

**Homework 1**  
MATH 166 - Fall 2024  
Tufts University, Department of Mathematics  
Due: September 12, 2024

1. BOOK QUESTIONS

Wasserman: Chapter 1: #4, #8; Chapter 2: #14; Chapter 3: #7; Chapter 4: #3

2. SUPPLEMENTAL QUESTION (EXPERIMENTALLY VERIFYING THE LAW OF LARGE NUMBERS)

The following may be performed in any scripting environment you prefer (MATLAB, R, Python, Julia,...)

- (a) For  $n = 10, 20, 30, \dots, 10000$ , sample  $n$  i.i.d. samples from  $\mathcal{N}(0, 1)$  i.e. the random variable  $X$  with density  $f_X(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)$ . Let  $\bar{x}_n$  be the corresponding sample average. Plot  $\bar{x}_n$  as a function of  $n$ . Describe the behavior as  $n$  increases. What does the Law of Large Numbers suggest will happen as  $n \rightarrow \infty$ ?
- (b) For  $n = 10, 20, 30, \dots, 10000$ , sample  $n$  i.i.d. samples from the Cauchy distribution, i.e. the random variable  $X$  with density  $f_X(x) = \frac{1}{\pi(1+x^2)}$ . Let  $\bar{x}_n$  be the corresponding sample average. Plot  $\bar{x}_n$  as a function of  $n$ . Describe the behavior as  $n$  increases. What does the Law of Large Numbers suggest will happen as  $n \rightarrow \infty$ ?