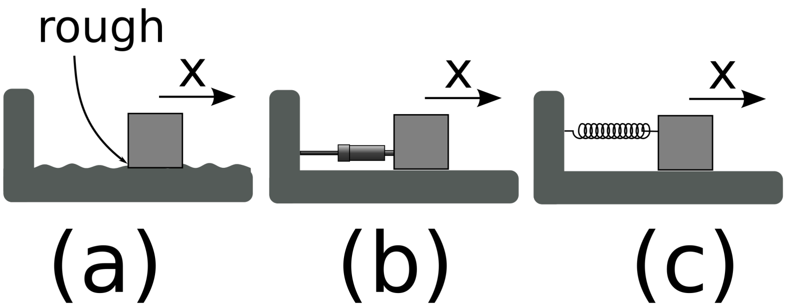
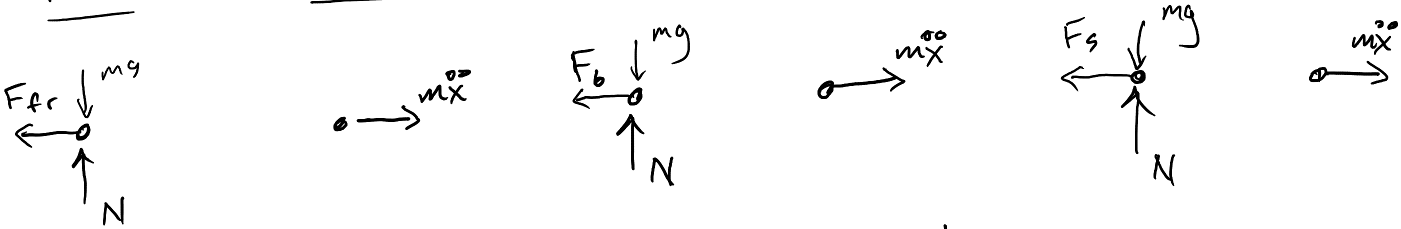


(a) FBD KD

(b) FBD KD

(c) FBD KD



[e o m] equation of motion

Initial values

(a) $ma = -\mu m g \Longrightarrow a = -\mu g$ ✓

(b) $a = -\frac{b}{m} v$

(c) $a = -\frac{k}{m} x$

$x(0) = 0 \text{ m}$

$v(0) = \dot{x}(0) = 10 \text{ m/s}$

solution (solve 2nd order ODE)

(a) kinematic eqn $\Longrightarrow \frac{d^2x}{dt^2} = -\mu g$

$$\frac{dx}{dt} = -\mu g t + \dot{x}_0$$

$$x = -\frac{\mu g}{2} t^2 + \dot{x}_0 \cdot t + x_0$$

find stopping point

$v = 0 = v_0 - \mu g t_{stop}$

(b) $a = -\frac{b}{m} v$ $\xleftarrow{bv} \bullet \; = \; \bullet \xrightarrow{ma}$

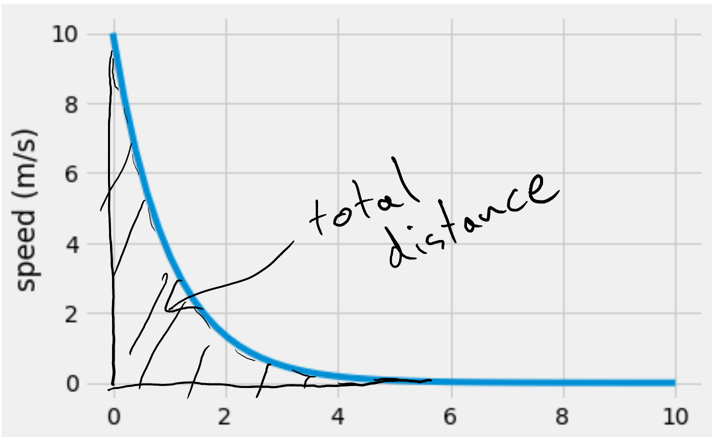
$$\frac{dv}{dt} = -\frac{b}{m} \cdot v$$

$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{b}{m} \int_0^t dt$$

$$\ln(v) - \ln(v_0) = -\frac{b}{m}(t - 0)$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{b}{m} t$$

$$\boxed{v(t) = v_0 e^{-b/m\, t}}$$



$$\frac{dx}{dt} = v_0 e^{-b/m\, t}$$

$$\int_{x_0}^{x} dx = \int_0^t v_0 e^{-b/m\, t} dt$$

$$x - x_0 = -v_0 \cdot \frac{m}{b}\left(e^{-b/m\, t} - e^{-b/m \cdot 0}\right)$$

$$x(t) = v_0 \frac{m}{b}\left(1 - e^{-b/m\, t}\right)$$

$x(t) = ?$

example

$$\frac{d}{dt}\left(e^{2t}\right) = 2e^{2t}$$

$$\int e^{2t} dt = \frac{1}{2} e^{2t} + C$$

$$\ddot{x} = -\frac{b}{m}\dot{x} \implies \begin{aligned} x &= Ae^{\lambda t} \\ \dot{x} &= A\lambda e^{\lambda t} \\ \ddot{x} &= A\lambda^2 e^{\lambda t} \end{aligned}$$

$$\cancel{A}\lambda^2 e^{\lambda t} = -\frac{b}{m}\cancel{A}\lambda e^{\lambda t}$$

$$\lambda(\lambda + b/m) = 0 \qquad \lambda = 0,\ -b/m$$

$$x(t) = A e^{-b/m\, t} + B$$

2 eqns × 2 unknowns

$$x(0) = A + B = 0$$
$$\dot{x}(0) = -b/m\, A = 10$$

$$\overset{kx}{\longleftarrow}\!\bullet \quad = \quad \bullet\!\overset{m\ddot{x}}{\longrightarrow}$$

$$\ddot{x} = -\frac{k}{m}x \qquad \begin{aligned} x(t) &= A\sin\omega t + B\cos\omega t \\ \dot{x}(t) &= A\omega\cos\omega t - B\omega\sin\omega t \\ \ddot{x}(t) &= -A\omega^2\sin\omega t - B\omega^2\cos\omega t \end{aligned}$$

$$-\omega^2(A s\omega t + B c\omega t) = -\frac{k}{m}(A s\omega t + B c\omega t)$$

$$\omega^2 = \frac{k}{m}$$