Homework 3

Due February 16th

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1. Suppose that an observer at rest with respect to the fixed distant stars sees an isotropic distribution of stars. That is, in any solid angle $d\Omega$ he sees $dN = N(d\Omega/4\pi)$ stars, where N is the total number of stars he can see.

   Suppose now that another observer (whose rest frame is S') is moving at a relativistic velocity v in the e_x direction. What is the distribution of stars seen by this observer? Specifically, what is the distribution function $P(\theta', \phi')$ such that the number of stars seen by this observer in her solid angle $d\Omega'$ is $P(\theta', \phi')d\Omega'$? Check to see that $\int_{sphere} P(\theta', \phi')d\Omega' = N$, and check that $P(\theta', \phi') \to \frac{N}{4\pi}$ as $v \to 0$. Where will the observer see the stars "bunch up"?

2. Compton Scattering.

   (a) A photon of wavelength λ hits a stationary electron (mass m_e) and comes off with a wavelength λ' at an angle θ. Derive the expression

   $$\lambda' - \lambda = (h/m_e)(1 - \cos\theta) .$$

   (b) When a photon scatters off a charged particle which is moving with a speed very nearly that of light, the photon is said to have undergone an inverse Compton scattering. Consider an inverse Compton scattering in which a charged particle of rest mass m and total mass-energy (as seen in the lab frame) $E \gg m$, collides head-on with a photon of frequency ν ($h\nu \ll m$). What is the maximum energy the particle can transfer to the photon?

   (c) If space is filled with black-body radiation of temperature $3^\circ K$ and contains cosmic ray protons of energies up to 10^{20} eV, how much energy can a proton of energy 10^{20} eV transfer to a 3° K photon?

3. Relativistic Rocket

   (a) If a rocket has engines that give it a constant acceleration of 1 g (relative to its instantaneous inertial frame), and the rocket starts from rest near the earth, how far from the earth (as measured in the earth's frame) will the rocket be in 40 years as measured on the earth? How far after 40 years as measured in the rocket?

(b) Compute the proper time for the occupants of a rocket ship to travel 30,000 light years from the Earth to the center of the galaxy. Assume they maintain an acceleration of 1 g for half the trip and decelerate at 1 g for the remaining half.

(c) What fraction of the initial mass of the rocket can be payload in the previous part? Assume an ideal rocket that converts rest mass into radiation and ejects all of the radiation out of the back with 100% efficiency and perfect collimation.

4. A new force field $F^\mu(x^\nu)$ is discovered which induces a 4-acceleration $a^\mu \equiv du^\mu/d\tau = m^{-1}F^\mu(x^\nu)$ on a particle of mass m, at position x^ν. Show that this force is not consistent with special relativity.