Homework 10

Due April 26th

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1. Which of the following are correct tensor equations:
   (a) $a_{\alpha\beta} = b_{\beta\alpha}$
   (b) $a^{\gamma}{}_{\alpha\beta} = b_{\alpha\beta} c^{\alpha\gamma}$
   (c) $F_{\mu\nu} = F^{\mu\nu}$
   (d) $G_{\alpha\alpha} = H^{\beta}_{\beta}$
   (e) $g_{\alpha\beta} A^{\alpha\beta} = 1$
   (f) $\sum_{i=1}^{d} A_{ii} = 1$
   (g) $\det A = 1$
   (h) $A_{\alpha\beta} B^{\gamma\delta} C_{\gamma} = D_{\alpha} E^{\mu\delta} F_{\mu} G_{\beta}$
   (i) $T^{\mu\nu} = T^{\mu\lambda}$

2. Consider the vector located at $r = 4M$, $\theta = \pi/4$, $\phi = t = 1.2345$ in the geometry outside of a black hole with components in the usual coordinatization of Schwarzchild geometry, $\vec{v} = (v^t, v^r, v^\theta, v^\phi) = (1, 2, 3, 4)$. How long is $\vec{v}$? What are the components of $\tilde{v}$? Answer the same questions but now for the vector $\vec{u} = (1, 2, 3, 4)$ located at $r = 10M$, $\theta = \phi = t = 5\pi/6$.

3. Calculate the Laplacian in the geometry outside of a rotating black hole with mass M and angular momentum J in the usual Boyer-Lindquist coordinates that we used in class. Does this reduce to the Laplacian for Schwarzchild as $J \to 0$ and then for spherical coordinates for $M \to 0$?