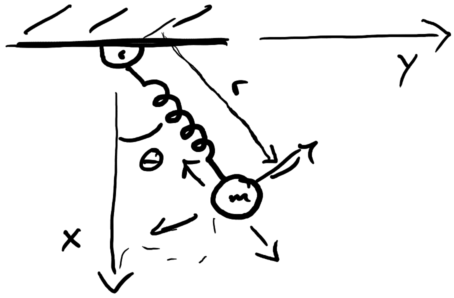
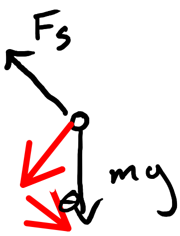
**3.23** Consider the simple planar pendulum discussed at length in this chapter, only now, rather than a solid massless rod, suppose the pendulum bob is attached to a spring with spring constant $k$ (Figure 3.45). Note that the spring does not bend or twist.

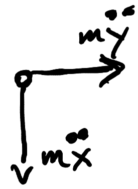
a. How many degrees of freedom are there in this problem?
b. What coordinates would you use to describe the configuration of the pendulum bob?



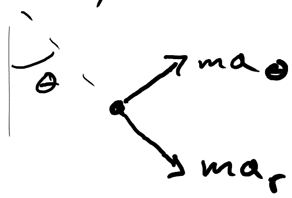

FBD




KD x, y




KD r, θ




eoms

$\Sigma F_r$ ① $\quad -F_s + mg\cos\theta = m(\ddot r - r\dot\theta^2)$

$\Sigma F_\theta$ ② $\quad -mg\sin\theta = m(2\dot r\dot\theta + r\ddot\theta)$

$$\Longrightarrow \quad -\frac{k}{m}(r-l_0) + g\cos\theta + r\dot\theta^2 = \ddot r$$
$$-g\sin\theta - 2\dot r\dot\theta = r\ddot\theta$$

eoms

$$\begin{matrix} ① \\ ② \end{matrix} \begin{bmatrix} \ddot r \\ r\ddot\theta \end{bmatrix} = \begin{bmatrix} r\dot\theta^2 + g\cos\theta - \frac{k}{m}r + \frac{kl_0}{m} \\ 2\dot r\dot\theta - g\sin\theta \end{bmatrix}$$

Bonus Math

$\theta << 1$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$
$$\sin\theta = \theta + \frac{\theta^3}{3!} - \frac{\theta^5}{5!} + \dots$$

linear approx

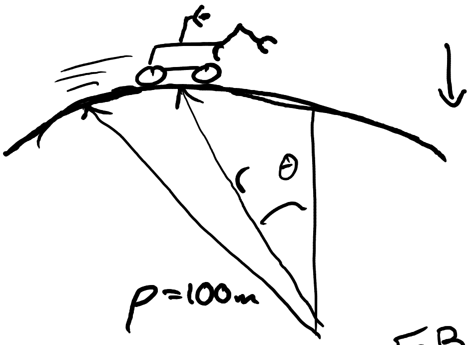
HOTs

linear eom $\rightarrow$ $\ddot r = \cancel{r\dot\theta^2}^{\,0} + g - \frac{k}{m}r + \frac{kl_0}{m}$

$r\ddot\theta = 2\cancel{\dot r}^{\,0}\dot\theta - g\sin\theta$

spring eqn $\rightarrow$ $\ddot{r} + \frac{k}{m} r = g + \frac{k l_0}{m}$

pendulum eqn $\rightarrow$ $\ddot{\theta} + \frac{g}{r} \theta = 0$

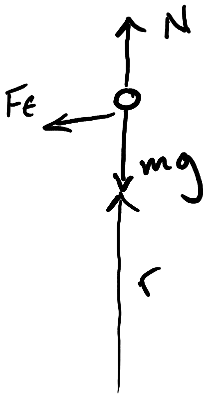



$m = 200\,kg$

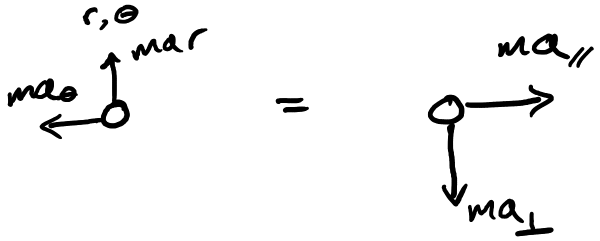
$\downarrow g = 1.6\ m/s^2$

$\frac{v^2}{R} = ?$

FBD




KD $_{r,\theta}$ KD




$a_{\parallel} = \frac{dv}{dt}$

$a_{\perp} = \frac{v^2}{R}$