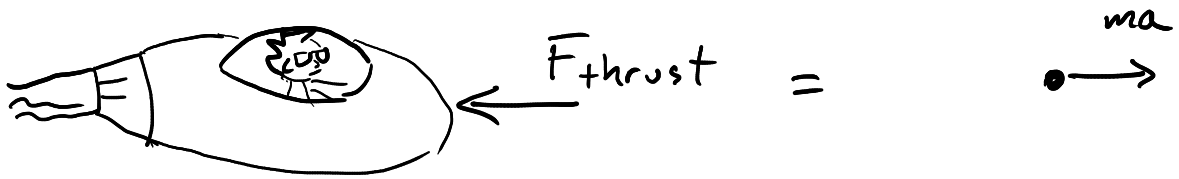


- 4.2 Consider a 10^4 kg spaceship in deep space. Suppose the spaceship's thruster produces 100 N of constant thrust. If the spaceship is moving at 50 m/s and the thruster is anti-aligned with the spaceship's velocity, how long should the thruster fire to bring the spaceship to rest with respect to absolute space?



$$\underbrace{\Delta(mv)}_{\text{change in momentum}} = \underbrace{F \Delta t}_{\text{Impulse}} \leftarrow \text{if } a = cst$$

$$O \xrightarrow{mv_0} + \int \xleftarrow{F} dt = O \xrightarrow{mv_f=0}$$

Impulse - Momentum Diagram

$$(10^4 \text{ kg}) \cdot 50 \text{ m/s} - 100 \text{ N} \cdot \Delta t = 0$$

$$\Delta t = \frac{100 \cdot 50 \text{ E2 } \frac{\text{kg m}}{\text{s}}}{100 \text{ N}} = 50 \text{ E2 s}$$

- 4.6 Suppose frame $B = (O', \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3)$ is traveling in the \mathbf{e}_1 direction at a constant speed of v_0 with respect to stationary frame $\mathcal{I} = (O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, as shown in Figure 4.19. Mass m_p is connected to point O' by a spring with spring constant k and rest length r_0 . The spring can freely pivot about O' . Assume that the positions of O and O' are the same at time $t = 0$.

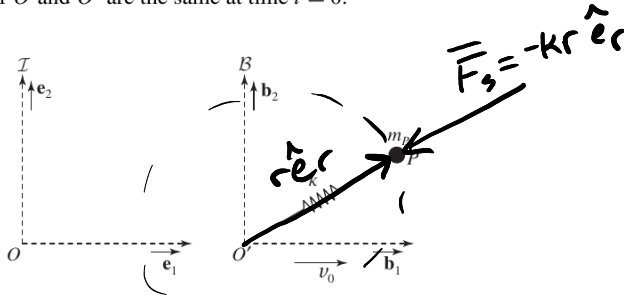


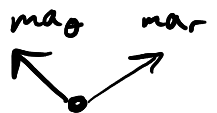
Figure 4.19 Problem 4.6.

- Using the coordinates of your choice, find the position $\mathbf{r}_{p/O'}$ and velocity $\mathcal{I}\mathbf{v}_{p/O'}$ of the mass with respect to O' in \mathcal{I} . [HINT: Introduce a polar frame at O' .]
- Find the position $\mathbf{r}_{p/O}$ and velocity $\mathcal{I}\mathbf{v}_{p/O}$ of the mass with respect to O in \mathcal{I} .
- Draw a free-body diagram for mass m_p . (There is no gravity in this problem.)
- Find the angular momentum $\mathcal{I}\mathbf{h}_{p/O'}$ of the mass with respect to O' in \mathcal{I} .
- Show that the angular momentum of the mass with respect to O' in \mathcal{I} is conserved, but the angular momentum with respect to O in \mathcal{I} is not.

FBD



KD



$$\Sigma \bar{\mathbf{M}}_{O'} = \bar{\mathbf{r}}_p \times \bar{\mathbf{F}}_s$$

$$\Sigma \bar{\mathbf{M}}_{O'} = r\hat{\mathbf{e}}_r \times -kr\hat{\mathbf{e}}_r = \mathbf{0}$$

$$-kr^2 \hat{\mathbf{e}}_r \times \hat{\mathbf{e}}_r$$



$$\Sigma \bar{\mathbf{M}}_{O'} = \mathbf{0} = \frac{d}{dt}(\bar{\mathbf{h}}_{O'})$$

$$\bar{\mathbf{h}}_{O'} = \bar{\mathbf{r}}_p \times (m\bar{\mathbf{v}}_p)$$

$$\bar{\mathbf{v}}_p = v_r\hat{\mathbf{e}}_r + v_\theta\hat{\mathbf{e}}_\theta = \dot{r}\hat{\mathbf{e}}_r + r\dot{\theta}\hat{\mathbf{e}}_\theta$$

$$\bar{\mathbf{h}}_{O'} = m(r\dot{r}\hat{\mathbf{e}}_r \times \hat{\mathbf{e}}_r + r \cdot r\dot{\theta}\hat{\mathbf{e}}_r \times \hat{\mathbf{e}}_\theta)$$

$$\boxed{\bar{\mathbf{h}}_{O'} = m r^2 \dot{\theta} \hat{\mathbf{b}}_3}$$

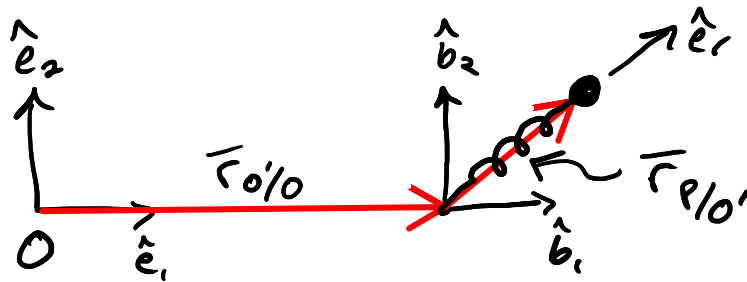
general formula for angular momentum of point mass

$$\bar{M}_O = \frac{d}{dt}(\bar{h}_O)$$

$$0 = \frac{d}{dt}(m r^2 \dot{\theta})$$

$$m r^2 \dot{\theta} = \text{constant}$$

$$r_o^2 \dot{\theta}_o = r^2 \dot{\theta} \Rightarrow \dot{\theta} = \frac{r_o^2 \dot{\theta}_o}{r^2}$$



$$\bar{r}_{P/O} = \bar{r}_{O'/O} + \bar{r}_{P/O'}$$

$$\Sigma \bar{M}_O = \bar{r}_{P/O} \times \bar{F}_s$$

$$= (\bar{r}_{O'/O} + \bar{r}_{P/O'}) \times (-kr \hat{e}_r)$$

$$= \underbrace{\bar{r}_{O'/O} \times -kr \hat{e}_r} + \cancel{r \hat{e}_r \times -kr \hat{e}_r}^0$$

$$= (vt \hat{b}_1) \times (-kr [\cos \theta \hat{b}_1 + \sin \theta \hat{b}_2])$$

$$\Sigma M_O = -vtkr \sin \theta \hat{b}_3 \neq 0$$

- 9.14 A uniformly dense marble of mass $m = 0.05 \text{ kg}$ and radius $R = 0.01 \text{ m}$ is released from rest at the top of a ramp and rolls without slipping down the ramp and off a table, as shown in Figure 9.33. Find the distance d from the foot of the table where the marble lands on the floor. The ramp height is $h_1 = 0.2 \text{ m}$, and its width is $w_1 = 0.4 \text{ m}$. The marble rolls a distance of $w_2 = 0.15 \text{ m}$ on the table, which has height $h_2 = 1 \text{ m}$. [HINT: Consider both the rotational and translational motion of the rolling marble.]

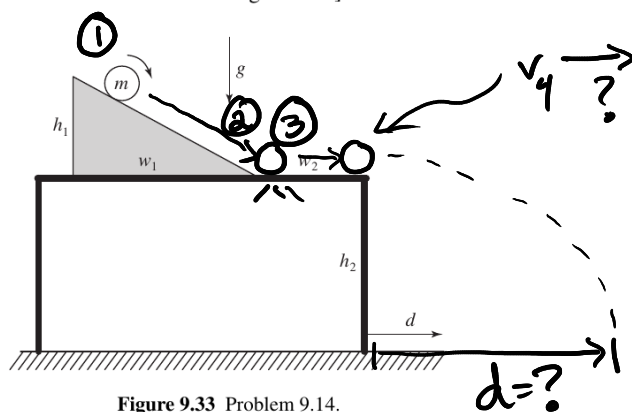
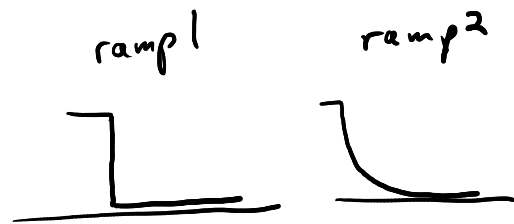
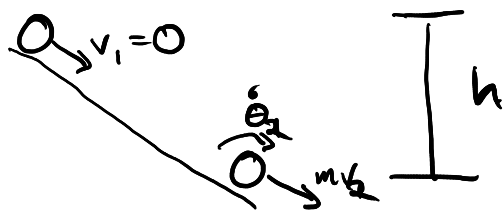


Figure 9.33 Problem 9.14.



$$m\vec{v}_0 + \int \vec{F} dt = m\vec{v}_f$$

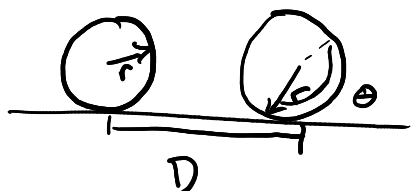
① → ②



$$mgh = T_2$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$$

$$I_{\text{sphere}} = \frac{2}{5}mr^2$$



$$D = r\theta$$

$$v = r\dot{\theta}$$

[rolling w/o slipping]

$$mgh_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}\frac{2}{5}mR^2\left(\frac{v_2}{R}\right)^2$$

$$= \frac{1}{2}mv_2^2\left(1 + \frac{2}{5}\right)$$

$$v_2 = \sqrt{\frac{2gh}{1 + \frac{2}{5}}}$$

$$\frac{v_2(w/I)}{v_2(w/o I)} = \frac{\sqrt{\frac{2gh}{1+2/5}}}{\sqrt{\frac{2gh}{1}}} = \sqrt{\frac{1/5}{1}} = \sqrt{\frac{5}{7}}$$