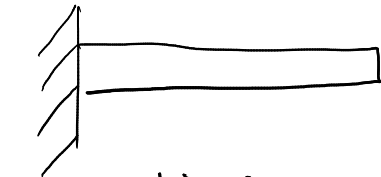


$$\begin{array}{r}
 6 \\
 -2 \\
 -2 \\
 -1 \\
 \hline
 1 \text{ DOF}
 \end{array}$$

Constraints

Fixed \rightarrow 3 constraints

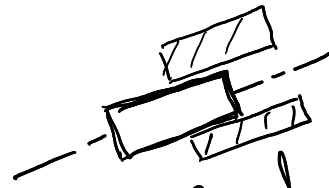


pin or hinge

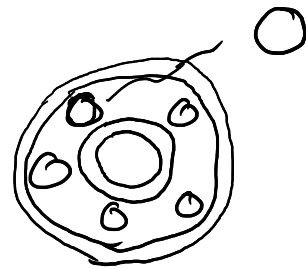
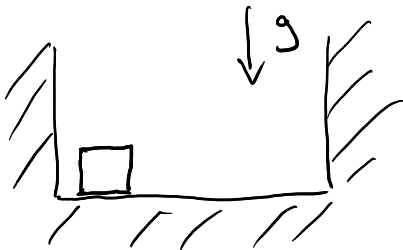
2 constraints



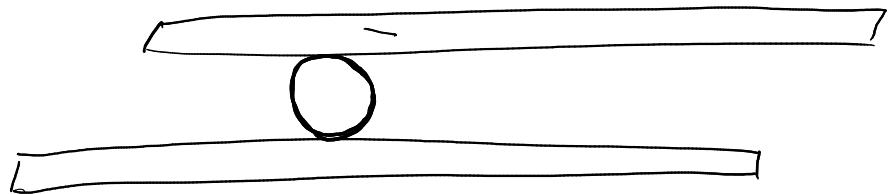
prismatic constraint

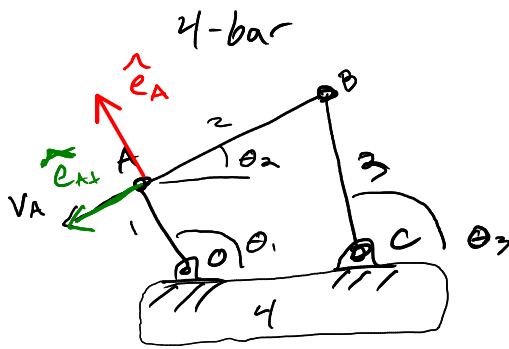


fixed rotation
& fixed path



ball bearings \rightarrow 1 DOF





$$l_1 = 0.25 \text{ m}, l_2 = 1 \text{ m}, l_3 = 1 \text{ m}$$

$$dx = 1 \text{ m and } dy = 0 \text{ m}$$

at time, t , link 1 (OA) is rotating at 10 rad/s. The positions of the pins are as follows

$$r_0 = 0\hat{i} + 0\hat{j} [\text{m}]$$

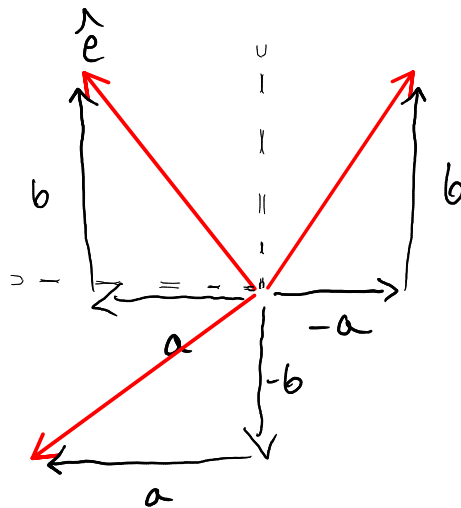
$$r_A = -0.203\hat{i} + 0.1459\hat{j} [\text{m}]$$

$$r_B = 0.494\hat{i} + 0.863\hat{j} [\text{m}]$$

$$r_C = 1\hat{i} + 0\hat{j} [\text{m}]$$

What are the rotation rates for links 2 and 3 (AB and BC, respectively)

$$\dot{\theta}_2 \quad \dot{\theta}_3$$



$$\bar{V}_A = 2.5 \text{ m/s} \left(-\frac{0.1459}{0.25} \hat{i} - \frac{0.203}{0.25} \hat{j} \right)$$

$$\bar{V}_C = 0\hat{i} + 0\hat{j} \text{ m/s}$$

piston-crank

know

$$\dot{\theta}_1 = 10 \text{ rad/s}$$

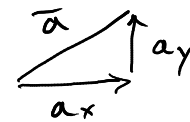
$$\bar{r}_A = -0.203\hat{i} + 0.1459\hat{j} \text{ m}$$

$$|\bar{r}_A| = 0.25 \text{ m}$$

$$\checkmark V_A = 2.5 \text{ m/s} = l_1 \dot{\theta}_1 \text{ m/s}$$

$$\hat{e}_A = \frac{-0.203}{0.25} \hat{i} + \frac{0.1459}{0.25} \hat{j}$$

$$\hat{e}_{A+} = -\frac{0.1459}{0.25} \hat{i} - \frac{0.203}{0.25} \hat{j}$$

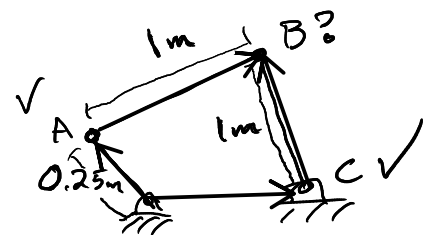


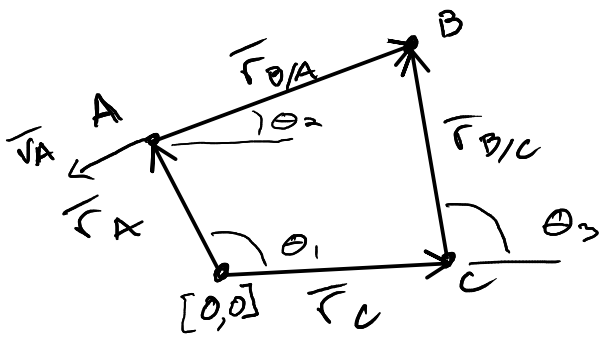
$$\cos \theta = \frac{a_x}{|\bar{a}|}$$

$$\sin \theta = \frac{a_y}{|\bar{a}|}$$

$$\hat{e} = \cos \theta \hat{i} + \sin \theta \hat{j}$$

$$\hat{e}_\perp = -\sin \theta \hat{i} + \cos \theta \hat{j}$$





$$v_A = l_1 \dot{\theta}_1 \hat{e}'_A$$

$$\bar{v}_{B/A} = l_2 \dot{\theta}_2 \hat{e}'_{B/A}$$

$$\bar{v}_{B/C} = l_3 \dot{\theta}_3 \hat{e}'_{B/C}$$

position
ctrs $\frac{d}{dt} \left(\bar{r}_A + \bar{r}_{B/A} = \bar{r}_C + \bar{r}_{B/C} \right) \leftarrow ?$

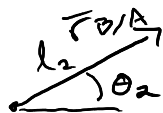
velocity
ctrs $\bar{v}_A + \bar{v}_{B/A} = \cancel{\bar{v}_C} + \bar{v}_{B/C}$

$$\bar{v}_A = 2.5 \text{ m/s} \left(-\frac{0.1459}{0.25} \hat{i} - \frac{0.203}{0.25} \hat{j} \right)$$

$$\bar{v}_C = 0 \hat{i} + 0 \hat{j} \text{ m/s}$$



$$\bar{r}_A = l_1 (\cos \theta_1 \hat{i} + \sin \theta_1 \hat{j})$$



$$\bar{r}_{B/A} = l_2 (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j})$$



$$\bar{r}_{B/C} = l_3 (\cos \theta_3 \hat{i} + \sin \theta_3 \hat{j})$$



$$\bar{r}_C = d \hat{i}$$

ctrs
on
position $\Rightarrow l_1 (\cos \theta_1 \hat{i} + \sin \theta_1 \hat{j}) + l_2 (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j})$
 $=$
 $l_3 (\cos \theta_3 \hat{i} + \sin \theta_3 \hat{j}) + d \hat{i}$

$$x \rightarrow l_1 \cos \theta_1 + l_2 \cos \theta_2 = l_3 \cos \theta_3 + dx$$

$$y \rightarrow l_1 \sin \theta_1 + l_2 \sin \theta_2 = l_3 \sin \theta_3$$

note

$$\frac{d}{dt}(l \cos \theta) = \frac{d}{d\theta} \frac{d\theta}{dt} (l \cos \theta) = -l \sin \theta \left(\frac{d\theta}{dt} \right)$$

$$= -l \dot{\theta} \sin \theta$$

CHAIN RULE!!

$$v_x \rightarrow -l_1 \dot{\theta}_1 \sin \theta_1 - l_2 \dot{\theta}_2 \sin \theta_2 = -l_3 \dot{\theta}_3 \sin \theta_3$$

find $\dot{\theta}_2 + \dot{\theta}_3$

$$v_y \rightarrow l_1 \dot{\theta}_1 \cos \theta_1 + l_2 \dot{\theta}_2 \cos \theta_2 = l_3 \dot{\theta}_3 \cos \theta_3$$

$$\dot{\theta}_1 = 10 \text{ rad/s}$$

$$r_0 = 0\hat{i} + 0\hat{j} [\text{m}]$$

SOHCAHTOA

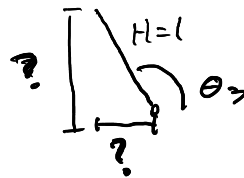
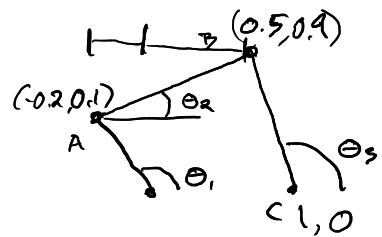
$$\cos \theta_1 = \frac{-0.203}{0.25} \quad \sin \theta_1 = \frac{0.14}{0.25}$$

$$r_A = -0.203\hat{i} + 0.1459\hat{j} [\text{m}]$$

$$r_B = 0.494\hat{i} + 0.863\hat{j} [\text{m}]$$

$$\cos \theta_2 = \frac{0.494 + 0.203}{1} \quad \sin \theta_2 = \frac{0.8}{1} \quad r_C = 1\hat{i} + 0\hat{j} [\text{m}]$$

$$\cos \theta_3 = \frac{0.494 - 1}{1} \quad \sin \theta_3 = \frac{0.9}{1}$$



$$l_1 \dot{\theta}_1 \sin \theta_1 + l_2 \dot{\theta}_2 \sin \theta_2 = l_3 \dot{\theta}_3 \sin \theta_3$$

$$\dot{\theta}_3 = \frac{l_1 \dot{\theta}_1 \sin \theta_1 + l_2 \dot{\theta}_2 \sin \theta_2}{l_3 \sin \theta_3}$$

$$l_1 \dot{\theta}_1 \cos \theta_1 + l_2 \dot{\theta}_2 \cos \theta_2 = l_3 \cos \theta_3 \left(\frac{l_1 \dot{\theta}_1 \sin \theta_1 + l_2 \dot{\theta}_2 \sin \theta_2}{l_3 \sin \theta_3} \right)$$

$$\dot{\theta} = \omega = \frac{d\theta}{dt}$$

?

equation of motion
is
ODE

centripetal
accel

$$a_c = \frac{v^2}{R}$$

is one component
of intrinsic accel.

$$a = \frac{dv}{dt} \hat{e}_{\tan} + \frac{v^2}{R} \hat{e}_{\perp}$$

$$\ddot{\theta} = \dot{\omega} = -\frac{g}{L} \sin\theta$$

[pendulum
eqn]