Homework 4

Due February 22nd

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1. Accelerated Reference Frame

   (a) Transform the line element of special relativity from the usual (t, x, y, z) rectangular coordinates to new coordinates (t', x', y', z') related by

   $$\begin{aligned} t &= \left(\frac{c}{g} + \frac{x'}{c}\right) \sinh\left(\frac{gt'}{c}\right) \\ x &= c\left(\frac{c}{g} + \frac{x'}{c}\right) \cosh\left(\frac{gt'}{c}\right) - \frac{c^2}{g} \\ y &= y' \qquad z = z' \end{aligned}$$

   for a constant g with the dimensions of acceleration.

   (b) For $gt'/c \ll 1$, show that this corresponds to a transformation to a uniformly accelerated frame in Newtonian mechanics.

   (c) Show that a clock at rest in this frame at $x' = h$ runs fast compared to a clock at rest at $x' = 0$ by a factor $(1 + gh/c^2)$. How is this related to the equivalence principle idea?

2. The Earth is approximately 5 billion years old. How much younger are the rocks at the center of the Earth than at the surface? If equal abundances of uranium-238 were present at the formation of the Earth, how much more of it would be present at the center of the earth than the surface? Assume the density of the Earth is constant.

3. Assume a spaceship of length l is undergoing rigid motion such that the back of the spaceship has constant acceleration g. To an observer at rest, are the front and back of the spaceship accelerating at the same rate? If not (spoiler alert...) what is the difference in acceleration? How large could the spaceship theoretically be and still undergo this kind of rigid motion?