ECE302 Project 2: Estimation Techniques

Contents

Scenario 1

An implementation of a Bayes MMSE and Linear MMSE estimators of the random variable Y from the random variable X where X = Y + W. Here Y ˜ U(-1,1) and W ˜ U(-2,2). Note that in this project, N represents the number of samples taken from the respective distributions.

```
clear;
clc;
close all;

N = 10000;

Y1 = random('Uniform',-1,1,N,1);
W = random('Uniform',-2,2,N,1);
X1 = Y1 + W;
```

Bayes MMSE Estimator

Here we take $\hat{y} = E[Y|X = x]$. Calculating this condition expectation leads to:

```
latex
```

\usepackage{mathworks} \[\hat{y} = \begin{cases} 0 & x\leq 0 \\ \frac{100-x}{100} & 0\leq x\leq 100 \\ 0 & 100\leq x \end{cases} \]

```
/latex
```

```
Y1est = zeros(N,1);
for i = 1:N
    if X1(i) < -1
        Y1est(i) = .5 + .5*X1(i);
    elseif X1(i) < 1
        Y1est(i) = 0;
    else
        Y1est(i) = -.5 + .5*X1(i);
    end
end
```

```
empmmse1 = mean((Y1 - Y1est).^2);

actmmse1 = .25;

Y1est = (1/5)*X1;

emplmmse1 = mean((Y1 - Y1est).^2);

actlmmse1 = 4/15;

table([empmmse1; emplmmsel],[actmmse1; actlmmse1],'VariableNames', ...
    ["Empirical MSE","Theorhetical MSE"],'RowNames', ...
    ["MMSE","LMMSE"])
```

```
Unrecognized function or variable 'emplmmsel'.

Error in proj2 (line 55)
table([empmmse1; emplmmsel],[actmmse1; actlmmse1],'VariableNames', ...
```

```
m = 5;
n = 5;

Y2est = zeros(N,m);
emplmmse2 = zeros(1,m);
actlmmse2 = zeros(1,m);
leg = strings(1,m);

for j = 1:m
    muY2 = 1;
    varY2 = j;
    varR = j;

    Y2 = random('Normal',muY2,sqrt(varY2),N,1);
    R = random('Normal',muR,sqrt(varR),N,n);
    X2 = Y2 + R;

    CXX = varY2*ones(n) + diag(varR*ones(1,n));
    CXY = varY2*ones(n,1);

    a = CXX\CXY;

    a0 = muY2 - dot(a,muY2*ones(n,1));

    Y2est(:,j) = a0 + dot(repmat(a',N,1),X2,2);
```

```
    emplmmse2(j) = mean((Y2 - Y2est(:,j)).^2);

    actlmmse2(j) = varY2 - CXY'*a;

    leg(j) = "\sigma_Y^2 = " + j + ", \sigma_R^2 = " + j;
end

sz = 25;

figure;
for j = 1:m
    scatter(emplmmse2(j),actlmmse2(j),sz,j,'filled','DisplayName',leg(j));
    hold on;
end
xlabel("Empirical LMMSE");
ylabel("Theorhetical LMMSE");
legend('location','northwest');
grid on;
title("LMMSE with " + n + " observations");
```