Homework 1

Due February 1st

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1) Prove that for a triangle of area A on the surface of a sphere of radius R, the sum of the interior angles is:

$$\sum(\text{Int.Angles}) = \pi + \frac{A}{R^2} \tag{1}$$

2) Consider the following coordinate transformation from familiar rectangular coordinates (x, y), labeling points in the plane to a new set of coordinates (μ, ν):

$$x = \mu\nu, \qquad y = \frac{1}{2}\left(\mu^2 - \nu^2\right) \tag{2}$$

a. Sketch the curves of constant μ and constant ν in the xy plane.

b. Transform the line element $dS^2 = dx^2 + dy^2$ into (μ, ν) coordinates.

c. Do the curves of constant μ intersect at right angles?

d. Find the equation of a circle of radius r centered at the origin in terms of μ and ν.

e. Calculate the ratio of the circumference to the diameter of a circle using (μ, ν) coordinates. Do you get the correct answer?

3) The surface of the Earth is not a perfect sphere. The polar radius of the Earth, 6357 km, is slightly less than the mean equatorial radius, 6378 km. Suppose the surface of the Earth is modeled by an axisymmetric surface with line element $dS^2 = a^2(d\theta^2 + f^2(\theta)\, d\phi^2)$ where

$$f(\theta) = \sin\theta\,(1 + \epsilon\sin^2\theta) \tag{3}$$

for some small ϵ. What values of a and ϵ would best reproduce the known polar and equatorial radii?