Homework 12

due May 10th

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1. For what constant κ do the Euler-Lagrange equations of the following action:
$$S[g_{\mu\nu}] = \int d^4x\sqrt{-g}\,(\kappa R + \mathcal{L}_M)$$
using the relation
$$-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_M\right)}{\delta g^{\mu\nu}} = T_{\mu\nu}$$
yield the canonical Einstein Field Equations,
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

2. Show via the Nöther procedure that spacetime translation invariance implies the conservation of a rank two tensor and express this tensor in terms of the Lagrangian for a scalar field:
$$S[\phi] = \int d^4x\mathcal{L}(\phi(x), \partial\phi(x)) \tag{1}$$
Is the resulting tensor necessarily symmetric? Any guesses as to why?[1]

3. Complete the following two calculations that were skipped in class:

   (a) Calculate G_{tt} for the static weakfield metric
$$ds^2 = -(1+2\phi)dt^2 + (1-2\phi)(dx^2 + dy^2 + dz^2)$$

   (b) Calculate the $G_{\hat{t}\hat{t}}$, $G_{\hat{r}\hat{r}}$, $G_{\hat{\phi}\hat{\phi}}$, and $G_{\hat{\theta}\hat{\theta}}$ for the FRW metric, with spatial curvature k
$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2 d\phi^2)\right)$$

   (c) Calculate $G_{\hat{t}\hat{t}}$ and $G_{\hat{r}\hat{r}}$ for the ansatz we made in calculating the Schwarzchild metric
$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega$$

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[1] This problem is slightly outside the scope of the course, just do your best. I fear that the lack of dynamics requirement at NYU is handicapping students interested in graduate school. For a complete solution to this problem see chapter 2 of Peskin and Schröder's book on quantum field theory.