4.2 Consider a 10^4 kg spaceship in deep space. Suppose the spaceship's thruster produces 100 N of constant thrust. If the spaceship is moving at 50 m/s and the thruster is anti-aligned with the spaceship's velocity, how long should the thruster fire to bring the spaceship to rest with respect to absolute space?

$\bullet \xrightarrow{ma}$

$$a = \frac{F_{thrust}}{m}$$

$$\Delta v = a \Delta t$$

Impulse-Momentum

$$\bullet \xrightarrow{m v_0} + \int \xleftarrow{F_{thrust}} dt = \bullet \xrightarrow{m v_f = 0}$$

$$F \cdot \Delta t$$

$$(100E2 \cdot 50)\frac{kg \cdot m}{s} - 100\,N \cdot \Delta t = 0$$

$$\Delta t = \frac{100 \cdot 50E2}{100\,N} \cdot s$$

4.6 Suppose frame $\mathcal{B} = (O', \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3)$ is traveling in the $\mathbf{e}_1$ direction at a constant speed of v_0 with respect to stationary frame $\mathcal{I} = (O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, as shown in Figure 4.19. Mass m_P is connected to point O' by a spring with spring constant k and rest length r_0. The spring can freely pivot about O'. Assume that the positions of O and O' are the same at time $t = 0$.

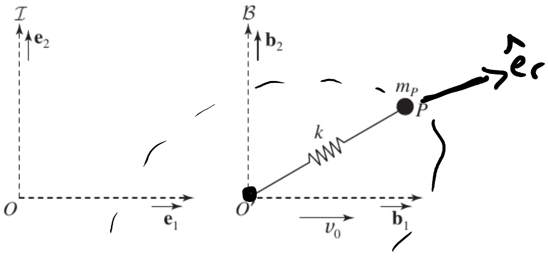


Figure 4.19 Problem 4.6.

a. Using the coordinates of your choice, find the position $\mathbf{r}_{P/O'}$ and velocity ${}^{\mathcal{I}}\mathbf{v}_{P/O'}$ of the mass with respect to O' in $\mathcal{I}$. [HINT: Introduce a polar frame at O'.]
b. Find the position $\mathbf{r}_{P/O}$ and velocity ${}^{\mathcal{I}}\mathbf{v}_{P/O}$ of the mass with respect to O in $\mathcal{I}$.
c. Draw a free-body diagram for mass m_P. (There is no gravity in this problem.)
d. Find the angular momentum ${}^{\mathcal{I}}\mathbf{h}_{P/O'}$ of the mass with respect to O' in $\mathcal{I}$.
e. Show that the angular momentum of the mass with respect to O' in $\mathcal{I}$ is conserved, but the angular momentum with respect to O in $\mathcal{I}$ is not.

FBD

F_s

$$\sum \overline{M}_{o'} = \overline{r}_{P/o'} \times \overline{F}_s$$

$$\overline{r}_{P/o'} = r\,\hat{e}_r$$

$$\overline{F}_s = -kr\,\hat{e}_r$$

$$\sum \overline{M}_{o'} = -kr^2\, \cancel{\hat{e}_r \times \hat{e}_r}^{\,0} = 0$$

$$\sum M_{o'} = \frac{d}{dt}\left(\overline{h}_{o'}\right) = \frac{d}{dt}\left(\overline{r}_{P/o'} \times m\overline{v}_P\right)$$

$$\overline{r}_{P/o'} \times m\overline{v}_P = r\hat{e}_r \times m\left(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\right)$$

$$= mr\dot{r}\,\cancel{\hat{e}_r \times \hat{e}_r}^{\,0} + mr^2\dot{\theta}\,\hat{e}_r \times \hat{e}_\theta$$

$\hat{k} \to \hat{e}_r$, e_θ

$$\boxed{\overline{h}_{o'} = mr^2\dot{\theta}\,\hat{b}_3}$$

general def'n of angular momentum of particle

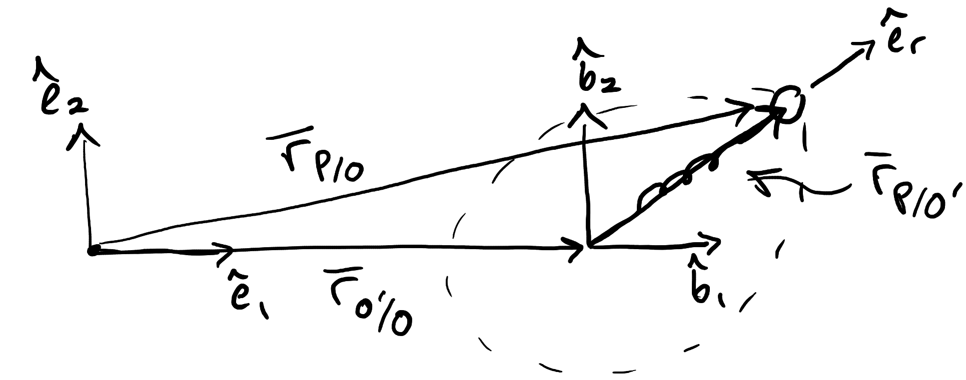
$$= mrv_\theta$$

$$0 = \frac{d}{dt}\left(mr^2\dot{\theta}\right)$$

$$\boxed{mr^2\dot{\theta} = \text{constant}}$$

✓

$$\boxed{r_0^2\dot{\theta}_0 = \frac{\cancel{m}}{\cancel{m}} r^2\dot{\theta}} \implies \dot{\theta} = \frac{r_0^2\dot{\theta}_0}{r^2}$$



$$\sum \bar{M}_O = \bar{r}_{P/O} \times \bar{F}_s = (\bar{r}_{O'/O} + \bar{r}_{P/O'}) \times -kr\hat{e}_r$$

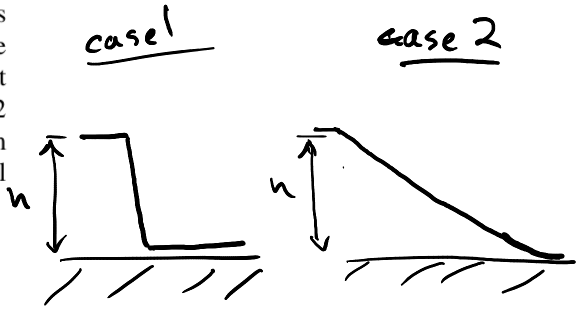
$$\sum \bar{M}_O = \bar{r}_{O'/O} \times -kr\hat{e}_r$$

$$\sum M_O = (vt\,\hat{b}_1) \times (-kr\hat{e}_r)$$

$$= (vt\,\hat{b}_1) \times (-kr)(\cos\theta\,\hat{b}_1 + \sin\theta\,\hat{b}_2)$$

$$\boxed{\sum M_O = -krvt\sin\theta\,\hat{b}_3}$$

9.14 A uniformly dense marble of mass $m = 0.05$ kg and radius $R = 0.01$ m is released from rest at the top of a ramp and rolls without slipping down the ramp and off a table, as shown in Figure 9.33. Find the distance d from the foot of the table where the marble lands on the floor. The ramp height is $h_1 = 0.2$ m, and its width is $w_1 = 0.4$ m. The marble rolls a distance of $w_2 = 0.15$ m on the table, which has height $h_2 = 1$ m. [HINT: Consider both the rotational and translational motion of the rolling marble.]



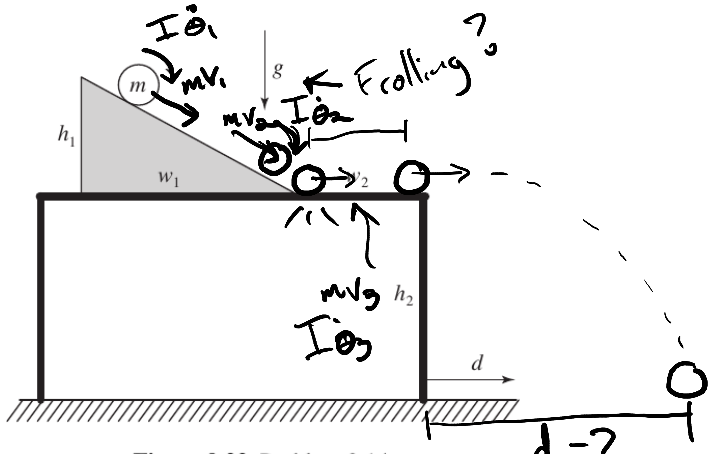
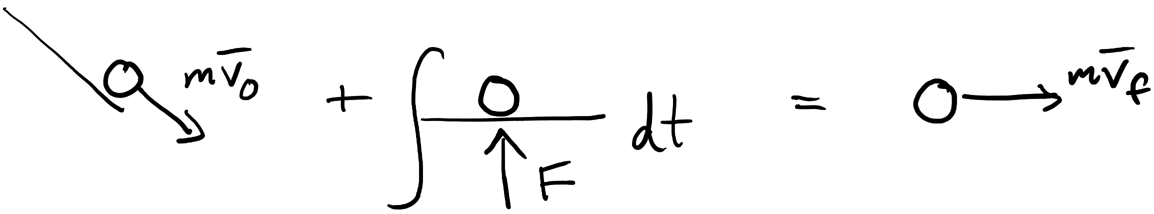


Figure 9.33 Problem 9.14.



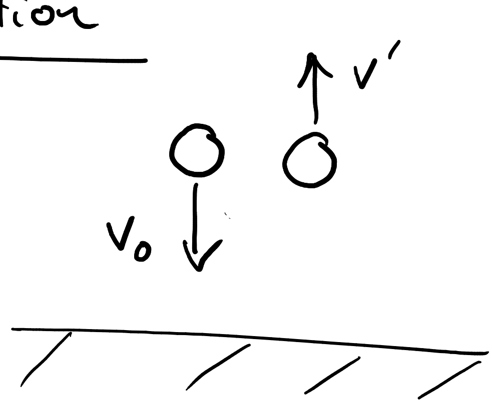
① → ② $\boxed{mgh_1 = T_2}$

② → ③ energy not conserved

calc d w/ Kinematic equs

coefficient of restitution

$$e = \frac{-v'}{V_0}$$



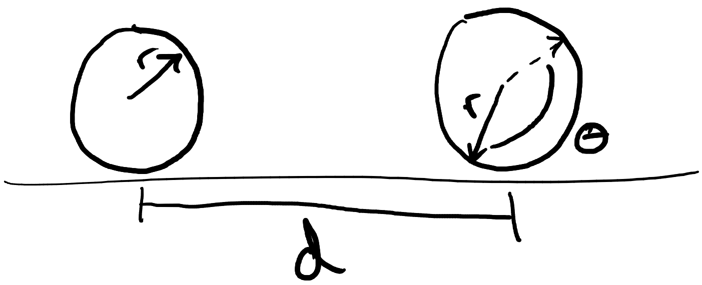
$$1 = \frac{-v'}{V_0}$$ because $\frac{1}{2}mV_0^2 + W = \frac{1}{2}m(v')^2$

$W = 0$

$0 = \frac{-v'}{v_0}$ $\qquad$ $\frac{1}{2} m v_0^2 + W = 0$

$W = -\frac{1}{2} m v_0^2$

$mgh_1 = \frac{1}{2} m v_2^2 + \frac{1}{2} I \dot{\theta}_2^2$ $\qquad$ $r\dot{\theta} = v$

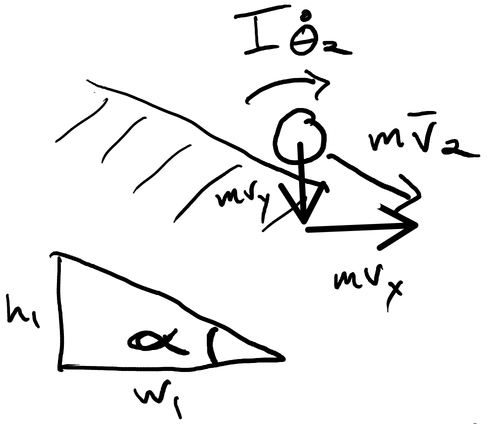


$d = r\theta$
$v = r\dot{\theta}$ [rolling w/o slipping]

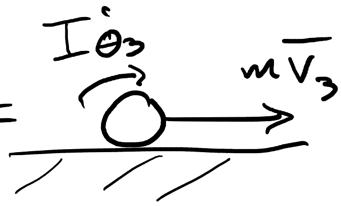
$\rightarrow$ $mgh_1 = \frac{1}{2} m v_2^2 + \frac{1}{2}\left(\frac{2}{5} m r^2\right) \cdot \left(\frac{v_2}{r}\right)^2$

$mgh_1 = \frac{1}{2} m v_2^2 \left(1 + \frac{2}{5}\right)$

$v_2 = \pm\sqrt{\frac{2gh}{1 + 2/5}} = \pm\sqrt{\frac{10}{7} gh}$

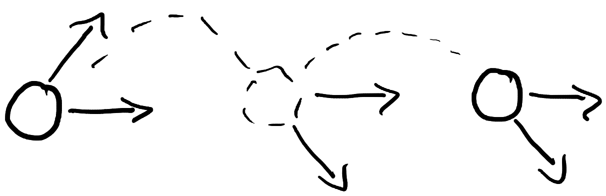


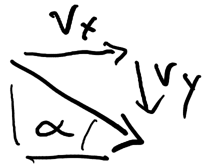
$+ \int$ 

$dt =$ 

$mv_y + F\Delta t = 0$

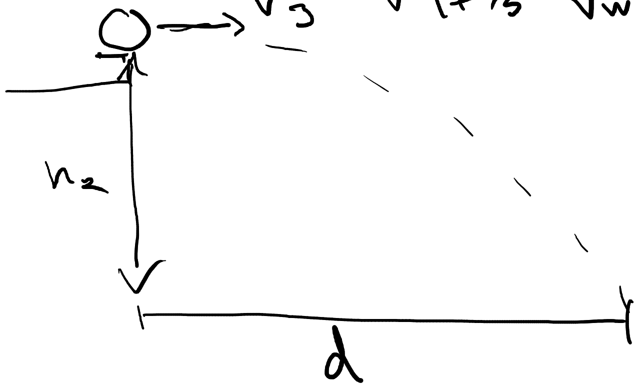
$mv_x + 0 = m\bar{v}_3$





$$V_2 \cdot \frac{w}{\sqrt{w^2+h_1^2}} = V_3$$

$$v_3 = \sqrt{\frac{2gh}{1+2/5}} \cdot \frac{w}{\sqrt{w^2+h^2}}$$



$$d = v_3 \cdot t$$

$$h_2 = \frac{g t^2}{2}$$