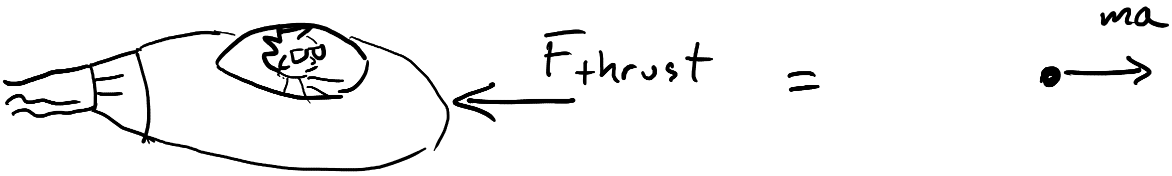
4.2 Consider a 10^4 kg spaceship in deep space. Suppose the spaceship's thruster produces 100 N of constant thrust. If the spaceship is moving at 50 m/s and the thruster is anti-aligned with the spaceship's velocity, how long should the thruster fire to bring the spaceship to rest with respect to absolute space?



$$\underbrace{\Delta(mv)}_{\text{change in momentum}} = \underbrace{F\Delta t}_{\text{Impulse}} \leftarrow \text{if } a = cst$$

$$\circ\overset{mv_0}{\longrightarrow} + \int \overset{F}{\longleftarrow} dt = \circ\overset{mv_f = 0}{\longrightarrow}$$

Impulse - Momentum Diagram

$$(10E3\,kg)\cdot 50\,\tfrac{m}{s} - 100\,N\cdot\Delta t = 0$$

$$\Delta t = \frac{\cancel{100}\cdot 50E2\ \frac{kg\,m}{s}}{\cancel{100}\ N} = 50E2\ s$$

4.6 Suppose frame $\mathcal{B} = (O', \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3)$ is traveling in the $\mathbf{e}_1$ direction at a constant speed of v_0 with respect to stationary frame $\mathcal{I} = (O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, as shown in Figure 4.19. Mass m_P is connected to point O' by a spring with spring constant k and rest length r_0. The spring can freely pivot about O'. Assume that the positions of O and O' are the same at time $t = 0$.

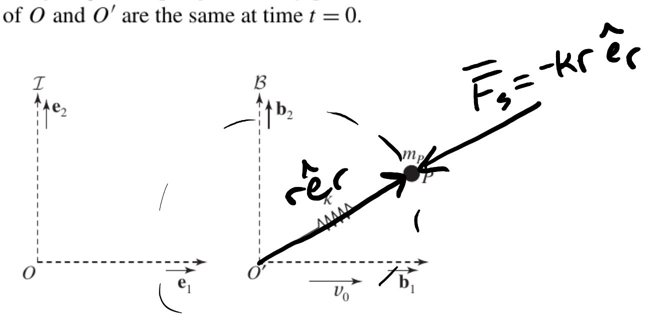
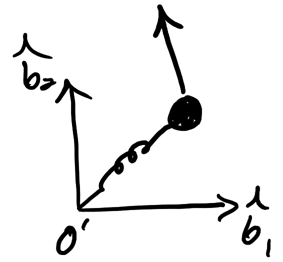
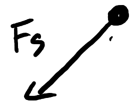


Figure 4.19 Problem 4.6.



a. Using the coordinates of your choice, find the position $\mathbf{r}_{P/O'}$ and velocity ${}^{\mathcal{I}}\mathbf{v}_{P/O'}$ of the mass with respect to O' in $\mathcal{I}$. [HINT: Introduce a polar frame at O'.]
b. Find the position $\mathbf{r}_{P/O}$ and velocity ${}^{\mathcal{I}}\mathbf{v}_{P/O}$ of the mass with respect to O in $\mathcal{I}$.
c. Draw a free-body diagram for mass m_P. (There is no gravity in this problem.)
d. Find the angular momentum ${}^{\mathcal{I}}\mathbf{h}_{P/O'}$ of the mass with respect to O' in $\mathcal{I}$.
e. Show that the angular momentum of the mass with respect to O' in $\mathcal{I}$ is conserved, but the angular momentum with respect to O in $\mathcal{I}$ is not.

FBD



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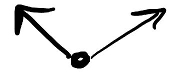
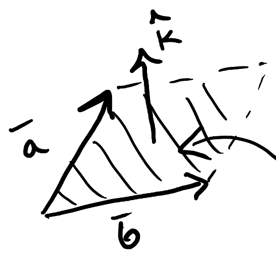


$$\Sigma \bar{M}_{O'} = \bar{r}_p \times \bar{F}_s$$

$$\Sigma \bar{M}_{O'} = r\hat{e}_r \times -kr\hat{e}_r = \bar{0}$$

$$-kr^2\,\hat{e}_r \times \hat{e}_r$$



$$\text{Area}\,\hat{k} = \bar{a} \times \bar{b}$$

$$\Sigma \bar{M}_{O'} = \bar{0} = \frac{d}{dt}\left(\bar{h}_{O'}\right)$$

$$\bar{h}_{O'} = \bar{r}_p \times (m\bar{v}_p)$$

$$\bar{v}_p = v_r\hat{e}_r + v_\theta\hat{e}_\theta = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\bar{h}_{O'} = m\big(r\dot{r}\,\underbrace{\cancel{\hat{e}_r \times \hat{e}_r}}_{0} + r \cdot r\dot{\theta}\,\hat{e}_r \times \hat{e}_\theta\big)$$

$$\boxed{\bar{h}_{O'} = mr^2\dot{\theta}\,\hat{b}_3}$$

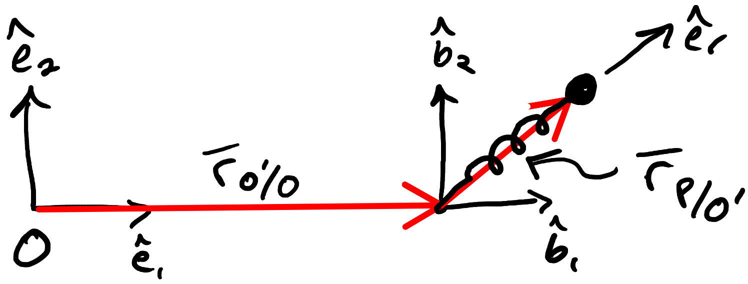
general formula for angular momentum of point mass

$$\bar{M}_{o'} = \frac{d}{dt}(\bar{h}_{o'})$$

$$0 = \frac{d}{dt}\left(m r^2 \dot{\theta}\right)$$

$$m r^2 \dot{\theta} = \text{constant}$$

$$r_0^2 \dot{\theta}_0 = r^2 \dot{\theta} \implies \dot{\theta} = \frac{r_0^2 \dot{\theta}_0}{r^2}$$



$$\bar{r}_{P/O} = \bar{r}_{O'/O} + \bar{r}_{P/O'}$$

$$\sum \bar{M}_O = \bar{r}_{P/O} \times \bar{F}_s$$

$$= \left(\bar{r}_{O'/O} + \bar{r}_{P/O'}\right) \times \left(-kr\,\hat{e}_r\right)$$

$$= \underbrace{\bar{r}_{O'/O} \times -kr\hat{e}_r}_{} + \cancel{r\hat{e}_r \times -kr\hat{e}_r}^{\,0}$$

$$= \left(vt\,\hat{b}_1\right) \times \left(-kr\left[\cos\theta\,\hat{b}_1 + \sin\theta\,\hat{b}_2\right]\right)$$

$$\sum M_O = -vtkr\sin\theta\,\hat{b}_3 \neq 0$$

9.14 A uniformly dense marble of mass $m = 0.05$ kg and radius $R = 0.01$ m is released from rest at the top of a ramp and rolls without slipping down the ramp and off a table, as shown in Figure 9.33. Find the distance d from the foot of the table where the marble lands on the floor. The ramp height is $h_1 = 0.2$ m, and its width is $w_1 = 0.4$ m. The marble rolls a distance of $w_2 = 0.15$ m on the table, which has height $h_2 = 1$ m. [HINT: Consider both the rotational and translational motion of the rolling marble.]

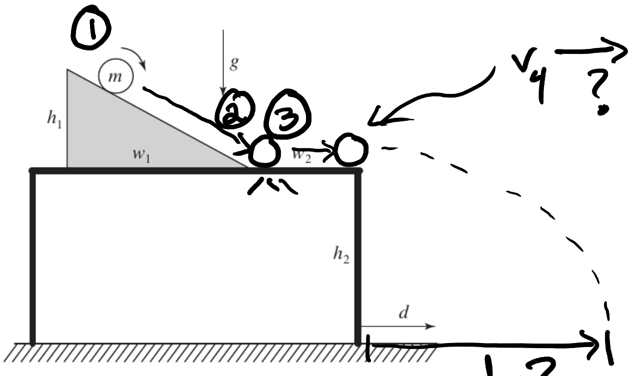
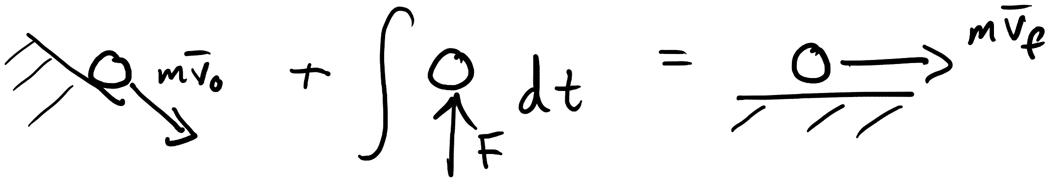
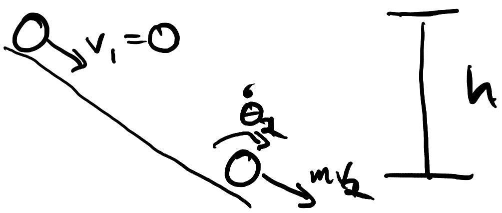


Figure 9.33 Problem 9.14.

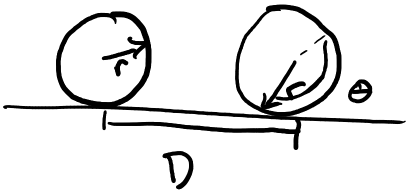


1 → 2



$$mgh = \underbrace{T_2}_{\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2}$$

$$I_{sphere} = \tfrac{2}{5} m r^2$$

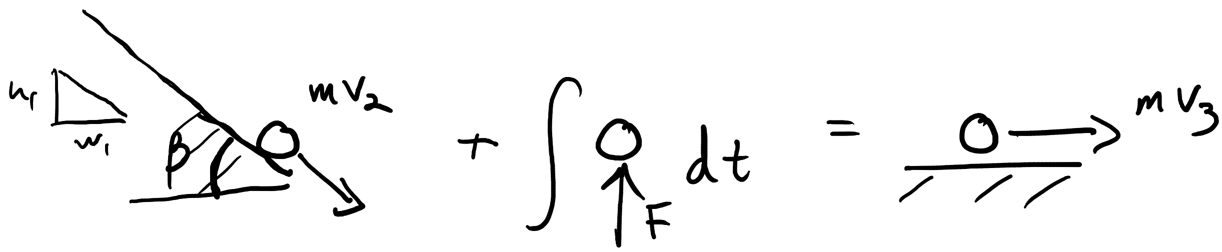


$D = r\theta$
$v = r\dot{\theta}$

[rolling w/o slipping]

$$mgh_1 = \tfrac{1}{2} m v_2^2 + \tfrac{1}{2}\,\tfrac{2}{5} m \cancel{r^2} \cdot \left(\frac{v_2}{\cancel{r}}\right)^2$$
$$= \tfrac{1}{2} m v_2^2 \left(1 + \tfrac{2}{5}\right)$$
$$v_2 = \sqrt{\frac{2gh}{1 + \frac{2}{5}}}$$

$$\frac{v_2(w/I)}{v_2(w/o\,I)} = \frac{\sqrt{\frac{2gh}{1+2/5}}}{\sqrt{\frac{2gh}{1}}} = \sqrt{\frac{1/(7/5)}{1/1}} = \sqrt{\frac{5}{7}}$$



$$v_2 = \sqrt{\frac{2gh}{1+2/5}}$$

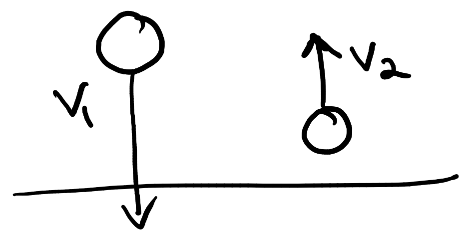
?

$= \cos\beta$ where $\tan\beta = \frac{h_1}{w_1}$

$$mv_x + 0 = mv_{3x} \Rightarrow \boxed{v_{3x} = \sqrt{\frac{2gh}{1+2/5}} \cdot \frac{w_1}{\sqrt{w_1^2+h_1^2}}}$$

$$mv_y + \int F\,dt = mv_{3y}$$

Coefficient of restitution ✱ Physics Phun Note



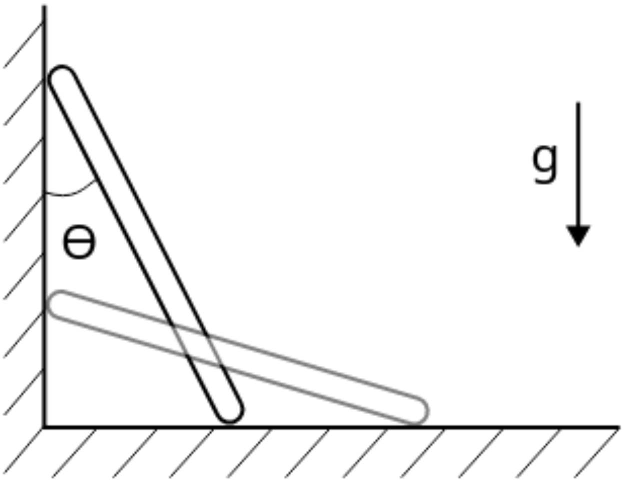
$$e = -\frac{v_2}{v_1} = 0 - 1$$

assume $e = 0$ by time leaving table

$$\frac{v_{3y}}{v_y} = 0 = e$$

$$\sqrt{\frac{10gh}{7}} \cdot \frac{w_1}{\sqrt{w_1^2+h_1^2}} = v_3$$

$$d = v_3 t$$
$$h_2 = g\frac{t^2}{2}$$



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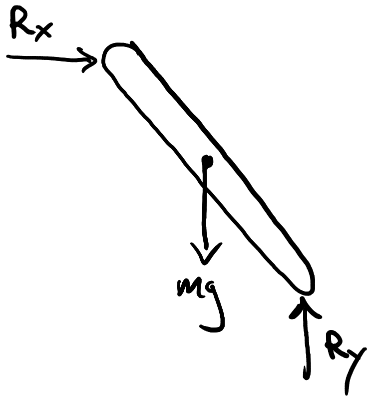

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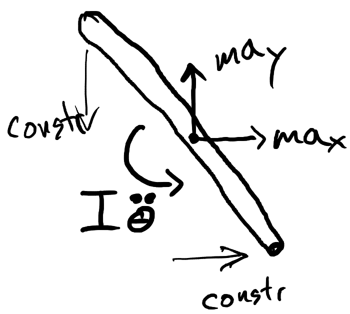


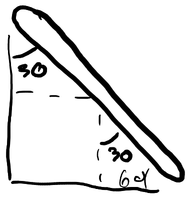
$\theta(0) = 0 \qquad \dot{\theta}(0) = 0$

$\dot{\theta}(t_1) = \frac{\pi}{6} \qquad \dot{\theta}(t_1) = ?$

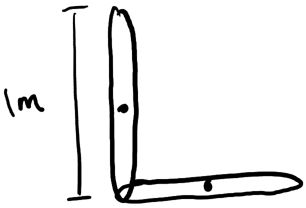
cons. of energy

$T_1 + V_1 + \cancel{W_{1\to 2}}^{0} = T_2 + V_2$

$T_1 = 0 \qquad T_2 = \frac{1}{2}mv^2 + \frac{1}{2}I\dot{\theta}^2$

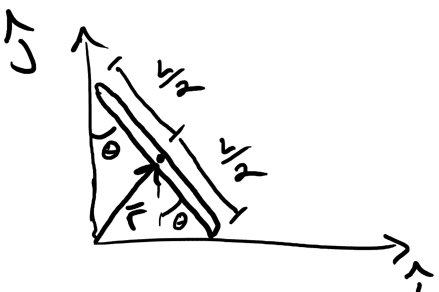


$V_1 = mgh_1 \qquad V_2 = mgh_2$



$h_1 = \frac{L}{2} \qquad h_2 = \frac{L}{2}\cos\frac{\pi}{6}$

$$mg\frac{L}{2}\left(1 - \cos\frac{\pi}{6}\right) = \frac{1}{2}mv_2^2 + \frac{1}{2}I\dot{\theta}_2^2$$



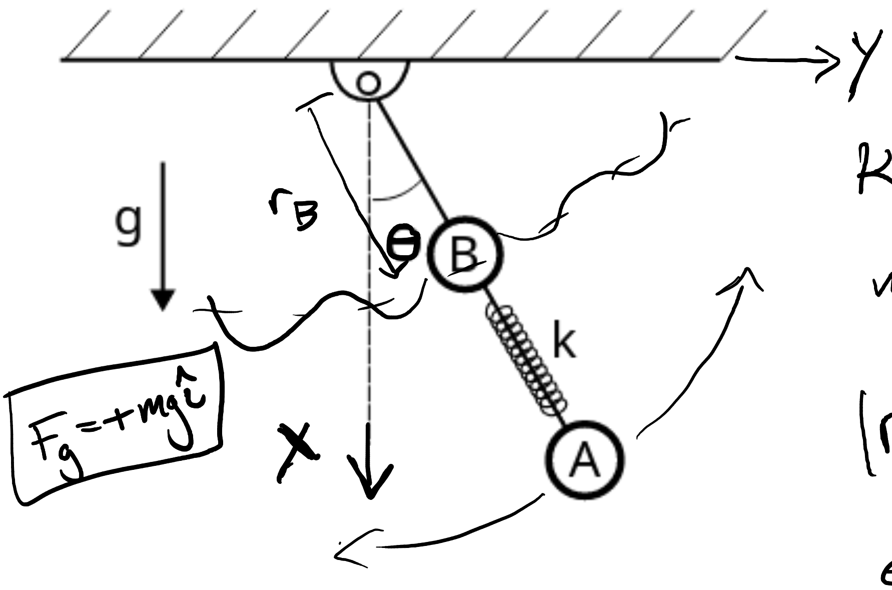
chain rule

$$\frac{d}{dt}(\sin\theta) = \frac{d\theta}{dt}\frac{d}{d\theta}(\sin\theta)$$

$$\bar{r} = \frac{L}{2}\sin\theta\,\hat{\imath} + \frac{L}{2}\cos\theta\,\hat{\jmath}$$

$$\bar{v} = \frac{L}{2}\dot{\theta}\left(\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}\right)$$

$$mg\frac{L}{2}\left(1-\cos\frac{\pi}{6}\right)=\frac{1}{2}m\left(\frac{L^2}{4}\dot{\theta}^2\right)+\frac{1}{2}\frac{mL^2}{12}\dot{\theta}^2$$



$K = 1000\,N/m$

$m_A = m_B = 1\,kg$

$|r_A| = 1m = L$

$\theta_A = \theta_B$

2 DOF

$\underbrace{r_B, \theta}$ + constants L

$$\bar{r}_A = L(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) = L\hat{e}_r$$

$$\bar{r}_B = r_B(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) = r_B\hat{e}_r$$

$$\bar{V}_A = L\dot{\theta}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) = L\dot{\theta}\hat{e}_\theta$$

$$\bar{V}_B = \dot{r}_B(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$
$$r_B\dot{\theta}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) = \dot{r}_B\hat{e}_r + r_B\dot{\theta}\hat{e}_\theta$$

$$T = \tfrac{1}{2}m_A V_A^2 + \tfrac{1}{2}m_B V_B^2$$
$$= \tfrac{1}{2}m_A(L\dot{\theta})^2 + \tfrac{1}{2}m_B(\dot{r}_B^2 + (r_B\dot{\theta})^2)$$

$$V = m_A g h_A + m_B g h_B + \tfrac{1}{2}k(r_B - 0.5)^2$$

$$\tfrac{1}{2}L^2\dot{\theta}^2 + \tfrac{1}{2}\dot{r}_B^2 + \tfrac{1}{2}r_B^2\dot{\theta}^2 = gL + gr_B - \tfrac{1}{2}k\left(r_B - \tfrac{1}{2}\right)^2$$