

Homework 12

due May 10th

1. For what constant κ do the Euler-Lagrange equations of the following action:

$$S[g_{\mu\nu}] = \kappa \int d^4x \sqrt{-g} R$$

yield the canonical Einstein Field Equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

2. Show via the Nöther procedure that spacetime translation invariance implies the conservation of a rank two tensor and express this tensor in terms of the Lagrangian for a scalar field:

$$S[\phi] = \int d^4x \mathcal{L}(\phi(x), \partial\phi(x)) \quad (1)$$

Is the resulting tensor necessarily symmetric? Any guesses as to why?¹

3. Complete the following two calculations that were skipped in class:

- (a) Calculate G_{tt} for the static weakfield metric

$$ds^2 = -(1 + 2\phi)dt^2 + (1 - 2\phi)(dx^2 + dy^2 + dz^2)$$

- (b) Calculate the $G_{\hat{t}\hat{t}}$, $G_{\hat{r}\hat{r}}$, $G_{\hat{\phi}\hat{\phi}}$, and $G_{\hat{\theta}\hat{\theta}}$ for the FRW metric, with spatial curvature k

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$$

- (c) Calculate $G_{\hat{t}\hat{t}}$ and $G_{\hat{r}\hat{r}}$ for the ansatz we made in calculating the Schwarzschild metric

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2d\Omega$$

¹This problem is slightly outside the scope of the course, just do your best. I fear that the lack of dynamics requirement at NYU is handicapping students interested in graduate school. For a complete solution to this problem see chapter 2 of Peskin and Schröder's book on quantum field theory.