Internet Appendix

Supplemental Materials for Financial Constraints, Sectoral Heterogeneity, and the Cyclicality of Investment

1 Appendix: Data Description

1.1 Quarterly Financial Report

The main source of aggregate data is the Quarterly Financial Report for Manufacturing Corporations (QFR). This survey dates back to World War II, when it was administered by the Office of Price Administration. The Census Bureau has been responsible for administering the survey since 1982. These data series are used as input to macroeconomic aggregates such as corporate profits. The QFR sample, which includes approximately 10,000 firms in a given quarter, is chosen based on asset sizes reported in corporate tax returns; any firm with more than \$250,000 in domestic assets is eligible for inclusion, and any firm with more than \$250 million is included in the sample with certainty. Firms who reside between these thresholds are chosen randomly with the goal of obtaining a representative sample and are included for 8 consecutive quarters with one-eighth of the sample replaced each quarter.

Historical data dating back to 1947 are available for download from the Census Bureau's website.[1] At the time of the first draft of this paper in February 2019, publicly available data from before 1987 were only be available in physical publications or microfilm. Using these physical copies, I digitized the data going back to 1966Q1. This process consisted of mostly manual entry and occasional use of optical character recognition (OCR) software when available. To ensure that the data series were digitized correctly, I have checked that aggregating the component series by either size or sector add up to the correct total in each quarter.

Each physical publication includes observations for the current quarter as well as the four preceding quarters. With few exceptions most of the data series were digitized from

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[1] https://www.census.gov/econ/qfr/

the publications in Q1 of each year. Using these five level observations, I calculated the four implied quarterly growth rates, giving me a series of growth rates. By using growth rates calculated within each release, I avoid problems from comparing levels before and after methodological changes (including changes in accounting procedures in 1973 and industry reclassifications in 1984 and 2001). I then applied these growth rates to the levels of the most recent releases, effectively taking the original growth paths and shifting them to the most up-to-date level.

Beyond adjusting for revisions, I have to account for the fact that the data are not seasonally adjusted and in nominal terms. I use calendar quarter fixed effects for each regression in my default specification to address these seasonality concerns, though the results are robust to using the Census Bureau's X-13ARIMA-SEATS seasonal adjustment process or a four-quarter moving average. I deflate the stock of net property, plant, and equipment using the nonresidential fixed investment price index. Sales for each sector are deflated using the manufacturing output price deflator for that sector after linearly interpolating it to a quarterly frequency. All other variables are deflated using the GDP price index.

The respondents are aggregated by sector as well as asset size. The data consist of eight nominal asset "buckets": under $5 million, $5-10 million, $10-25 million, $25-50 million, $50-100 million, $100-250 million, $250-1,000 million, and $1+ billion. One issue with using the size data is that the cutoffs are in nominal values and fixed over time; a firm with $50 million in assets in 1967 is much larger relative to the size of the total manufacturing sector than a firm with $50 million assets in 2007. One way to address this is to combine many of the smaller bins into one "small" classification. For my baseline specification, I follow Crouzet (2017) in classifying all of the firms with less than $1 billion in nominal assets as being "small". An alternative approach uses percentiles of sales. This is the approach used in Gertler and Gilchrist (1994) (who use a 30% threshold) and Kudlyak and Sánchez (2017) (who use 25%). My results are robust to calcuating the size cutoffs in this way.

Industries are classified by the Census Bureau based on sources of revenue. As part of its submission, each company in the survey reports a breakdown of gross receipts by source industry. To be in the scope of the QFR manufacturing sample, a firm must have manufacturing as its largest source of gross receipts. Once a corporation is assigned to the manufacturing sector, it is categorized into a subsector based on its largest share of *manufacturing* receipts. For example, if a firm has 40% of its revenue from manufacturing and 30% each from mining and retail trade, then the firm would be classified in the manufacturing sector. If 60% of the firm's manufacturing activity was conducted in the machinery subsector and 40% in the chemicals subsector, then the activities of the entire corporation would be assigned to the machinery subsector. These classifications are reviewed periodically and

changed as needed for as long as the corporation remains in the sample.

To provide further evidence that the QFR data are in line with other measures of the capital stock, I can compare them to fixed asset data from the Bureau of Economic Analysis (BEA). These data provide end-of-year estimates of the value of total fixed assets for both the durable and nondurable manufacturing sectors. Figure 1 shows the year-over-year changes in the BEA measure compared to the Q4/Q4 changes in the QFR data and suggests that the two data series are capturing the same fundamental investment behavior. The correlations between the BEA and QFR measures are high for the total series (0.87) as well as both the durable (0.83) and nondurable (0.81) subseries, suggesting that the QFR data can be appropriately described as a higher-frequency and more detailed version of the BEA fixed asset data.
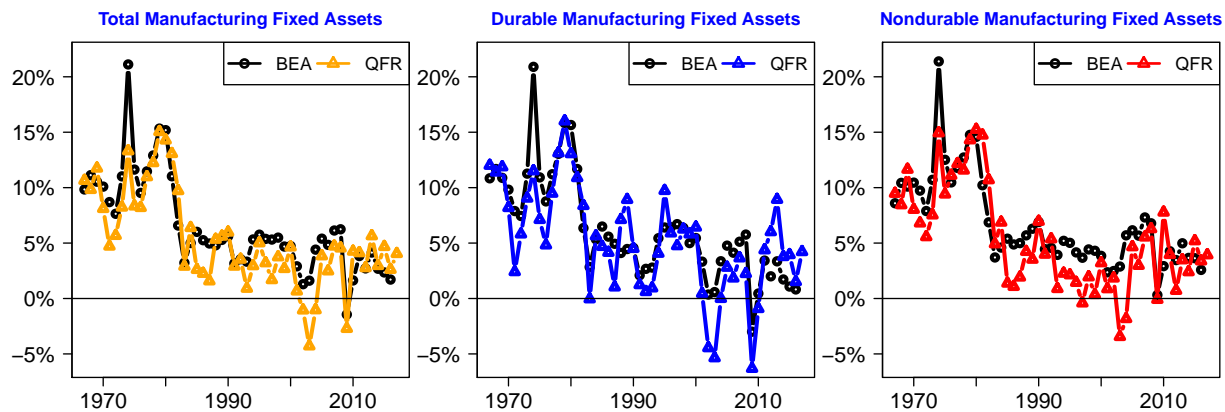

Figure 1: Y/Y % Changes in BEA and QFR Fixed Asset Measures

Note: This figure compares the yearly percent changes in the BEA and QFR measures of the nominal fixed asset stock for the manufacturing sector. The QFR numbers are shown as the year-over-year change in the fourth quarter of each year for comparison to the BEA data (which are at an annual frequency and recorded at year-end).

1.2 Building Permit Data

This section describes the building permit data used in the paper. Building permits are required when undertaking new construction, and this information is publicly available through local municipalities. Dodge Analytics[2] collects this information, which includes the type of structure and a cost estimate used for tax purposes.

Obtaining accurate cost information is important for local permit issuing authorities because more expensive construction projects are assessed greater permit fees. Section 108.3 of the 2018 International Existing Building Code states:

"The applicant for a permit shall provide an estimated permit value at time of application. Permit valuations shall include total value of work including materials and labor for which the permit is being issued, such as electrical, gas, mechanical, plumbing equipment, and permanent systems. If, in the opinion of the code official, the valuation is underestimated on the application, the permit shall be denied unless the applicant can show detailed estimates to meet the approval of the code official. Final building permit valuation shall be set by the code official."[3]

These are used as guidelines by local municipalities and form the foundation of permit procedures in most cases. They are generally taken at face value by the issuing agencies. In some cases jurisdictions will also include their own terms and requirements. Rather than relying on contractor estimates, many municipalities establish a fixed formula determining the cost per square foot based on the type of construction.[4]

In general, contractors have incentives to underestimate how much projects will cost given that these estimates for the basis for permit fees and certain types of taxes. Some municipalities will require a contractor to submit a signed affidavit showing the final construction costs for tax purposes, and Dodge Analytics will often follow up with contractors to obtain final valuations as part of its data collection process, but it is likely that many of the permits included in their data are ultimately based on the initial estimates provided by contractors before work has started. In practice these institutional features can certainly lead to variation in the valuations of similar projects across municipalities, but they are likely to wash out when aggregating up to the national level and comparing these totals over time.

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[2]https://www.construction.com/

[3]https://codes.iccsafe.org/content/IEBC2018/CHAPTER-1-SCOPE-AND-ADMINISTRATION?site_type=public

[4]Boulder, Colorado is an example of such a county. Their valuation table can be found here: https://bouldercolorado.gov/links/fetch/23187

1.3 Compustat Data

All of the variable definitions are standard and follow the literature closely, especially Jeenas (2019) and Ottonello and Winberry (2020). I use the nonresidential fixed investment price index to deflate the capital stock and the GDP price index to deflate all other variables. I use data starting in 1985 to avoid changes with sampling composition before that. In line with my analysis of aggregate data, I only consider monetary shocks that occur up to 2004.

- **Manufacturing**: My main analysis focuses on the manufacturing sector. I define a firm to be in the manufacturing sector if it is classified as being in manufacturing according to either the SIC (codes starting with 20-39) or NAICS (codes starting with 31-33). These can be classified into durable or nondurable producers according to the following sectors:

	SIC	NAICS
Durable	24-25, 32-40	33, 321, 327
Nondurable	20-23, 26-31	31, 322-326

  To match the definitions used in the QFR data as closely as possible, I classify firms as durable or nondurable according to the following procedure:

  1. Firms are classified as durable producers if they have a durable NAICS code as defined above.

  2. If a firm has no NAICS code but has a durable SIC code as defined above, I define it as durable.

  3. In rare instances, the NAICS and SIC codes suggest different sectors; this occurs because a small number of industries have been reclassified over time. In these cases I use the NAICS classification.

- **Investment**: This variable denotes the capital stock of each firm at the end of the quarter. As the initial entry I use the firm's first observation of *Property, Plant, and Equipment (Gross)*, which is item 118 and denoted *PPEGTQ* in the Compustat database. From this initial level, I add the quarterly change in *Property, Plant, and Equipment (Net)*, which is item 42 and denoted *PPENTQ*. I use this method because there are many more observations of the net measure than the gross measure of each firm's capital stock. If a firm is missing a single value of *PPENTQ* between two nonmissing values, I linearly impute it using the observations on either side. For instances of two or more consecutive missing values for a firm, no imputation is done.

I only consider investment "runs" of least 40 consecutive quarterly observations after imputation in my main analysis.

- **Total and net current leverage**: I define leverage as the sum of current liabilities ($DLCQ$, item 45) and long-term debt ($DLTTQ$, item 71) divided by firm size as measured total assets (ATQ, item 44). I standardize leverage by subtracting its mean and dividing by its standard deviation across the entire sample. I define net current leverage as the ratio of short-term assets minus short-term liabilities to total assets $\left(\frac{ACTQ-LCTQ}{ATQ}\right)$.

- **Dropped observations**: To minimize the effects of outliers and reporting errors, I exclude firm-quarter observations with any of the following features:

    1. A ratio of acquisitions ($AQCY$) to assets (ATQ) larger than 5%.

    2. An investment rate (defined as $\frac{k_t - k_{t-1}}{k_{t-1}}$) in the top or bottom 0.5 percent of the distribution.

    3. A leverage ratio greater than 10 or a net current leverage ratio either above 10 or below -10.

    4. Changes in quarterly real sales of more than 100% or less than -100%.

Summary statistics are shown in Table 1.

	All Manufacturing		Nondurable		Durable	
Variable	Δk_t	Assets	Δk_t	Assets	Δk_t	Assets
Mean	0.012	$1,766	0.013	$2,784	0.011	$1,207
Median	-0.002	$111	-0.001	$149	-0.003	$98
Std. Dev.	0.126	$8,650	0.134	$11,979	0.122	$5,699

Table 1: Summary Statistics for Manufacturing Firms in Compustat

Note: These statistics cover only manufacturing firms in Compustat from 1985-2008. Assets are deflated using the GDP price index and expressed in millions of 2009 dollars. Δk_t refers to the change in the log level of property, plant, and equipment net of depreciation (NPPE) deflated by the nonresidential fixed investment price index. Statistics for changes in NPPE are calculated across all firm-quarters while the ones for assets are calculated as the time average of the cross sectional value in each quarter.
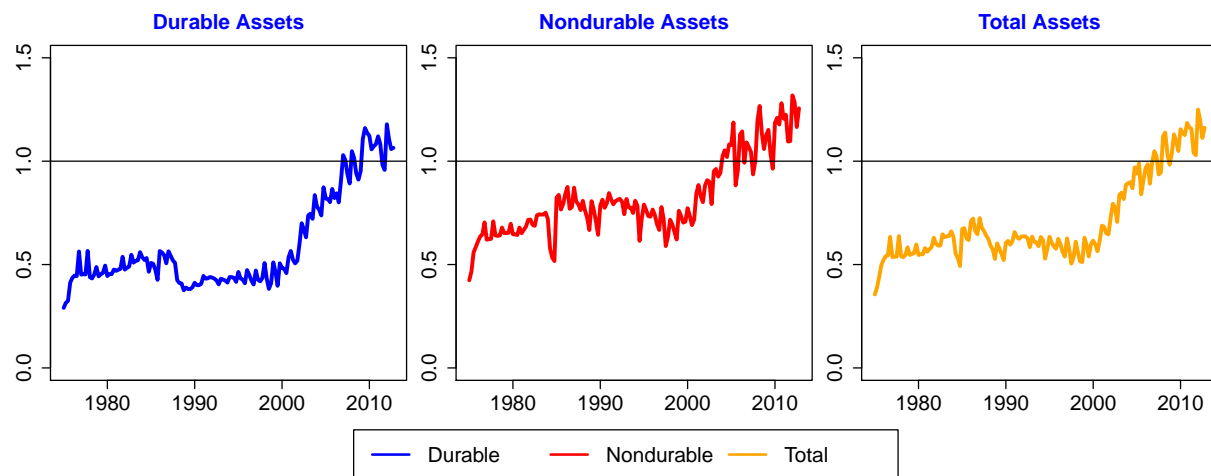
Figure 2: Ratio of Compustat to QFR Measures of Total Assets

Note: This figure shows the ratio of total assets in manufacturing firms in Compustat to total assets for all manufacturing firms from the QFR in each quarter.
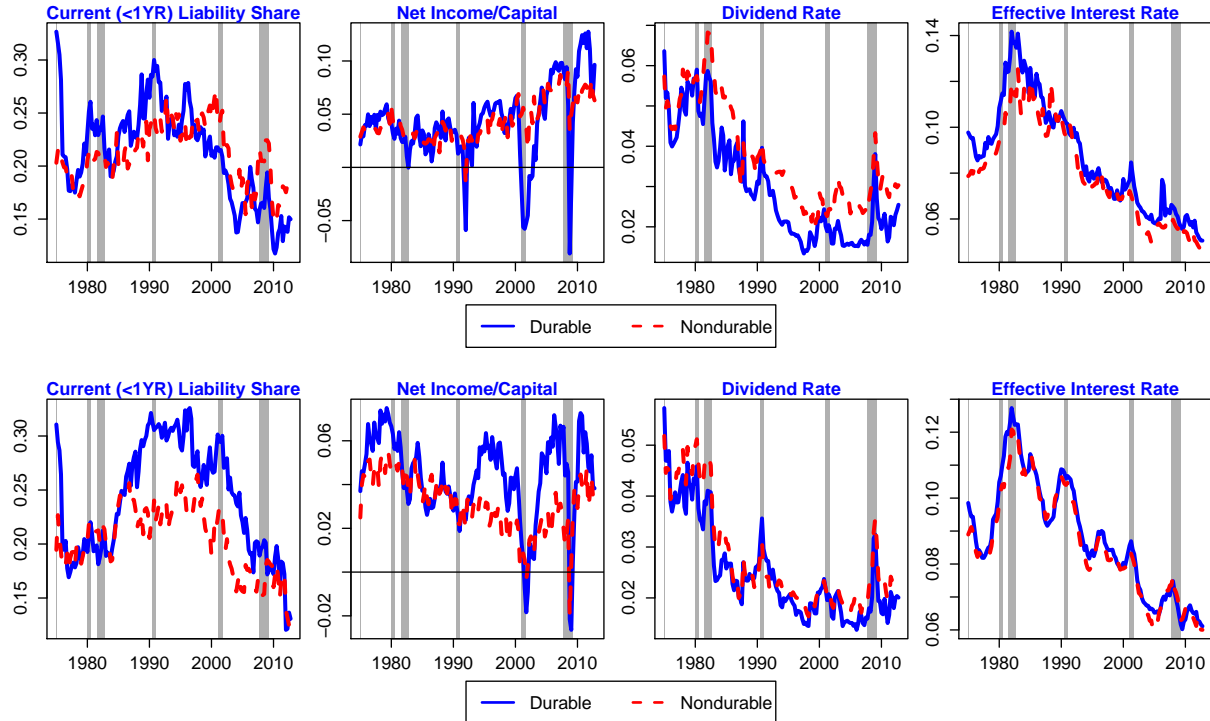
Figure 3: Mean (Top) and Median (Bottom) Compustat Financial Ratios

Note: This figure shows versions of the financial ratios used in the paper calculated from Compustat. The figures in the top row are calculated after winsorizing the top and bottom 1% of observations of each ratio and then weighted by total liabilities (for the short-term liability ratio), the total capital stock (for the net income-capital stock ratio), total equity (for the dividend rate), and total debt (for the effective interest rate). The figures in the bottom row are medians and are not winsorized.

1.4   Monetary Shocks

I use as a measure of exogenous monetary policy shocks the series generated by Coibion (2012) that extends the original work of Romer and Romer (2004). This methodology uses the FOMC Greenbook forecasts, which are a crucial and high-quality source of information for FOMC participants, to represent the Fed's information set. These forecasts are used as the input for a forward-looking Taylor Rule similar to the one below, and the shocks are taken to be the series of residuals ϵ_t^m.

$$\Delta i_t = \beta i_{t-1} + \sum_k \phi_x^k E_t x_{t+k} + \sum_k \phi_\pi^k E_t \pi_{t+k} + \epsilon_t^m \tag{1}$$

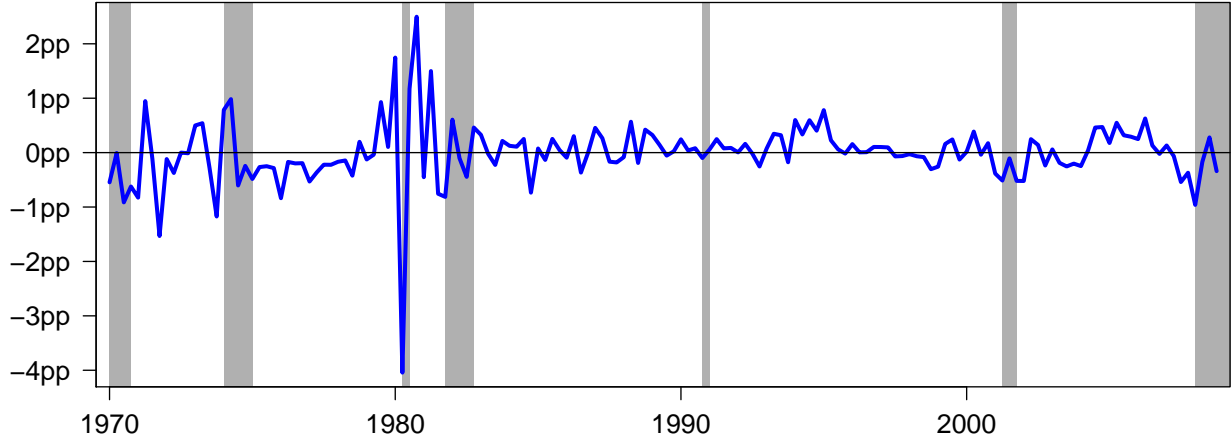The time series of shocks is shown in Figure 4 below.

Figure 4: Time Series of Monetary Shocks

Note: This figure shows the monetary shock series used in my analysis. The shock series I use is developed in Romer and Romer (2004) and extended in Coibion (2012). Positive values correspond to contractionary shocks.

9

2 Robustness Checks and Extensions for Main Results

This section outlines robustness checks and shows some extensions using QFR, user cost, and Compustat data.

2.1 QFR Results

This section discusses a variety of robustness checks and extensions of my main results. My findings are robust to using a VAR specification instead of a local projection approach; using shocks identified in the manner of Gertler and Karadi (2015) instead of R&R-style shocks; alternative controls; and using the manufacturing investment price index for each sector from the BEA fixed asset data to deflate the capital stock instead of the nonresidential fixed investment price deflator.

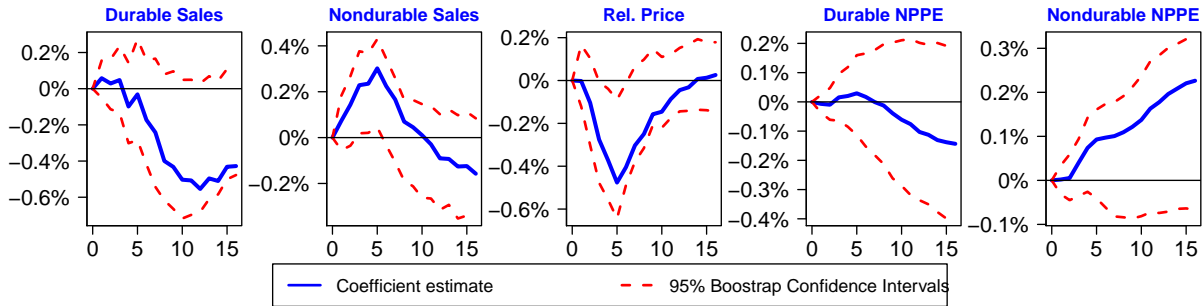### 2.1.1 Alternative Specification: Vector Autoregression

Figure 5: VAR Impulse Responses to Contractionary MP Shock (95% CI)

Note: This figure shows the impulse response to a one standard deviation contractionary FFR shock estimated from a standard recursive SVAR with a constant, linear time trend, seasonal fixed effects, and the following variable ordering: real durable sales, real nondurable sales, the relative price of manufacturing investment to manufacturing output, the real durable capital stock, the real nondurable capital stock, and the Federal Funds Rate. The FFR is in levels and all other data series are in logs. The data span 1970-2004 to match the baseline specification. Bootstrapped 95% confidence intervals are calculated based on 250 draws.
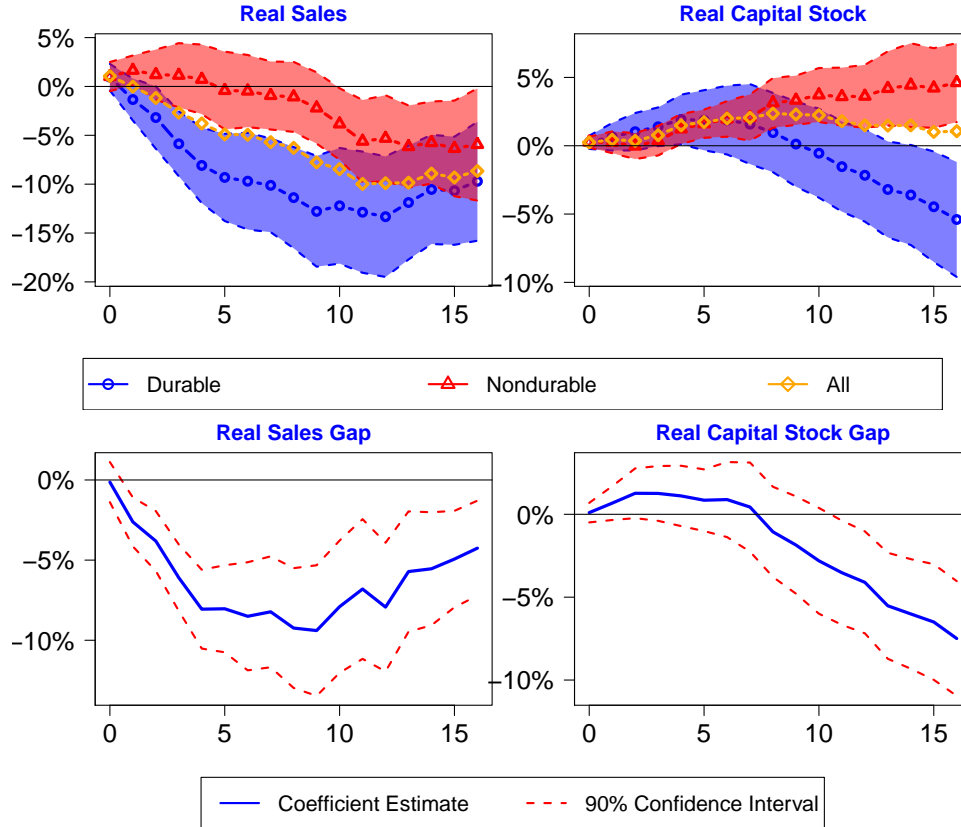
2.1.2 Gertler-Karadi Shocks

Figure 6: IRFs (top) and Estimated Gaps (bottom) using Gertler-Karadi Shocks

Note: This figure shows the coefficient estimates γ_h^i from Equation 1 of the main paper using the shocks identified in Gertler and Karadi (2015). The top row shows the responses of NPPE, which is measured by the QFR item "Stock of Property, Plant, and Equipment Net of Depreciation" and deflated using the NIPA nonresidential fixed investment price index, and sales, which is the QFR sales measure deflated by the NIPA manufacturing output price index for each sector. The bottom row shows the estimated log difference between each measure: $y_t \equiv \log(X_t^D) - \log(X_t^N)$. 90% confidence intervals are calculated using Newey-West standard errors. Regressions include shocks from 1975-2004 and outcomes through 2008.

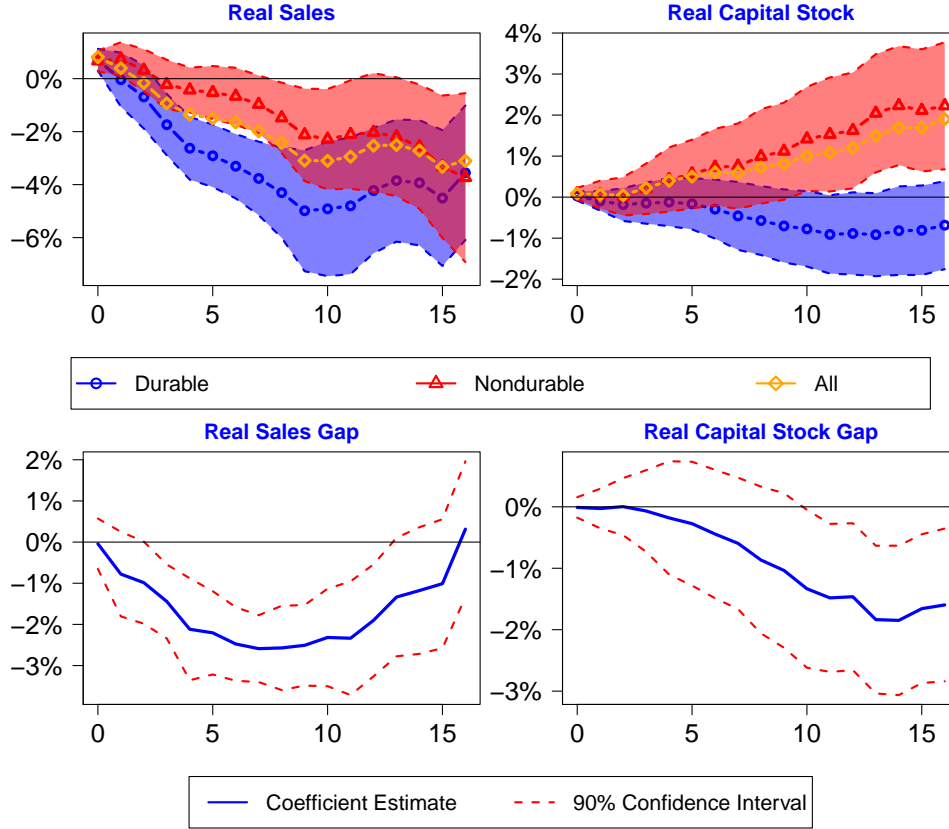### 2.1.3 Number of Autoregressive Lags of Dependent Variable

Figure 7: IRFs with 4 Lags of Dependent Variable

Note: This figure shows the coefficient estimates γ_h^i from Equation 1 of the main paper but modified to include 4 lags of the dependent variable. The top row shows the responses of NPPE, which is measured by the QFR item "Stock of Property, Plant, and Equipment Net of Depreciation" and deflated using the NIPA nonresidential fixed investment price index, and sales, which is the QFR sales measure deflated by the NIPA manufacturing output price index for each sector. The bottom row shows the estimated log difference between each measure: $y_t \equiv \log(X_t^D) - \log(X_t^N)$. 90% confidence intervals are calculated using Newey-West standard errors. Regressions include shocks from 1970-2004 and outcomes through 2008.
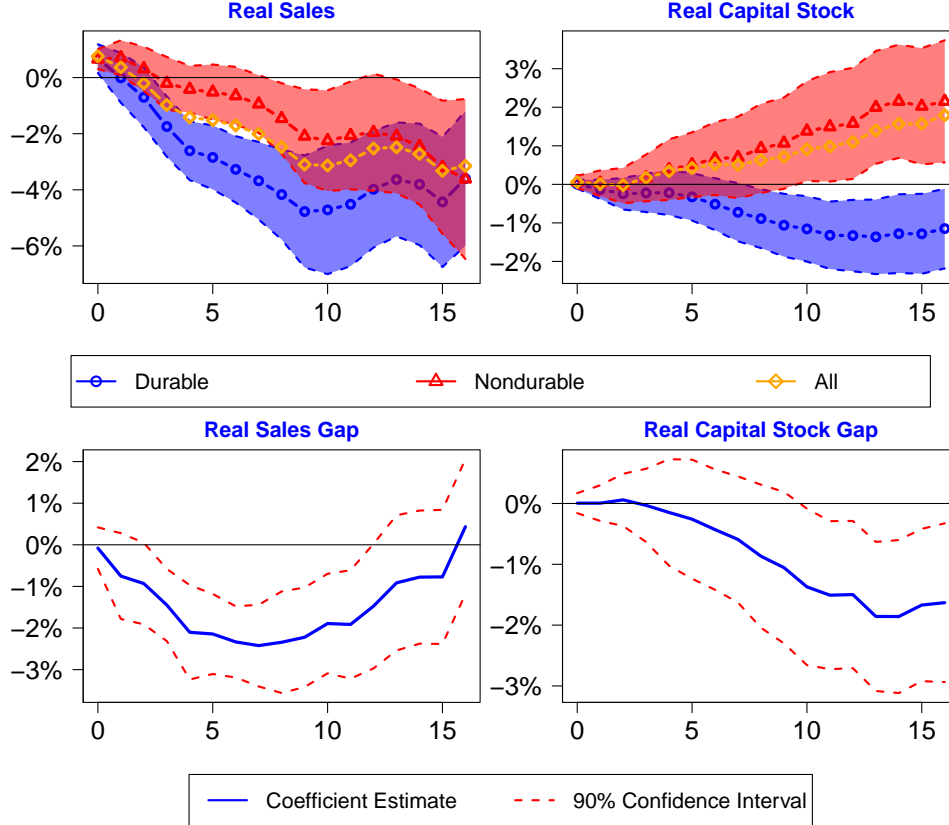
Figure 8: IRFs with 8 Lags of Dependent Variable

Note: This figure shows the coefficient estimates γ_h^i from Equation 1 of the main paper but modified to include 8 lags of the dependent variable. The top row shows the responses of NPPE, which is measured by the QFR item "Stock of Property, Plant, and Equipment Net of Depreciation" and deflated using the NIPA nonresidential fixed investment price index, and sales, which is the QFR sales measure deflated by the NIPA manufacturing output price index for each sector. The bottom row shows the estimated log difference between each measure: $y_t \equiv \log(X_t^D) - \log(X_t^N)$. 90% confidence intervals are calculated using Newey-West standard errors. Regressions include shocks from 1970-2004 and outcomes through 2008.
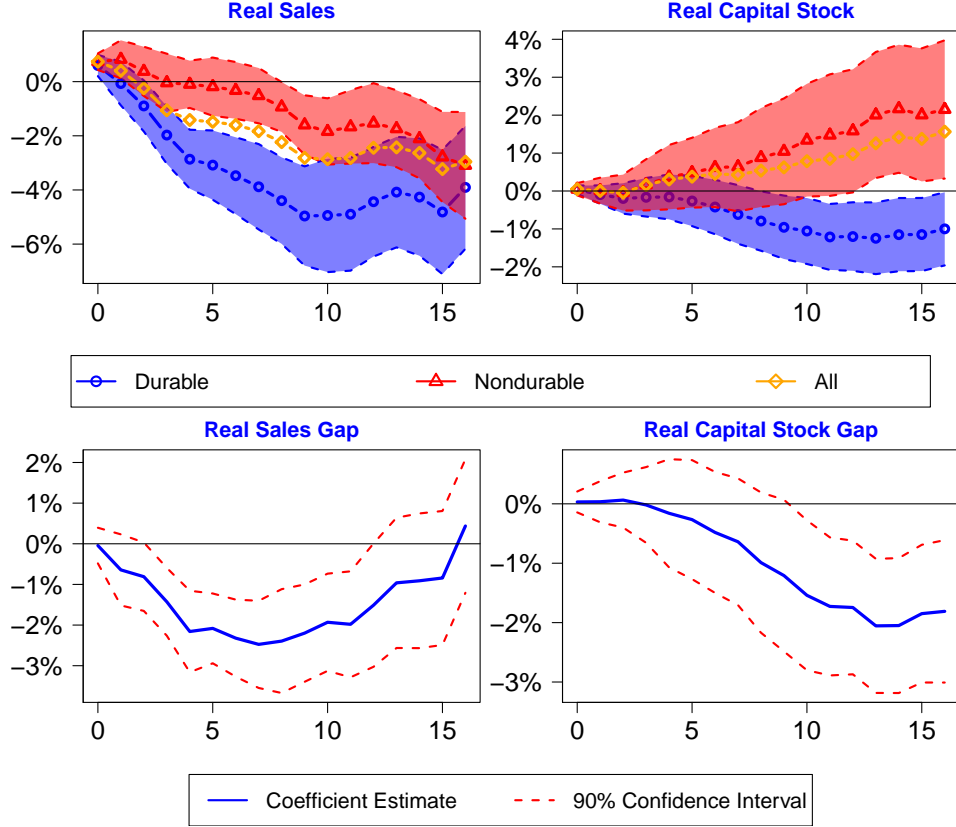
Figure 9: IRFs with 16 Lags of Dependent Variable

Note: This figure shows the coefficient estimates γ_h^i from Equation 1 of the main paper but modified to include 16 lags of the dependent variable. The top row shows the responses of NPPE, which is measured by the QFR item "Stock of Property, Plant, and Equipment Net of Depreciation" and deflated using the NIPA nonresidential fixed investment price index, and sales, which is the QFR sales measure deflated by the NIPA manufacturing output price index for each sector. The bottom row shows the estimated log difference between each measure: $y_t \equiv \log(X_t^D) - \log(X_t^N)$. 90% confidence intervals are calculated using Newey-West standard errors. Regressions include shocks from 1970-2004 and outcomes through 2008.
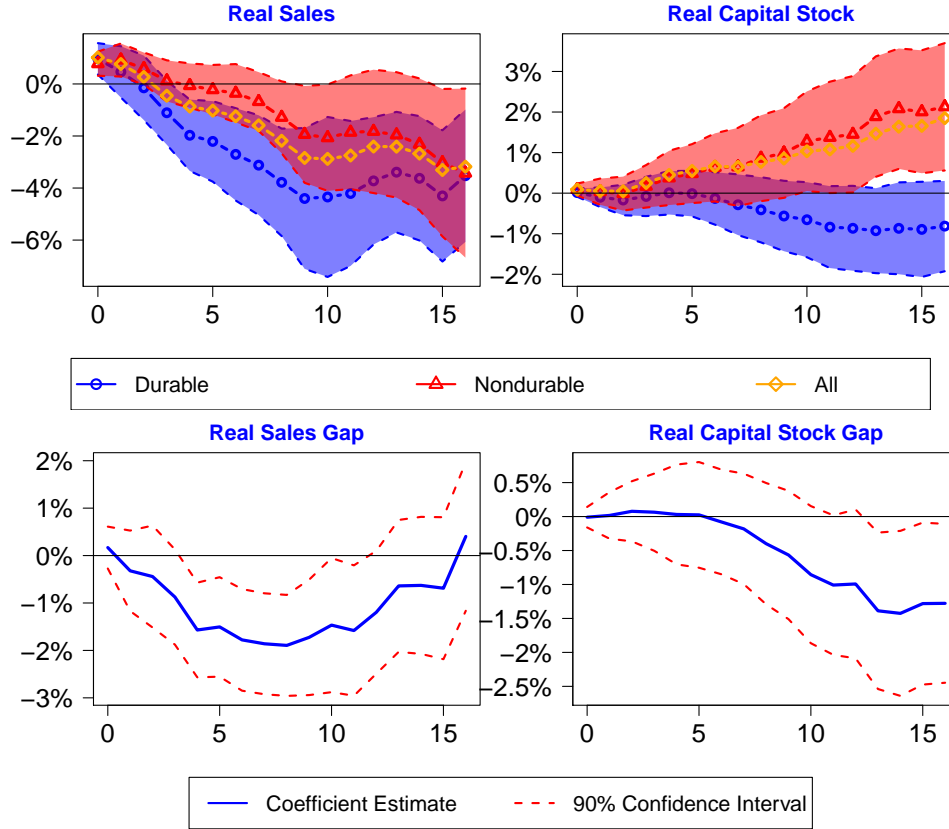
2.1.4   Alternative Controls

Figure 10: IRFs Excluding Lagged Real GDP Growth as Control

Note: This figure shows the coefficient estimates γ_h^i from Equation 1 of the main paper but excludes lagged real GDP growth as a control. The top row shows the responses of NPPE, which is measured by the QFR item "Stock of Property, Plant, and Equipment Net of Depreciation" and deflated using the NIPA nonresidential fixed investment price index, and sales, which is the QFR sales measure deflated by the NIPA manufacturing output price index for each sector. The bottom row shows the estimated log difference between each measure: $y_t \equiv \log(X_t^D) - \log(X_t^N)$. 90% confidence intervals are calculated using Newey-West standard errors. Regressions include shocks from 1970-2004 and outcomes through 2008.
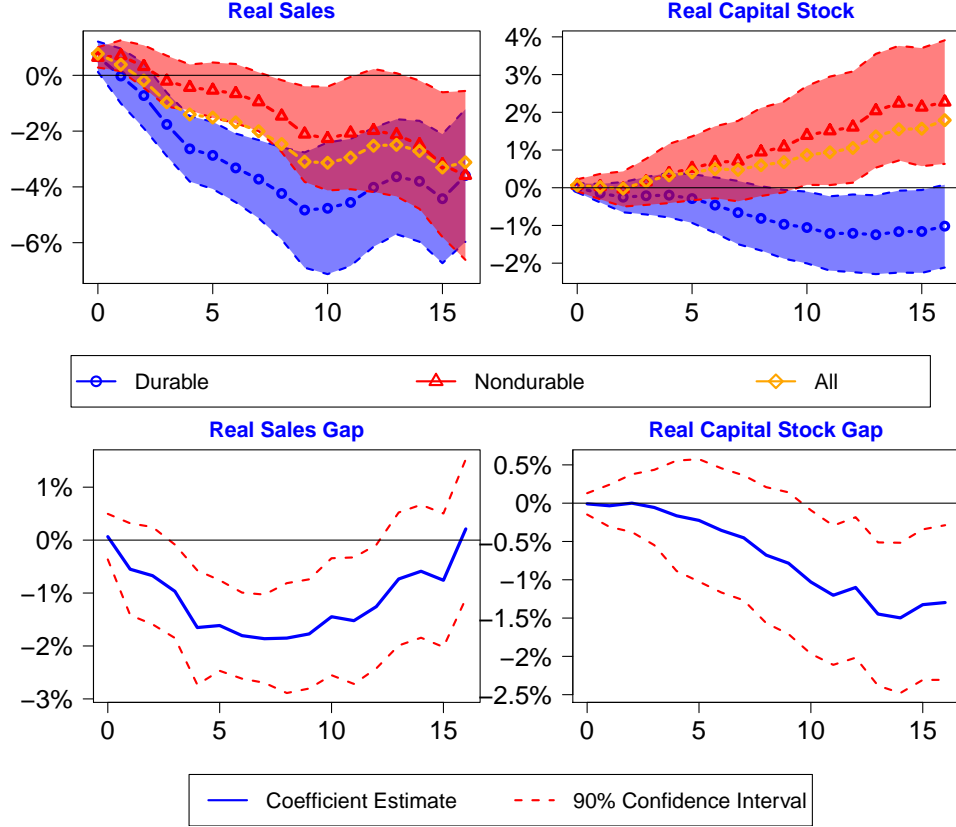
Figure 11: IRFs Excluding Lagged Shock as Control

Note: This figure shows the coefficient estimates γ_h^i from Equation 1 of the main paper but excludes ϵ_{t-1}^m as a control. The top row shows the responses of NPPE, which is measured by the QFR item "Stock of Property, Plant, and Equipment Net of Depreciation" and deflated using the NIPA nonresidential fixed investment price index, and sales, which is the QFR sales measure deflated by the NIPA manufacturing output price index for each sector. The bottom row shows the estimated log difference between each measure: $y_t \equiv \log(X_t^D) - \log(X_t^N)$. 90% confidence intervals are calculated using Newey-West standard errors. Regressions include shocks from 1970-2004 and outcomes through 2008.

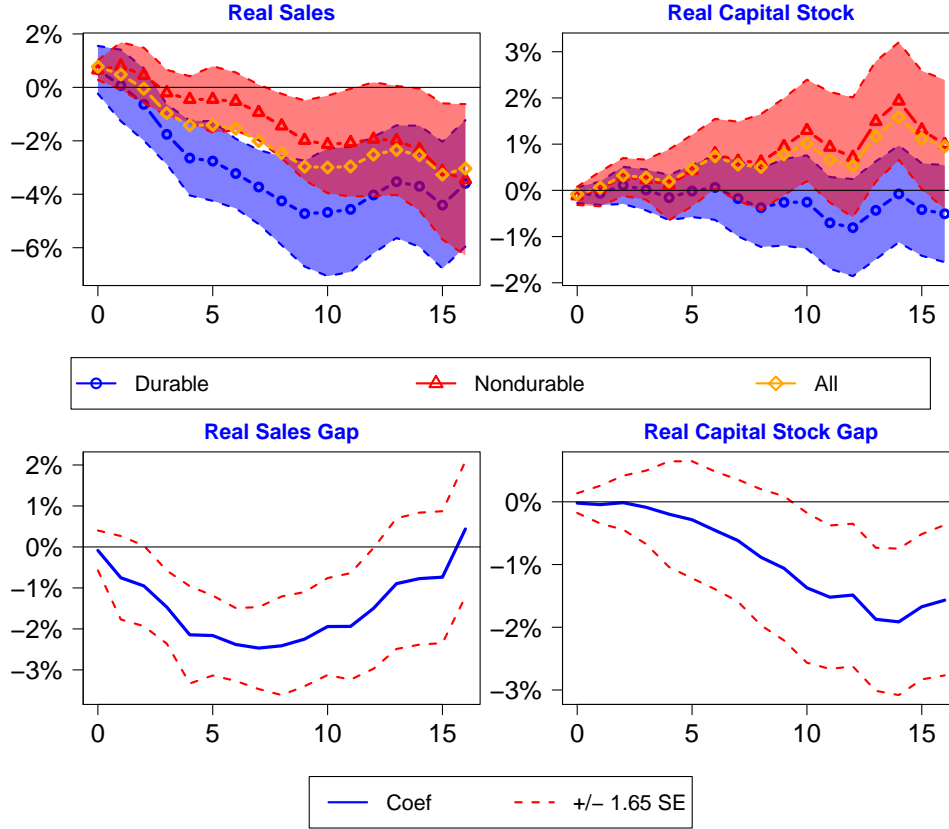### 2.1.5 BEA Sector-Specific Investment Price Deflators

Figure 12: IRFs to 100bp Contractionary Shock Using BEA Investment Deflators (90% CI)

Note: This figure shows the coefficient estimates γ_h^i from Equation 1 of the main paper. The top row shows the responses of NPPE, which is measured by the QFR item "Stock of Property, Plant, and Equipment Net of Depreciation" deflated using the BEA sector-specific investment price index for each sector, and sales, which is the QFR sales measure deflated by the NIPA manufacturing output price index for each sector. The bottom row shows the estimated log difference between each measure:
$y_t \equiv \log(X_t^D) - \log(X_t^N)$. 90% confidence intervals are calculated using Newey-West standard errors. Regressions include shocks from 1970-2004 and outcomes through 2008.

2.2 BEA Results

The main paper analyzed the responses of the total manufacturing sector capital stock as measured by BEA fixed asset data and compared it to the responses of other sectors. It is possible to break the BEA data down further to analyze the effects of durable and nondurable manufacturers separately, but these estimates are significantly noisier due to the linear interpolation. The top row of Figure 13 shows the estimated impulse responses using the exact same specification as in my baseline results, with each sector's responses estimated independently. There are two major differences relative to the QFR results. The first is that the coefficient estimates are much smaller, with peak effects of less than 0.5%. The second is that the standard error estimates are much wider relative to the coefficient estimate. Much of the variation in monetary policy shocks occurs at the quarterly frequency, but the BEA will smooth these changes across all quarters in the year so that the effects become smaller and less significant.

Despite the issues with interpolation, directly estimating the gaps between the durable and nondurable sector in the BEA data provides support for my main findings. The bottom row of Figure 13 shows significant differences between the responses of durable and nondurable producers starting around three years after the shock for total fixed assets as well as both structures and equipment. This provides support for my main findings using the QFR data. They also suggest that the differential behavior of the capital stock is not driven by any one particular type of fixed asset. Figure 14 repeats this analysis for the BEA's investment measure and finds generally similar (though noisier) results.
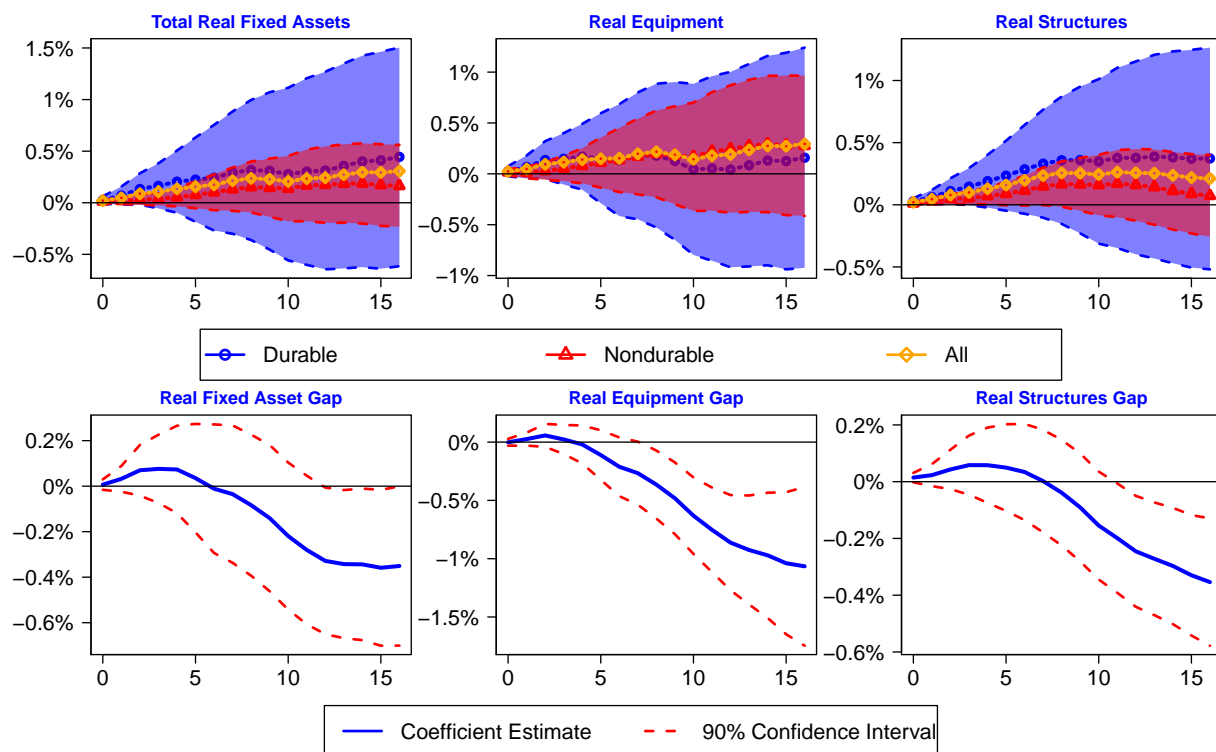
Figure 13: IRFs to 100bp Contractionary Shock, BEA Fixed Asset Measures (90% CI)

Note: This figure shows the responses of fixed assets to monetary shocks based on a linear quarterly interpolation of the annual BEA real fixed asset data using the same estimating equation as in my baseline results. The top row shows the directly estimated impulse responses for each sector for total fixed assets as well as breakdowns for equipment and structures. The bottom row shows the estimated log difference between each measure: $y_t \equiv \log(X_t^D) - \log(X_t^N)$ for $X \in \{Total, Equipment, Structures\}$. Regressions include shocks from 1970-2004 and outcomes through 2008. 90% confidence intervals are calculated using Newey-West standard errors.
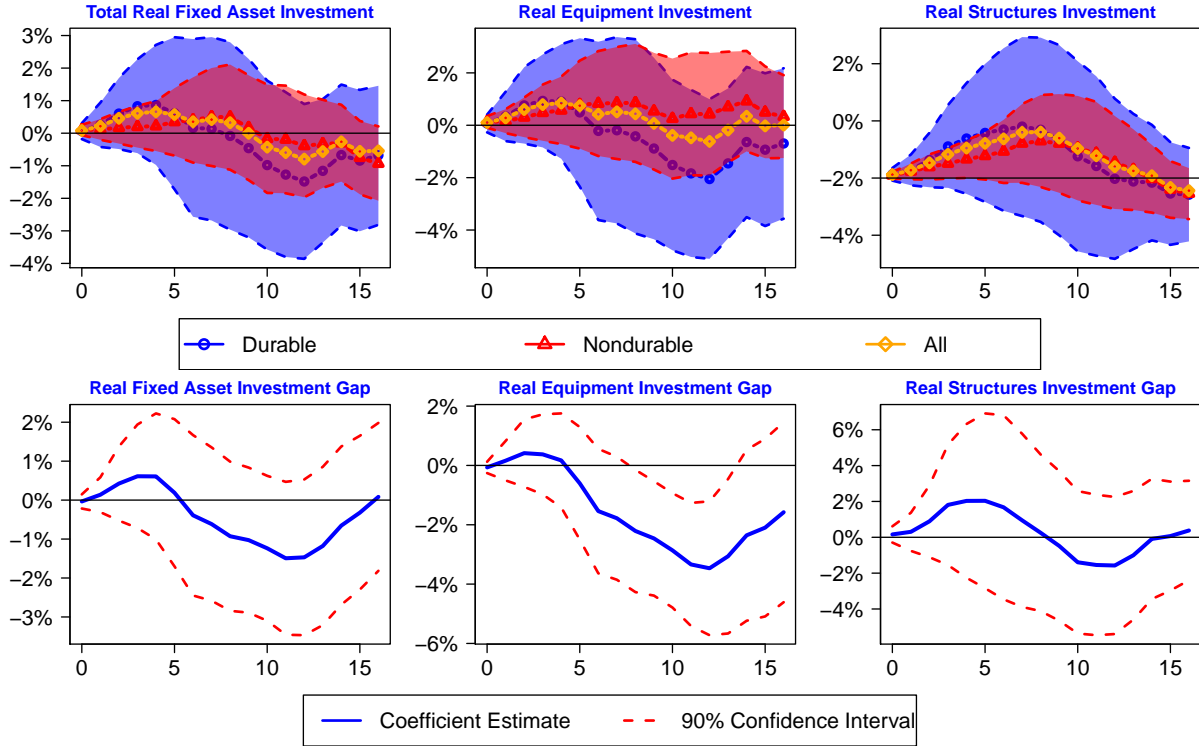
Figure 14: IRFs to 100bp Contractionary Shock, BEA Investment Measures (90% CI)

Note: This figure shows the responses of investment to monetary shocks based on a linear quarterly interpolation of the annual BEA real fixed asset investment data using the same estimating equation as in my baseline results. The top row shows the directly estimated impulse responses for each sector for total fixed assets as well as breakdowns for equipment and structures. The bottom row shows the estimated log difference between each measure: $y_t \equiv \log(X_t^D) - \log(X_t^N)$ for $X \in \{Total, Equipment, Structures\}$. Regressions include shocks from 1970-2004 and outcomes through 2008. 90% confidence intervals are calculated using Newey-West standard errors.

2.3 User Cost Results

Effective interest rate measures are not directly observable in the QFR prior to 1998. To obtain the sector-specific interest rates I use in my main analysis, I use data from Compustat. To calculate these interest rates, I first calculate the rate of interest expenses to total debt using the WRDS financial ratio suite. I Winsorize the top and bottom 1% of observations and then calculate a mean for each sector in each quarter weighted by total debt.

Because these observations are only available starting in 1975, I retroactively apply the change in yields on AAA bonds between 1970 and 1975 to get a series running back to 1970. This effectively assumes that the spread between each sector's average borrowing rate and the AAA yield was constant over this five-year window, though the results are extremely similar if I start the regressions after the Compustat data are available. One issue with this approach is that the Compustat calculations only give the *average* interest rate, whereas the *marginal* rate is the one relevant for the user cost calculation. In practice this does not appear to make a large difference, however, as user costs estimated using AAA or BAA bond yields as well as the Federal Funds Rate all yield similar results.

The estimated interest rates are shown in Figure 15. In general, the average interest rates calculated from Compustat behave quite similarly across sectors; contractionary monetary shocks lead to small on-impact effects that gradually increase to a peak of about 50bp between two and three years after the shock, consistent with the idea that it takes time for higher marginal rates to increase average rates as debt based on old rates expires and new debt is issued. The bottom panel shows that while the effect on the Federal Funds Rate is large, the effects on corporate bond yields are more muted. Aside from a slightly larger increase on impact, the responses are very similar to those measured in the Compustat data.
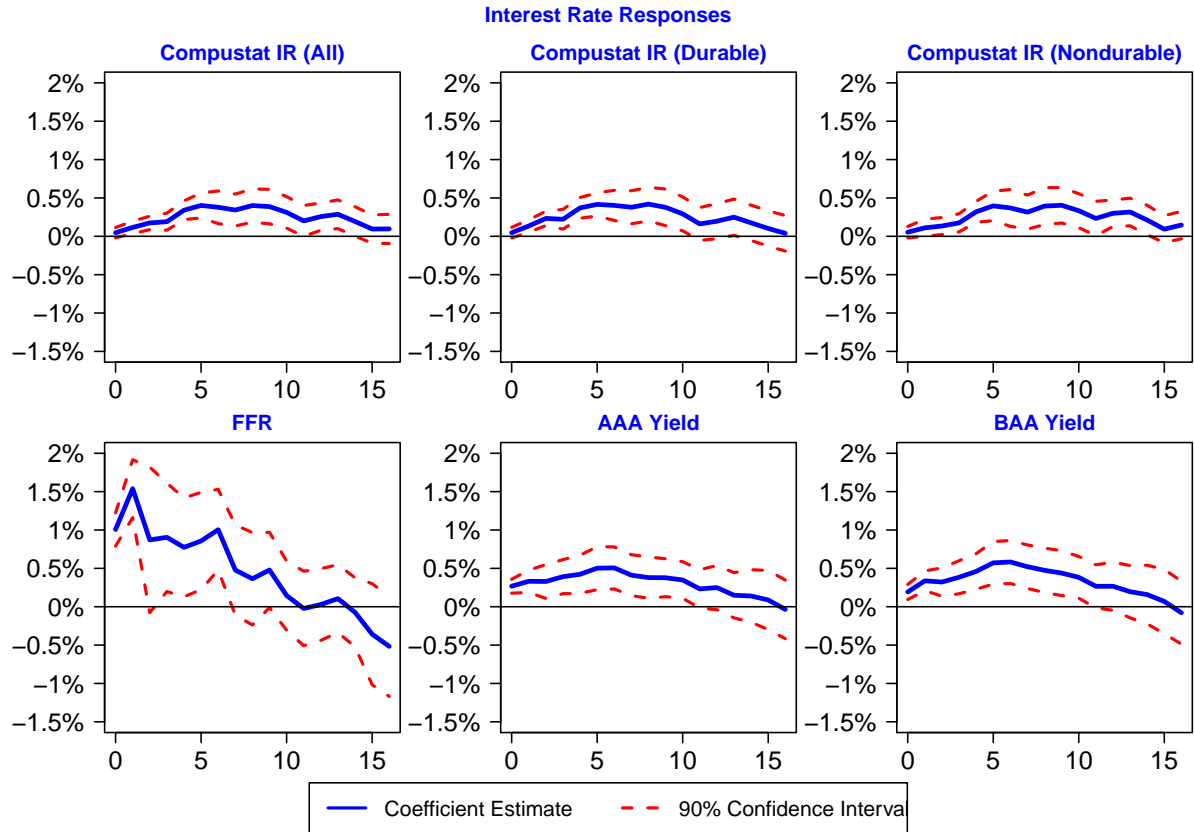
Figure 15: Impulse Responses to 100bp Contractionary Monetary Shock (90% CI)

Note: This figure shows the responses of interest rates to monetary shocks. The top row shows average interest rates calculated from Compustat as total interest payments divided (and weighted) by total debt after winsorizing the top and bottom 1% of observations. Because Compustat interest rates are only available starting in 1975, I calculate levels by retroactively applying changes in the AAA yield rate from 1970-1975. The bottom row shows the responses of the Federal Funds Rate as well as AAA and BAA corporate bond yields.

2.4 Compustat Results

2.4.1 Compustat Aggregates

Figure 16 shows the coefficient estimate from aggregating the capital stock measures in Compustat and estimating the effects of a monetary shock using the baseline estimating equation from the main paper. The top panel shows that the estimated effects of a monetary shock are quite similar for both manufacturing and nonmanufacturing firms, which show persistent declines of up to about 5%. The bottom panel shows that the decline in the manufacturing capital stock is driven by the durable sector, which experiences a decline of up to almost 10% in the years following the shock, while the nondurable sector shows a much smaller decline that quickly returns to its original level.
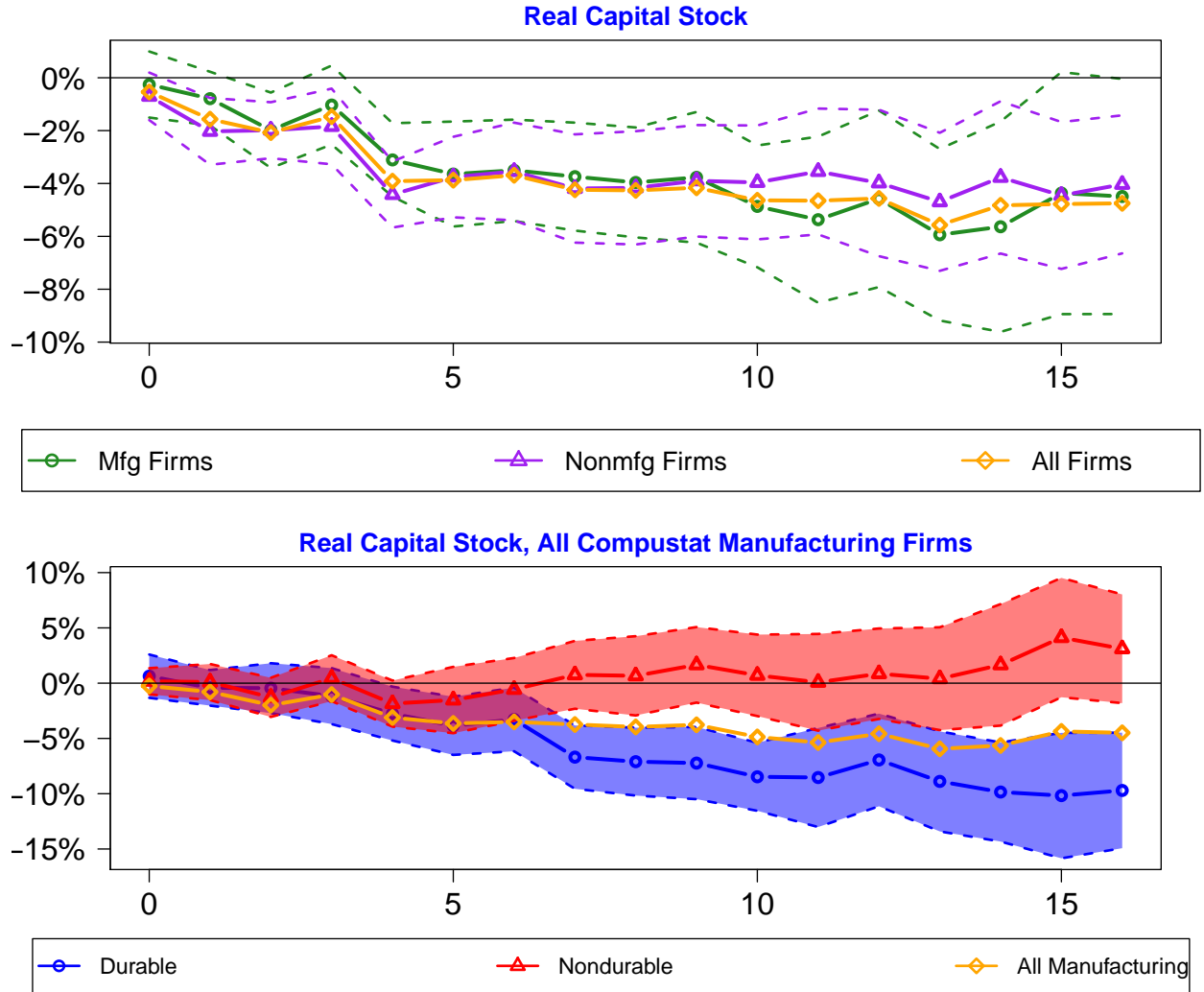
Figure 16: IRFs for Aggregated Compustat Data (90% CI)

Note: This figure shows the coefficient estimates γ_h^i from Equation 1 of the main paper, which correspond to the effects of a 100bp contractionary monetary shock. The dependent variable is the four-quarter moving average of the aggregate capital stock across Compustat firms deflated by the NIPA nonresidential fixed investment price index. "Nonmanufacturing" firms include all firms outside of manufacturing, finance, insurance, real estate, public administration, and utility sectors. 90% confidence intervals are calculated using Newey-West standard errors. Because the Compustat data are less reliable prior to 1985 and my baseline specification includes six autoregressive lags, regressions include shocks from 1986Q3-2004Q4 and outcomes through 2008Q4.

2.4.2   Compustat Panel

The panel specification I use is similar to Jeenas (2019) and Ottonello and Winberry (2020):

$$\Delta k_{j,t+h} = \alpha_{j,h} + \alpha_{cq,h} + \alpha_{fq,h} + \beta Z_{t-1} + \Omega Y_{t-1} + \gamma_h \epsilon_t^m + \nu_{j,t+h} \tag{2}$$

$\Delta k_{j,t+h}$ is the cumulative change in the log of the real capital stock between time $t-1$ and time $t+h$ so that $h=0$ corresponds to the same quarter at which the shock hits and $h=16$ corresponds to four years after. ϵ_t^m is the same R&R-style monetary policy shock used in the aggregate analysis. $\alpha_{j,h}$ is a firm fixed effect, $\alpha_{cq,h}$ is a calendar quarter fixed effect, $\alpha_{fq,h}$ is a fiscal quarter fixed effect, Z_{t-1} is a vector of firm-level controls including normalized leverage, log assets, sales growth, and the current share of assets, and Y_{t-1} is a vector of lagged aggregate controls including CPI inflation, GDP growth, and the unemployment rate. A linear time trend is included to maintain consistency with the main aggregate specification. The estimates of γ_h for all horizons up to $h=16$ are shown in Figure 17 with 90% confidence intervals for durable, nondurable, and all manufacturing firms.

The coefficient estimates, which show the estimated response of the capital stock of the average manufacturing firm in Compustat, are positive and highly significant throughout most of the four years following the shock for all manufacturing firms, with a peak increase of more than 9%. This increase is driven primarily by the durable sector, which shows a peak increase of over 11%, while the response of the nondurable sector is more muted.

The seemingly contradictory behavior of the average and aggregate responses can be explained by the behavior of the largest firms. Figure 18 shows the estimates based on only firm-quarter observations with at least $10bn in nominal assets. This group includes fewer than 300 firms over the period from 1985-2008 but accounts for the majority of the capital stock in Compustat; its share rose from just over 50% in 1985 to more than 85% by the end of 2012. These results show significant estimated declines in the capital stocks for all manufacturing firms, with the decline driven by nondurable producers. The next section uses QFR data broken down by both sector and size to show how they can account for these firm-level findings.

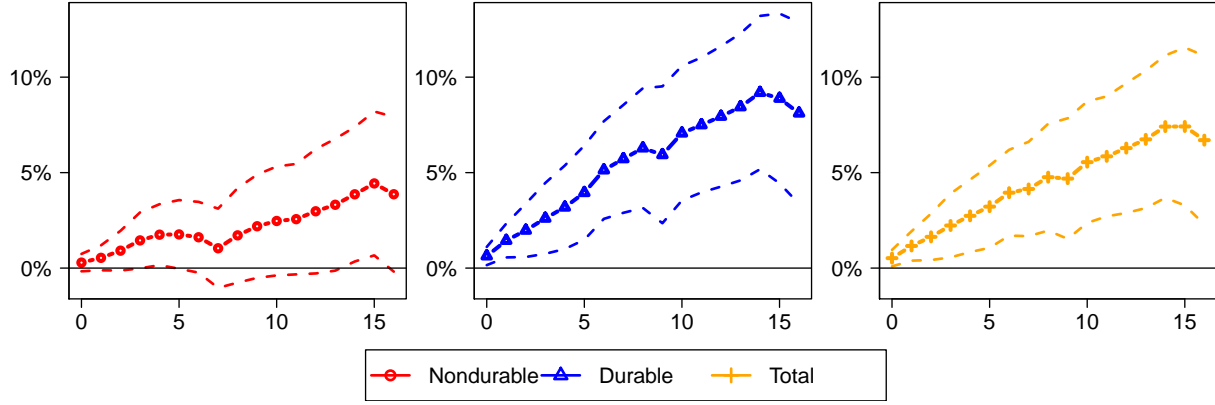**Average NPPE Stock Response for Compustat Manufacturing Firms**

Figure 17: IRFs Using Compustat Data for Manufacturing Firms, Separate Regressions (90% CI)

Note: This figure shows the coefficient estimates γ_h^i from Equation 1 of the main paper for all manufacturing firms in Compustat. Capital stocks are deflated using the nonresidential fixed investment price deflator. Data include shocks from 1985-2004 and outcomes through 2008. Data definitions and sample construction are described in 1.3. Dashed lines indicated 90% confidence intervals calculated using Driscoll-Kraay standard errors.

Average NPPE Stock Response for Large Compustat Manufacturing Firms
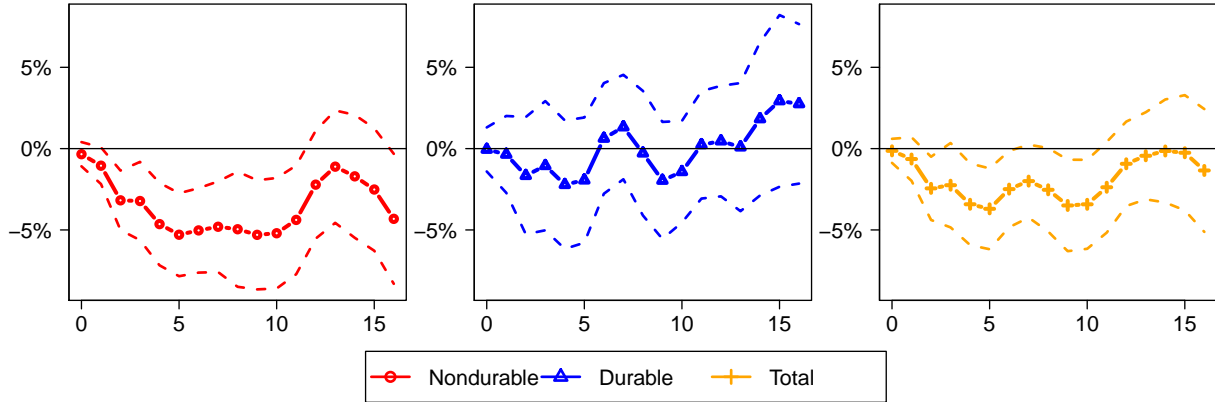
Figure 18: IRFs Using Compustat Data for Large Manufacturing Firms, Separate Regressions (90% CI)

Note: This figure shows the coefficient estimate γ_h based on Equation 2 for manufacturing firms in Compustat. Capital stocks are deflated using the nonresidential fixed investment price deflator. This specification includes only firm-quarter observations with at least \$10bn in nominal assets. Data include shocks from 1985-2004 and outcomes through 2008. Data definitions and sample construction are described in 1.3. Dashed lines indicated 90% confidence intervals calculated using Driscoll-Kraay standard errors.

2.4.3 Comparing Compustat and QFR Results

The results for the QFR firms with over $1bn in assets are shown in Figure 19. These firms provide the closest comparison to the aggregates available from Compustat. The increase in the capital stock of large nondurable firms in the QFR matches closely with the results of all Compustat firms as well as the subset that excludes those with foreign operations. While there is a modest increase in the durable capital stock of the largest QFR firms during the first two years after the shock, there is a decline by the end of the response horizon that lines up well with the Compustat aggregates.

While there is a decline in the aggregate capital stock for the largest firms in the QFR, Figure 20 shows that there is a large increase- almost 10% by the end of the response horizon- in the capital stock of the QFR firms which have fewer than $1bn in assets. The result is a modest increase in the aggregate manufacturing capital stock.

While it is perhaps surprising that the smaller firms in the QFR are driving the increase, this is a relatively modern phenomenon. This can be seen by fixing a response horizon and estimating a rolling regression of the capital stock responses across firms of different sizes/sectors at that horizon. This facilitates comparison of the responsiveness for different types of firms over time. These results are shown in Figure 21. They show that the kinds of firms that decreased their capital stock in response to monetary shocks during the early part of the sample—small firms and durable producers— instead responded by increasing their capital stocks during the later part of the sample. This is consistent with the idea that financial constraints have eased over time.
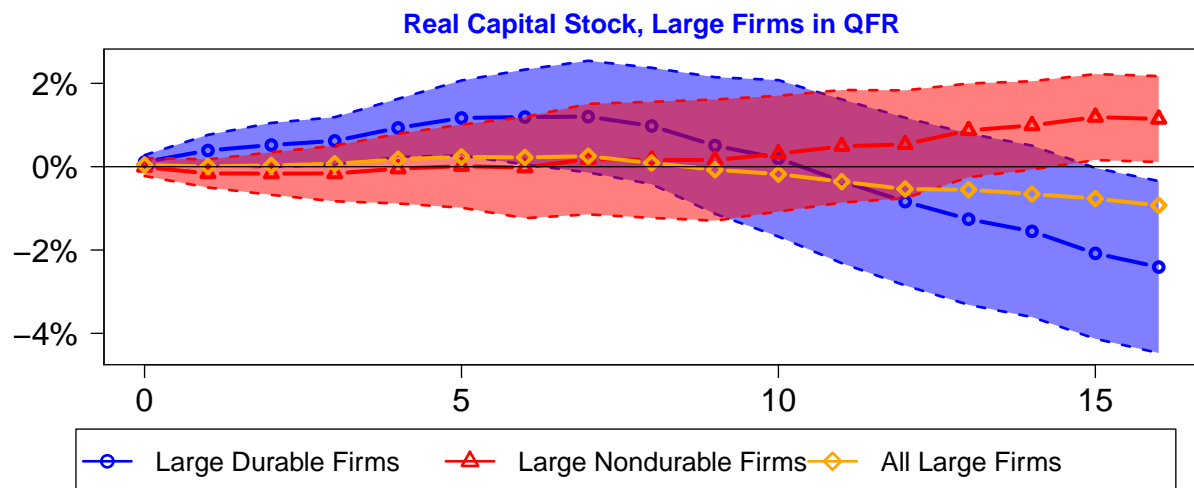
Real Capital Stock, Large Firms in QFR

Figure 19: IRFs for QFR Firms with >$1bn in Assets (90% CI)

Real Capital Stock
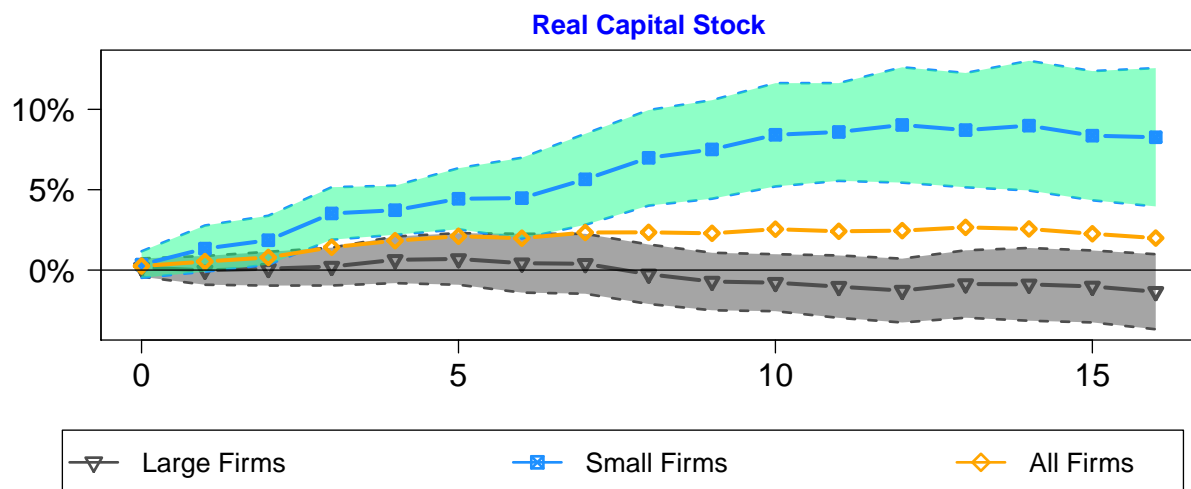
Figure 20: IRFs for QFR Firms by Size (90% CI)

Note: This figure shows the coefficient estimates γ_h^i from the baseline empirical specification, which correspond to the effects of a 100bp contractionary monetary shock. The dependent variable is the QFR NPPE measure deflated by the nonresidential fixed investment price index. "Large" firms are defined as those with >1$bn in nominal assets, while "small" firms include all others. 90% confidence intervals are calculated using Newey-West standard errors. To ensure comparability with the Compustat aggregates, regressions include shocks from 1986Q3-2004Q4 and outcomes through 2008Q4.

Figure 21: Estimates from Rolling Regressions (90% CI)

Note: This figure shows the estimated response of the real manufacturing capital stock using the baseline specification with a rolling 20-year window usin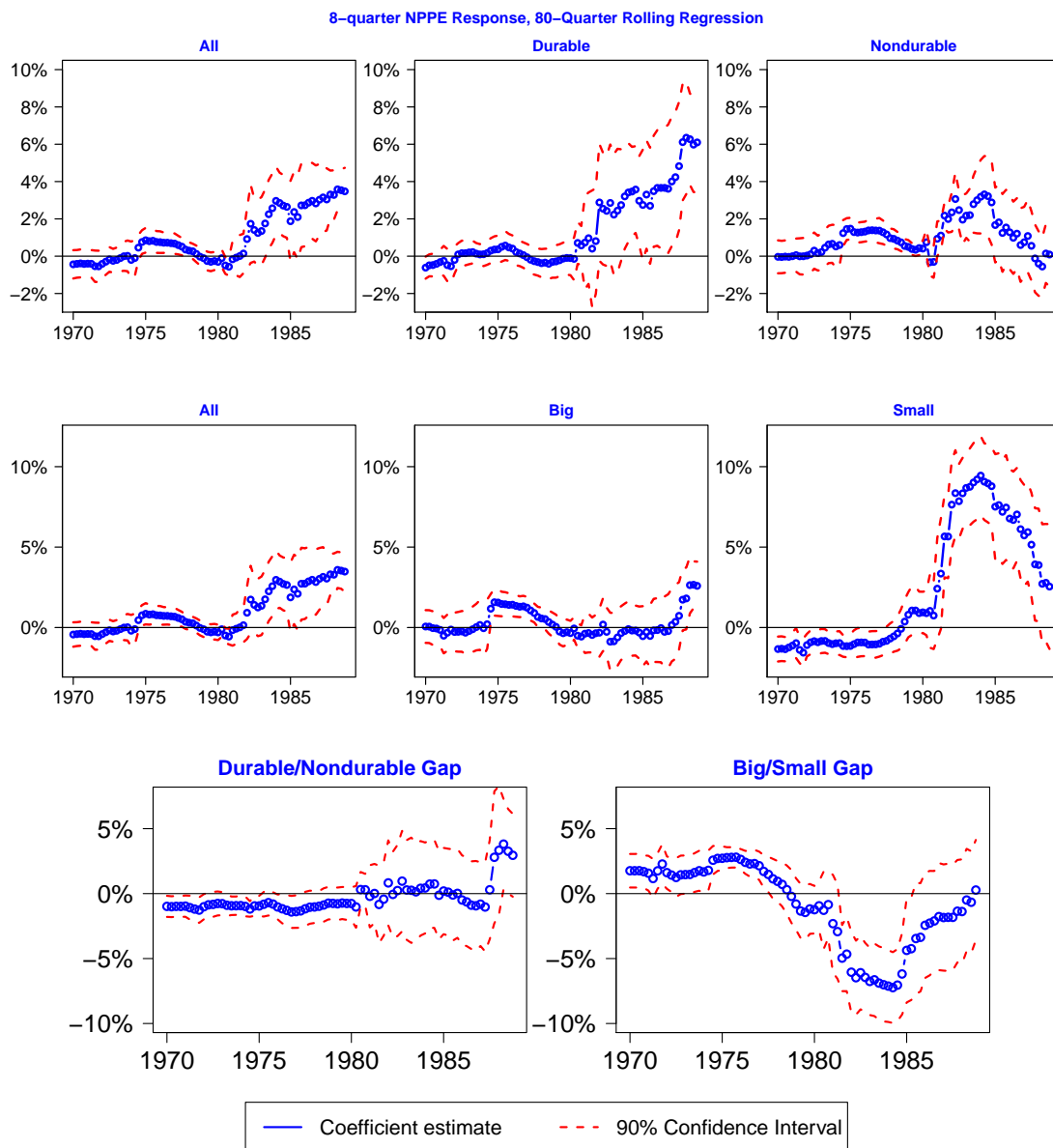g Equation **??**. The horizon is fixed at 8 quarters after the shock and the date on the horizontal axis corresponds to the start date of the regression, e.g. the coefficient estimate for 1980 is estimated using data from 1980-2000. The starting years on the horizontal axis range from 1970-1988; they include shocks through 2006 and outcomes through 2008. The top row shows the estimates split by sector. The middle row shows the estimates split by firm size, with "large" firms defined as those with at least \$1bn in nominal assets and "small" firms including all others. The bottom row shows the estimated effects on the log difference across sectors and sizes: $\log(K^D) - \log(K^N)$ for the left panel and $\log(K^{Large}) - \log(K^{Small})$ for the right. 90% confidence intervals are calculated using Newey-West standard errors.

3 Model

This section shows the parameter values used in the model and the entire set of equilibrium conditions along with several robustness checks and extensions. I show that the main results are robust to forcing durable producers to borrow at the risk-free rate instead of at zero net interest and that the model is still able to generate qualitatively similar results even in the case of equally sticky prices in both sectors. Finally, a simple corporate finance model is used to provide theoretical justification for the fact that durable producers are more financially constrained, and I show that the solution to this model is an "investment multiplier" that takes on the same functional form as the one used in the paper's New Keynesian model.

3.1 Parameter Values

Parameter	Value	Description
β_S, β_B	0.99, 0.98	Discount factors
ω	0.5	Share of savers
η	0.8	Nondurable consumption share
ρ_w, μ_w	0.5, 0.1	Wage rigidity and markup
h	0.9	Habit formation
ν	4	Labor disutility scale
m, ξ	0.7, 0.1	Borrowing limits
ϕ_D, ϕ_N	0, 58.25	Price adjustment costs
θ_D, θ_N	2	Investment adjustment costs
δ_D, δ_K	0.02, 0.03	Depreciation rates
ϵ_D, ϵ_N	11	Substitution elasticities
α_D, α_N	0.33	Capital shares
ϕ_π, ρ	1.5, 0.9	Taylor Rule

Table 2: Parameter Values

3.2 Full Set of Equilibrium Conditions

This section shows the set of equations which fully characterize the solution to the model. After plugging in the household's demand curve, the full Lagrangian can be formulated as below. ξ^N is set sufficiently high such that the borrowing constraint does not bind for nondurable producers and thus $\mu_t^N = 0$; as a result, the sector-specific superscripts are omitted in the body of the paper.

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta_S^t \frac{\lambda_{S,t}}{\lambda_{S,0}} \left\{ p_t^j(i) \left(\frac{p_t^j(i)}{P_t^j} \right)^{-\epsilon_j} Y_t^j - w_t N_t^j - p_t^D I_t^j - \frac{\phi_j}{2} \left(\Pi_t^j(i) - 1 \right)^2 Y_t^j(i) \right.$$

$$+ mk_t^j \left[I_t^j \left(1 - \frac{\theta_j}{2} \left(\frac{I_t^j}{I_{t-1}^j} - 1 \right)^2 \right) + (1-\delta_j)K_t^j - K_{t+1}^j \right] + \mu_t^j \left[\xi^j p_t^D K_t^j - w_t N_t^j - p_t^D I_t^j \right]$$

$$\left. + mc_t^j \left[A_t(K_t^j)^{\alpha_j}(N_t^j)^{1-\alpha_j} - Y_t^j(i) \right] \right\} \tag{3}$$

The full set of equilibrium conditions are as follows:

$$\eta \left(\frac{1}{C_{B,t}^N - hC_{B,t-1}^N} - h\beta_B E_t \left[\frac{1}{C_{B,t+1}^N - hC_{B,t}^N} \right] \right) = \lambda_{B,t} p_t^N \tag{4}$$

$$w_t = \left(\frac{\nu H_{B,t}^{\chi}}{\lambda_{B,t}} (1+\mu^w) \right)^{1-\rho_w} \left(\frac{w_{t-1}}{\Pi_t} \right)^{\rho_w} \tag{5}$$

$$\lambda_{B,t} p_t^D = \frac{(1-\eta)}{D_{B,t}} + m\psi_t p_t^D \lambda_{B,t} + \beta_B E_t \left[\lambda_{B,t+1} p_{t+1}^D (1-\delta_D) \right] \tag{6}$$

$$(1+i_t)\psi_t = 1 - \beta_B E_t \left[\frac{\lambda_{B,t+1}(1+i_t)}{\lambda_{B,t}\Pi_{t+1}} \right] \tag{7}$$

$$D_{B,t} = C_{B,t}^D + (1-\delta_D)D_{B,t-1} \tag{8}$$

$$(1+i_t)B_{B,t} = mp_t^D D_{B,t} \tag{9}$$

$$p_t^N C_{B,t}^N + p_t^D C_{B,t}^D + \frac{(1+i_{t-1})B_{B,t-1}}{\Pi_t} = B_{B,t} + w_t H_{B,t}^D + w_t H_{B,t}^N \tag{10}$$

31

$$\eta \left(\frac{1}{C_{S,t}^N - hC_{S,t-1}^N} - h\beta_S E_t \left[\frac{1}{C_{S,t+1}^N - hC_{S,t}^N} \right] \right) = \lambda_{S,t} p_t^N \tag{11}$$

$$w_t = \left(\frac{\nu H_{S,t}^\chi}{\lambda_{S,t}} (1 + \mu^w) \right)^{1-\rho_w} \left(\frac{w_{t-1}}{\Pi_t} \right)^{\rho_w} \tag{12}$$

$$\lambda_{S,t} p_t^D = \frac{(1-\eta)}{D_{S,t}} + \beta_S E_t \left[\lambda_{S,t+1} p_{t+1}^D (1 - \delta_D) \right] \tag{13}$$

$$D_{S,t} = C_{S,t}^D + (1 - \delta^D) D_{S,t-1} \tag{14}$$

$$\lambda_{S,t} = \beta_S E_t \left[\frac{\lambda_{S,t+1}(1 + i_t)}{\Pi_{t+1}} \right] \tag{15}$$

$$w_t(1 + \mu_t) = (1 - \alpha^D) mc_t^D A_t (K_t^D)^{\alpha_D} (H_t^D)^{-\alpha_D} \tag{16}$$

$$w_t = (1 - \alpha^N) mc_t^N A_t (K_t^N)^{\alpha_N} (H_t^N)^{-\alpha_N} \tag{17}$$

$$(1 + \mu_t) p_t^D = mk_t^D \left[1 - \frac{\theta_D}{2} \left(\frac{I_t^D}{I_{t-1}^D} - 1 \right)^2 - \theta_D \left(\frac{I_t^D}{I_{t-1}^D} - 1 \right) \left(\frac{I_t^D}{I_{t-1}^D} \right) \right]$$
$$+ \beta_S E_t \left[mk_{t+1}^D \theta_D \left(\frac{I_{t+1}^D}{I_t^D} - 1 \right) \left(\frac{I_{t+1}^D}{I_t^D} \right) \right] \tag{18}$$

$$p_t^D = mk_t^N \left[1 - \frac{\theta_N}{2} \left(\frac{I_t^N}{I_{t-1}^N} - 1 \right)^2 - \theta_N \left(\frac{I_t^N}{I_{t-1}^N} - 1 \right) \left(\frac{I_t^N}{I_{t-1}^N} \right) \right]$$
$$+ \beta_S E_t \left[mk_{t+1}^N \theta_N \left(\frac{I_{t+1}^N}{I_t^N} - 1 \right) \left(\frac{I_{t+1}^N}{I_t^N} \right) \right] \tag{19}$$

$$mk_t^D = \beta_S E_t \left[\left(\frac{\lambda_{S,t+1}}{\lambda_S} \right) \left(A_{t+1} \alpha_N K_{t+1}^{D \ \alpha_D - 1} H_{t+1}^{D \ 1 - \alpha_D} mc_{t+1}^D + mk_{t+1}^D (1 - \delta_K) \right) + \xi p_{t+1}^D \mu_{t+1} \right] \tag{20}$$

$$mk_t^N = \beta_S E_t \left[\left(\frac{\lambda_{S,t+1}}{\lambda_S} \right) \left(A_{t+1} \alpha_N K_{t+1}^{N \ \alpha_N - 1} H_{t+1}^{N \ 1 - \alpha_N} mc_{t+1}^N + mk_{t+1}^N (1 - \delta_K) \right) \right] \tag{21}$$

$$w_t H_t^D + p_t^D I_t^D = \xi p_t^D K_t^D \tag{22}$$

$$\left[(1-\epsilon_D)p_t^D + \epsilon_D mc_t^D\right] - \phi_D \left(\Pi_t^D - 1\right)\Pi_t^D + \beta_S\phi_D E_t\left[\left(\frac{\lambda_{S,t+1}}{\lambda_{S,t}}\right)\left(\Pi_{t+1}^D - 1\right)\Pi_{t+1}^D\left(\frac{Y_{t+1}^D}{Y_t^D}\right)\right] = 0 \tag{23}$$

$$\left[(1-\epsilon_N)p_t^N + \epsilon_N mc_t^N\right] - \phi^N \left(\Pi_t^N - 1\right)\Pi_t^N + \beta_S\phi_N E_t\left[\left(\frac{\lambda_{S,t+1}}{\lambda_{S,t}}\right)\left(\Pi_{t+1}^N - 1\right)\Pi_{t+1}^N\left(\frac{Y_{t+1}^N}{Y_t^N}\right)\right] = 0 \tag{24}$$

$$Y_t^D = A_t\left(K_t^D\right)^{\alpha_D}\left(H_t^D\right)^{1-\alpha_D} \tag{25}$$

$$Y_t^N = A_t\left(K_t^N\right)^{\alpha_N}\left(H_t^N\right)^{1-\alpha_N} \tag{26}$$

$$K_{t+1}^D = (1-\delta_K)K_t^D + I_t^D\left[1 - \frac{\theta_D}{2}\left(\frac{I_t^D}{I_{t-1}^D} - 1\right)^2\right] \tag{27}$$

$$K_{t+1}^N = (1-\delta_K)K_t^N + I_t^N\left[1 - \frac{\theta_N}{2}\left(\frac{I_t^N}{I_{t-1}^N} - 1\right)^2\right] \tag{28}$$

$$\omega H_{S,t}^D + (1-\omega)H_{B,t}^D = H_t^D \tag{29}$$

$$\omega H_{S,t}^N + (1-\omega)H_{B,t}^N = H_t^N \tag{30}$$

$$\omega C_{S,t}^D + (1-\omega)C_{B,t}^D = C_t^D \tag{31}$$

$$\omega C_{S,t}^N + (1-\omega)C_{B,t}^N = C_t^N \tag{32}$$

$$\omega D_{S,t} + (1-\omega)D_{B,t} = D_t \tag{33}$$

$$\omega B_{S,t} + (1-\omega)B_{B,t} = 0 \tag{34}$$

$$K_t^D + K_t^N = K_t \tag{35}$$

$$C_t^D + I_t^D + I_t^N + \frac{\phi_D}{2}\left(\Pi^D - 1\right)^2 Y_t^D = Y_t^D \tag{36}$$

$$C_t^N + \frac{\phi_N}{2}\left(\Pi^N - 1\right)^2 Y_t^N = Y_t^N \tag{37}$$

$$A_t = A_{t-1}^{\rho^A} \exp\left(e_t^A\right) \tag{38}$$

$$\beta_S(i + i_t) = (\beta_S(i_{t-1}))^\rho \left(\Pi_t^{\phi_\Pi}\right)^{1-\rho} \exp(e_t^M) \tag{39}$$

$$\Pi_t^D = \frac{p_t^D}{p_{t-1}^D}\Pi_t \tag{40}$$

$$\Pi_t^N = \frac{p_t^N}{p_{t-1}^N}\Pi_t \tag{41}$$

$$1 = (p_t^N)^\eta (p_t^D)^{1-\eta} \tag{42}$$

3.3 Model with Interest Rate Wedge

In the baseline model, firms are able to borrow at zero net interest and thus are not directly affected by changes in the cost of capital. This is a conservative assumption, as increases in the cost of capital will exacerbate the constraints faced by durable producers. In practice, as shown in Figure 15, contractionary monetary shocks have about the same effects on the interest rates of durable and nondurable producers. As a result, sectoral differences in the empirical user cost responses are driven mostly by changes in relative prices.

The model can easily be modified to allow for borrowing costs of constrained firms to increase in response to the monetary contraction. In this alternate setup, the equilibrium conditions for households and nondurable producers are unchanged; the only difference is that durable producers now have to pay interest (at the risk-free rate) on the funds they borrow to purchase capital and labor. The modified equations are:

$$w_t(1 + \mu_t)(1 + i_t) = (1 - \alpha^D)mc_t^D A_t(K_t^D)^{\alpha^D}(H_t^D)^{-\alpha^D} \tag{43}$$

$$(1 + \mu_t)p_t^D(1 + i_t) = mk_{D,t}\left[1 - \frac{\theta_D}{2}\left(\frac{I_{D,t}}{I_{D,t-1}} - 1\right)^2 - \theta_D\left(\frac{I_{D,t}}{I_{D,t-1}} - 1\right)\left(\frac{I_{D,t}}{I_{D,t-1}}\right)\right] \\ + \beta_S E_t\left[mk_{t+1}^D\theta_D\left(\frac{I_{D,t+1}}{I_{D,t}} - 1\right)\left(\frac{I_{D,t+1}}{I_{D,t}}\right)\right] \tag{44}$$

$$(1 + i_t)\left(w_t H_t^D + p_t^D I_t^D\right) = \xi p_t^D K_t^D \tag{45}$$

The impulse responses incorporating these modifications are shown in Figure 23 and are virtually indistinguishable from the baseline results because, as in the data, interest rates are relatively small drivers of user cost compared to the relative price of investment.

3.4 Model with Sticky Prices in Both Sectors

Even though the assumption that durable prices are more flexible is supported by existing empirical work, my results do not depend on it. Calibrations which use the baseline non-durable price stickiness for both sectors ($\phi_D = \phi_N = 58.25$) lead to an increase in investment in both sectors in response to the contractionary shock, but this depends on the calibration of the other parameters. Even with this higher degree of price stickiness, the model is able to generate the appropriate responses of investment in the case of tighter financial constraints (setting $\xi = 0.04$ instead of its baseline value of 0.1). The IRFs are shown in Figure 22. This suggests that imposing equal degrees of price stickiness will not automatically lead to behavior inconsistent with the main mechanisms described in my paper.
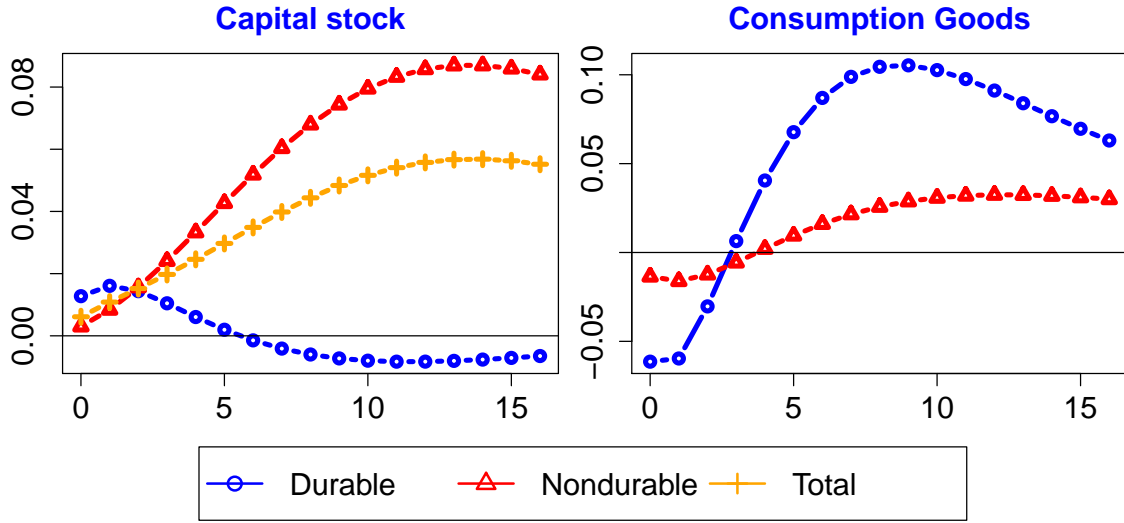
Figure 22: Model IRFs with Sticky Prices in Both Sectors

Note: This figure shows the impulse responses to a 100bp contractionary monetary shock when the price stickiness of both sectors is set to be $\phi_N = \phi_D = 58.25$ and the financial constraint parameter is set to $\eta = 0.4$. All other parameters remain the same as in Table 2.
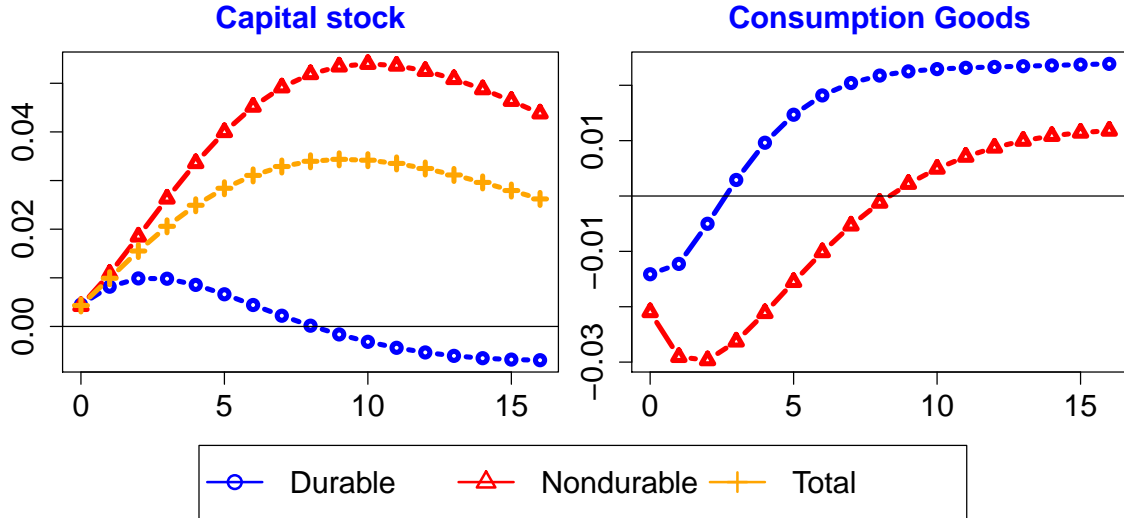

Figure 23: Model IRFs with Interest Rate Wedge

Note: This figure shows the impulse responses to a 100bp contractionary monetary shock when durable producers must pay the risk-free rate on their loans. The modified equilibrium conditions are described in Section 3.3. All parameters remain the same as in Table 2.

3.5 Additional Model Figures

This section shows several additional impulse responses from the model. A comparison of the responses of consumption and investment is shown in Figure 24. The left column reproduces the analysis from the main paper by showing the responses of the capital stock of each type of producer in both the model (the top row) and the data (the bottom row). The right column shows the responses of nondurable consumption expenditure and the stock of consumer durable goods. The bottom-right panel shows that the responses of both nondurable consumption and the stock of durable goods decline by about 1% before returning to their pre-shock level. The model matches the responses of nondurable consumption relatively well, though with a larger peak effect. The on-impact decline in the stock of durable goods matches the peak effect seen in the data, though it recovers quickly and ends up stabilizing at a level above where it started. This is due to the combination of the persistent drop in prices and the lack of adjustment frictions on the part of consumer durables.
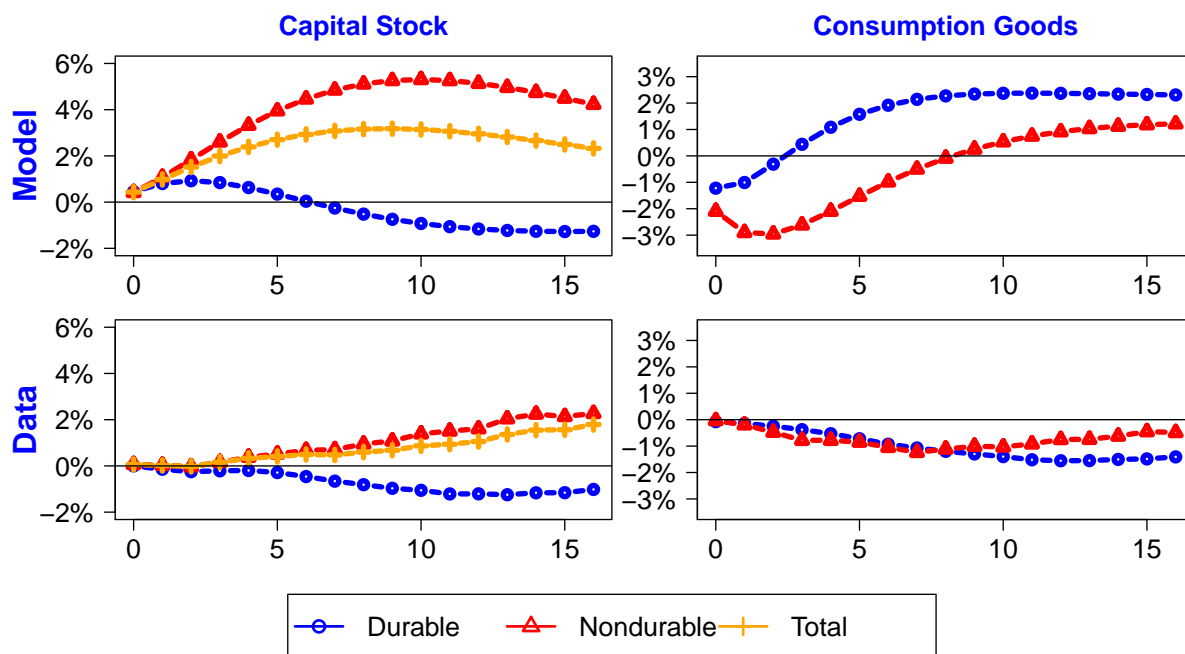

Figure 24: Model and Data IRFs to Contractionary MP Shock

Note: The top row shows the model responses to a 100bp contractionary monetary shock for the capital stock (left side) as well as the level of nondurable consumption and stock of consumer durable goods (right side). The bottom row shows the empirical estimates to a 100bp contractionary monetary policy shock. Empirical measures of consumer durables are reported as stocks while consumer nondurables are reported as expenditure flows. The capital stock estimates on the left are the same as those shown in the baseline empirical results. The nondurable consumption series comes from the BEA's real personal consumption expenditure data. The data for the stock of consumer durable goods come from the BEA's real fixed asset accounts and do not include housing.

3.6 Theoretical Basis for Lending Frictions

The New Keynesian model used in the paper treated durable goods producers as exogenously subject to financing constraints. This section outlines a plausible theoretical mechanism that would endogenously lead to such frictions: the fact that durable producers face more volatile demand for their product due to its longevity.

I use the workhorse model developed in in Tirole (2010) to analyze this mechanism for several reasons. First, the model is simple, tractable, and allows for analytic results. Second, the model is flexible enough to easily incorporate a stylized type of demand volatility. Finally, the solution to the model is an "investment multiplier" which says that the amount of funds that a firm is able to raise is a linear function of the value of its assets, which is the same functional form as the one found in the paper's more elaborate New Keynesian model. While not all of the parameters which determine the investment multiplier in this simplified context have direct counterparts in the larger model, this section provides justification for my choice of working capital constraint and allows for some insightful comparative static exercises.

3.6.1 The Simple Model

There is a risk-neutral entrepreneur with sole access to the technology to produce their good. The production of the project is a function of the investment X and effort $e \in \{l, h\}$ put into it. The entrepreneur has net worth A that can be invested in the project; if he wishes to invest $X > A$, he must borrow $L = X - A$ from the banking sector, which is perfectly competitive and risk neutral.

The financing of the projects is non-trivial due to the presence of a moral hazard problem. If the entrepreneur exerts high effort e_h, the project succeeds with probability p_h and produces according to the linear "production" function RX where R is the productivity or return of the project and $X \in [0, \infty)$. If the entrepreneur exerts low effort, the project succeeds with probabilty $p_l < p_h$ and the entrepreneur receives private benefits proportional to the level of investment BX. I assume that $p_h R > 1 > p_l R + B$, which tells us that the project is only NPV positive on a per-unit basis in the case of high effort, and $p_h R < 1 + \frac{p_h B}{\Delta p}$, which leads to a bounded quantity of investment.

Because effort is not observable the contract cannot directly reward the entrepreneur for working hard, so it must be set up in an incentive-compatible manner to prevent them from running away with the money. This means that the entrepreneur must have enough "skin in the game" such that their private benefit from working hard exceeds their gains from shirking. The contracting problem will have individual rationality (IR) constraints for both the borrower and lender and an incentive compatibility (IC) constraint for the borrower.

Formally, their problem will be to split the investment X and total expected successful return R into separate pieces for both the lenders and borrowers. Incentive compatibility will require that the expected gain for the producer exceeds the private benefit of shirking:

$$R_b(e_h)X \geq R_b(e_l)X \implies p_h R_b X \geq p_l R_b X + BX \implies R_b X \geq \frac{BX}{\Delta p} \tag{46}$$

Here I've defined $\Delta p \equiv (p_h - p_l)$ to be the improvement in success probability that results from hard work. Because the per-unit net return of the project $(p_h R - 1)$ is greater than 1, the constrained investors will always have incentives to invest more in the project and they will only be limited by the set of contracts agreeable to the bank. Thus, their IC constraint will bind $(R_b X = \frac{BX}{\Delta p})$ and their IR constraint will be slack. The positive net return will result in constrained investors optimally pledging their full wealth A to the project so that $L = X - A$.

I now write the IR constraint for the bank knowing that the optimal contract will induce high effort on the part of the firm and that shirking will not be observed in equilibrium. I also allow for an outside option of investing their funds to earn a risk-free gross interest rate of $(1 + i)$:

$$(1+i)L \geq p_h R_l X \implies (1+i)(X - A) \geq p_h [RX - R_b X] \tag{47}$$

The second inequality holds because the lender's return can be written as the total return minus the portion promised to the borrower. Because the entrepreneurs have market power in this setup, the IR constraint will bind for the bank and they will receive expected net returns of zero in equilibrium. Thus, combining equations (1) and (2) leads to following condition:

$$(i+i)(X - A) = p_h [RX - R_b X] \implies (1+i)(X - A) = p_h \left[RX - \frac{BX}{\Delta p} \right]$$

$$\implies X \left[1 - \frac{p_h}{1+i} \left(R - \frac{B}{\Delta p} \right) \right] = A \implies X = \left(\frac{1}{1 - \frac{p_h}{1+i} \left[R - \frac{B}{\Delta p} \right]} \right) A \tag{48}$$

Re-write the utility function as a linear function of X and then plug in the investment

multiplier derived above to write the borrower's net utility as follows:

$$U^B = (p_h R - 1)X = \left(\frac{p_h R - 1}{1 - \frac{p_h}{1+i}\left[R - \frac{B}{\Delta p}\right]} \right) A \tag{49}$$

Because all firms have constant returns to scale and the project has positive NPV, they will always want to invest as much as possible. The model solution will be an "investment multiplier" k that reflects the return of the project, the outside interest rate, the project's probaiblity of success, and the severity of the moral hazard problem. In this setup, because R is known by both parties before the investment is sunk, the contract can be interpreted as either debt or equity.

3.6.2 Implications of Demand Volatility

A simple way to extend the model to allow for durable goods to have more volatile demand is to treat the parameter R as a random variable that is realized after financing is obtained but prior to effort being exerted. In this setup the per-unit returns to investment can be thought of as the price of the good being sold; in this context, durable producers face more volatile returns because their good is longer-lived, and this longevity makes intertemporal substitution easier and leads to a more volatile price. In this section I show that the combination of volatile returns and equity contracts will cause the investment multiplier to decrease in the case of a mean-preserving spread in the return.

The simplest illustration of how demand volatility can influence terms of equity is in the discrete case. Instead of being deterministic as in the previous section, the return $\tilde{R}$ is now a random variable that is realized after investment has been sunk but before effort has been exerted. It takes on a value of R_0 with probability θ and R_1 with probability $1 - \theta$. Define the expected return $\bar{R} \equiv \theta R_0 + (1 - \theta)R_1$. In expectation the investment project is NPV positive in the case of high effort: $p_h \bar{R} > 1 > p_l \bar{R} + B$. As a result, the entrepreneur will want to exert effort when the high return R_h is realized. If R_0 is realized, there is no surplus to be gained from exerting effort since $p_l R_0 + B > p_l R_0$, so the entrepreneur will slack.[5]

If the borrower could credibly commit to working hard regardless of the realization of $\tilde{R}$, then they would be able to promise a higher return to the lender and receive more financing. However, because the bank knows that the entrepreneur will not exert effort if

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[5]Once financed, the funds can only be allocated toward the project. This prevents the entrepreneur from simply "running away with the money" and earning a net return of 1 if R_0 is realized, which would be higher than the expected value of shirking on the project.

R_0 is realized, they will internalize this outcome when making their lending decision and subsequently reduce the available quantity of funds. In this sense it is the bank who bears the downside risk to bad realizations of $\tilde{R}$ while the entrepreneur captures the upside. It is this fundamental asymmetry that allows volatility to exacerbate financial constraints even when all agents are risk neutral.

The optimal equity contract will involve the borrower receiving a share γ of the proceeds of the project regardless of outcome. If R_0 is realized, the borrower will find it optimal not to exert effort, and the gross expected return will be $p_l R_0$. If R_1 is realized, the borrower will find it optimal to exert effort, and the gross return will be $p_h R_1$. The incentive compatibility condition requires $\gamma p_h R_1 \geq \gamma p_l R_1 + B \implies \gamma = \frac{B}{R_1 \Delta p}$.

The lender will receive a per-unit share of $1 - \gamma$ of the per-unit return of the project, which can be written $\hat{R} \equiv \theta p_l R_0 + (1-\theta) p_h R_1$. Their IR constraint requires that they receive in expectation enough to keep them indifferent between investing and earning the risk-free rate: $(1+i)(X - A) = X(1 - \gamma)\hat{R}$. Plugging in the borrower's IC constraint yields the model's solution:

$$(1+i)(X - A) = X \left(1 - \frac{B}{\Delta p R_1}\right) \hat{R} \implies X = \left[\frac{1}{1 - \left(\frac{1 - \frac{B}{\Delta p R_1}}{(1+i)} \hat{R}\right)}\right] A \qquad (50)$$

If $\theta = 0$, then $\hat{R} = p_h R_1$, and the solution collapses to that of the previous section. In this deterministic case, the borrowers would expect to receive $(1 - \gamma) p_h \bar{R}$. In the presence of moral hazard, however, the fact that the borrowers will not exert effort if R_0 is realized prevents the lender from earning this return. Instead, they earn $(1 - \gamma)\hat{R}$. This difference can be written:

$$p_h \bar{R} - \hat{R} = p_h \left(\theta R_0 + (1 - \theta) R_1\right) - (\theta p_l R_0 + (1 - \theta) p_h R_1) = \theta \Delta p R_0 \qquad (51)$$

As long as $R_0 > 0$, this difference will be positive, which means that the investment multiplier will be larger in the deterministic case even when the expected returns are the same. The fact that the benefits of shirking only accrue to the borrower and not the lender lead to a lower investment multiplier for more volatile projects.

3.6.3 Relationship to DSGE Model

In these models the solution is an investment multiplier of the form $X = \xi A$ where X was the amount of funds obtained by the entrepreneur and invested in the project, A is the value of the entrepreneur's assets pledged toward the project, and ξ is the multiplier that links the two. If $\xi_i > \xi_j$, then firm i is able to obtain a greater amount of financing for the same initial level of assets, and thus firm i can be interpreted as less financially constrained than firm j. The previous section showed that $\xi_{baseline} > \xi_{volatile}$, showing that firms facing a mean-preserving spread in the volatility of their expected returns would be able to obtain a smaller financing multiplier:

$$\left(\frac{1}{1 - \frac{p_h}{1+i}\left[\bar{R} - \frac{B}{\Delta p}\right]} \right) > \left(\frac{1}{1 - \left(\frac{1 - \frac{B}{\Delta p R_1}}{(1+i)}\hat{R} \right)} \right) \tag{52}$$

The conceptual link between this simple model and the more complex DSGE model in the body of the paper is quite clear. In that model, the main borrowing constraint for durable producers was:

$$w_t H_t^D + p_t^D I_t^D = \xi p_t^D K_t^D \tag{53}$$

The total amount invested in the "project" each period- which in this case corresponds to the production of durable goods- is simply the total expenditure on labor and capital, so $X = w_t H_t^D + p_t^D I_t^D$. The total amount of assets available to the producer each period is simply the value of their capital stock, so $A = p_t^D k_t^D$. Putting these together, this becomes $X = \xi A$, which is precisely the same functional form as in the baseline model.

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