

(a) [e o m] equation of motion  
 $ma = -\mu mg \Rightarrow a = -\mu g \checkmark$

Initial values

(b)  $a = -\frac{b}{m}v$

(c)  $a = -\frac{k}{m}x$

$x(0) = 0 \text{ m}$

solution (solve 2<sup>nd</sup> order ODE)  $v(0) = \dot{x}(0) = 10 \text{ m/s}$

(a) kinematic eqn  $\Rightarrow \frac{d^2x}{dt^2} = -\mu g$

find stopping point  
 $v = 0 = v_0 - \mu g t_{\text{stop}}$

$\frac{dx}{dt} = -\mu g t + \dot{x}_0$

$x = -\frac{\mu g}{2} t^2 + \dot{x}_0 \cdot t + x_0$

(b)

$$a = -\frac{b}{m} v$$

$$\overleftarrow{bv} = \overrightarrow{ma}$$

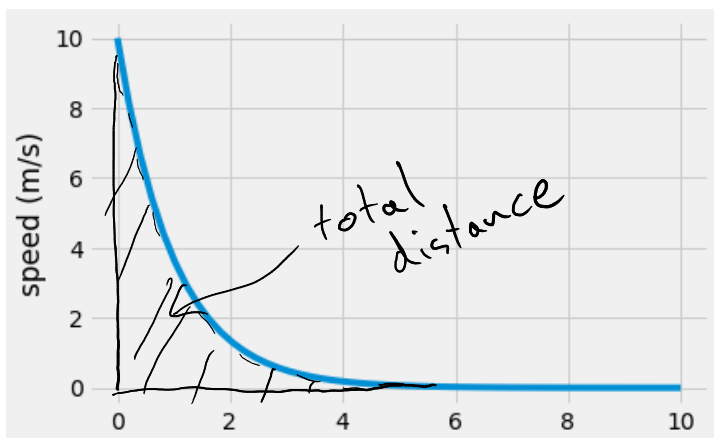
$$\frac{dv}{dt} = -\frac{b}{m} \cdot v$$

$$\int_{v_0}^v \frac{dv}{v} = -\frac{b}{m} \int_0^t dt$$

$$\ln(v) - \ln(v_0) = -\frac{b}{m}(t - 0)$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{b}{m}t$$

$$v(t) = v_0 e^{-\frac{b}{m}t}$$



$$\frac{dx}{dt} = v_0 e^{-\frac{b}{m}t}$$

$$\int_{x_0}^x dx = \int_0^t v_0 e^{-\frac{b}{m}t} dt$$

$$x - x_0 = -v_0 \cdot \frac{m}{b} \left( e^{-\frac{b}{m}t} - e^{-\frac{b}{m} \cdot 0} \right)$$

$$x(t) = v_0 \frac{m}{b} (1 - e^{-\frac{b}{m}t})$$

$$x(t) = ?$$

example

$$\frac{d}{dt}(e^{2t}) = 2e^{2t}$$

$$\int e^{2t} dt = \frac{1}{2} e^{2t} + C$$

$$\ddot{x} = -\frac{b}{m} \dot{x} \Rightarrow$$

$$x = A e^{\lambda t}$$

$$\dot{x} = A \lambda e^{\lambda t}$$

$$\ddot{x} = A \lambda^2 e^{\lambda t}$$

$$\cancel{A} \lambda^2 e^{\lambda t} = -\frac{b}{m} \cancel{A} \lambda e^{\lambda t}$$

$$\lambda(\lambda + \frac{b}{m}) = 0 \quad \lambda = 0, -\frac{b}{m}$$

$$x(t) = A e^{-\frac{b}{m}t} + B$$

2 eqns x 2 unknowns

$$x(0) = A + B = 0$$

$$\dot{x}(0) = -\frac{b}{m}A = 10$$

$$\overset{kx}{\leftarrow} = \overset{m\ddot{x}}{\rightarrow}$$

$$\ddot{x} = -\frac{k}{m} x$$

$$x(t) = A \sin \omega t + B \cos \omega t$$

$$\dot{x}(t) = A \omega \cos \omega t - B \omega \sin \omega t$$

$$\ddot{x}(t) = -A \omega^2 \sin \omega t - B \omega^2 \cos \omega t$$

$$-\omega^2 (A \sin \omega t + B \cos \omega t) = -\frac{k}{m} (A \sin \omega t + B \cos \omega t)$$

$$\omega^2 = \frac{k}{m}$$