3.1 Express $\mathbf{r}_{P/Q}$ shown in Figure 3.34 using vector components in frame $\mathcal{A}$. Now express $\mathbf{r}_{P/Q}$ using vector components in frame $\mathcal{B}$.

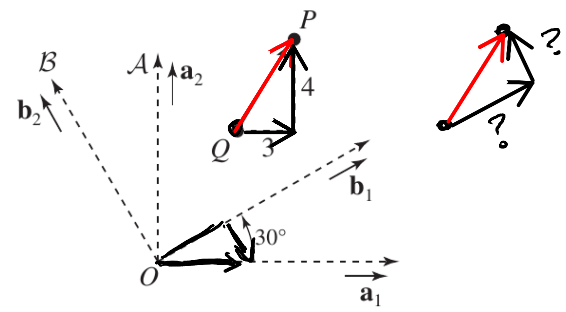


Figure 3.34 Problem 3.1.

$$\bar{r}_{P/Q} = 3\,\hat{a}_1 + 4\,\hat{a}_2 = ?\,\hat{b}_1 + ?\,\hat{b}_2$$

$$\rightarrow \quad \uparrow \qquad \nearrow \quad \nwarrow$$

$$\bar{r}_{P/Q} = 3(\cos\theta\,\hat{b}_1 - \sin\theta\,\hat{b}_2) + 4(\sin\theta\,\hat{b}_1 + \cos\theta\,\hat{b}_2)$$

$$\bar{r}_{P/Q} = (3\cos\theta + 4\sin\theta)\,\hat{b}_1 + (4\cos\theta - 3\sin\theta)\,\hat{b}_2$$

$$\hat{a}_1 = \underline{\cos\theta}\,\hat{b}_1 + \underline{-\sin\theta}\,\hat{b}_2$$

$$\hat{a}_2 = \underline{\sin\theta}\,\hat{b}_1 + \underline{\cos\theta}\,\hat{b}_2$$

$$\hat{b}_1 = \cos\theta\,\hat{a}_1 + \sin\theta\,\hat{a}_2$$

$$\hat{b}_2 = -\sin\theta\,\hat{a}_1 + \cos\theta\,\hat{a}_2$$

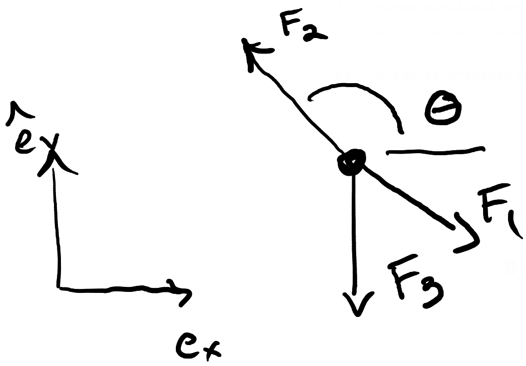
Linear Algebra

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}_{\text{rotation matrix}} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix}$$

3.2 Three forces act on particle P: $\mathbf{F}_1 = \boxed{2\mathbf{e}_x} - 5\mathbf{e}_y$ N, $\mathbf{F}_2 = \boxed{10(\cos\theta}\,\mathbf{e}_x + \sin\theta\,\mathbf{e}_y)$ N, and $\mathbf{F}_3 = (5 - 4\sqrt{6})\mathbf{e}_y$ N. If the resultant force is zero, find θ and the force vector $\mathbf{F}_2$. Sketch the three forces acting on P.

FBD

KD



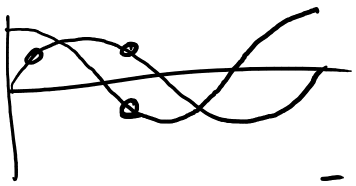
$=$

$\bullet = 0 = ma$

$$\Sigma F_x = 0 = 2 + 10\cos\theta$$

$$\Sigma F_y = 0 = -5 + 5 - 4\sqrt{6} + 10\sin\theta$$

$$\Sigma F_x = \underbrace{\bar{F}_1 \cdot \hat{e}_x}_{2} + \underbrace{\bar{F}_2 \cdot \hat{e}_x}_{0} + \underbrace{\bar{F}_3 \cdot \hat{e}_x}_{10\cos\theta}$$



$$\frac{-2}{10} = \cos\theta$$

$$\frac{4\sqrt{6}}{10} = \sin\theta$$

$$\boxed{\theta = 101^\circ}$$

$$\tan\theta = -\frac{2}{4\sqrt{6}}$$

3.2 Three forces act on particle P: $\mathbf{F}_1 = 2\mathbf{e}_x - 5\mathbf{e}_y$ N, $\mathbf{F}_2 = 10(\cos\theta\,\mathbf{e}_x + \sin\theta\,\mathbf{e}_y)$ N, and $\mathbf{F}_3 = (5 - 4\sqrt{6})\mathbf{e}_y$ N. If the resultant force is zero, find θ and the force vector $\mathbf{F}_2$. Sketch the three forces acting on P.

3.3 Consider frames $\mathcal{I} = (O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ and $\mathcal{A} = (O, \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$, where $\mathbf{e}_3 = \mathbf{a}_3$, as shown in Figure 3.35. Find the position of P with respect to O in the following coordinates:

a. Cartesian coordinates in $\mathcal{I}$, $(x, y)_{\mathcal{I}}$.
b. Cartesian coordinates in $\mathcal{A}$, $(a_1, a_2)_{\mathcal{A}}$.
c. Polar coordinates in $\mathcal{I}$, $(r, \theta)_{\mathcal{I}}$.
d. Polar coordinates in $\mathcal{A}$, $(\rho, \beta)_{\mathcal{A}}$.

$\bar{r}_P = r\,\hat{e}_r$

$\bar{r}_P = r(\cos\theta\,\hat{e}_1 + \sin\theta\,\hat{e}_2)$

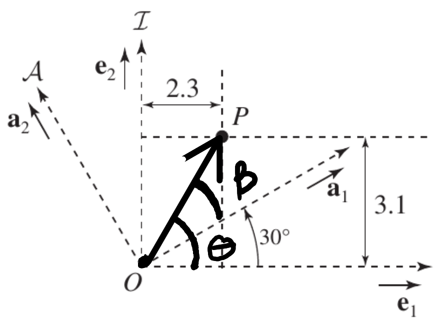


Figure 3.35 Problem 3.3.

(a) $x = 2.3 \quad y = 3.1 \qquad \bar{r}_P = x\hat{e}_1 + y\hat{e}_2$

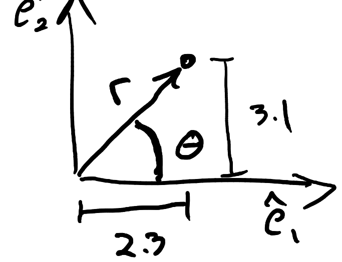
(b) $x' = 3.5 \quad y' = 1.5 \qquad \bar{r}_P = x'\hat{a}_1 + y'\hat{a}_2$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 30° & \sin 30° \\ -\sin 30° & \cos 30° \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix} = \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \end{bmatrix}$$

(c) $r = \sqrt{2.3^2 + 3.1^2}$

$\theta = \tan^{-1}\left(\frac{3.1}{2.3}\right) = 53°$



(d) $r = r_c$

$\beta = 23° = 53° - 30°$ ✓