Homework 11

Due May 3rd

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1. Show that the homogeneous Maxwell's equations reduce to $dF = 0$. (The homogeneous ones are the ones without sources.)

2. Recall when introducing Killing vectors, we showed that a metric that was independent of coordinate x^1, had a killing vector

$$\xi^\alpha = (0, 1, 0, 0) \tag{1}$$

   (a) Show by explicit calculation that

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$

   This is known as Killing's Equation, and is actually the definition of a Killing vector. For the more mathematically inclined, this equation simply says that when the metric is Lie dragged in the direction ξ, nothing happens. That's the fancy differential geometry definition of a 'symmetry' of a manifold; expressed as

$$\pounds_\xi g = 0 \tag{2}$$

   (b) Show that for the 3D Cartesian plane, $ds^2 = dx^2 + dy^2 + dz^2$, the three Killing vectors $\partial/\partial x$, $\partial/\partial y$ and $\eta = -y(\partial/\partial x) + x(\partial/\partial y)$ all satisfy Killing's equation.

   (c) Show that the rotational symmetry about a point that is not the origin corresponds to a Killing vector that is a linear combination of these three.

3. Calculate the Riemann curvature of the metric

$$ds^2 = -(1 + gx)^2 dt^2 + dx^2 + dy^2 + dz^2 \tag{3}$$

   You should get 0. This means that a uniform gravitational field is not a true gravitational field (there are no tidal forces, so if everyone is in free-fall, there's no difference between this and empty space). Find the coordinate transformation that puts this metric into the usual Minkowski form.

4. Describe the geodesic deviation of two neighboring geodesics in circular orbit around a black hole in the Schwarzchild geometry.