^2$ 과 $\sum_{i=1}^n a_{ij}^2$ 의 구분.
- 합의 제곱

```
A
```

```
##     Col
## Row  c1 c2 c3 c4 c5
##   r1 80 75 90 83 70
##   r2 70 83 90 75 80
```

```
sum(A[, 1])
```

```
## [1] 150
```

```
sum(A[, 1])^2
```

```
## [1] 22500
```

```
sum(A[, 2])
```

```
## [1] 158
```

```
sum(A[, 2])^2
```

```
## [1] 24964
```

```
sum(A[, 3])
```

```
## [1] 180
```

```
sum(A[, 3])^2
```

```
## [1] 32400
```

```
sum(A[, 4])
```

```
## [1] 158
```

```
sum(A[, 4])^2
```

```
## [1] 24964
```

```
sum(A[, 5])
```

```
## [1] 150
```

```
sum(A[, 5])^2
```

```
## [1] 22500
```

```
colSums(A)^2
```

```
##    c1    c2    c3    c4    c5
## 22500 24964 32400 24964 22500
```

- 제곱의 합

```
A[, 1]
```

```
## r1 r2
## 80 70
```

```
A[, 1]^2
```

```
##   r1   r2
## 6400 4900
```

```
sum(A[, 1]^2)
```

```
## [1] 11300
```

```
A[, 2]
```

```
## r1 r2
## 75 83
```

```
A[, 2]^2
```

```
##   r1   r2
## 5625 6889
```

```
sum(A[, 2]^2)
```

```
## [1] 12514
```

```
A[, 3]
```

```
## r1 r2
## 90 90
```

```
A[, 3]^2
```

```
##   r1   r2
## 8100 8100
```

```
sum(A[, 3]^2)
```

```
## [1] 16200
```

```
A[, 4]
```

```
## r1 r2
## 83 75
```

```
A[, 4]^2
```

```
##   r1   r2
## 6889 5625
```

```
sum(A[, 4]^2)
```

```
## [1] 12514
```

```
A[, 5]
```

```
## r1 r2
## 70 80
```

```
A[, 5]^2
```

```
##   r1   r2
## 4900 6400
```

```
sum(A[, 5]^2)
```

```
## [1] 11300
```

```
A^2
```

```
##     Col
## Row    c1   c2   c3   c4   c5
##   r1 6400 5625 8100 6889 4900
##   r2 4900 6889 8100 5625 6400
```

```
colSums(A^2)
```

```
##    c1    c2    c3    c4    c5
## 11300 12514 16200 12514 11300
```

행렬의 정의

통계학 점수와 전체 평점

- 열 방향으로 읽어들여 차원 정하기

```
M1 <- matrix(c(80, 75, 90, 83, 70, 3.0, 3.5, 4.0, 3.1, 2.2), ncol=2)
M1
```

```
##      [,1] [,2]
## [1,]   80  3.0
## [2,]   75  3.5
## [3,]   90  4.0
## [4,]   83  3.1
## [5,]   70  2.2
```

```
str(M1)
```

```
##  num [1:5, 1:2] 80 75 90 83 70 3 3.5 4 3.1 2.2
```

- 통계학 점수와 전체평점, 즉 열 벡터 단위로 읽어서 합치기

```
stat.score <- c(80, 75, 90, 83, 70)
GPA <- c(3.0, 3.5, 4.0, 3.1, 2.2)
M2 <- cbind(stat.score, GPA)
M2
```

```
##      stat.score GPA
## [1,]         80 3.0
## [2,]         75 3.5
## [3,]         90 4.0
## [4,]         83 3.1
## [5,]         70 2.2
```

```
dimnames(M2)[[1]] <- paste("student", 1:5, sep="")
M2
```

```
##          stat.score GPA
## student1         80 3.0
## student2         75 3.5
## student3         90 4.0
## student4         83 3.1
## student5         70 2.2
```

```
str(M2)
```

```
##  num [1:5, 1:2] 80 75 90 83 70 3 3.5 4 3.1 2.2
##  - attr(*, "dimnames")=List of 2
##   ..$ : chr [1:5] "student1" "student2" "student3" "student4" ...
##   ..$ : chr [1:2] "stat.score" "GPA"
```

- 각 학생의 통계학점수와 전체평점을 모아서 행 단위로 읽어서 합치기

```
M3 <- matrix(c(80, 3.0, 75, 3.5, 90, 4.0, 83, 3.1, 70, 2.2), ncol=2, byrow=TRUE)
M3
```

```
##      [,1] [,2]
## [1,]   80  3.0
## [2,]   75  3.5
## [3,]   90  4.0
## [4,]   83  3.1
## [5,]   70  2.2
```

- $r \times c$ 행렬의 표시

```
dim(M3)
```

```
## [1] 5 2
```

```
nrow(M3)
```

```
## [1] 5
```

```
ncol(M3)
```

```
## [1] 2
```

- 2×3 행렬의 예시

```
A <- matrix(c(3, -2, -1, 6, 8, 4), nrow=2)
A
```

```
##      [,1] [,2] [,3]
## [1,]    3   -1    8
## [2,]   -2    6    4
```

- 대각행렬

```
D <- diag(c(-3, 5, -6))
D
```

```
##      [,1] [,2] [,3]
## [1,]   -3    0    0
## [2,]    0    5    0
## [3,]    0    0   -6
```

- 주의 사항

```
diag(3)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

```
diag(3, nrow=1)
```

```
##      [,1]
## [1,]    3
```

- 하삼각행렬과 상삼각행렬

```
T.lower <- matrix(c(1, 0, 0, 0, 3, 0, 0, 0, 3, -1, 3, 0, 4, 2, 5, 6), nrow=4)
T.lower
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    3    3    4
## [2,]    0    0   -1    2
## [3,]    0    0    3    5
## [4,]    0    0    0    6
```

```
Matrix::isTriangular(T.lower)
```

```
## [1] TRUE
## attr(,"kind")
## [1] "U"
```

```
T.upper <- matrix(c(3, -2, 5, 0, 0, 5, -4, 2, 0, 0, 3, 7, 0, 0, 0, 0), nrow=4)
T.upper
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    3    0    0    0
## [2,]   -2    5    0    0
## [3,]    5   -4    3    0
## [4,]    0    2    7    0
```

```
Matrix::isTriangular(T.upper)
```

```
## [1] TRUE
## attr(,"kind")
## [1] "L"
```

- Transition Probability Matrix

```
P <- matrix(c(0.2, 0.4, 0.8, 0.6), nrow=2)
P
```

```
##      [,1] [,2]
## [1,]  0.2  0.8
## [2,]  0.4  0.6
```

```
rowSums(P)
```

```
## [1] 1 1
```

1.3 벡터와 스칼라

- 열 벡터와 행 벡터

```
x <- c(3, -2, 0, 1)
is.vector(x)
```

```
## [1] TRUE
```

```
is.matrix(x)
```

```
## [1] FALSE
```

```
dim(x)
```

```
## NULL
```

```
length(x)
```

```
## [1] 4
```

```
str(x)
```

```
##  num [1:4] 3 -2 0 1
```

```
x.mat <- matrix(x, ncol=1)
x.mat
```

```
##      [,1]
## [1,]    3
## [2,]   -2
## [3,]    0
## [4,]    1
```

```
is.vector(x.mat)
```

```
## [1] FALSE
```

```
is.matrix(x.mat)
```

```
## [1] TRUE
```

```
dim(x.mat)
```

```
## [1] 4 1
```

```
length(x.mat)
```

```
## [1] 4
```

```
str(x.mat)
```

```
##  num [1:4, 1] 3 -2 0 1
```

```
t(x.mat)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    3   -2    0    1
```

```
is.vector(t(x.mat))
```

```
## [1] FALSE
```

```
is.matrix(t(x.mat))
```

```
## [1] TRUE
```

```
dim(t(x.mat))
```

```
## [1] 1 4
```

```
length(t(x.mat))
```

```
## [1] 4
```

```
str(t(x.mat))
```

```
##  num [1, 1:4] 3 -2 0 1
```

- 작업 파일 저장

```
save.image(file="chapter_01_contents.rda")
```