ECE-210-A HW6

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This homework concerns digital filters and the Z-transform.

1. For this question, you will work with the discrete-time system described by the following transfer function:

$$H(z) = \frac{\frac{1}{2} + \frac{2}{3}z + \frac{3}{7}z^2}{3 + \frac{1}{3}z + \frac{1}{2}z^3}$$

   (a) Store the transfer function in coefficient vectors `b` and `a` for the numerator and denominator, respectively.

   Be **very** careful setting this up. See `help zplane` to see the format that `zplane` expects the polynomial functions to be in. (Hint: you may need to left-pad one of `a` or `b` with zero(s).)

   (b) Plot the poles and zeros of this system using a MATLAB function and the transfer function coefficients. List the ROCs (regions of convergence) and state whether each ROC is causal and/or stable.

   (c) Compute the poles and zeros of this system using a MATLAB function. Do these poles and zeros match what you saw in the previous question? (*Optional.* Calculate the poles and zeros by hand and verify once again that these are correct.)

   (d) Compute the first 32 samples of the impulse response of this transfer function using a MATLAB function and save this to a variable `h`.

   (e) Generate a 64-length vector $x[n] = \left(-\frac{5}{6}\right)^n$, $0 \leq n < 64$. This is an arbitrary discrete-time signal. Show the result of applying the filter with transfer function $H(z)$ to this signal by:

      i. Using `filter`.

      ii. Convolution in the time-domain with the impulse response.

      iii. *(Optional.)* Multiplication in the z-domain. Show your work. (Note that you can use `impz` to take inverse Z-transform of a rational function.)

   Truncate the filtered signals to the first 32 samples before plotting them. Do your output signals match? (Note that `conv` will produce a vector of different length than x – this is inconsequential and you can truncate it to the correct length.)

2. We're going to be exploring a filter by placing its poles and zeros in specific locations. Last question you converted a system description from its transfer function (tf form) to its zeros and poles (zp(k) form). Now we will do the opposite.

   (a) Compute numerator and denominator vectors for a transfer function with $k = 0.1$ and these poles and zeros:

   Poles: $0.7375 \exp(\pm 0.7601j)$, $0.9589 \exp(\pm 1.2360j)$

   Zeros: $\exp(\pm 2.056j)$, $\exp(\pm 1.4261j)$

   (b) Create a pole-zero plot using the numerator and denominator vectors and a MATLAB function.

   (c) Use `freqz` as shown in lecture to find the frequency response of this filter. This will return two vectors: H (complex frequency response vector) and ω (digital frequencies corresponding to H).

   Do not use the default plot created by `freqz` – you will be manually creating the plot. (The default plot is shown if you do not use the return value of `freqz`, so make sure to save the values `[H, w]` for the next question.)

   (d) H is complex: it has a magnitude and a phase. Plot these in two subplots similar to what was shown in lecture. Make sure to:

   - Convert the magnitude gain (magnitude of H) in decibels and label accordingly.
   - Plot the delay (phase of H) in degrees. Use `unwrap` to make it look nicer. (Note that for this particular filter, there are jagged edges even with `unwrap` because it is a sharp filter. However, without unwrap you get some additional unwanted jagged edges.)
   - Show units in axis labels. (Remember that the frequency vector w is in digital units of radians, not radians per second.)
   - Remember that the frequency vector w goes only from 0 to π. Properly scale and limit and label the x-axis using `xlim`, `xticks`, and `xticklabels`.

   (e) Using the information from the frequency response, what kind of filter is this (high/low/band-pass)? Can you see this from the pole-zero plot as well?

   (f) *(Optional.)* Choose a sampling frequency f_s and apply the filter to sinusoids at $f \in \{0.1f_s/2, 0.5f_s/2, 0.9f_s/2\}$ and see that this indeed attenuates frequencies as you expect.