In this homework, you will explore the topics of the past two lectures: state space and the symbolic toolbox.

1. Here you will check the functionality of ***ss2tf*** against the symbolic toolbox through this continuous state space system:

$$A = \begin{bmatrix} -4 & 4 \\ -2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

   - First check (show me) to make sure the real part of all of the eigenvalues are negative, thus making the system stable.
   - Next check the norm of e^{At} over a large enough time t and show in a plot that it decays to 0. Caution: the regular ***exp*** function is not appropriate for matrices, find a function which is.
   - Now use ***ss2tf*** and return the transfer function numerator and denominator.
   - Check your work by using the formula $H(s) = C(sI - A)^{-1}B + D$ with a symbolic variable s.
   - Extract the numerator and denominator of the symbolic rational function using a single function. Then convert these to polynomial vectors using another single function.

2. Consider the system of differential equations below:

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x - y$$

Solve the system using the symbolic toolbox's ***dsolve*** function using four *different* initial conditions (so you should have four sets of solutions):

$$x(0) = 1, y(0) = 1$$

$$x(0) = -1, y(0) = 1$$
$$x(0) = 1, y(0) = -1$$
$$x(0) = -1, y(0) = -1$$

Then plot all four curves on the same plot from $t = 0$ to $t = 100$ using ***fplot*** (use a legend). It should be a spiral.

3. (a) Create a function which takes in a matrix A and plots (use ***semilogy***) $||e^{At}||_F$ and $e^{\alpha t}$ on the same plot for $0 \leq t \leq 100$ (you can sample t at integer values) where $\alpha = max(Re(\Lambda(A)))$ (maximum real part of the eigenvalues of A) and $|| \cdot ||_F$ is the Frobenius norm. In your main script call this function on a random 10x10 matrix with entries sampled from the standard normal distribution minus 2 times the identity. Check out what happens when α is positive or negative and check out what happens when the eigenvalue which α corresponds to is complex and write what happens in a comment.

   (b) Create a function which takes in a matrix A and plots (use ***semilogy***) $||A^n||_F$ and ρ^n on the same plot for $0 \leq n \leq 100$ (you can sample n at integer values) where $\rho = max(|\Lambda(A)|)$ (maximum magnitude of the eigenvalues of A) and $|| \cdot ||_F$ is the Frobenius norm. In your main script call this function on a random 10x10 matrix with entries sampled from a normal distribution with mean 0, variance 0.1. Check out what happens when ρ is less than or greater than 1 and check out what happens when the eigenvalue which ρ corresponds to is complex and write what happens in a comment.