Homework 4
MATH 166 - Fall 2024
Tufts University, Department of Mathematics
Instructor: James M. Murphy
Due: October 3, 2024

1. Book Questions

Wasserman: Chapter 9: #2, #4, #6 (a)-(c);

2. Supplemental Question (Bias and MLE)

Let $x_1, x_2, \ldots, x_n$ be i.i.d. samples from a random variable X that is Gaussian with parameters $\mu \in \mathbb{R}$ and $\sigma^2 > 0$.

(a) Show the MLE estimators for μ, σ^2 are

$$\hat{\mu}_n = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \hat{\sigma}_n^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2.$$

(b) Show that the MLE estimator $\hat{\sigma}_n^2$ for σ^2 is s.t. $\mathbb{E}(\hat{\sigma}_n^2) = \frac{n-1}{n} \cdot \sigma^2$, and is thus biased.

(c) Verify (b) empirically as follows. Let $n = 3, 4, ..., 100$. For each n value, generate 100 i.i.d. samples of size n from $\mathcal{N}(0, 1)$. For each sample, compute the MLE estimate for σ^2; then, average across the 100 trials; this average may be thought of as an estimate for $\mathbb{E}(\hat{\sigma}_n^2)$. Plot your average MLE as a function of n and describe the behavior.