MATH 166 Statistics - Fall 2024
Tufts University, Department of Mathematics
Instructor: James M. Murphy
Practice Exam 1

Instructions: This exam has four questions and is out of a total of 100 points. Each question is worth 25 points. No graphing calculators, books, or notes are allowed. Be sure to show all work for all problems. No credit will be given for answers without work shown. If you do not have enough room in the space provided you may use additional paper. Be sure to clearly label each problem and attach them to the exam. You have 120 minutes. Good luck! :-)

Your Printed Name: ______________________________

Problem	Score
1	
2	
3	
Total	

Academic Honesty Certification:

I certify that I have taken this exam without the aid of unauthorized people or objects.

Signature: ______________________________ Date: ____________________

Question 1

Let $X_1, X_2, \ldots, X_n$ be a sequence of i.i.d. Gaussians with parameters μ, σ^2. Let $\bar{X}_n = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} X_i$ be their empirical average.

(a) Compute the expected value and variance of $\bar{X}_n$.

(b) Use your result in (a) to show $\bar{X}_n$ is a consistent estimator for μ. ***Hint:*** *You may use Chebyshev's inequality, which tells us that for any random variable X and any $\epsilon > 0$, $\mathbb{P}(|X - \mathbb{E}(X)| > \epsilon) \leq \frac{1}{\epsilon^2}\mathrm{Var}(X)$.*

Question 2

Let X be a uniform random variable on $(0, \theta)$ for some unknown parameter $\theta > 0$. Let $x_1, \ldots, x_n$ be an i.i.d. sample from X. Recall that the maximum likelihood estimate for θ is $\hat{\theta} = \max_{i=1,\ldots,n} x_i$.

(a) What does it mean for a general estimator $\hat{\theta}$ of θ to be unbiased?

(b) Argue that with $\hat{\theta}$ as above, $\mathbb{P}(\hat{\theta} < \theta) = 1$.

(c) Argue that $\hat{\theta}$ as above is biased. *Hint: use (b).*

Question 3

Let X be a Gaussian random variable with unknown parameter $\mu > 0$ and known parameter $\sigma^2 = 1$; recall this means X has density

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2}\right).$$

Given $x_1, \ldots, x_n$, estimate μ using maximum likelihood.