Homework 8

Due April 5th

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1. The Schwarzchild line element in spherical coordinates should be seared into your brains by now, but just in case it isn't:

$$ds^2_{sch} = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega$$

   (a) Calculate the line element ds^2 after the following redefinition of t:

$$T = t + 2M\ln\left|\frac{r}{2M} - 1\right|$$

   (b) A large class of metrics in general relativity (including the Schwarzchild metric) can be rewritten in the form:

$$g^{\mu\nu} = \eta^{\mu\nu} - 2\, h(x^\mu)\, l^\mu l^\nu$$

   with $\eta^{\mu\nu}$ being the flat Minkowski metric. What is l^μ for the Schwarzchild metric?

   (c) Calculate $g_{\mu\nu}l^\mu l^\nu$ and $\eta_{\mu\nu}l^\mu l^\nu$. These coordinates are another example infalling coordinates for the Schwarzchild metric (the Kerr metric can also be rewritten in this way!). These coordinates are ideal for performing certain tasks like calculating the metric of a black hole, flung at you close to the speed of light. The geometry is encoded in the vector l^μ so by knowing how these transform, one can calculate the metric of a boosted black hole.

2. (a) An observer falls feet first into a Schwarzchild black hole looking down at her feet. Is there ever a moment when she cannot see her feet? For instance, can she see her feet when her head is crossing the horizon? If so, what radius does she see them at? Does she ever see her feet hit the singularity at $r = 0$ assuming she remains intact until her head reaches that radius? Analyze these questions with an Eddington-Finkelstein or Kruskal diagram.

   (b) *Is it dark inside a black hole?* An observer outside sees a star collapsing to a black hole become dark. But would it be dark inside a black hole assuming a collapsing star continues to radiate at a steady rate as measured by an observer on its surface?

3. Once across the even horizon of a black hole, what is the *longest* proper time the observer can spend before being destroyed in the singularity?

4. The singularity of a Schwarzchild black hole is a moment in time, not a point in space. What about the Kerr metric? Is it a moment at all? Show that the singularity of a Schwarzchild black hole is indeed a moment, and derive the geometry of the (true, not coordinate) Kerr singularity. (*Hint*: set $\rho = dt = 0$)