Chapter 5 Lab

coop711

2015-10-29

```
## [1] "adjoint" "cofactor" "minor"
```

5.

5.1

$$(a) A = \begin{bmatrix} 6 & 13 \\ 5 & 12 \end{bmatrix}$$

```
(A1 \leftarrow matrix(c(6, 5, 13, 12), 2))
```

```
## [,1] [,2]
## [1,] 6 13
## [2,] 5 12
```

minor(A1, 1, 1)

[1] 12

minor(A1, 2, 1)

[1] 13

minor(A1, 1, 2)

[1] 5

minor(A1, 2, 2)

[1] 6

cofactor(A1, 1, 1)

[1] 12

cofactor(A1, 2, 1)

```
## [1] -13
cofactor(A1, 1, 2)
## [1] -5
cofactor(A1, 2, 2)
## [1] 6
A1[1, 1]*cofactor(A1, 1, 1) + A1[2, 1]*cofactor(A1, 2, 1)
## [1] 7
A1[1, 2]*cofactor(A1, 1, 2) + A1[2, 2]*cofactor(A1, 2, 2)
## [1] 7
det(A1)
## [1] 7
adjoint(A1)
   [,1] [,2]
## [1,] 12 -13
## [2,] -5 6
1/det(A1) * adjoint(A1)
              [,1]
                        [,2]
## [1,] 1.7142857 -1.8571429
## [2,] -0.7142857 0.8571429
solve(A1)
             [,1]
                      [,2]
## [1,] 1.7142857 -1.8571429
## [2,] -0.7142857 0.8571429
A1 %*% solve(A1)
```

```
## [,1] [,2]
 ## [1,] 1 0
## [2,] 0 1
 solve(A1) %*% A1
 ## [,1] [,2]
 ## [1,] 1 0
 ## [2,] 0 1
(b) B = \begin{bmatrix} 3 & -4 \\ 7 & 14 \end{bmatrix}
 (B1 \leftarrow matrix(c(3, 7, -4, 14), 2))
 ## [,1] [,2]
 ## [1,] 3 -4
## [2,] 7 14
 minor(B1, 1, 1)
 ## [1] 14
 minor(B1, 2, 1)
 ## [1] -4
 minor(B1, 1, 2)
 ## [1] 7
 minor(B1, 2, 2)
 ## [1] 3
 cofactor(B1, 1, 1)
 ## [1] 14
 cofactor(B1, 2, 1)
```

```
## [1] 4
cofactor(B1, 1, 2)
## [1] -7
cofactor(B1, 2, 2)
## [1] 3
B1[1, 1]*cofactor(B1, 1, 1) + B1[2, 1]*cofactor(B1, 2, 1)
## [1] 70
B1[1, 2]*cofactor(B1, 1, 2) + B1[2, 2]*cofactor(B1, 2, 2)
## [1] 70
det(B1)
## [1] 70
adjoint(B1)
   [,1] [,2]
## [1,] 14 4
## [2,] -7
1/det(B1) * adjoint(B1)
      [,1]
                  [,2]
## [1,] 0.2 0.05714286
## [2,] -0.1 0.04285714
solve(B1)
      [,1]
              [,2]
## [1,] 0.2 0.05714286
## [2,] -0.1 0.04285714
B1 %*% solve(B1)
```

```
## [,1] [,2]
 ## [1,] 1 0
 ## [2,] 0 1
 solve(B1) %*% B1
               [,1] [,2]
 ## [1,] 1.000000e+00
 ## [2,] -5.551115e-17
 round(solve(B1) %*% B1, digits = 2)
 ## [,1] [,2]
 ## [1,] 1 0
 ## [2,] 0 1
5.2
 A1 %*% B1
 ## [,1] [,2]
 ## [1,] 109 158
 ## [2,] 99 148
 solve(A1 %*% B1)
             [,1] [,2]
 ## [1,] 0.3020408 -0.322449
 ## [2,] -0.2020408 0.222449
 solve(B1)
 ## [,1] [,2]
 ## [1,] 0.2 0.05714286
 ## [2,] -0.1 0.04285714
 solve(A1)
```

```
## [2,] -0.7142857 0.8571429
```

 $[,1] \qquad [,2]$

[1,] 1.7142857 -1.8571429

solve(B1) %*% solve(A1)

```
## [,1] [,2]
## [1,] 0.3020408 -0.322449
## [2,] -0.2020408 0.222449
```

5.3

(a)
$$A = \begin{bmatrix} -1 & 3 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 4 \end{bmatrix}$$

```
(A3 \leftarrow matrix(c(-1, 0, 1, 3, 2, 0, 0, 1, 4), 3))
```

```
## [,1] [,2] [,3]
## [1,] -1 3 0
## [2,] 0 2 1
## [3,] 1 0 4
```

minor(A3, 1, 1)

[1] 8

minor(A3, 2, 1)

[1] 12

minor(A3, 3, 1)

[1] 3

minor(A3, 1, 2)

[1] -1

minor(A3, 2, 2)

[1] -4

minor(A3, 3, 2)

[1] -1

minor(A3, 1, 3)

```
## [1] -2
minor(A3, 2, 3)
## [1] -3
minor(A3, 3, 3)
## [1] -2
cofactor(A3, 1, 1)
## [1] 8
cofactor(A3, 2, 1)
## [1] -12
cofactor(A3, 3, 1)
## [1] 3
cofactor(A3, 1, 2)
## [1] 1
cofactor(A3, 2, 2)
## [1] -4
cofactor(A3, 3, 2)
## [1] 1
cofactor(A3, 1, 3)
## [1] -2
cofactor(A3, 2, 3)
```

[1] 3 cofactor(A3, 3, 3) ## [1] -2 A3[1, 1]*cofactor(A3, 1, 1) + A3[2, 1]*cofactor(A3, 2, 1) + A3[3, 1]*cofactor(A 3, 3, 1)## [1] -5 A3[1, 2]*cofactor(A3, 1, 2) + A3[2, 2]*cofactor(A3, 2, 2) + A3[3, 2]*cofactor(A 3, 3, 2)## [1] -5 A3[1, 3]*cofactor(A3, 1, 3) + A3[2, 3]*cofactor(A3, 2, 3) + A3[3, 3]*cofactor(A 3, 3, 3) ## [1] -5 det(A3) ## [1] -5 adjoint(A3) ## [,1] [,2] [,3] ## [1,] 8 -12 3 ## [2**,**] 1 - 41 ## [3,] -2 3 -2 1/det(A3) * adjoint(A3) ## [,1] [,2] [,3]## [1,] -1.6 2.4 -0.6 ## [2,] -0.2 0.8 -0.2## [3,] 0.4 -0.6 0.4

solve(A3)

```
## [,1] [,2] [,3]

## [1,] -1.6 2.4 -0.6

## [2,] -0.2 0.8 -0.2

## [3,] 0.4 -0.6 0.4
```

```
A3 %*% solve(A3)
```

```
## [,1] [,2] [,3]

## [1,] 1.000000e+00 0 0.000000e+00

## [2,] -5.551115e-17 1 -5.551115e-17

## [3,] 0.000000e+00 0 1.000000e+00
```

round(A3 %*% solve(A3), digits = 2)

```
## [,1] [,2] [,3]
## [1,] 1 0 0
## [2,] 0 1 0
## [3,] 0 0 1
```

solve(A3) %*% A3

(b)
$$B = \begin{bmatrix} 10 & 6 & -1 \\ 6 & 5 & 4 \\ -1 & 4 & 17 \end{bmatrix}$$

$$(B3 \leftarrow matrix(c(10, 6, -1, 6, 5, 4, -1, 4, 17), 3))$$

```
## [,1] [,2] [,3]
## [1,] 10 6 -1
## [2,] 6 5 4
## [3,] -1 4 17
```

minor(B3, 1, 1)

```
## [1] 69
```

```
minor(B3, 2, 1)
```

```
## [1] 106
```

```
minor(B3, 3, 1)
## [1] 29
minor(B3, 1, 2)
## [1] 106
minor(B3, 2, 2)
## [1] 169
minor(B3, 3, 2)
## [1] 46
minor(B3, 1, 3)
## [1] 29
minor(B3, 2, 3)
## [1] 46
minor(B3, 3, 3)
## [1] 14
cofactor(B3, 1, 1)
## [1] 69
cofactor(B3, 2, 1)
## [1] -106
cofactor(B3, 3, 1)
## [1] 29
```

```
cofactor(B3, 1, 2)
## [1] -106
cofactor(B3, 2, 2)
## [1] 169
cofactor(B3, 3, 2)
## [1] -46
cofactor(B3, 1, 3)
## [1] 29
cofactor(B3, 2, 3)
## [1] -46
cofactor(B3, 3, 3)
## [1] 14
B3[1, 1]*cofactor(B3, 1, 1) + B3[2, 1]*cofactor(B3, 2, 1) + B3[3, 1]*cofactor(B
3, 3, 1)
## [1] 25
B3[1, 2]*cofactor(B3, 1, 2) + B3[2, 2]*cofactor(B3, 2, 2) + B3[3, 2]*cofactor(B
3, 3, 2)
## [1] 25
B3[1, 3]*cofactor(B3, 1, 3) + B3[2, 3]*cofactor(B3, 2, 3) + B3[3, 3]*cofactor(B
3, 3, 3)
## [1] 25
det(B3)
```

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```
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## [1] 25
adjoint(B3)
## [,1] [,2] [,3]
## [1,] 69 -106 29
## [2,] -106 169 -46
## [3,] 29 -46 14
1/det(B3) * adjoint(B3)
        [,1] [,2] [,3]
## [1,] 2.76 -4.24 1.16
## [2,] -4.24 6.76 -1.84
## [3,] 1.16 -1.84 0.56
solve(B3)
   [,1] [,2] [,3]
##
## [1,] 2.76 -4.24 1.16
## [2,] -4.24 6.76 -1.84
## [3,] 1.16 -1.84 0.56
B3 %*% solve(B3)
                                        [,3]
                [,1]
                            [,2]
## [1,] 1.000000e+00 1.998401e-15 7.771561e-16
## [2,] -3.552714e-15 1.000000e+00 4.440892e-16
## [3,] 3.552714e-15 0.000000e+00 1.000000e+00
round(B3 %*% solve(B3), digits = 2)
## [,1] [,2] [,3]
## [1,] 1 0 0
## [2,]
        0
             1
                   0
## [3,] 0 0
solve(B3) %*% B3
                          [,2]
               [,1]
## [1,] 1.000000e+00 -3.552714e-15 3.552714e-15
## [2,] 2.220446e-15 1.000000e+00 -3.552714e-15
## [3,] 2.553513e-15 4.440892e-16 1.000000e+00
```

```
round(solve(B3) %*% B3, digits = 2)
```

```
## [,1] [,2] [,3]
## [1,] 1 0 0
## [2,] 0 1 0
## [3,] 0 0 1
```

(c)
$$C = (1/10) \begin{bmatrix} -0 & -10 & 0 \\ -6 & 0 & -8 \\ 8 & 0 & -6 \end{bmatrix}$$

```
(C3 \leftarrow (1/10)*matrix(c(0, -10, 0, -6, 0, -8, 8, 0, -6), 3))
```

```
## [,1] [,2] [,3]
## [1,] 0 -0.6 0.8
## [2,] -1 0.0 0.0
## [3,] 0 -0.8 -0.6
```

```
minor(C3, 1, 1)
```

```
## [1] 0
```

```
minor(C3, 2, 1)
```

[1] 1

minor(C3, 3, 1)

[1] 0

minor(C3, 1, 2)

[1] 0.6

minor(C3, 2, 2)

[1] 0

minor(C3, 3, 2)

[1] 0.8

```
minor(C3, 1, 3)
## [1] 0.8
minor(C3, 2, 3)
## [1] 0
minor(C3, 3, 3)
## [1] -0.6
cofactor(C3, 1, 1)
## [1] 0
cofactor(C3, 2, 1)
## [1] -1
cofactor(C3, 3, 1)
## [1] 0
cofactor(C3, 1, 2)
## [1] -0.6
cofactor(C3, 2, 2)
## [1] 0
cofactor(C3, 3, 2)
## [1] -0.8
cofactor(C3, 1, 3)
## [1] 0.8
```

```
cofactor(C3, 2, 3)
## [1] 0
cofactor(C3, 3, 3)
## [1] -0.6
C3[1, 1]*cofactor(C3, 1, 1) + C3[2, 1]*cofactor(C3, 2, 1) + C3[3, 1]*cofactor(C
3, 3, 1)
## [1] 1
C3[1, 2]*cofactor(C3, 1, 2) + C3[2, 2]*cofactor(C3, 2, 2) + C3[3, 2]*cofactor(C
3, 3, 2)
## [1] 1
C3[1, 3]*cofactor(C3, 1, 3) + C3[2, 3]*cofactor(C3, 2, 3) + C3[3, 3]*cofactor(C
3, 3, 3)
## [1] 1
det(C3)
## [1] 1
adjoint(C3)
       [,1] [,2] [,3]
## [1,] 0.0 -1 0.0
## [2,] -0.6 0 -0.8
## [3,] 0.8
               0 -0.6
1/det(C3) * adjoint(C3)
##
      [,1] [,2] [,3]
## [1,] 0.0 -1 0.0
## [2,] -0.6
              0 -0.8
             0 -0.6
## [3,] 0.8
```

```
solve(C3)
```

```
## [,1] [,2] [,3]
## [1,] 0.0 -1 0.0
## [2,] -0.6 0 -0.8
## [3,] 0.8 0 -0.6
```

```
t(C3)
```

```
## [,1] [,2] [,3]
## [1,] 0.0 -1 0.0
## [2,] -0.6 0 -0.8
## [3,] 0.8 0 -0.6
```

C3 %*% solve(C3)

```
## [,1] [,2] [,3]
## [1,] 1 0 0
## [2,] 0 1 0
## [3,] 0 0 1
```

round(C3 %*% solve(C3), digits = 2)

```
## [,1] [,2] [,3]
## [1,] 1 0 0
## [2,] 0 1 0
## [3,] 0 0 1
```

```
solve(C3) %*% C3
```

```
## [,1] [,2] [,3]
## [1,] 1 0 0
## [2,] 0 1 0
## [3,] 0 0 1
```

5.4

(a)
$$\begin{bmatrix} 7 & 3 \\ 8 & 9 \end{bmatrix}$$

```
matrix(c(7, 8, 3, 9), 2)
```

```
## [,1] [,2]
## [1,] 7 3
## [2,] 8 9
```

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```
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det(matrix(c(7, 8, 3, 9), 2))
## [1] 39
adjoint(matrix(c(7, 8, 3, 9), 2))
## [,1] [,2]
## [1,] 9 -3
## [2,] -8 7
solve(matrix(c(7, 8, 3, 9), 2))
##
             [,1]
                        [,2]
## [1,] 0.2307692 -0.07692308
## [2,] -0.2051282 0.17948718
matrix(c(6, 8, 31, 29), 2)
## [,1] [,2]
## [1,] 6 31
## [2,]
         8
              29
det(matrix(c(6, 8, 31, 29), 2))
## [1] -74
adjoint(matrix(c(6, 8, 31, 29), 2))
   [,1] [,2]
## [1,] 29 -31
## [2,]
         -8 6
solve(matrix(c(6, 8, 31, 29), 2))
                         [,2]
             [,1]
## [1,] -0.3918919 0.41891892
## [2,] 0.1081081 -0.08108108
```

(c)
$$\begin{bmatrix} -7 & -4 \\ 3 & 1 \end{bmatrix}$$

```
matrix(c(-7, 3, -4, 1), 2)
```

```
## [,1] [,2]
## [1,] -7 -4
## [2,] 3 1
```

```
det(matrix(c(-7, 3, -4, 1), 2))
```

```
## [1] 5
```

```
adjoint(matrix(c(-7, 3, -4, 1), 2))
```

```
## [,1] [,2]
## [1,] 1 4
## [2,] -3 -7
```

```
solve(matrix(c(-7, 3, -4, 1), 2))
```

```
## [,1] [,2]
## [1,] 0.2 0.8
## [2,] -0.6 -1.4
```

(d)
$$\begin{bmatrix} 1 & 5 & -5 \\ 3 & 2 & -5 \\ 6 & -2 & -5 \end{bmatrix}$$

```
## [,1] [,2] [,3]
## [1,] 1 5 -5
## [2,] 3 2 -5
## [3,] 6 -2 -5
```

```
det(matrix(c(1, 3, 6, 5, 2, -2, -5, -5, -5), 3))
```

```
## [1] -5
```

```
## [,1] [,2] [,3]
## [1,] -20 35 -15
## [2,] -15 25 -10
## [3,] -18 32 -13
```

solve(matrix(c(1, 3, 6, 5, 2, -2, -5, -5, -5), 3))

```
## [,1] [,2] [,3]
## [1,] 4.0 -7.0 3.0
## [2,] 3.0 -5.0 2.0
## [3,] 3.6 -6.4 2.6
```

(e)
$$\begin{bmatrix} -3 & -3 & -2 \\ 2 & 5 & 3 \\ -6 & -7 & -4 \end{bmatrix}$$

```
matrix(c(-3, -3, -2, 2, 5, 3, -6, -7, -4), 3)
```

```
## [,1] [,2] [,3]
## [1,] -3 2 -6
## [2,] -3 5 -7
## [3,] -2 3 -4
```

```
det(matrix(c(-3, -3, -2, 2, 5, 3, -6, -7, -4), 3))
```

```
## [1] -5
```

```
adjoint(matrix(c(-3, -3, -2, 2, 5, 3, -6, -7, -4), 3))
```

```
## [,1] [,2] [,3]
## [1,] 1 -10 16
## [2,] 2 0 -3
## [3,] 1 5 -9
```

```
solve(matrix(c(-3, -3, -2, 2, 5, 3, -6, -7, -4), 3))
```

```
## [,1] [,2] [,3]
## [1,] -0.2 2 -3.2
## [2,] -0.4 0 0.6
## [3,] -0.2 -1 1.8
```

(f)
$$\begin{vmatrix} 2 & -5 & 1 \\ 1 & 1 & 4 \\ 3 & 0 & -2 \end{vmatrix}$$

```
matrix(c(2, -5, 1, 1, 1, 4, 3, 0, -2), 3)
```

```
## [,1] [,2] [,3]
## [1,] 2 1 3
## [2,] -5 1 0
## [3,] 1 4 -2
```

```
det(matrix(c(2, -5, 1, 1, 1, 4, 3, 0, -2), 3))
```

```
## [1] -77
```

```
adjoint(matrix(c(2, -5, 1, 1, 1, 4, 3, 0, -2), 3))
```

```
## [,1] [,2] [,3]
## [1,] -2 14 -3
## [2,] -10 -7 -15
## [3,] -21 -7 7
```

```
solve(matrix(c(2, -5, 1, 1, 1, 4, 3, 0, -2), 3))
```

```
## [,1] [,2] [,3]

## [1,] 0.02597403 -0.18181818 0.03896104

## [2,] 0.12987013 0.09090909 0.19480519

## [3,] 0.27272727 0.09090909 -0.09090909
```

(g)
$$\begin{vmatrix} -1 & -2 & 1 \\ -2 & 2 & -2 \\ 1 & -2 & -1 \end{vmatrix}$$

```
matrix(c(-1, -2, 1, -2, 2, -2, 1, -2, -1), 3)
```

```
## [,1] [,2] [,3]
## [1,] -1 -2 1
## [2,] -2 2 -2
## [3,] 1 -2 -1
```

```
det(matrix(c(-1, -2, 1, -2, 2, -2, 1, -2, -1), 3))
```

```
## [1] 16
```

```
adjoint(matrix(c(-1, -2, 1, -2, 2, -2, 1, -2, -1), 3))
```

```
## [,1] [,2] [,3]
## [1,] -6 -4 2
## [2,] -4 0 -4
## [3,] 2 -4 -6
```

```
solve(matrix(c(-1, -2, 1, -2, 2, -2, 1, -2, -1), 3))
```

```
## [,1] [,2] [,3]
## [1,] -0.375 -0.25 0.125
## [2,] -0.250 0.00 -0.250
## [3,] 0.125 -0.25 -0.375
```

(h)
$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

```
# install.packages("Matrix", repos = "https://cran.rstudio.com")
library(Matrix)
matrix(c(1, -2, 1, -2, 4, -2, 1, -2, 1), 3)
```

```
## [,1] [,2] [,3]
## [1,] 1 -2 1
## [2,] -2 4 -2
## [3,] 1 -2 1
```

```
det(matrix(c(1, -2, 1, -2, 4, -2, 1, -2, 1), 3))
```

```
## [1] 0
```

```
adjoint(matrix(c(1, -2, 1, -2, 4, -2, 1, -2, 1), 3))
```

```
## [,1] [,2] [,3]
## [1,] 0 0 0
## [2,] 0 0 0
## [3,] 0 0 0
```

```
# solve(matrix(c(1, -2, 1, -2, 4, -2, 1, -2, 1), 3))
rankMatrix(matrix(c(1, -2, 1, -2, 4, -2, 1, -2, 1), 3))
```

```
## [1] 1
## attr(,"method")
## [1] "tolNorm2"
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
## [1] 6.661338e-16
```

(i)
$$\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$

```
matrix(c(7, 4, -4, 4, 7, -4, -1, -1, 4), 3)
```

```
## [,1] [,2] [,3]
## [1,] 7 4 -1
## [2,] 4 7 -1
## [3,] -4 -4 4
```

```
det(matrix(c(7, 4, -4, 4, 7, -4, -1, -1, 4), 3))
```

```
## [1] 108
```

```
adjoint(matrix(c(7, 4, -4, 4, 7, -4, -1, -1, 4), 3))
```

```
## [,1] [,2] [,3]
## [1,] 24 -12 3
## [2,] -12 24 3
## [3,] 12 12 33
```

```
solve(matrix(c(7, 4, -4, 4, 7, -4, -1, -1, 4), 3))
```

```
## [,1] [,2] [,3]

## [1,] 0.2222222 -0.1111111 0.02777778

## [2,] -0.1111111 0.2222222 0.02777778

## [3,] 0.1111111 0.1111111 0.30555556
```

자료 저장

```
save.image("chapter_5_lab.rda")
```