Midterm

Due Friday March 25th

Please choose one of the following problems to do with your group. If you feel like you can't do either of them completely, but can do parts of both of them, mercy will be granted on your soul if you present partial completions of both questions, but attempting both is in no way necessary.

The exam is to be completed in groups of 4 or 5 (other groups helping each other is ok and encouraged). Only one final copy needs to be turned in for your group. Please submit in LaTeX. Other formats are allowed but frowned upon (akin to using the mostly minuses metric convention or liking the Matrix sequel more than the original. There was no third Matrix movie, that was a Dragon Ball Z version of the story of Christ).

1. *Interstellar*: Remember that Matthew McCoughangeyee version of Inception that came out a little bit ago? Let's do some calculations regarding that planet orbiting the black hole (if you've never seen the movie, no worries, it doesn't help you for this problem... or for most problems in life for that matter). Imagine you're in a spaceship in Keplerian orbit 1 AU outside of a static black hole with mass 62 $M_\odot$ (the resultant black hole of the gravitational wave detection event, but not spinning). At this distance your ship's orbit is truly Keplerian because spacetime is very flat there. You and your crew notice a planet orbiting the black hole at the inner-most stable orbit. Your mission is to go to this planet, send a signal from its surface back to the ship and come back. You must determine what the most efficient orbital trajectory is given the following constraints:

   (a) Time is very dilated this close to the black hole and you don't want to keep your crew waiting. Calculate the proper time of your trajectory and try and minimize it. Your crew must be alive when you get back to succeed.

   (b) You don't have infinite fuel, and as a matter of fact, you'd like to spend as little fuel as possible. Calculate the amount of rocket fuel you'll need to get to the planet and back along the orbital maneuver you choose. This part may require a bit of reading on orbital maneuvers and approximations may be needed to perform the calculation. If you make any approximations make sure to fully, quantitatively, justify them. Alternatively you can do numerical integration. Use whatever resource you'd like, but make sure you cite whatever you use; this should read like a lab report or a research paper. Also, keep in mind the optimal fuel path might not be the least proper time path. Balance these two constraints against each other and discuss.

   (c) Nowhere along your trajectory should you experience more than 10g. You also have to be alive when you get back to succeed (that is, don't just radially plunge into the planet, smashing onto its surface at half the speed of light.)

Please include the following information in your discussion of the mission: When you get to the planet to send the signal, at what angle do you have to point the transmitter so that the signal bends into your spacecraft? Make whatever approximations necessary but justify. If the planet looks like a blue water planet when you're on it, what color did it look like from the ship? Can you 'see' the planet from the ship at all in the visible spectrum? In the film, "1 hour on the planet is like 7 years back on the ship". How close to the event horizon would the planet have to be for this to be true? (in Interstellar the black hole was rotating so your calculation doesn't invalidate the film outright...just so you know, don't go writing a letter to Christopher Nolan or something). Tides on this planet looked to be hundreds of meters high. Assuming the near side of the planet is at the distance calculated in the last sentence, and the far side is 100km radially away from the black hole, how large are the tidal forces?

2. *Data Driven Geometry*: You're a two dimensional creature living in a little circular house (small but not a point) with a huge clocktower on top. You've been watching Lenny Susskind's YouTube lectures on general relativity and decide to try and understand your local geometry. You perform the following experiment. You take pieces of string of various lengths (measured in your 'paces' which is the only unit of length available to you), walk until the string is taut, and then circumambulate your house. Also, when you arrive at the maximum distance from your house for a given piece of string, you look back and count how many seconds tick off your home clocktower in precisely one minute of time according to your Flavor Flav necklace-clock.

   You determine what a 'circle around your house' is by walking directly away from your house until the string is taut and then pick the direction such that the string remains taut without breaking and continue walking in that direction until you arrive at where you started. The results of this experiment are tabulated below, as well as available on the website in digital form. Bare in mind that the relationship between 'paces' and actual distance, is a little fuzzy. You never make the same step twice (I think that's how that phrase goes) so there will be standard deviation error bars present in this data. Use this information to determine the spacetime geometry of your surface.

   Please include the following in your discussion: When you get to a certain distance away with your string, no matter what direction you step, the string becomes less taut. How is this possible? What does it mean about your world? What does you house look like from this point?

   Does your clock look any different? Does it become hard to 'see' your clock at certain distances from your house?

You're at home watching Netflix, drinking a MountainDew Gamefuel (Halo 2, limited edition) and you spill your drink. The liquid all rushes away from you out of your house, slowing down to an eventual stop, making a liquid ring around your house. How far away is that ring? What does this imply about your world? Does this line up with your analysis of the measurements of your clock? In light of this, do you have any suggestions as to what your 'world' might be? What further experiments (as well as expected theoretical results given your hypothesis) would you conduct?

Radius	Walk Length	Walk Length Uncertainty	Observed Time	Observed Time Uncertainty
200	1.28e+03	62.4	60	1.21
251	1.59e+03	78	62.5	1.22
315	1.9e+03	97.3	61.3	1.23
395	2.39e+03	121	60.9	1.25
496	2.97e+03	149	62.3	1.27
622	3.71e+03	183	64.3	1.31
781	4.49e+03	221	67.1	1.38
980	5.01e+03	261	74	1.47
1.23e+03	6.09e+03	296	80.3	1.6
1.54e+03	6.09e+03	314	80.5	1.68
1.94e+03	5.89e+03	294	76.4	1.59
2.43e+03	5.53e+03	247	72.9	1.44
3.05e+03	3.46e+03	187	64.2	1.32
3.83e+03	2.49e+03	113	61.4	1.24
4.8e+03	380	19.2	60.5	1.2