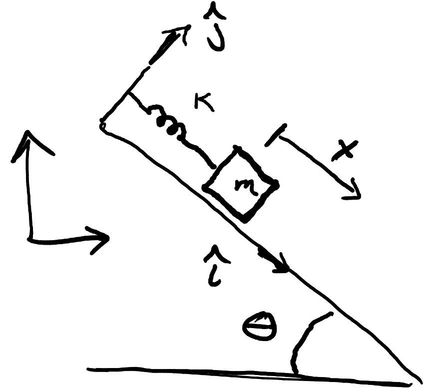
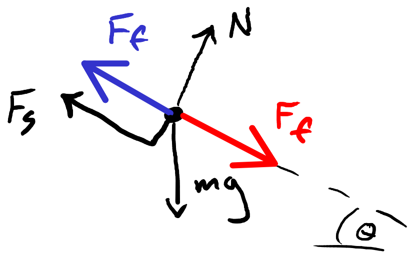
3.37 A mass m is on an inclined plane at angle θ, as shown in Figure 3.53. Attached to the mass is a spring of spring constant k that is fixed at the top of the incline. There is gravity in this problem.

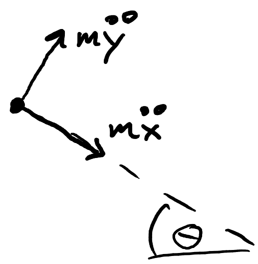
a. Assuming a coefficient of friction μ, find the equation of motion for this system using Cartesian coordinates in the $\mathcal{A}$ frame. [HINT: Use the sgn($\cdot$) function to extract the sign of the speed.]
b. Solve the equation of motion assuming no friction with initial conditions $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$.



FBD



KD

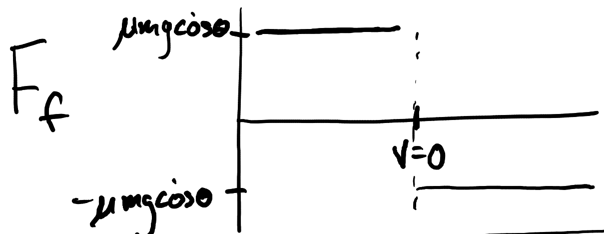


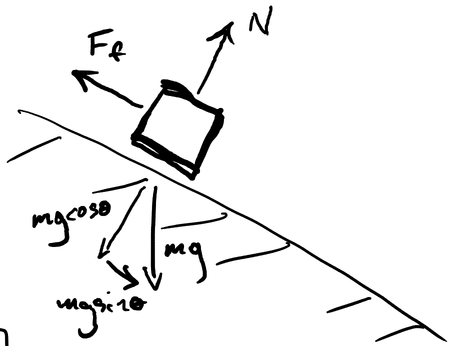
$-\text{sgn} \equiv -\frac{V}{|V|}$

$\text{sgn}(-10) = -1$

$\text{sgn}(3) = +1$

eom $\Rightarrow$ $m\ddot{x} = \begin{cases} \_\_\_\_\_\_\_\_ & ① \\ \_\_\_\_\_\_\_\_ & ② \end{cases}$





$$\boxed{F_f = -\mu mg\cos\theta \cdot \frac{V}{|V|}}$$

$$\boxed{F_s = -kx_s = -k(\Delta x) = -k(x_s - l_0)}$$

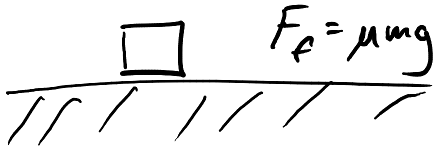
eom

$$m\ddot{x} = -kx - \mu mg\cos\theta \cdot \frac{V}{|V|} + mg\sin\theta$$

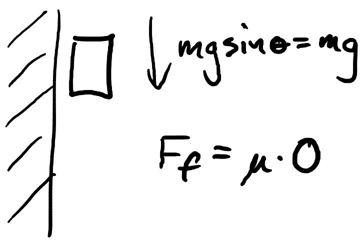
extreme cases

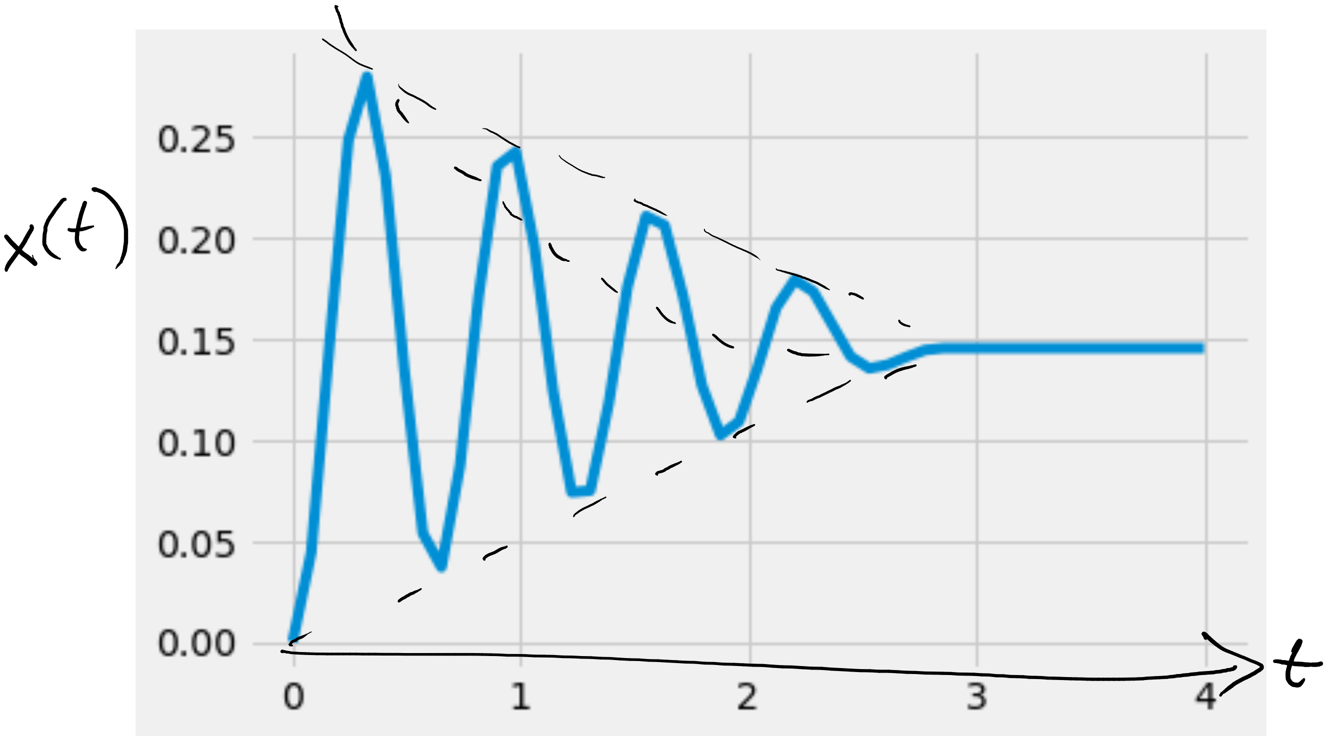
$\theta = 0$

$\rightarrow mg\sin\theta = 0$



$\theta = 90$






```
In [1]:  import numpy as np
         import matplotlib.pyplot as plt
         plt.style.use('fivethirtyeight')
```

```
In [7]:  def springmass(t, y):
             dy = np.zeros(len(y))
             dy[0] = y[1]
             dy[1] = 0*9.81*np.cos(np.pi/6)*y[1]/np.abs(y[1])
             dy[1] += -100*(y[0] - 0.1)
             dy[1] += 9.81*np.sin(np.pi/6)
             return dy
```

eom
w/ $\mu = 0$

```
In [8]:  from scipy.integrate import solve_ivp

         sol = solve_ivp(springmass, [0, 3], [0, 0.001],
                         t_eval=np.linspace(0,3))

         plt.plot(sol.t, sol.y[0])
```

```
Out[8]: [<matplotlib.lines.Line2D at 0x7f3c711020d0>]
```