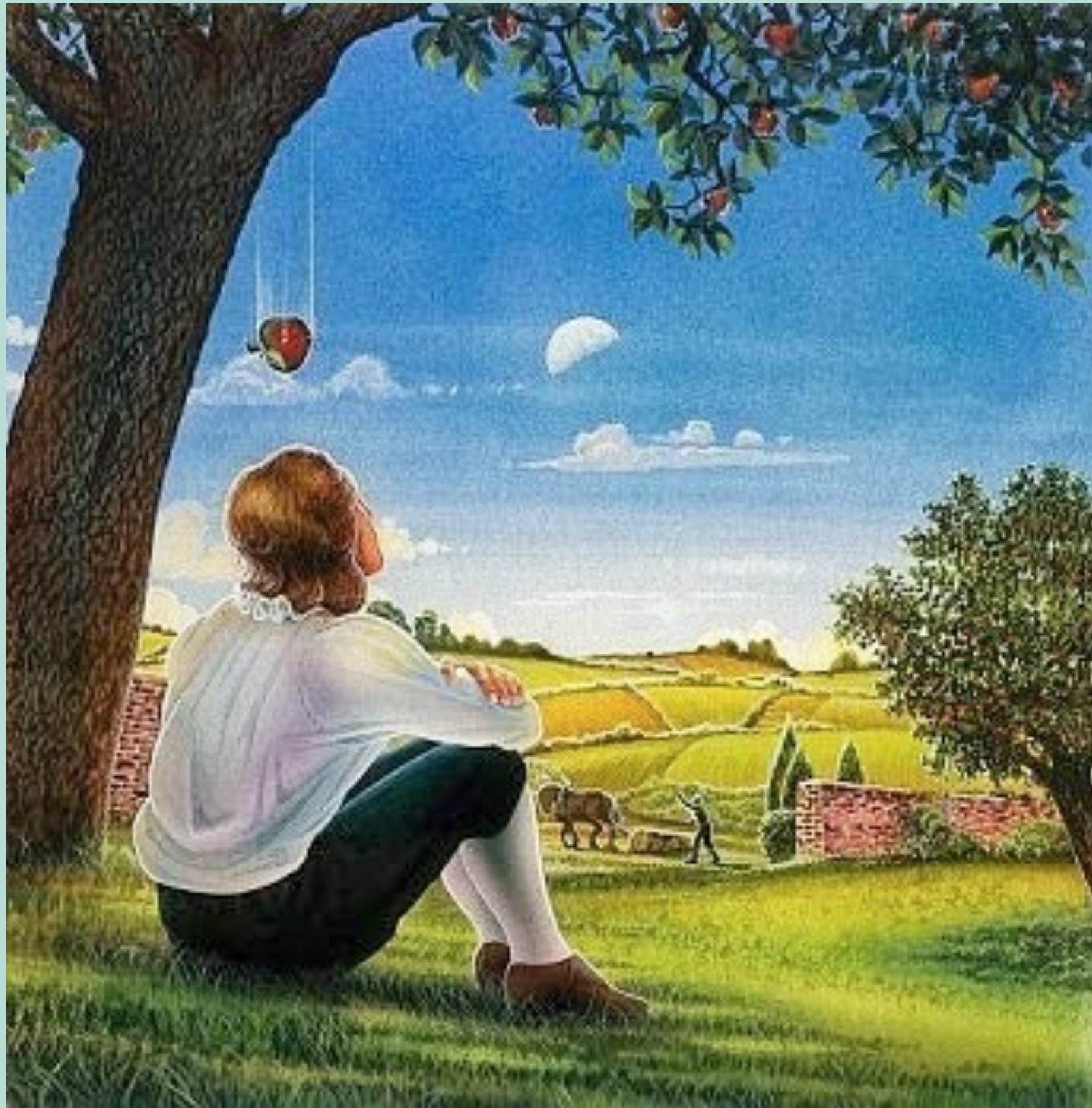


with your host:

Lecture 16: Gravity



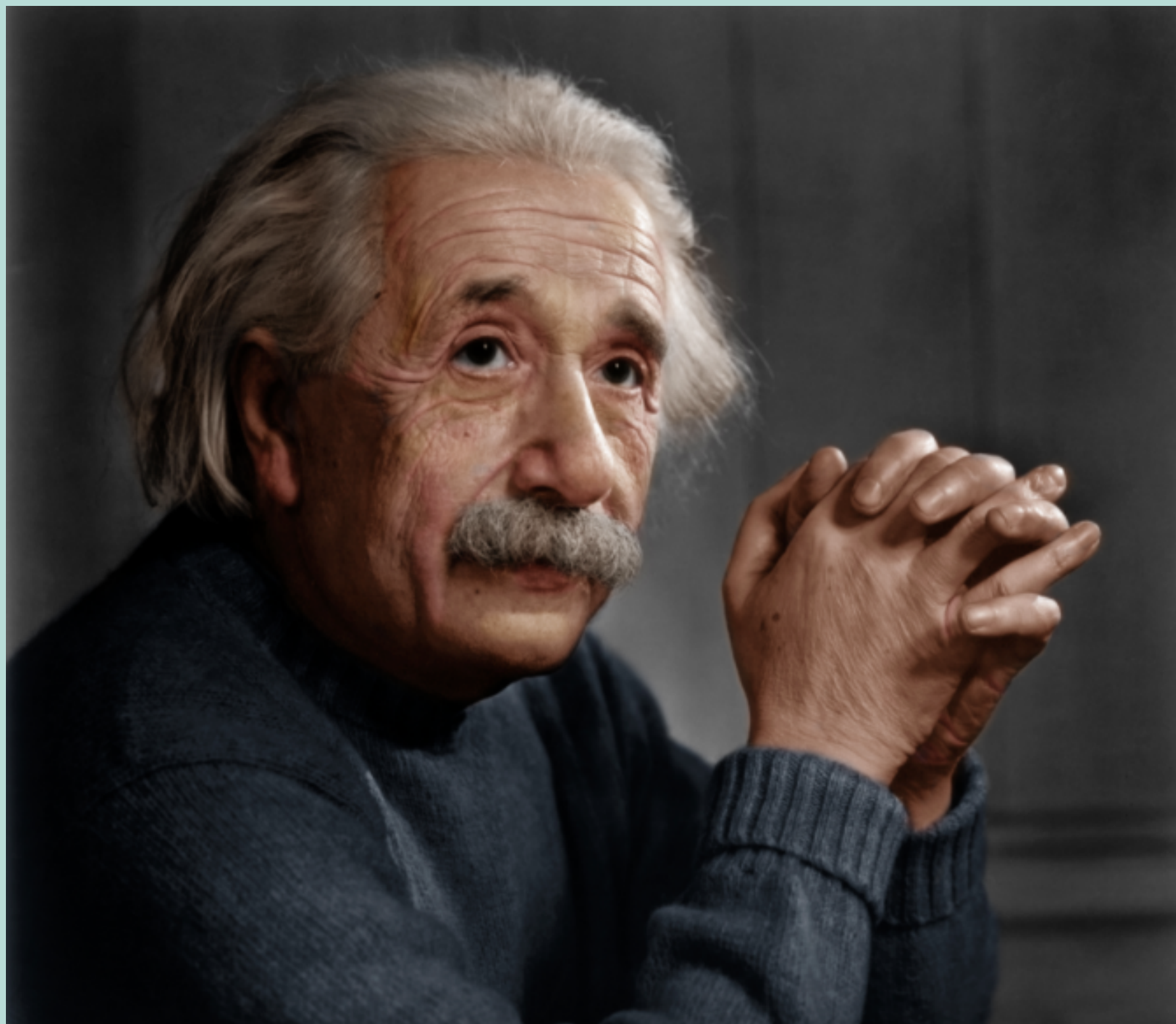
Coop



$$F = \frac{Gm_1m_2}{r^2}$$



- This worked remarkably well, from describing projectile motion on earth, to deriving Kepler's 3 Laws from first principles
- There is one subtlety however. "r" is a function of time, and "time" in Newtonian theory is universal

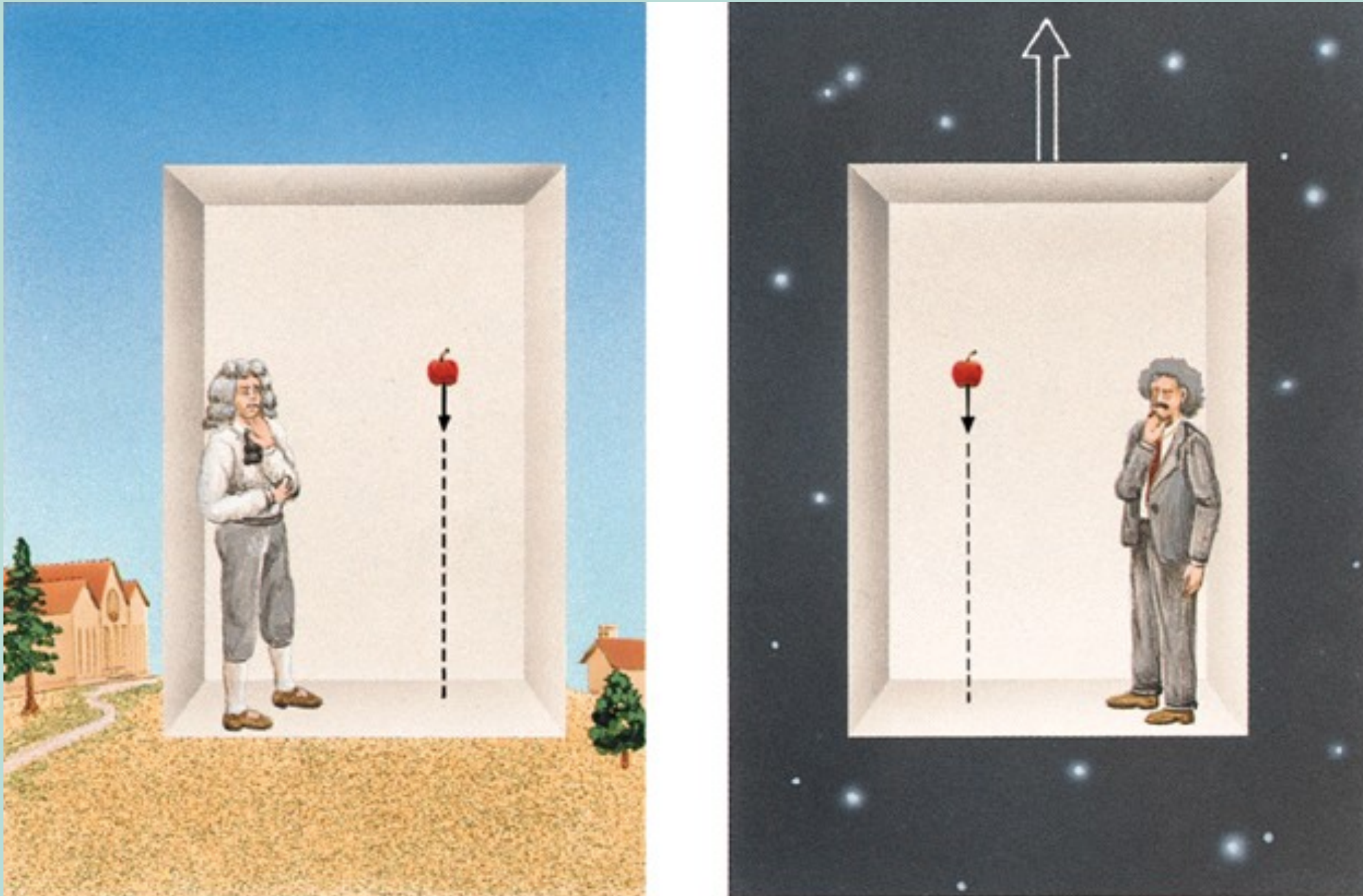


Relativity of Simultaneity

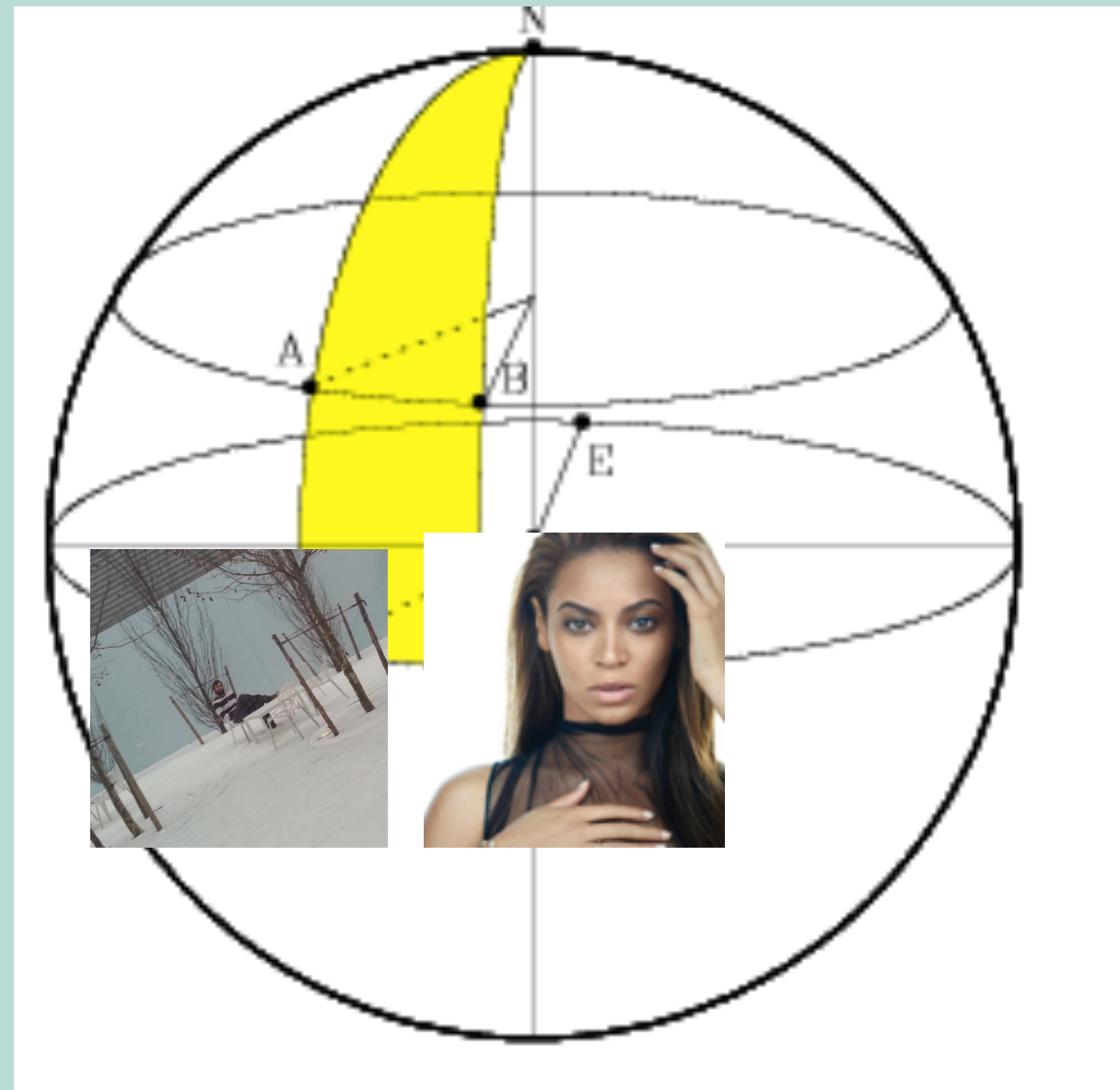
- Maxwell's equations introduce a fundamental speed of nature, that is independent of the speed of the laboratory, thus violating centuries of Galilean Relativity
- Einstein fully thinks through the consequence of this and realizes *time* must not be universal in the sense that people moving at different speeds with respect to each other *measure* time differently

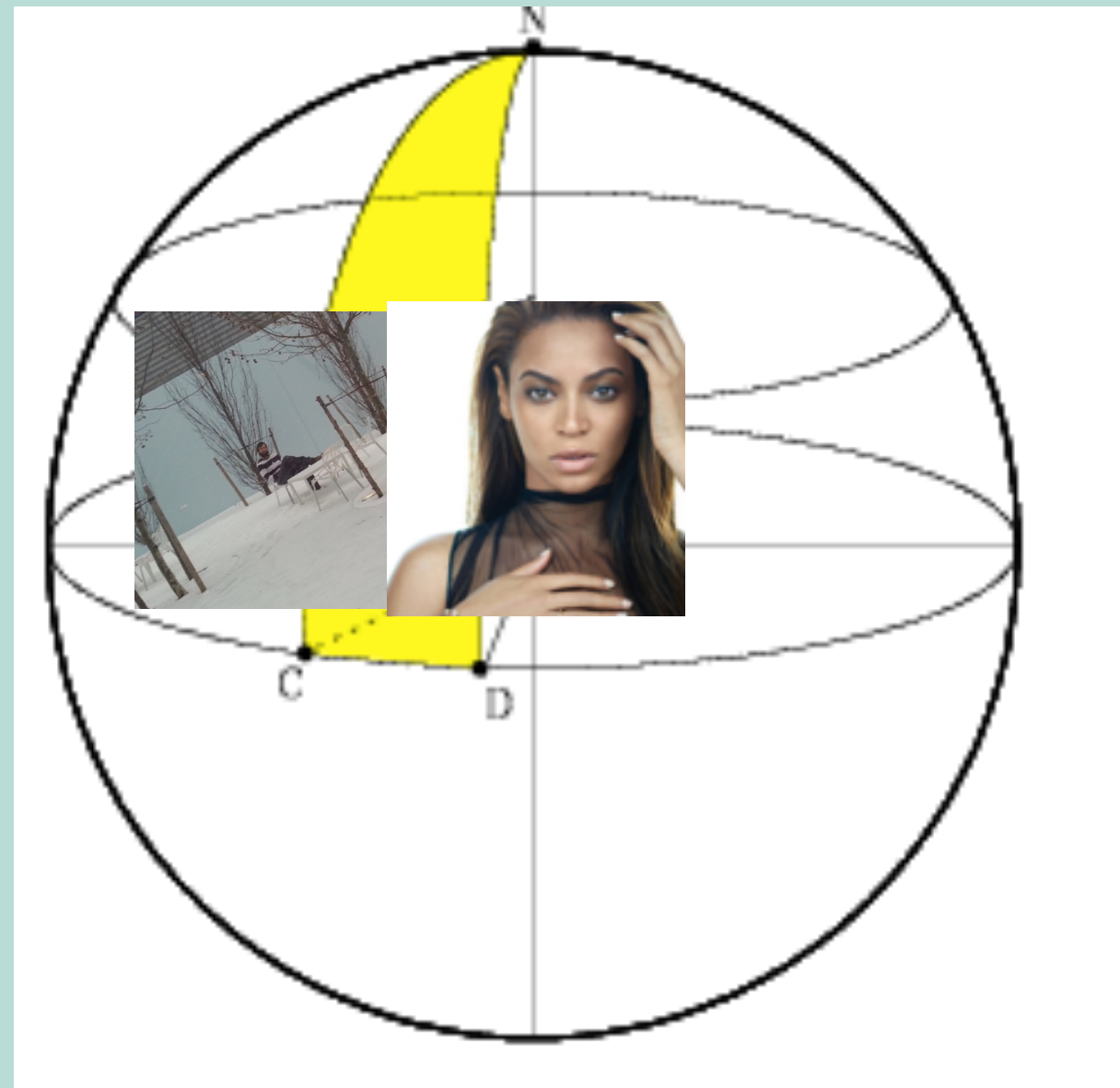
- Going back to Newton's theory of gravity, this means that it's not clear what the force due to gravity should be, since the force seems to be *instantaneous* which violates special relativity
- Note: the violations would be very small, so Newtonian gravity is still 'correct' in the sense that it applies to a very broad class of problems. Only rapidly spinning black holes and *extremely* precise planetary predictions require a new theory.

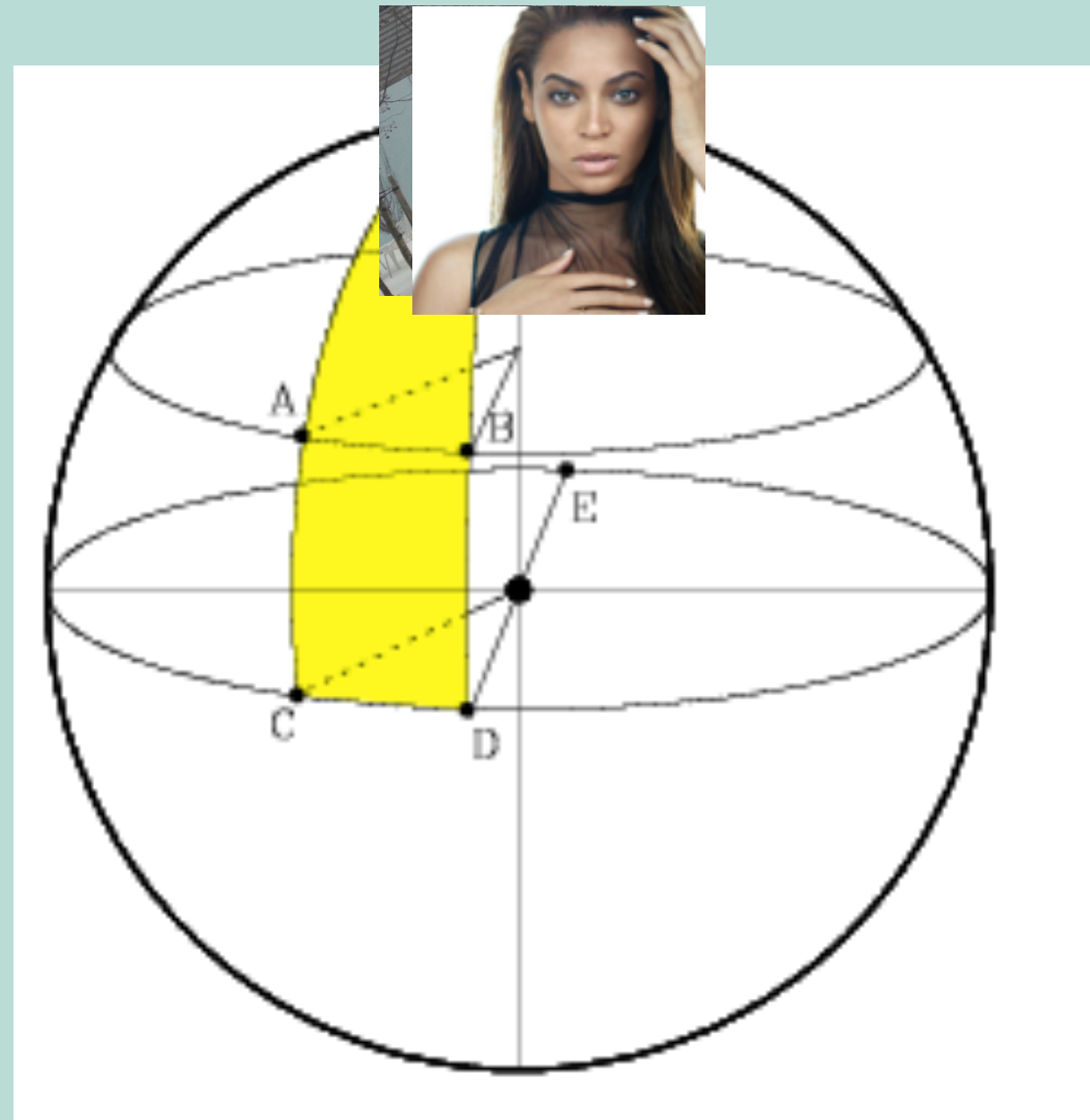
The Equivalence Principle




$$m_{\text{gravitational}} = m_{\text{inertial}}$$





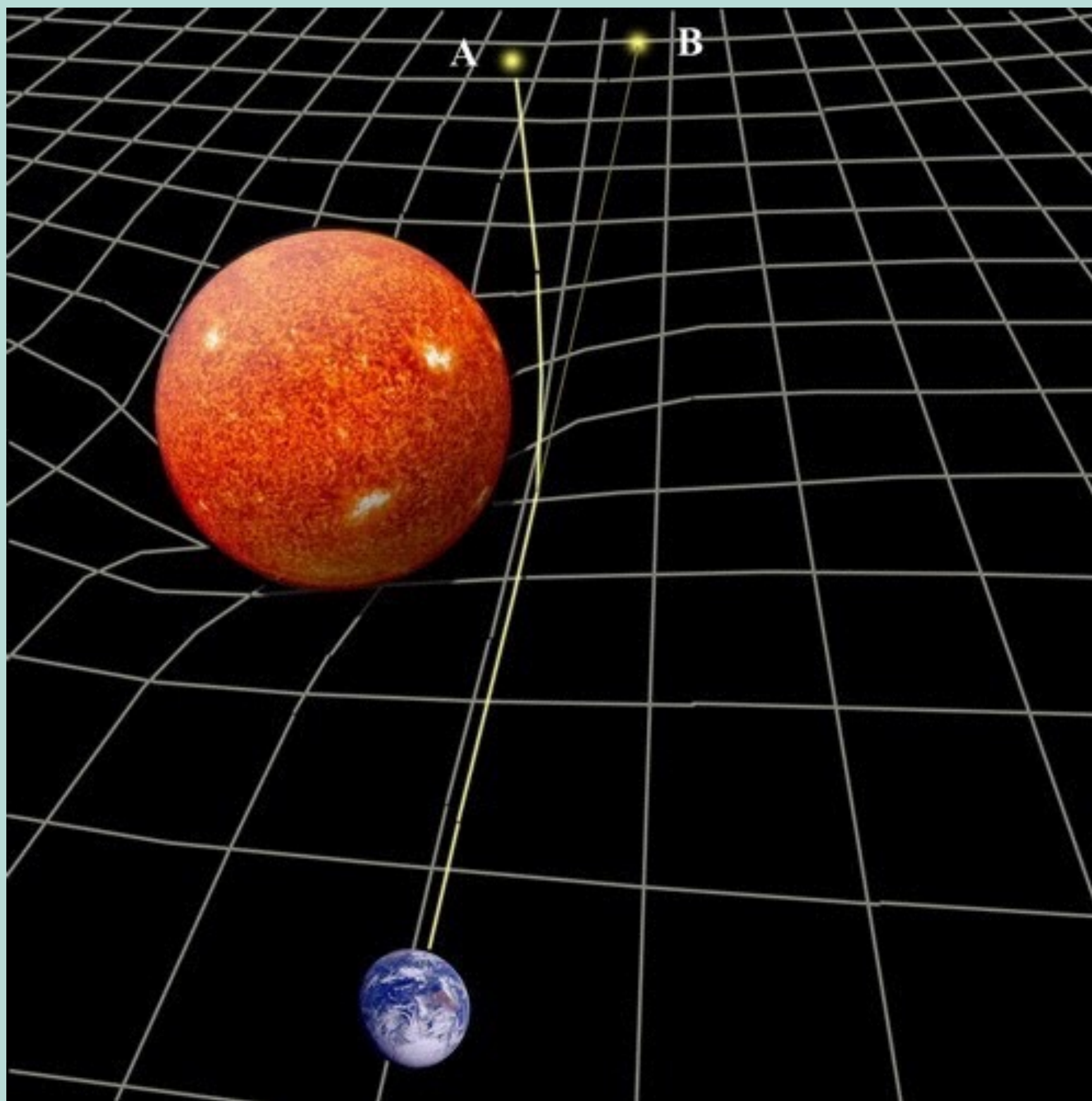


General Relativity

Gravity  Geometry

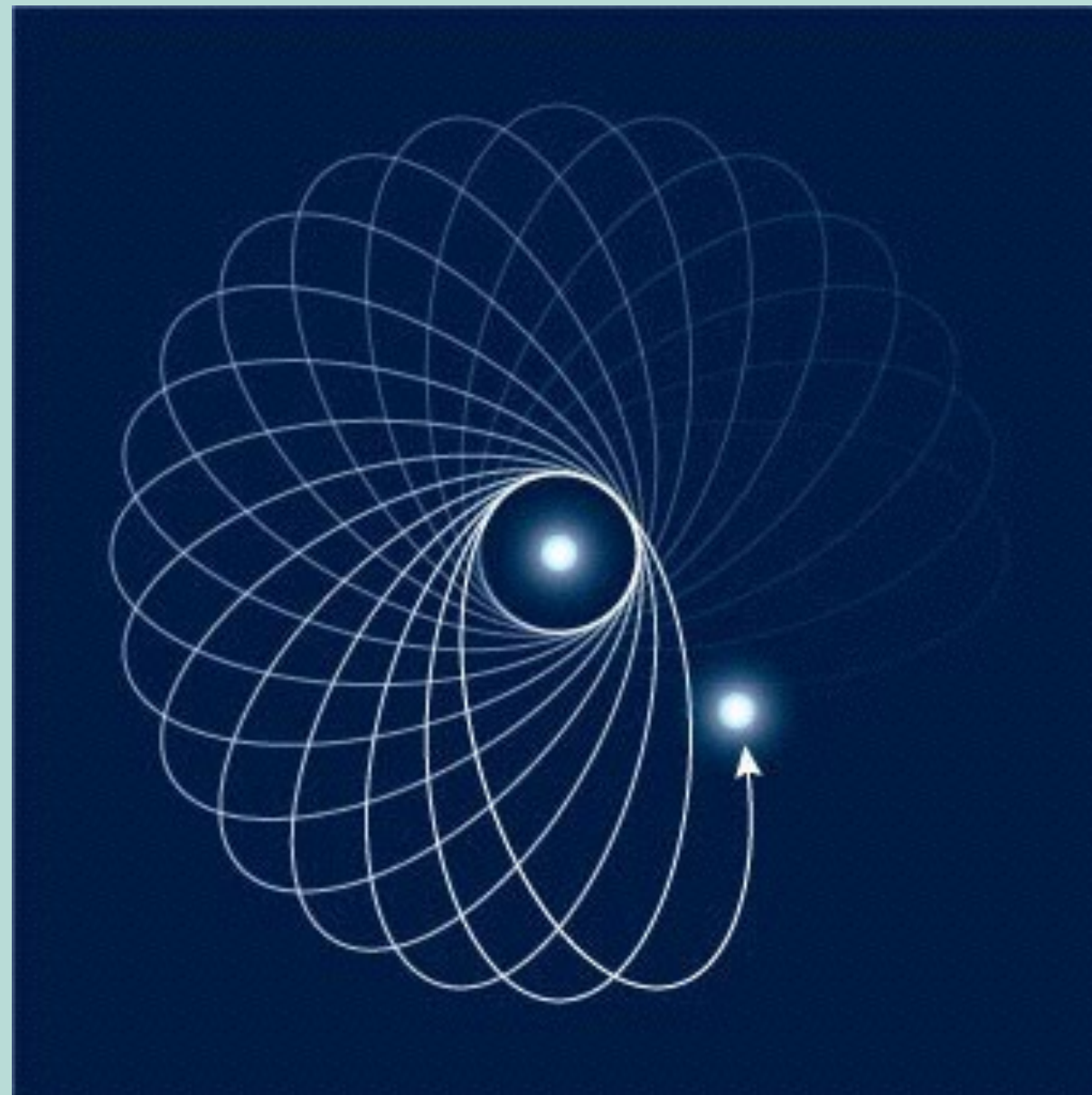
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi\mathcal{G}}{c^4}T_{\mu\nu}$$

Spacetime is bent by matter

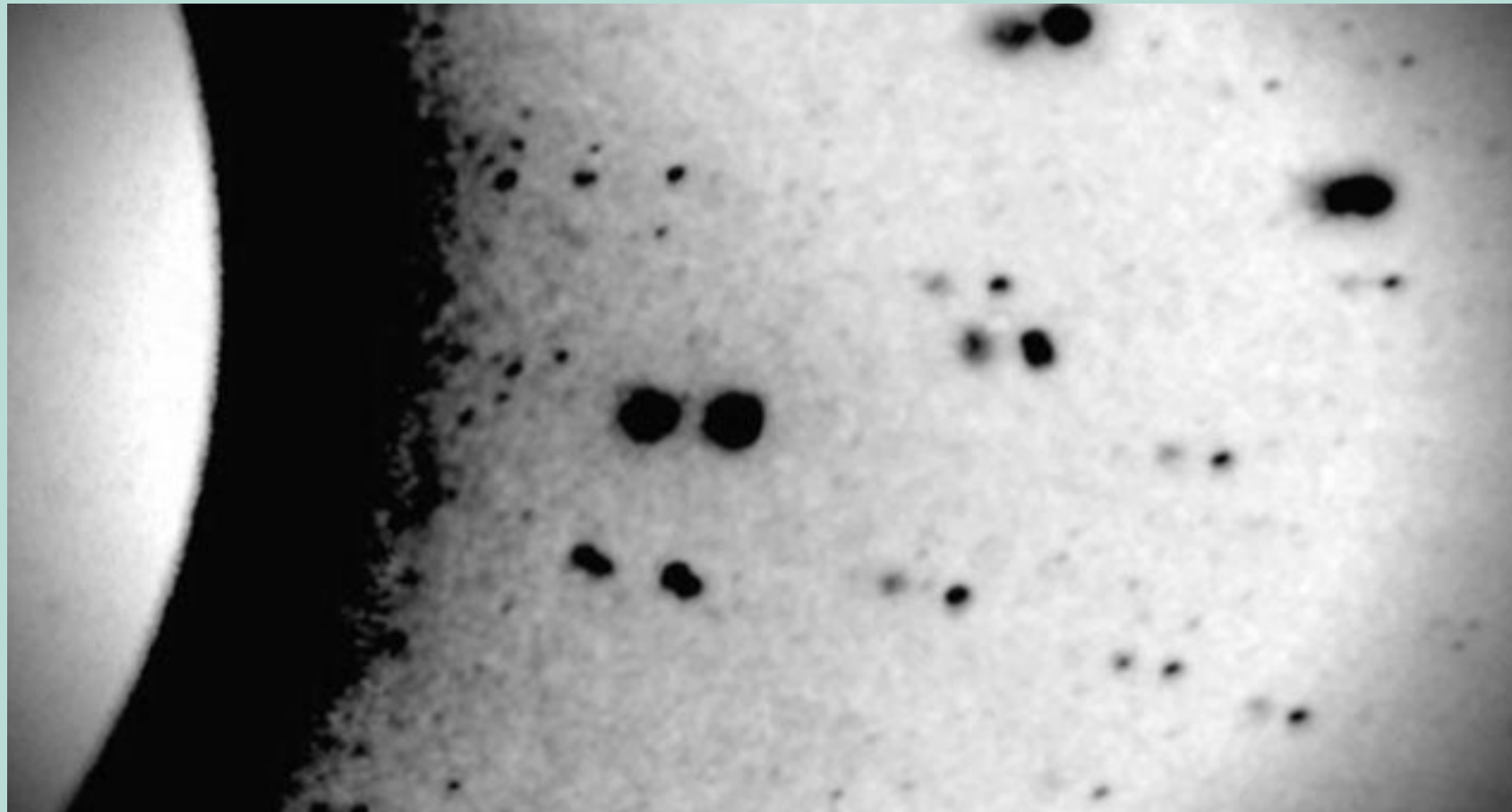


Evidence?

43''/century anomalous
precession of mercury



1919 Solar Eclipse

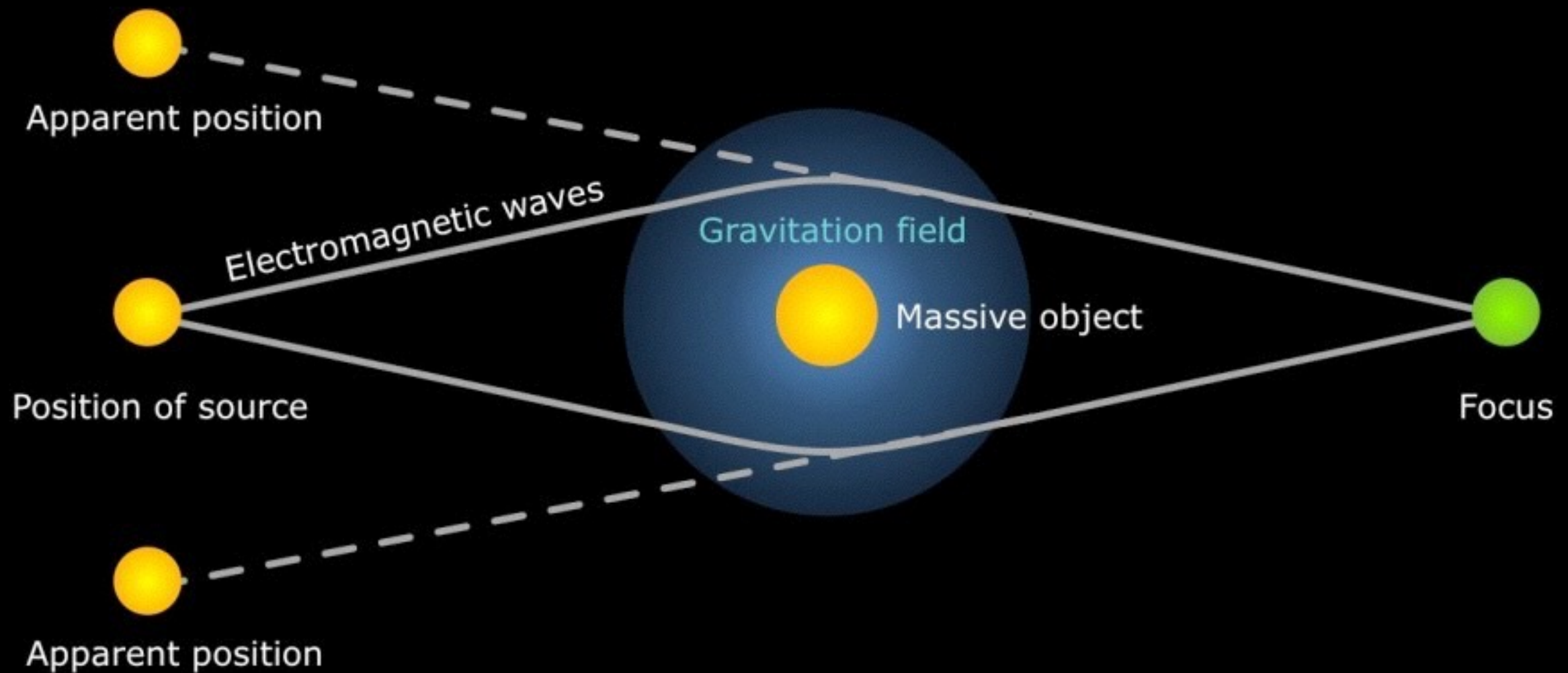


- For light, $m = 0$! But for Einstein, $E = mc^2$ so light doesn't need "mass" to feel the effects of gravity, just energy!

Gravitational Lensing



Gravitational Lensing



And one more striking
prediction...

Blackholes

- Interestingly, the notion of a “blackhole”, that is, a surface from which light cannot escape, is older than general relativity
- Recall, the escape velocity of Earth’s surface gravity is 11.2 km/s and this number is independent of mass! Newton believed that light were particles, so they should feel “g” of gravity like everything else
- The result of this calculation is that if light moves with speed c , then if its size smaller than:

$$R \leq \frac{2GM}{c^2} \quad \text{then the light would never escape...}$$

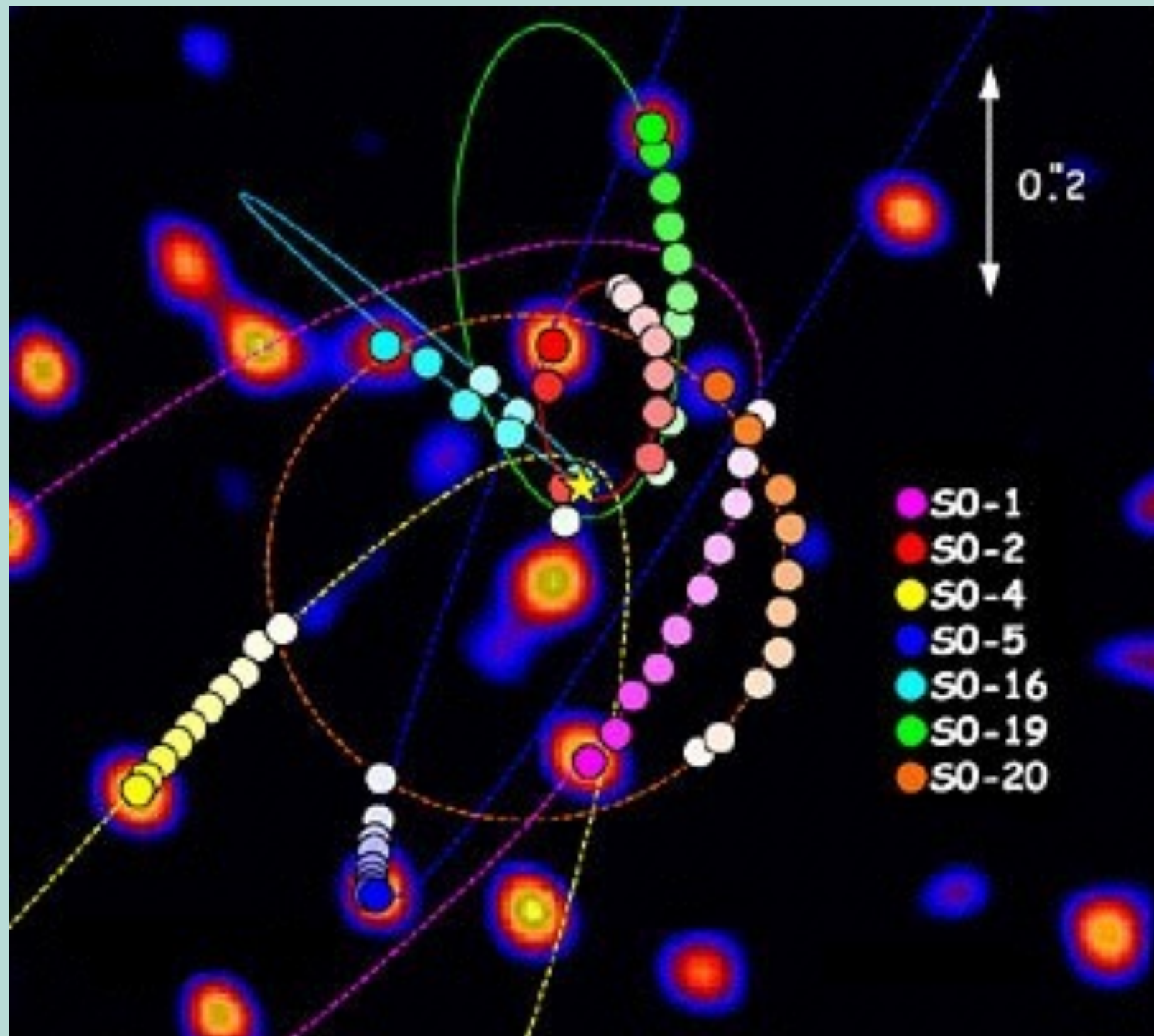
Blackholes

- In Einstein's theory, light doesn't have mass, and so the Newtonian calculation isn't relevant. However, its path is still bent by gravity and surprisingly the result is the same

$$R_{schwarz} = \frac{2\mathcal{G}M}{c^2}$$

- This distance is called the “Schwarzschild Radius” and when a mass M is compressed to within this radius, a spherical surface with radius $R_{schwarz}$ known as the “event horizon” forms which acts as a ‘causality barrier’, creating a one way street from the exterior to the interior of the black hole

Sagittarius A



4,000,000 suns of mass in “empty” space

Blackholes

- Black holes can also “rotate”. It’s hard to imagine what empty space rotating looks like, but one phenomenon which is to be expected is rotating black holes are *oblate* (remember Jupiter/Saturn?)
- Rotating blackholes are *incredibly* efficient at converting rest mass into energy. In terms of the $E = mc^2$ of available energy for a particle of mass ‘m’, Fusion turns mass into energy at an efficiency of 4%.
- Dropping something into a maximally rotating blackhole could be as high as 47% efficient!

Blackholes



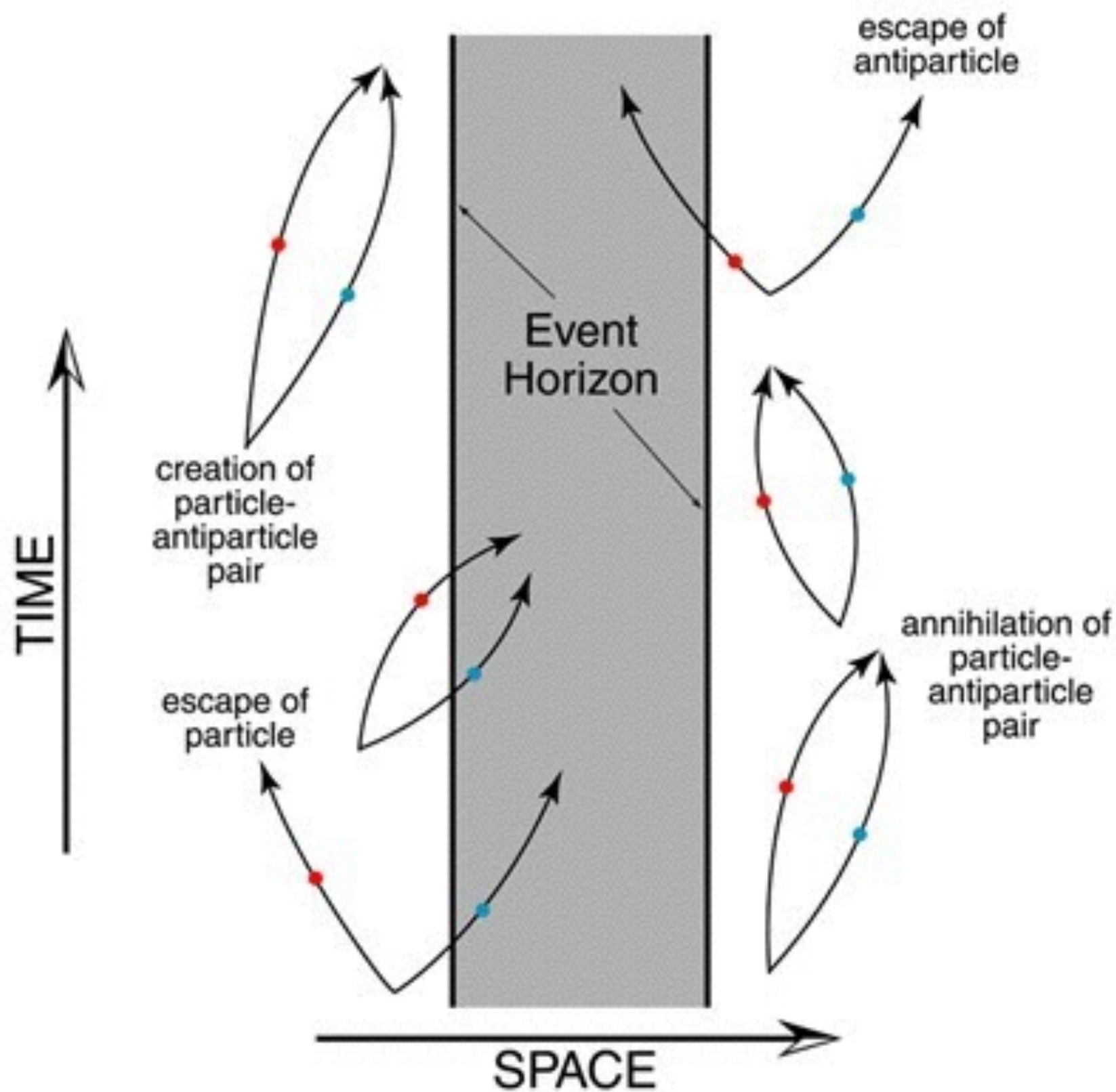
- This isn't merely a theoretical calculation, the most powerful persistent objects in the universe can only be explained by matter falling into a rapidly rotating black hole

Blackholes

- A truly bizarre phenomenon happens whenever you mix quantum mechanics with general relativity, Black holes seem to *radiate*. This is known as “Hawking Radiation”
- While this may seem like it violates the conservation of energy, the Heisenberg Uncertainty principle (remember? The thing responsible for electrons not falling into the nucleus!) allows for particles to be created out of the vacuum in pairs and eventually annihilate.

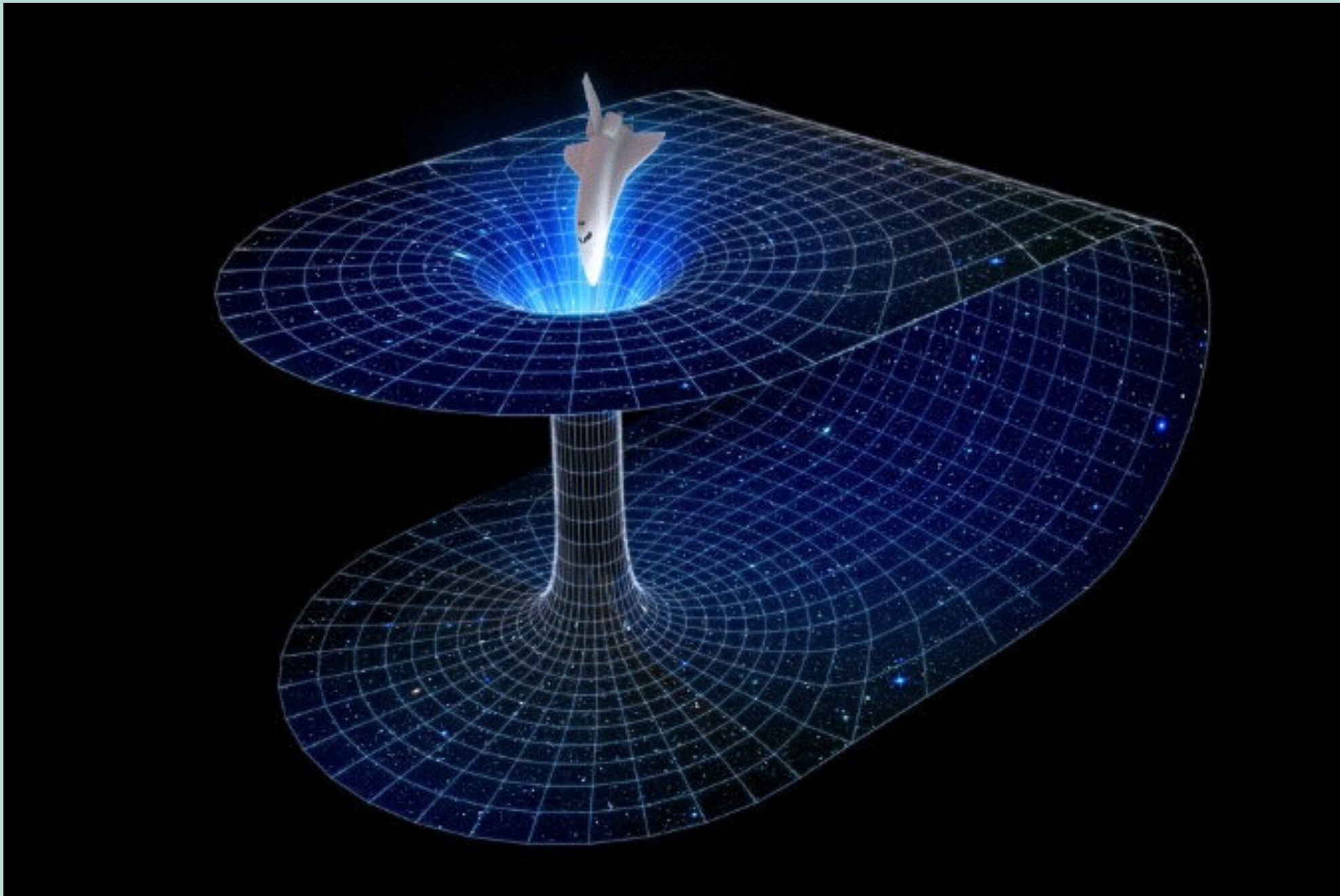
Blackholes

- This notion of “empty” space being a roiling sea of particles constantly appearing and disappearing was (and should have been) controversial in the early days of quantum mechanics but today we know this is a perfectly calculable phenomenon.
- The presence of a one-way surface in spacetime where once particles cross it, they can’t go back, implies that particle-antiparticle pairs created at the horizon might lead to one particle falling in and the other escaping the blackhole. These escaping particles are the source of Hawking radiation.



Wormholes

- Maybe an even stranger phenomenon is that of an “Einstein-Rosen Bridge” or more colloquially, a “Wormhole”
- Since matter distorts geometry, it is mathematically possible that distant regions of space could be connected by a narrow region of spacetime known as a wormhole
- No one has yet found a configuration of matter that would lead to a *traversable* wormhole, so the jury is out on intergalactic travel, but who knows! The theoretical prospects look slim though :/



Cosmology

- Another incredible outcome of Einstein's theory is that now we can start thinking about the dynamics of the *whole universe*
- While the universe looks incredibly complex at small scales ($\sim 0 - 10000000$ parsecs), the large scale distribution of matter is both **homogeneous** and **isotropic**



Friedmann, LeMaitre, Robertson, Walker

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

Cosmology

- This is an incredibly simple expression (as far as physics equations go) yet it contains a very powerful description of the entire history of the universe at large scales.
- If you run the ‘time’ parameter backwards you get a singular point at which the universe had no spatial extent, this point (in time!) we call “the big bang”

The Big Bang

- The term was coined pejoratively in 1949 by Fred Hoyle who preferred the 'steady state' theories of the universe
- One experimental flaw of this model was that it implied that today we should be swimming in a very cold bath of photons coming uniformly from all directions.

The Cosmic Microwave Background

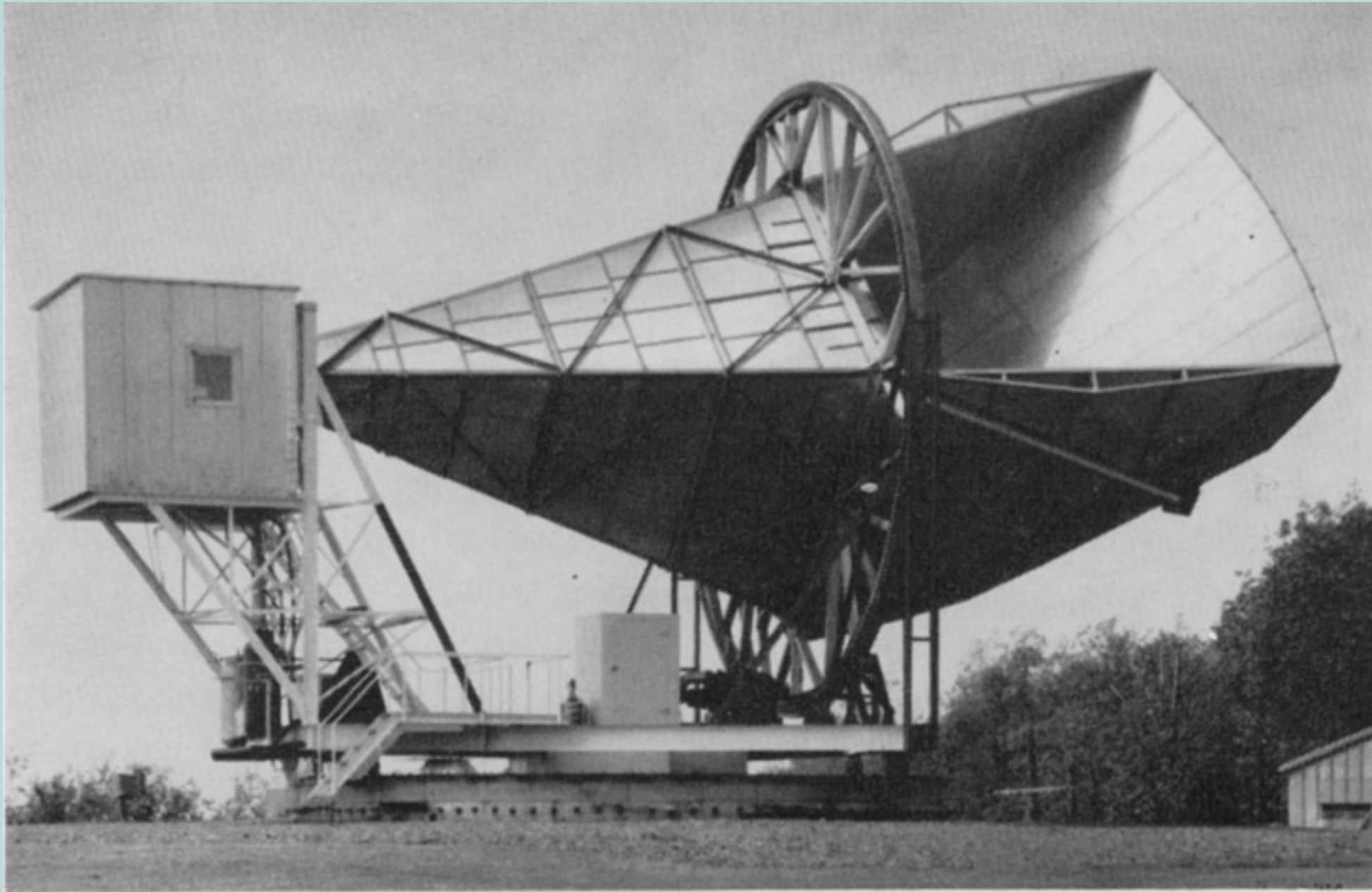
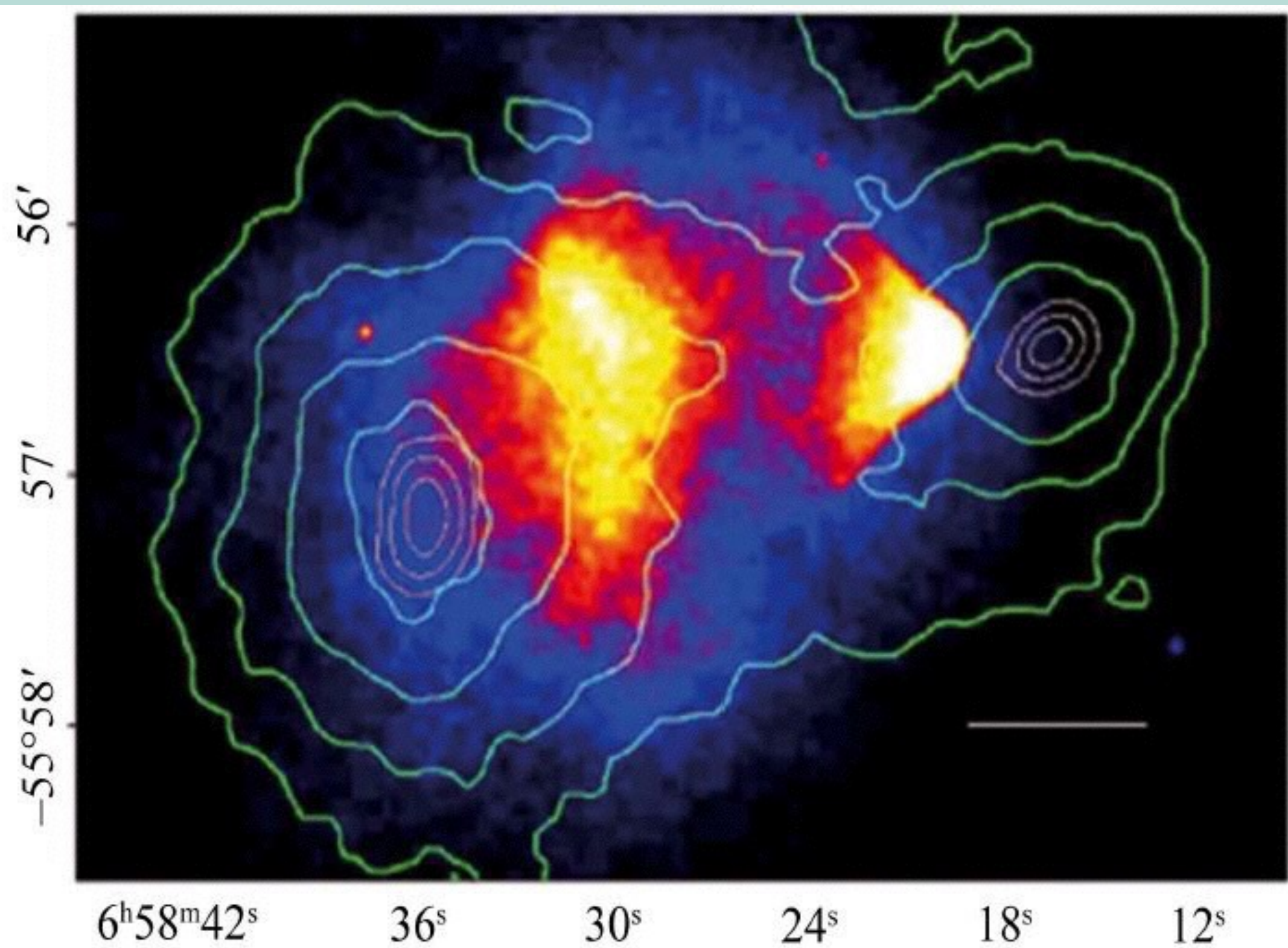


Fig. 2 — Horn-reflector antenna used in Project Echo experiment.

OPEN QUESTIONS

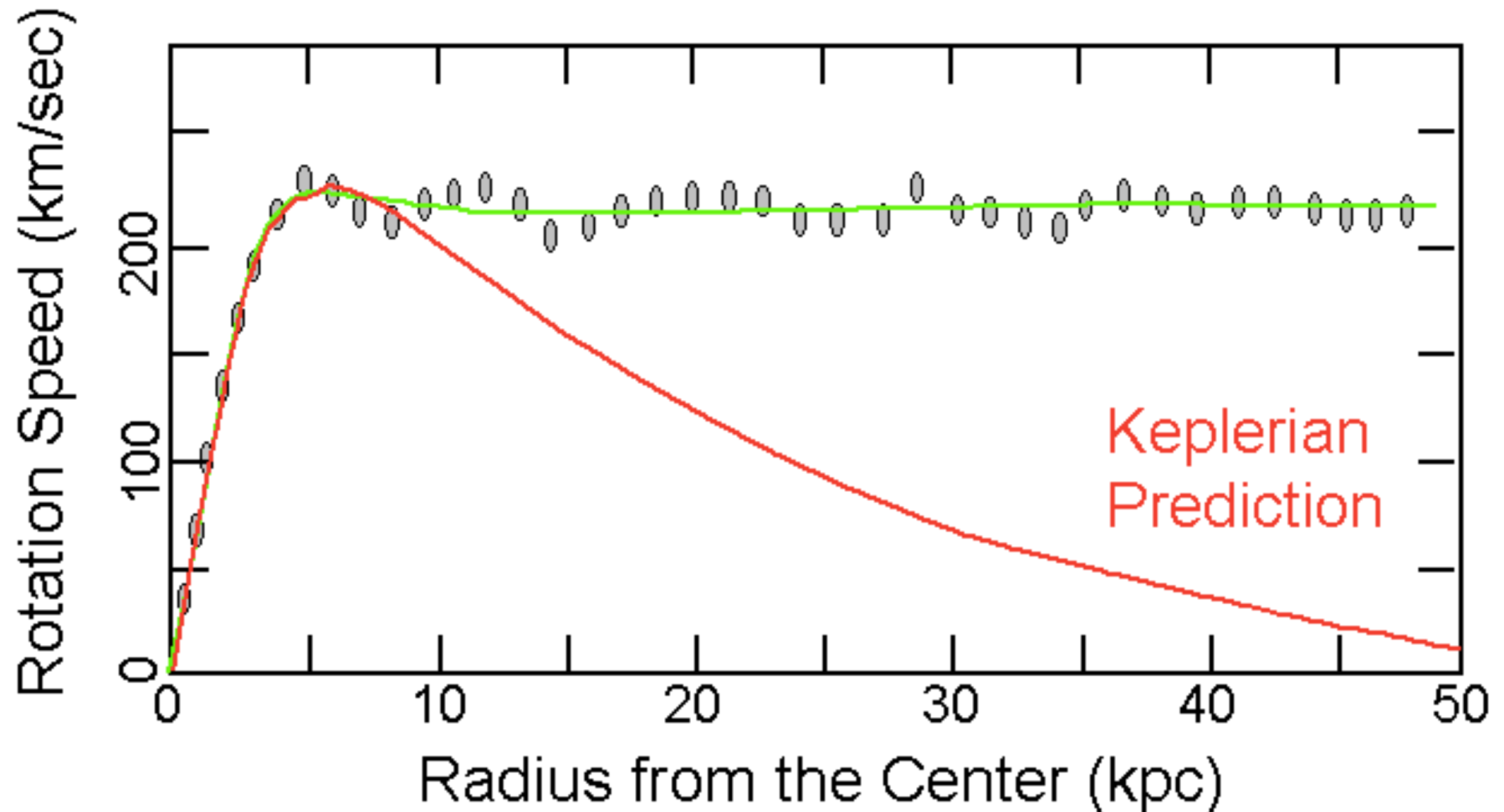
- There are many mysteries that remain in physics, but the cosmic mysteries of astronomy probably loom the largest. Solve any one of these mysteries and you can feel free to pick up your Nobel Prize at your earliest convenience.

“Dark Matter”



“Dark Matter”

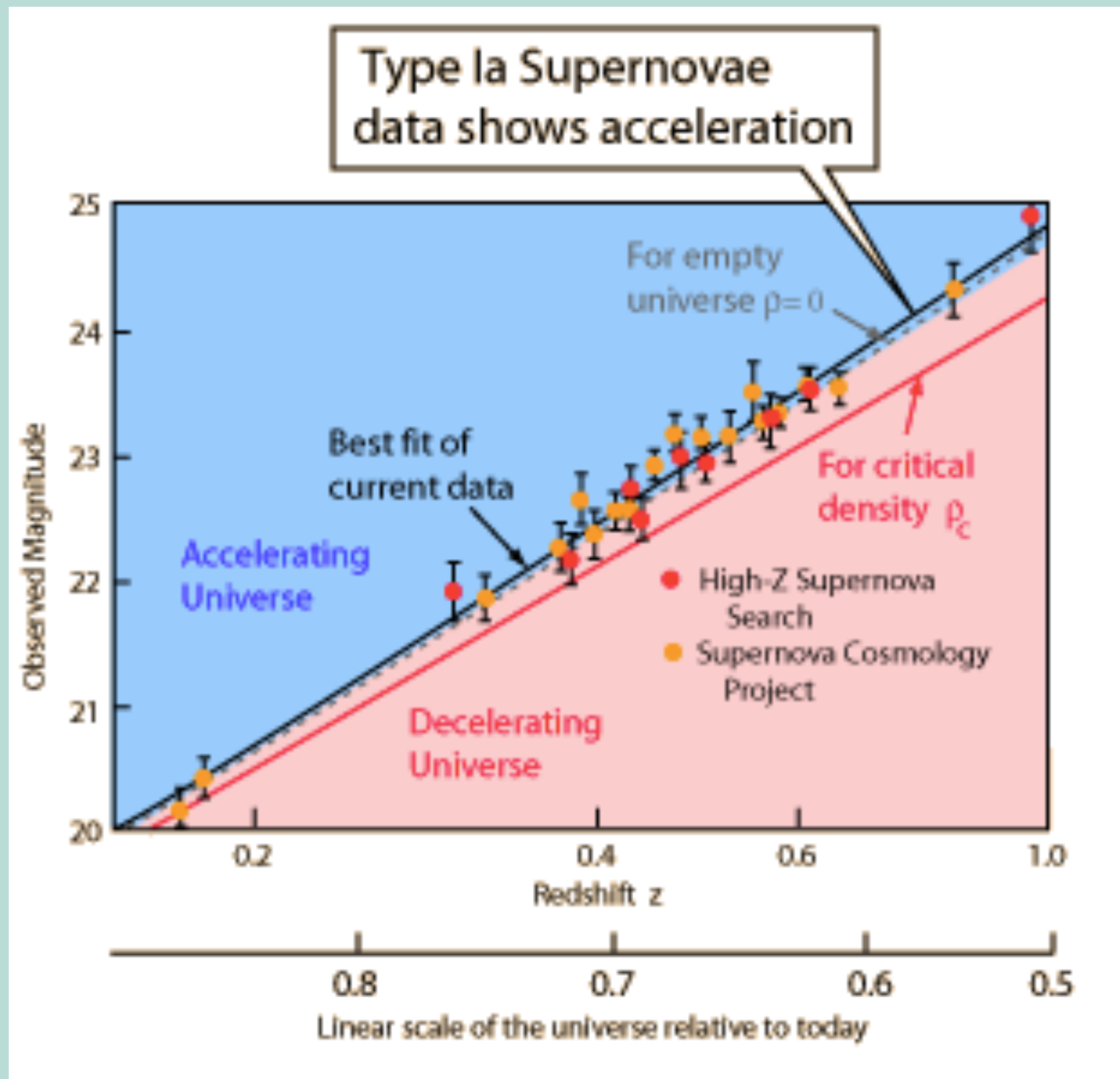
Observed vs. Predicted Keplerian



“Dark Matter”

- The age of the universe problem
- Structure formation
- Big Bang Nucleosynthesis
- Baryon Acoustic Oscillations

“Dark Energy”

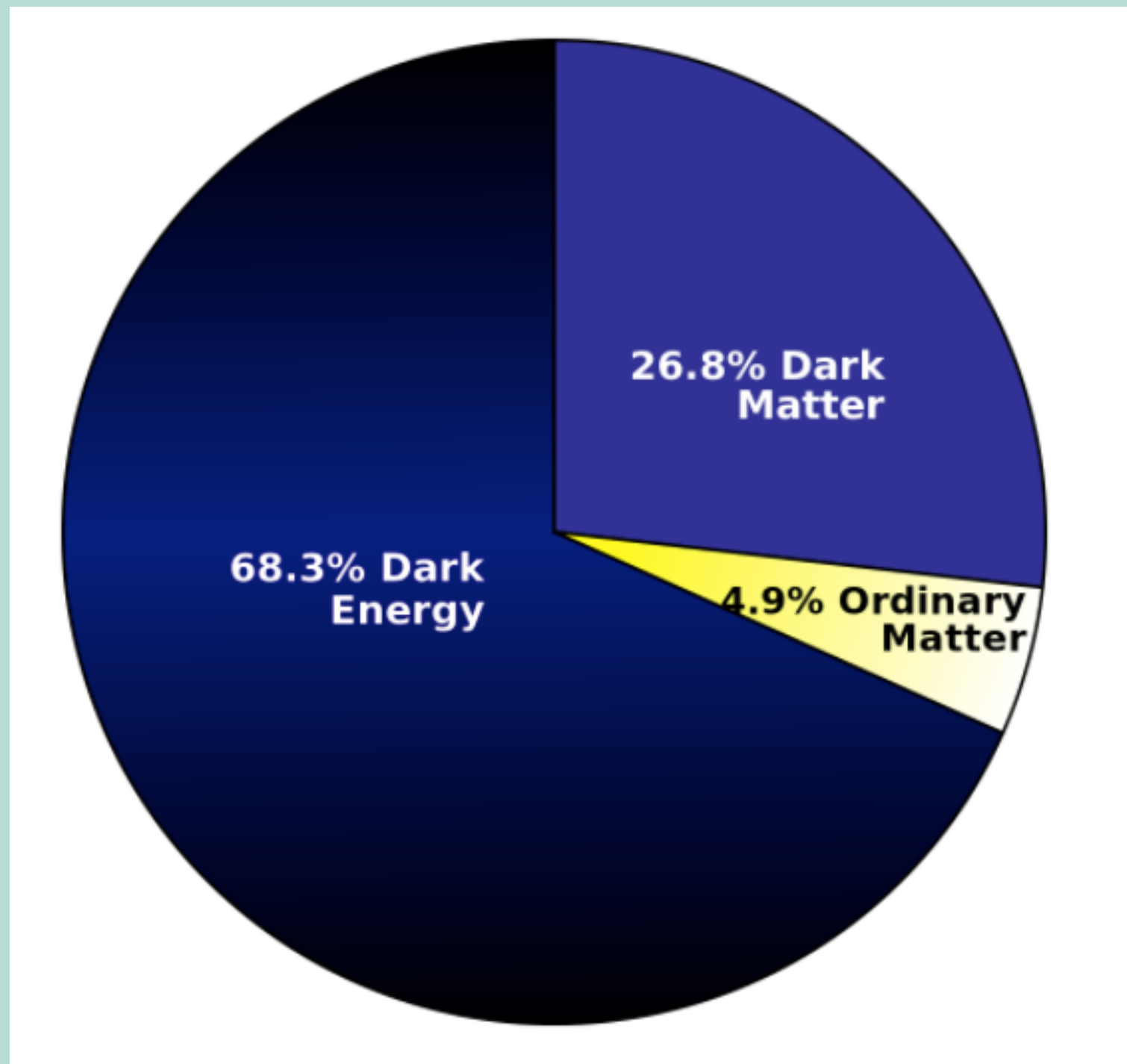


“Dark Energy”

- Dark energy is also known as the “cosmological constant problem” because the phenomena associated with dark energy can be accounted for by simply including a constant energy density throughout empty space. Quantum mechanics would seem to suggest such a thing, but how big should it be?

$$\frac{\Lambda}{M_{planck}^4} \sim 10^{-120}$$

Are these problems a big deal?



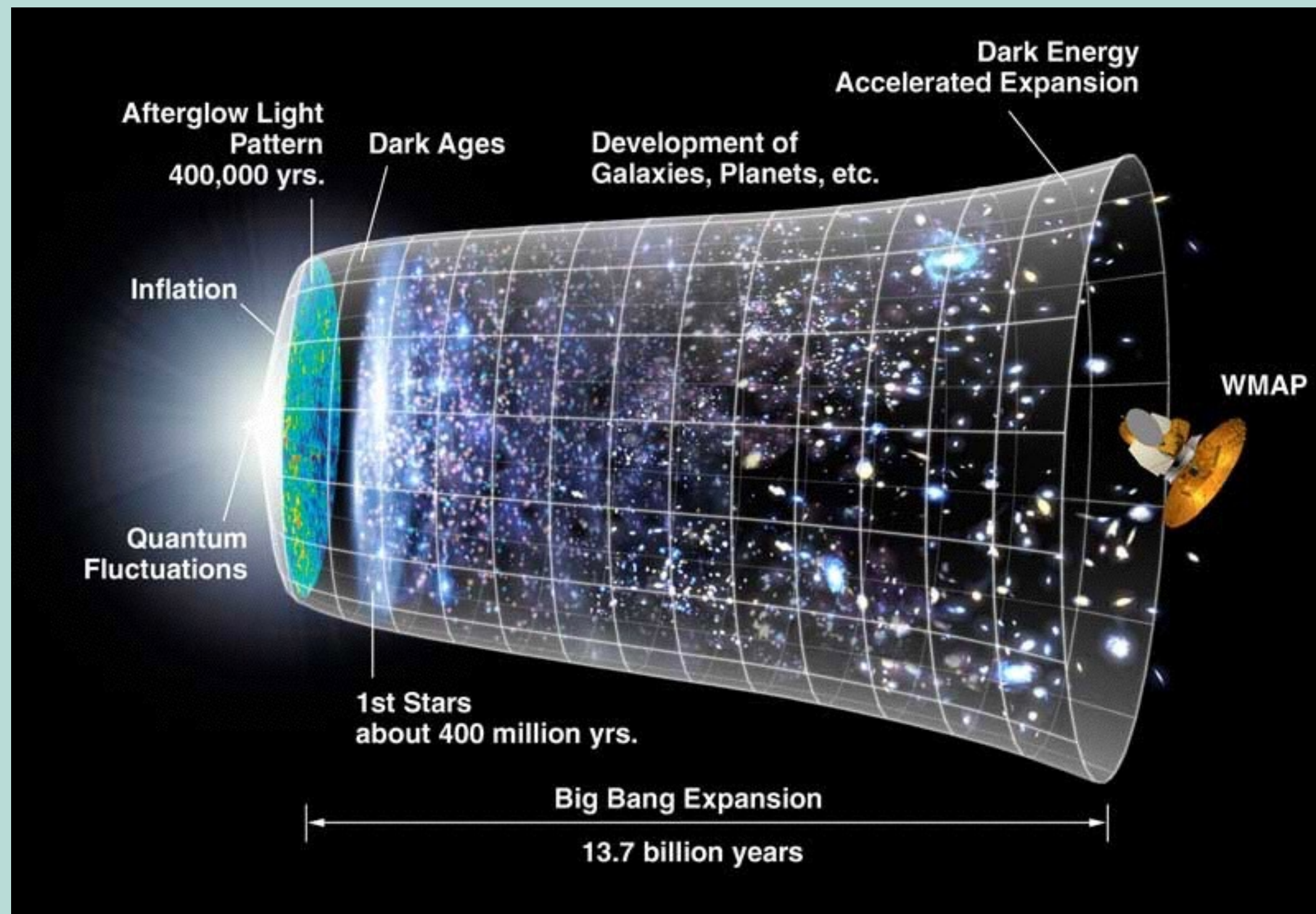
...yeah kinda

Inflation

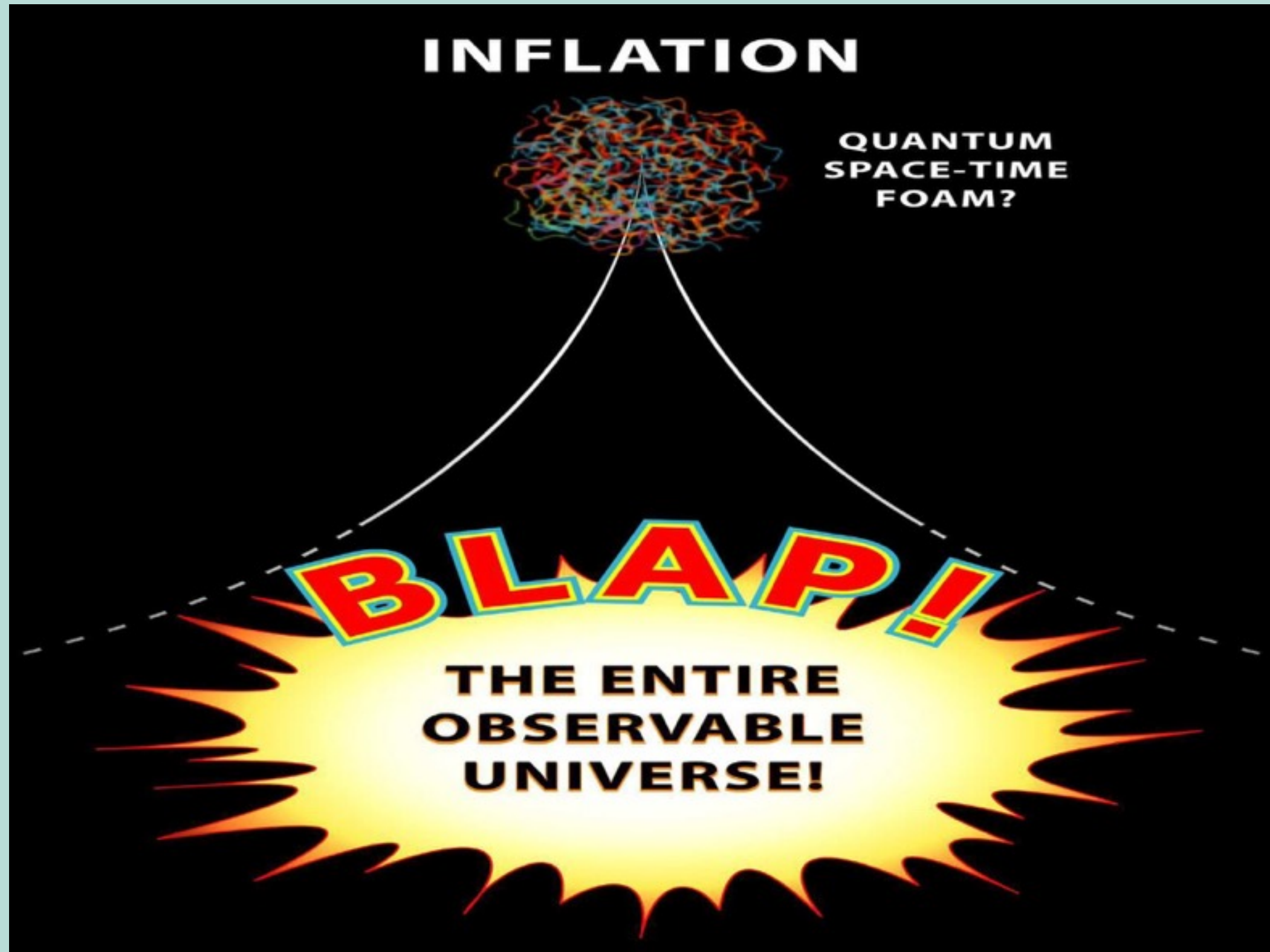
- Another interesting ‘problem’ is the curious fact that the universe is so symmetric and uniform.
- Religious sentiment might prefer a mysterious source of infinite energy perfectly tuned to create our beautiful earth in our highly symmetric universe.
- Scientists have learned over-time though, that when things seem highly tuned, there is typically a dynamical reason (see evolution)

Inflation

- A potential solution to this “problem” is that the theory of general relativity has in it a very simple mechanism for creating an accelerated expansion of space (remember the cosmological constant?)
- This exponential stretching of the universe would take any messy initial conditions and turn them into a perfectly smooth and uniform universe.



Science.



“Quantum Spacetime Foam”

- There are 2 theories of nature that apply to different sets of phenomena: Quantum Field Theory (small, fundamental particles) and General Relativity (macroscopic systems and high energy)
- We have no way to calculate the predictions made by these theories where they're both applicable. Luckily, the conditions where we would need such a theory haven't existed in the universe since the big bang, but if we want to understand the origin of inflation or the quantum nature of spacetime, we'll need to confront this deficiency.
- ‘String Theory’ is a paradigm which is teaching us a lot about what to expect from quantum gravity, but it's far from a tried and tested scientific theory.