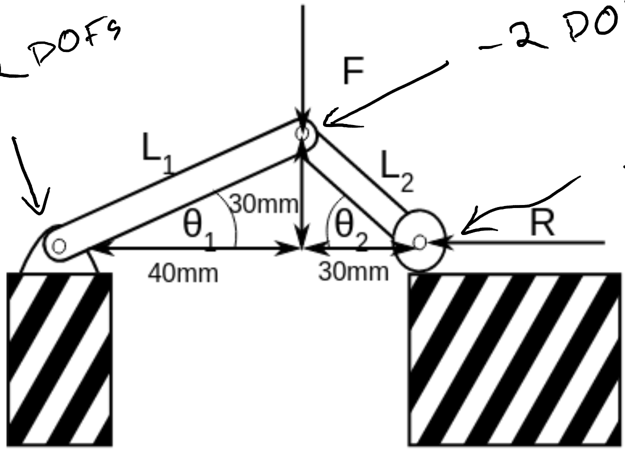
-2 DOFs

-2 DOFs



slides on x-axis
-1 DOF

6
-2
-2
-1

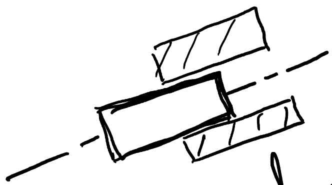
$\boxed{1 \text{ DOF}}$

Constraints

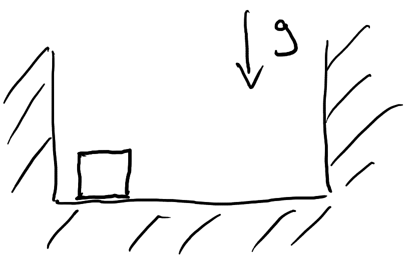
fixed 3 constraints

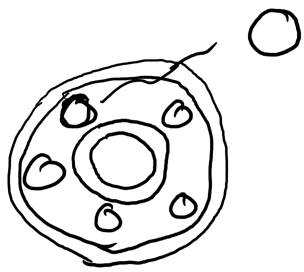
pin or hinge 2 constraints

prismatic constraint

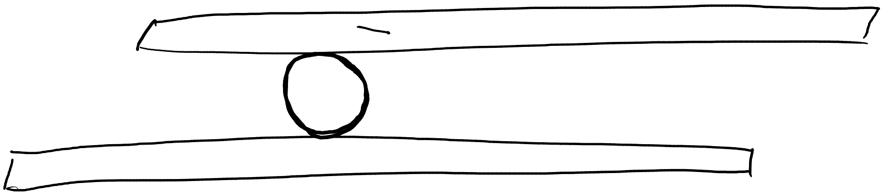


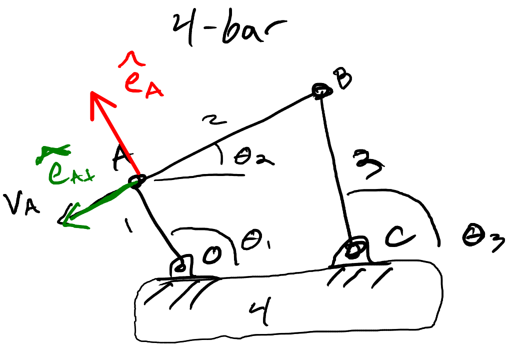
fixed rotation
& fixed path





ball bearings → 1 DOF





$l_1 = 0.25\ m,\ l_2 = 1\ m,\ l_3 = 1\ m$

$dx = 1\ m\ and\ dy = 0\ m$

at time, t, link 1 (OA) is rotating at 10 rad/s. The positions of the pins are as follows

$r_0 = 0\hat{i} + 0\hat{j}$ [m]

$r_A = -0.203\hat{i} + 0.1459\hat{j}$ [m]

$r_B = 0.494\hat{i} + 0.863\hat{j}$ [m]

$r_C = 1\hat{i} + 0\hat{j}$ [m]

What are the rotation rates for links 2 and 3 (AB and BC, respectively)

$\dot{\theta}_2$? $\dot{\theta}_3$?

piston-crank

know

$\dot{\theta}_1 = 10\ ^{rad}/_s$

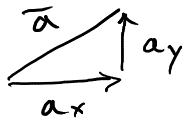
$\bar{r}_A = -0.203\hat{i} + 0.1459\hat{j}$ m

$|\bar{r}_A| = 0.25$ m

$\swarrow V_A = 2.5\ ^m/_s = l_1 \dot{\theta}_1\ ^m/_s$

$$\hat{e}_A = \frac{-0.203}{0.25}\hat{i} + \frac{0.1459}{0.25}\hat{j}$$

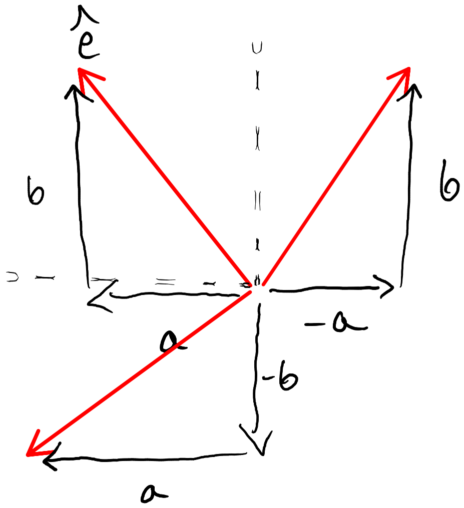
$$\hat{e}_{A\perp} = \frac{-0.1459}{0.25}\hat{i} - \frac{0.203}{0.25}\hat{j}$$



$$\cos\theta = \frac{a_x}{|\bar{a}|}$$

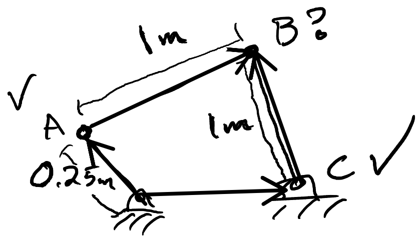
$$\sin\theta = \frac{a_y}{|\bar{a}|}$$

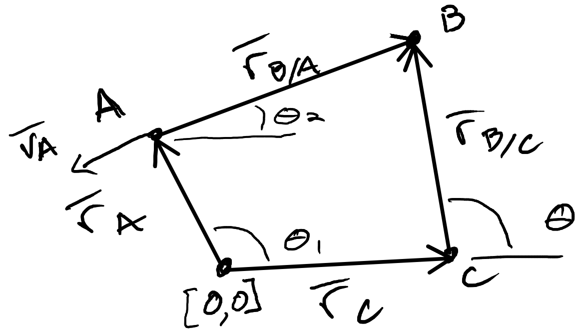
$$\boxed{\begin{aligned}\hat{e} &= \cos\theta\,\hat{i} + \sin\theta\,\hat{j} \\ \hat{e}_\perp &= -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\end{aligned}}$$



$$\bar{V}_A = 2.5\ ^m/_s\left(-\frac{0.1459}{0.25}\hat{i} - \frac{0.203}{0.25}\hat{j}\right)$$

$$\bar{V}_c = 0\hat{i} + 0\hat{j}\ ^m/_s$$





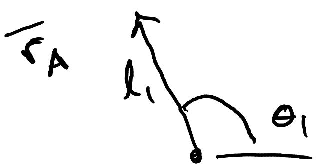
$$v_A = l_1 \dot\theta_1 \hat e_A'$$
$$\bar V_{B/A} = l_2 \dot\theta_2 \hat e_{B/A}'$$
$$\bar V_{B/C} = l_3 \dot\theta_3 \hat e_{B/C}'$$

position cstrs $\dfrac{d}{dt}\left(\bar r_A + \bar r_{B/A} = \bar r_c + \bar r_{B/C} \right)$?

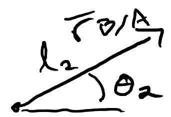
velocity cstrs $\bar V_A + \bar V_{B/A} = \underbrace{\cancel{\bar V_c}}_{0} + \bar V_{B/C}$

$$\bar V_A = 2.5 \tfrac{m}{s} \left(-\frac{0.1459}{0.25} \hat\imath - \frac{0.203}{0.25} \hat\jmath \right)$$

$$\bar V_c = 0 \hat\imath + 0 \hat\jmath \ \tfrac{m}{s}$$



$$\bar r_A = l_1 (\cos\theta_1 \hat\imath + \sin\theta_1 \hat\jmath)$$



$$\bar r_{B/A} = l_2 (\cos\theta_2 \hat\imath + \sin\theta_2 \hat\jmath)$$

$$\bar r_{B/C} = l_3 (\cos\theta_3 \hat\imath + \sin\theta_3 \hat\jmath)$$

$$\bar r_c = d_x \hat\imath$$

cstrs on position $\Longrightarrow$ $l_1 (\cos\theta_1 \hat\imath + \sin\theta_1 \hat\jmath) + l_2 (\cos\theta_2 \hat\imath + \sin\theta_2 \hat\jmath)$

$$=$$

$$l_3 (\cos\theta_3 \hat\imath + \sin\theta_3 \hat\jmath) + d_x \hat\imath$$

$x \rightarrow \quad l_1 \cos\theta_1 + l_2 \cos\theta_2 = l_3 \cos\theta_3 + d_x$

$y \rightarrow \quad l_1 \sin\theta_1 + l_2 \sin\theta_2 = l_3 \sin\theta_3$

note

$$\boxed{\frac{d}{dt}\left(l \cos\theta \right) = \frac{d}{d\theta}\frac{d\theta}{dt}\left(l \cos\theta \right) = -l \sin\theta \left(\frac{d\theta}{dt}\right) = -l\dot{\theta}\sin\theta}$$

CHAIN RULE!!

$V_x \rightarrow -l_1 \dot{\theta}_1 \sin\theta_1 - l_2 \underline{\dot{\theta}_2} \sin\theta_2 = -l_3 \underline{\dot{\theta}_3} \sin\theta_3$

find $\dot{\theta}_2$ + $\dot{\theta}_3$

$V_y \rightarrow \quad l_1 \dot{\theta}_1 \cos\theta_1 + l_2 \underline{\dot{\theta}_2} \cos\theta_2 = l_3 \underline{\dot{\theta}_3} \cos\theta_3$

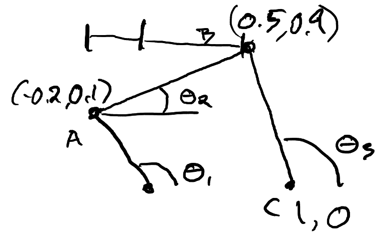
$\dot{\theta}_1 = 10 \text{ rad/s}$

$r_0 = 0\hat{\imath} + 0\hat{\jmath}$ [m]

$r_A = -0.203\hat{\imath} + 0.1459\hat{\jmath}$ [m]

$r_B = 0.494\hat{\imath} + 0.863\hat{\jmath}$ [m]

$r_C = 1\hat{\imath} + 0\hat{\jmath}$ [m]

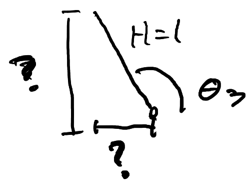


SOHCAHTOA

$\cos\theta_1 = \frac{-0.203}{0.25} \qquad \sin\theta_1 = \frac{0.14}{0.25}$

$\cos\theta_2 = \frac{0.494+0.203}{1} \qquad \sin\theta_2 = \frac{0.8}{1}$

$\cos\theta_3 = \frac{0.494-1}{1} \qquad \sin\theta_3 = \frac{0.9}{1}$



$l_1 \dot{\theta}_1 s\theta_1 + l_2 \dot{\theta}_2 s\theta_2 = l_3 \dot{\theta}_3 s\theta_3$

$\dot{\theta}_3 = \frac{l_1 \dot{\theta}_1 s\theta_1 + l_2 \dot{\theta}_2 s\theta_2}{l_3 s\theta_3}$

$l_1 \dot{\theta}_1 c\theta_1 + l_2 \dot{\theta}_2 c\theta_2 = l_3 c\theta_3 \left(\frac{l_1 \dot{\theta}_1 s\theta_1 + l_2 \dot{\theta}_2 s\theta_2}{l_3 s\theta_3} \right)$

$$\dot{\Theta} = \omega = \frac{d\theta}{dt}$$

?

equation of motion
is
ODE

$$\ddot{\Theta} = \dot{\omega} = -\frac{g}{L}\sin\theta$$

[pendulum
eom]

centripetal
accel

$$a_c = \frac{v^2}{R}$$

is one component
of intrisic accel.

$$a = \frac{dv}{dt}\hat{e}_{tan} + \frac{v^2}{R}\hat{e}_{\perp}$$