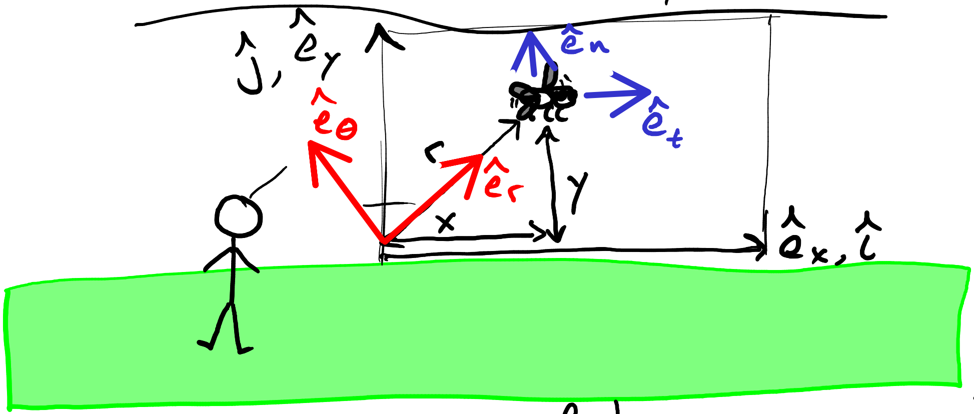
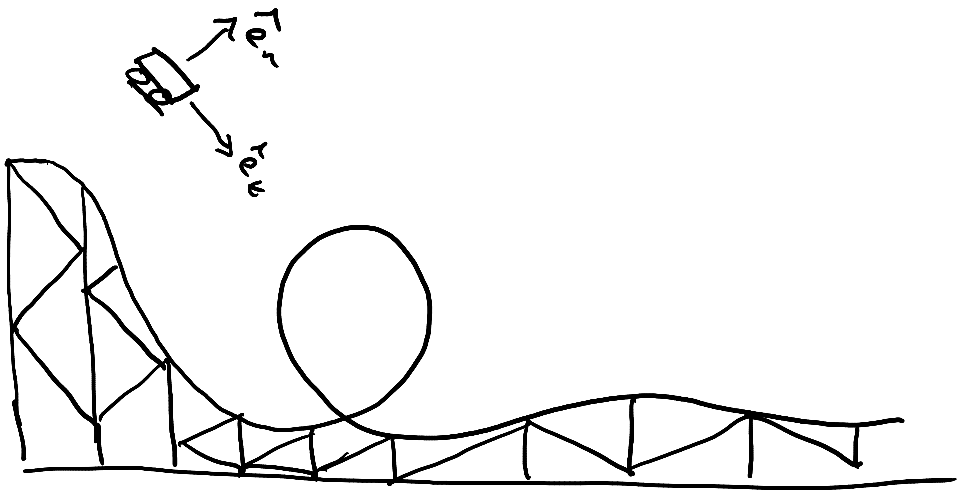
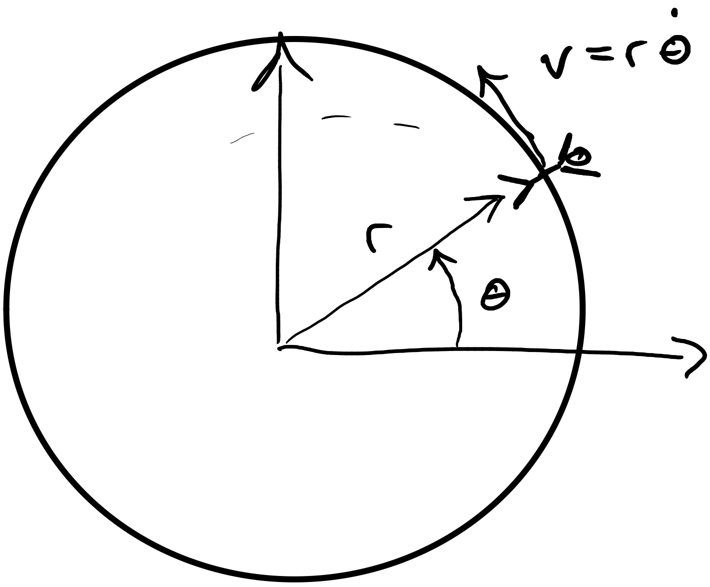
coordinate systems



	Cartesian	Polar	Intrinsic
position	$\bar{r} = x\hat{e}_x + y\hat{e}_y$	$= r\hat{e}_r$	$= 0\hat{e}_t + 0\hat{e}_n$
velocity	$\bar{v} = \dot{\bar{r}} = \dot{x}\hat{e}_x + \dot{y}\hat{e}_y$	$= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	$= v\hat{e}_t$
accel.	$\bar{a} = \dot{\bar{v}} = \ddot{x}\,\hat{e}_x + \ddot{y}\,\hat{e}_y$	$= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$	$= \frac{dv}{dt}\hat{e}_t + \frac{v^2}{R}\hat{e}_n$





$\theta = \omega t$

$\dot{\theta} = \omega$

$\dot{r} = 0$

$\ddot{r} = 0$

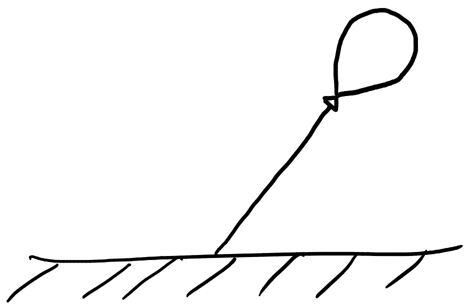
$$\bar{r} = r\hat{e}_r = r(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$\bar{v} = \overset{0}{\cancel{\dot{r}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})}} + r\dot{\theta}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$\bar{a} = -r\dot{\theta}^2(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) = \boxed{-\frac{v^2}{r}\hat{e}_r}$$

$$|\bar{v}| = r\dot{\theta} = v$$

$$r\dot{\theta}^2 = \frac{v^2}{r} = \frac{(r\dot{\theta})^2}{r}$$

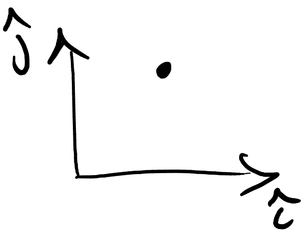


3.34

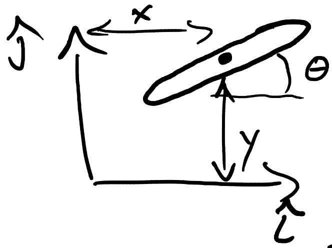
the balloon. Find the equations of motion of the balloon relative to the fixed point using the following steps:

a. Sketch a point-mass model of the balloon and string.
b. Choose coordinates and define reference frame(s).
c. Draw a free-body diagram of the balloon.
d. Derive the inertial kinematics of the balloon.
e. Write down Newton's second law for the balloon and solve for its equation(s) of motion.

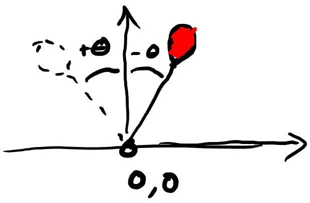
point mass vs rigid body



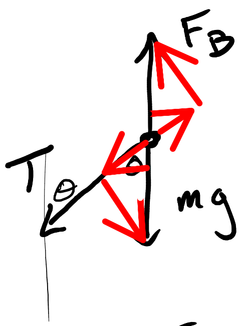
ΣF_x
ΣF_y



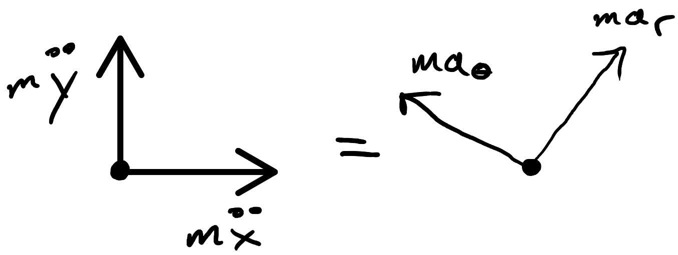
$$\Sigma F_x = m\ddot{x}$$
$$\Sigma F_y = m\ddot{y}$$
$$\Sigma M_G = I\ddot{\theta}$$



FBD



KD



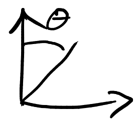
$$\Sigma F_x = -T\sin\theta = m\ddot{x}$$

$$\Sigma F_y = -T\cos\theta + F_B - mg = m\ddot{y}$$

$$\sum F_r = -T - mg\cos\theta + 2mg\cos\theta = m(-r\dot{\theta}^2)$$

$$\sum F_\theta = -mg\sin\theta + 2mg\sin\theta = m(r\ddot{\theta})$$

$$? \overset{eom}{\Longrightarrow}$$



$$\ddot{x} = -\frac{T}{m}\sin\theta$$

$$\ddot{y} = -\frac{T}{m}\cos\theta + g$$

$$x = r\sin\theta$$

$$y = r\cos\theta$$

$$\dot{x} = r\dot{\theta}\cos\theta$$

$$\dot{y} = -r\dot{\theta}\sin\theta$$

$$\ddot{x} = -r\dot{\theta}^2\sin\theta + r\ddot{\theta}\cos\theta$$

$$\ddot{y} = -r\dot{\theta}^2\cos\theta - r\ddot{\theta}\sin\theta$$