ECE-210-A HW2

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This homework will review numerical integration and differentiation using vectorized operations, some plotting operations. Remember the style guidelines from the previous assignment. For each question, either save the result to a variable or print out the result to the screen (don't suppress the result).

1. Now it's your turn to try out numerical integration and differentiation.

   (a) Create two vectors of length 100 and 1000, each containing evenly spaced samples of the function $g(t) = \arctan t$ for $0 \leq t \leq 2\pi$.

   (b) Approximate the derivatives of the vectors using the difference quotient method (i.e., use `diff` to take the difference between adjacent verbs and divide elementwise by Δt). Call this estimate $\hat{g}'(t)$.

   (c) Evaluate the analytical derivative $g'(t)$ over the same intervals of t.

   (d) Say we define the error of the estimate of the derivative to be a normalized mean-square error:

   $$\epsilon(g', \hat{g}') = \frac{1}{N} \sum_t (g'(t) - \hat{g}'(t))^2$$

   where N is the number of samples. Calculate the error for both estimates for the derivative – which is smaller?

   (Note: truncate/pad vectors as necessary.)

   (e) Approximate the integrals of your original vectors using `cumtrapz` and `cumsum`. This will give you four approximations of the integral of g. Repeat the steps in parts (c) and (d): evaluate the analytical antiderivative at the same t points, and calculate the error estimates for each of the four estimates of the integral.

   (Note: The antiderivative is $t \arctan t - \frac{1}{2} \ln\left(1 + t^2\right) + C$.)

   (f) Plot the best estimate for the integral. Title your plot.

   (g) *(Optional)* Explore the plotting API: Give the horizontal and vertical axes a label. Turn the grid on/off. Change the axis ticks. Subplots! We will explore more plotting functions in class soon.

   (h) *(Optional)* Integrate using Simpson's rule and compare results.

2. Perform the following matrix operations (without `for` loops). Save each result to a separate variable (i.e., don't alter A after creating it).

   (a) Generate the matrix:

   $$A = \begin{bmatrix} 10^0 & 10^1 & \dots & 10^4 \\ 10^5 & 10^6 & \dots & 10^9 \\ \vdots & \vdots & \ddots & \vdots \\ 10^{45} & 10^{46} & \dots & 10^{49} \end{bmatrix} \in M_{10\times 5}(\mathbb{R})$$

   (Hint: use `logspace` and `reshape`.)

   (b) Flip the third row of A left to right.

   (c) Create a column vector of the geometric means of each row. (Recall that the geometric mean of $x_1, x_2, \dots, x_n$ is $\sqrt[n]{x_1 x_2 \dots x_n}$. The `prod` function will probably be helpful.)

   (d) Create the submatrix $B \in M_{3\times 3}(\mathbb{R})$ such that $b_{ij} = a_{(i+5)(j+1)}$.

   (e) Delete rows 5-10 of A.

3. Create a matrix C such that

   $$c_{ab} = \frac{\exp(j(a+bj)) - \exp(-j(a+bj))}{2j}$$

   where $-2\pi \le a, b \le 2\pi$ and a and b increment by 10^{-2}. Generate this matrix in the following ways, and time each method using `tic`/`toc`. Write a comment describing the behavior your observations.

   (a) Using `for` loops and no pre-allocation.

   (b) Using `for` loops and pre-allocation.

   (c) Repeat parts (a) and (b), but flip the order of your loops.

   (d) Using `meshgrid`.

   (e) Using broadcasting.
   (Note: for future assignments, you may use either meshgrid or broadcasting. For this assignment, I want you to try both.)

   (Note: make sure to properly clear variables to make sure you see the effects of pre-allocation.)

   (Note: try plotting the real/imaginary parts and magnitude of this function. If you haven't identified it already, what function is this? Do you get the same result if you apply that function to $a + bj$ instead of using the provided equation?)

4. *(Optional)* Try these problems using Python and numpy/matplotlib.

5. *(Optional)* How does vectorization work (in MATLAB or in general)? Do a little research and write a few sentences explaining in your own words how it achieves speedup over `for` loops.