

**Homework 5**  
MATH 166 - Fall 2024  
Tufts University, Department of Mathematics  
Instructor: James M. Murphy  
Due: October 10, 2024

1. BOOK QUESTIONS

Wasserman: Chapter 9: #3 (in part (c), ignore the part about the parametric bootstrap. Hint: look at example 9.29), #5 (ignore the part about Fischer information)

SUPPLEMENTAL QUESTION 1 (ASYMPTOTIC UNBIASEDNESS)

We say an estimator  $\hat{\theta}_n$  is *asymptotically unbiased* for  $\theta$  if  $\lim_{n \rightarrow \infty} \mathbb{E}(\hat{\theta}_n) = \theta$ , where as usual the expectation is taken over the random sample. Let  $x_1, \dots, x_n$  be an i.i.d. sample from  $\text{Unif}(0, \theta)$ . Recall that the MLE estimator for  $\theta$  is  $\hat{\theta}_n = \max_{1 \leq i \leq n} x_i$ .

- (a) Show  $\hat{\theta}_n$  is biased for every  $n$ .
- (b) Show  $\hat{\theta}_n$  is asymptotically unbiased.

SUPPLEMENTAL QUESTION 2 (PROPERTIES OF KL DIVERGENCES)

The Kullback-Leibler distance is not a metric in the traditional sense. We will investigate some of its properties below. Let  $f, g$  be any probability density functions.

- (a) Show  $D_{KL}(f, f) = 0$ .
- (b) Show  $D_{KL}(f, g) \geq 0$  (Hint:  $\log\left(\frac{1}{y}\right) \geq 1 - y$  for all  $y$ ).