4.2 Consider a 10^4 kg spaceship in deep space. Suppose the spaceship's thruster produces 100 N of constant thrust. If the spaceship is moving at 50 m/s and the thruster is anti-aligned with the spaceship's velocity, how long should the thruster fire to bring the spaceship to rest with respect to absolute space?

$$\bullet \xrightarrow{ma}$$

$$a = \frac{F_{thrust}}{m}$$

$$\Delta v = a \Delta t$$

Impulse - Momentum

$$\bullet \xrightarrow{mv_0} + \int \xleftarrow{F_{thrust}} dt = \bullet \xrightarrow{mv_f = 0}$$

$$F \cdot \Delta t$$

$$(100E2 \cdot 50) \frac{kg \cdot m}{s} - 100\,N \cdot \Delta t = 0$$

$$\Delta t = \frac{100 \cdot 50E2}{100\,N} \cdot s$$

4.6 Suppose frame $\mathcal{B} = (O', \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3)$ is traveling in the $\mathbf{e}_1$ direction at a constant speed of v_0 with respect to stationary frame $\mathcal{I} = (O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, as shown in Figure 4.19. Mass m_P is connected to point O' by a spring with spring constant k and rest length r_0. The spring can freely pivot about O'. Assume that the positions of O and O' are the same at time $t = 0$.

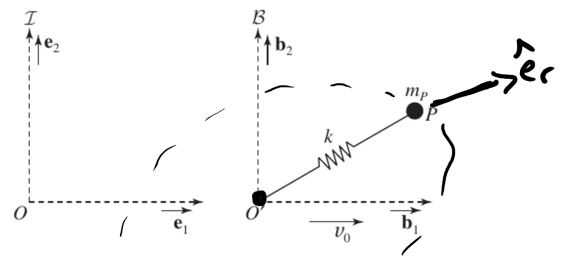


Figure 4.19 Problem 4.6.

a. Using the coordinates of your choice, find the position $\mathbf{r}_{P/O'}$ and velocity ${}^{\mathcal{I}}\mathbf{v}_{P/O'}$ of the mass with respect to O' in $\mathcal{I}$. [HINT: Introduce a polar frame at O'.]
b. Find the position $\mathbf{r}_{P/O}$ and velocity ${}^{\mathcal{I}}\mathbf{v}_{P/O}$ of the mass with respect to O in $\mathcal{I}$.
c. Draw a free-body diagram for mass m_P. (There is no gravity in this problem.)
d. Find the angular momentum ${}^{\mathcal{I}}\mathbf{h}_{P/O'}$ of the mass with respect to O' in $\mathcal{I}$.
e. Show that the angular momentum of the mass with respect to O' in $\mathcal{I}$ is conserved, but the angular momentum with respect to O in $\mathcal{I}$ is not.

FBD

$$\sum \bar{M}_{O'} = \bar{r}_{P/O'} \times \bar{F}_s$$

$$\bar{r}_{P/O'} = r\hat{e}_r$$

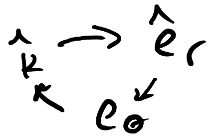
$$\bar{F}_s = -kr\hat{e}_r$$

$$\sum \bar{M}_{O'} = -kr^2 \cancel{\hat{e}_r \times \hat{e}_r}^{\,0} = 0$$

$$\sum M_{O'} = \frac{d}{dt}(\bar{h}_{O'}) = \frac{d}{dt}(\bar{r}_{P/O'} \times m\bar{v}_P)$$

$$\bar{r}_{P/O'} \times m\bar{v}_P = r\hat{e}_r \times m(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta)$$

$$= \cancel{mr\dot{r}\,\hat{e}_r \times \hat{e}_r}^{\,0} + mr^2\dot{\theta}\,\hat{e}_r \times \hat{e}_\theta$$



$$\boxed{\bar{h}_{O'} = mr^2\dot{\theta}\,\hat{b}_3}$$

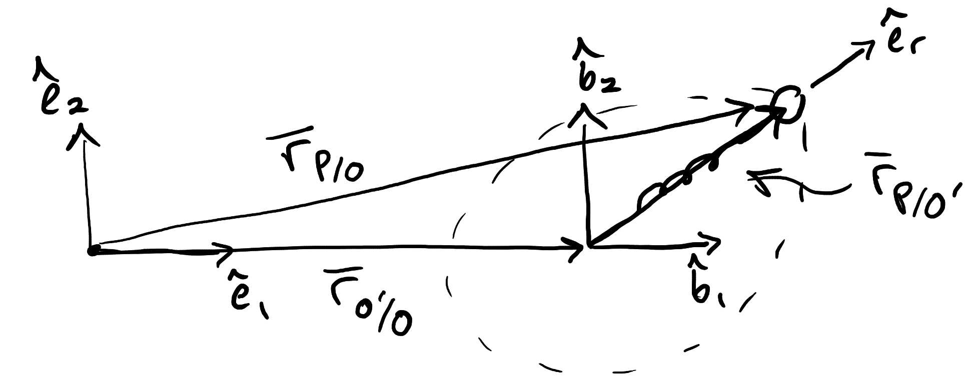
general def'n of angular momentum of particle

$$= mrv_\theta$$

$$0 = \frac{d}{dt}(mr^2\dot{\theta})$$

$$\boxed{mr^2\dot{\theta} = \text{constant}}$$ ✓

$$\boxed{r_0^2 \dot{\theta}_0 = \frac{\cancel{m}}{\cancel{m}} r^2 \dot{\theta}} \implies \dot{\theta} = \frac{r_0^2 \dot{\theta}_0}{r^2}$$



$$\sum \bar{M}_o = \bar{r}_{P/O} \times \bar{F}_s = (\bar{r}_{O'/O} + \bar{r}_{P/O'}) \times -kr\hat{e}_r$$

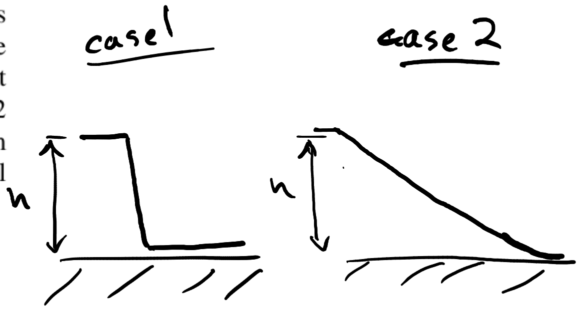
$$\sum \bar{M}_o = \bar{r}_{O'/O} \times -kr\hat{e}_r$$

$$\sum M_o = (vt\,\hat{b}_1) \times (-kr\hat{e}_r)$$

$$= (vt\,\hat{b}_1) \times (-kr)(\cos\theta\,\hat{b}_1 + \sin\theta\,\hat{b}_2)$$

$$\boxed{\sum M_o = -krvt\sin\theta\,\hat{b}_3}$$

9.14 A uniformly dense marble of mass $m = 0.05$ kg and radius $R = 0.01$ m is released from rest at the top of a ramp and rolls without slipping down the ramp and off a table, as shown in Figure 9.33. Find the distance d from the foot of the table where the marble lands on the floor. The ramp height is $h_1 = 0.2$ m, and its width is $w_1 = 0.4$ m. The marble rolls a distance of $w_2 = 0.15$ m on the table, which has height $h_2 = 1$ m. [HINT: Consider both the rotational and translational motion of the rolling marble.]



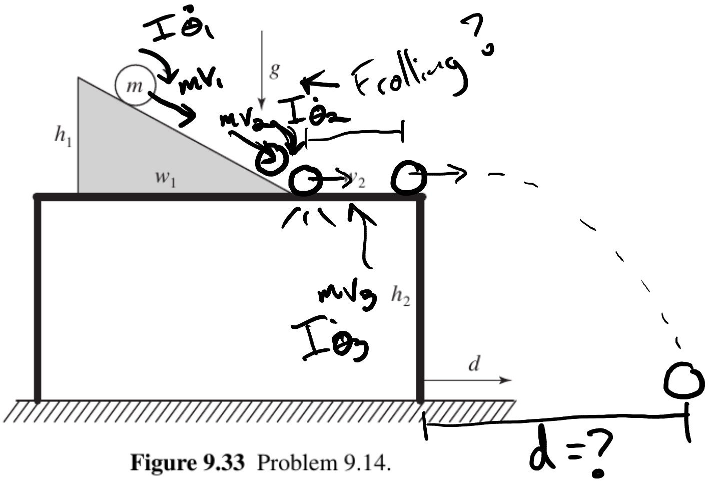
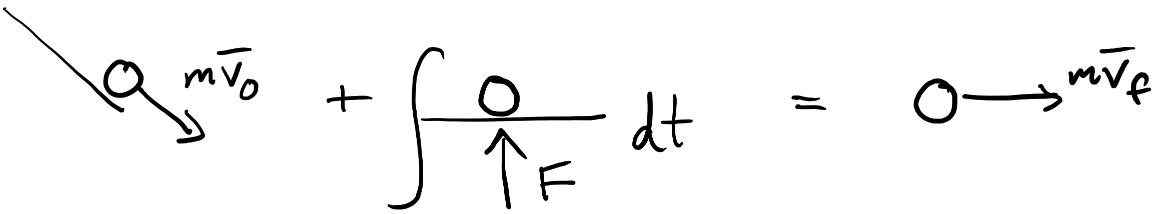


Figure 9.33 Problem 9.14.



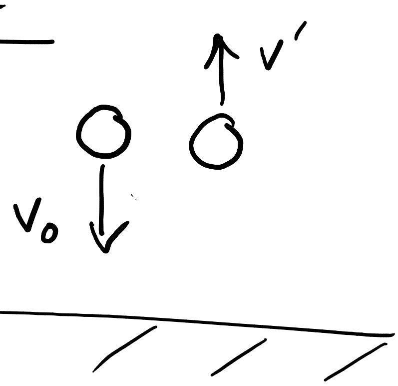
① → ② $\boxed{mgh_1 = T_2}$

② → ③ energy not conserved

calc d w/ Kinematic equs

coefficient of restitution

$$e = \frac{-v'}{v_0}$$



$$1 = \frac{-v'}{v_0}$$

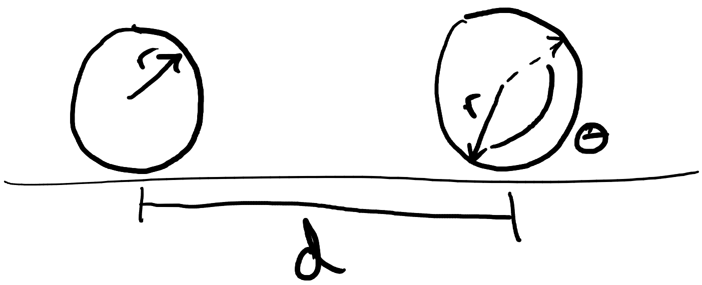
because $\frac{1}{2}mv_0^2 + W = \frac{1}{2}m(v')^2$

$W = 0$

$$0 = \frac{-v'}{v_0} \qquad \tfrac{1}{2} m v_0^2 + W = 0$$

$$W = -\tfrac{1}{2} m v_0^2$$

$$mgh_1 = \tfrac{1}{2} m v_2^2 + \tfrac{1}{2} I \dot{\theta}_2^2 \qquad r\dot{\theta} = v$$



$$\left.\begin{aligned} d &= r\theta \\ v &= r\dot{\theta} \end{aligned}\right. \quad \left[\text{rolling w/o slipping}\right]$$

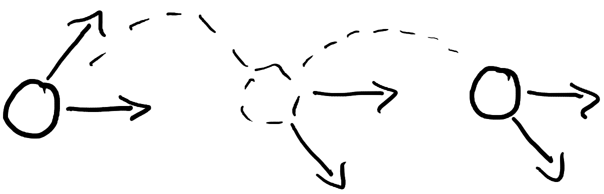
$$\rightarrow \quad mgh_1 = \tfrac{1}{2} m v_2^2 + \tfrac{1}{2}\left(\tfrac{2}{5} m r^2\right) \cdot \left(\frac{v_2}{r}\right)^2$$

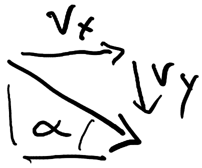
$$mgh_1 = \tfrac{1}{2} m v_2^2 \left(1 + \tfrac{2}{5}\right)$$

$$v_2 = \pm\sqrt{\frac{2gh}{1 + 2/5}} = \pm\sqrt{\tfrac{10}{7} gh}$$

$$mv_y + F\Delta t = 0$$

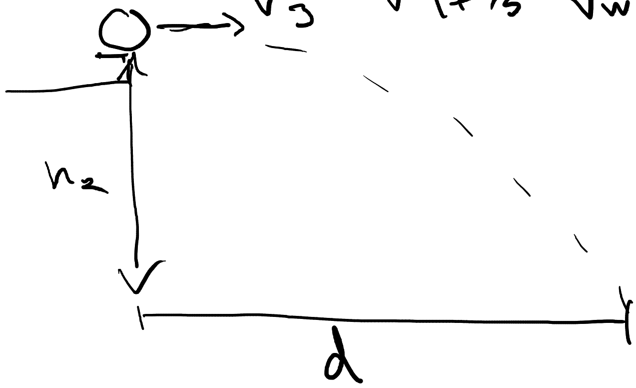
$$mv_x + 0 = m\bar{v}_3$$





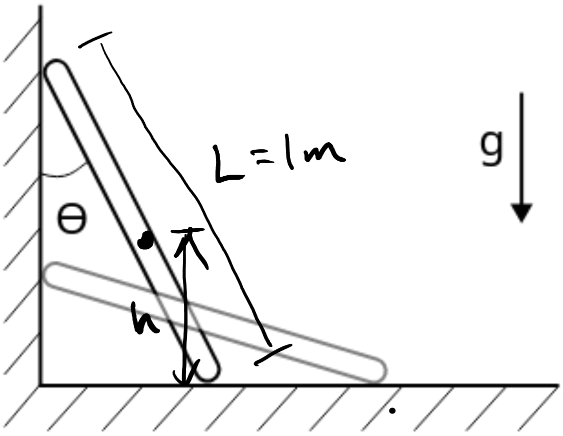
$$v_2 \cdot \frac{w}{\sqrt{w^2+h_1^2}} = v_3$$

$$v_3 = \sqrt{\frac{2gh}{1+2/5}} \cdot \frac{w}{\sqrt{w^2+h^2}}$$



$$d = v_3 \cdot t$$

$$h_2 = \frac{gt^2}{2}$$



$V = mgh$

$\theta(t=0) = 0 = \theta_1 \qquad \theta(t_1) = \pi/6 = \theta_2$

$\dot\theta(t=0) = 0 = \dot\theta_1 \qquad \dot\theta(t_1) = ? = \dot\theta_2$

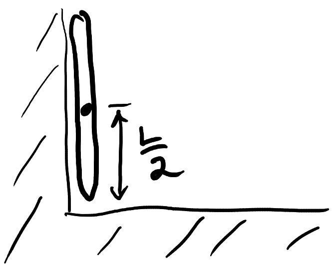
$$T = \tfrac{1}{2} m v^2 + \tfrac{1}{2} I \dot\theta^2$$

$T_1 + V_1 + \cancelto{0}{W_{1\to2}} = T_2 + V_2$ [cons. of energy]

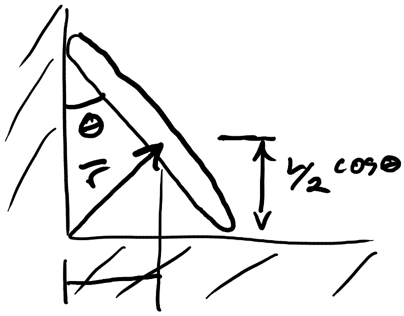
$T_1 = 0 \qquad T_2 = \tfrac{1}{2} m v_2^2 + \tfrac{1}{2} I \dot\theta_2^2$

$V_1 = mg\frac{L}{2} \qquad V_2 = mg\frac{L}{2}\cos\pi/6$

$h_1 = \frac{L}{2} \qquad h_2 = \frac{L}{2}\cos\theta$



$$\cancelto{0}{W_{1\to2}} + mg\frac{L}{2}\left(1 - \cos\frac{\pi}{6}\right) = \tfrac{1}{2} m v_2^2 + \tfrac{1}{2}\frac{mL^2}{12}\cdot\dot\theta_2^2$$

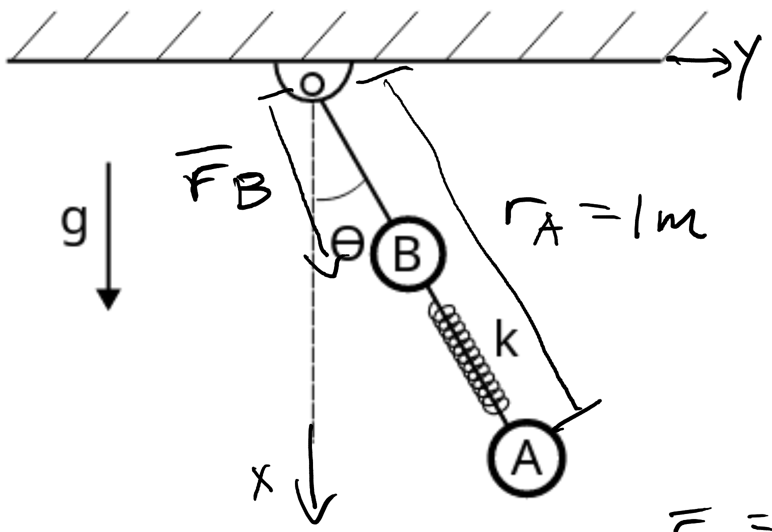


$$\bar{r} = \frac{L}{2}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$\bar{V} = \frac{L}{2}\dot{\theta}(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

$$\bar{V} \cdot \bar{V} = v^2 = \frac{L^2}{4}\dot{\theta}^2(\cos^2\theta + \sin^2\theta)$$

$$mg\frac{L}{2}\left(1 - \cos\frac{\pi}{6}\right) = \frac{1}{2}m\frac{L^2\dot{\theta}^2}{4} + \frac{1}{2}\frac{mL^2}{12}\dot{\theta}^2$$



Step 0

DOF = 2 DOF

$K = 1000 \, N/m$

$m_A = m_B = 1 \, kg$

$$\bar{r}_A = L(\cos\theta \,\hat{i} + \sin\theta \,\hat{j}) = L\hat{e}_r$$

$$\bar{r}_B = r_B(\cos\theta \,\hat{i} + \sin\theta \,\hat{j}) = r_B\hat{e}_r$$

$$\bar{V}_A = L\dot{\theta}(-\sin\theta \,\hat{i} + \cos\theta \,\hat{j}) = L\dot{\theta}\hat{e}_\theta$$

$$\bar{V}_B = r_B\dot{\theta}(-s\theta \,\hat{i} + c\theta \,\hat{j}) + \dot{r}_B(c\theta \,\hat{i} + s\theta \,\hat{j})$$

$$\bar{V}_B = r_B\dot{\theta}\,\hat{e}_\theta + \dot{r}_B\hat{e}_r$$

$$T = \tfrac{1}{2} m_A V_A^2 + \tfrac{1}{2} m_B V_B^2$$

$$V_A^2 = \bar{V}_A \cdot \bar{V}_A = L^2\dot{\theta}^2\overbrace{\cancel{(\sin^2\theta + \cos^2\theta)}}^{1}$$

$$V_B^2 = \left[(-r_B\dot{\theta}\sin\theta + \dot{r}_B\cos\theta)\hat{i} + (r_B\dot{\theta}\cos\theta\,\hat{j} + \dot{r}_B\sin\theta\,\hat{j})\right]^2$$

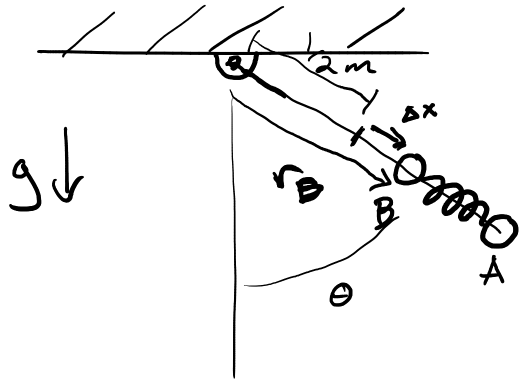
$$r_B^2\dot{\theta}^2\sin^2\theta + r_B^2\dot{\theta}^2\cos^2\theta$$

$$\dot{r}_B\cos^2\theta + \dot{r}_B\sin^2\theta$$

$$\overbrace{\cancel{-2r_B\dot{r}\cos\theta\sin\theta + 2r_B\dot{r}_B\cos\theta\sin\theta}}^{0}$$

$$V_B^2 = r_B^2\dot{\theta}^2 + \dot{r}_B^2$$

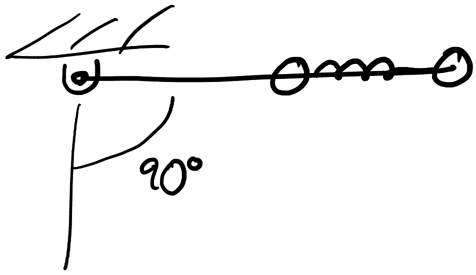
$$T = \tfrac{1}{2} m_A L^2\dot{\theta}^2 + \tfrac{1}{2} m_B(r_B^2\dot{\theta}^2 + \dot{r}_B^2)$$



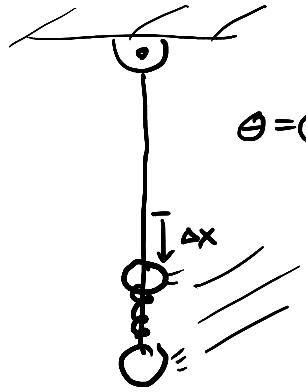
$$V = m_A g h_A + m_B g h_B + \tfrac{1}{2} k (\Delta x)^2$$

$$V = m_A g L\cos\theta + m_B g r_B \cos\theta + \tfrac{1}{2} k \left(r_B - \tfrac{1}{2}\right)^2$$

1



2



$$T_1 = 0$$

$$V_1 = -m_A g L \cos 90 - m_B g r_B \cos 90 + \tfrac{1}{2} k\left(\tfrac{1}{2} - \tfrac{1}{2}\right)^2$$

$$V_1 = 0$$

$$V_2 = -m_A g L \cos(0) - m_B g r_B \cos(0) + \tfrac{1}{2} k\left(r_B - \tfrac{1}{2}\right)^2$$

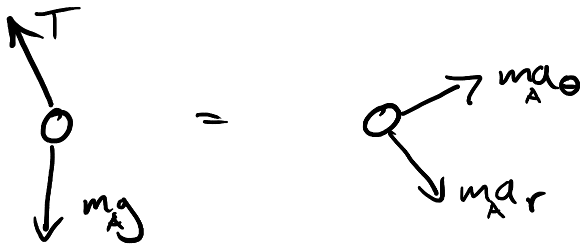
$$V_2 = -mgL - m_B g r_B + \tfrac{1}{2} k\left(r_B - \tfrac{1}{2}\right)^2$$

$$T_2 = \tfrac{1}{2} m_A L^2 \dot{\theta}^2 + \tfrac{1}{2} m_B\left(r_B^2 \dot{\theta}^2 + \dot{r}_B^2\right)$$

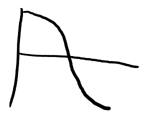
$$T_1 + V_1 = T_2 + V_2$$

$$0 = -m_A g L - m_B g r_B + \tfrac{1}{2} k\left(r_B - \tfrac{1}{2}\right)^2 + \tfrac{1}{2} m_A L^2 \dot{\theta}^2 + \tfrac{1}{2} m_B\left(r_B^2 \dot{\theta}^2 + \dot{r}_B^2\right)$$

missing info



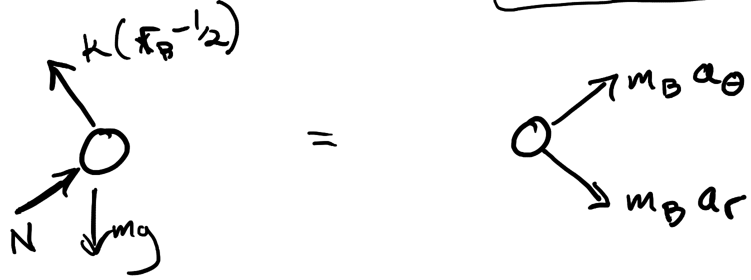
$$\ddot{\theta} = -\frac{g}{L}\sin\theta$$



$$\dot{\theta}\frac{d\dot{\theta}}{d\theta} = -\frac{g}{L}\sin\theta$$

$$\dot{\theta}^2 - \cancel{\dot{\theta}_0^2}^{\,0} = \frac{-2g}{L}\left(\cancel{\cos\theta}^{\,0} - \cancel{\cos\theta_0}^{\,1}\right)$$

$$\boxed{\dot{\theta} = \sqrt{\frac{2g}{L}} \quad @ \ \theta = 0^\circ}$$



$$-k(r_B - 1/2) + mg\cos\theta = m_B(\ddot{r} - r\dot{\theta}^2)$$

$$N - mg\sin\theta = m_B(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$m_B\ddot{r} + kr = \frac{k}{2} + mg\cos\theta + mr\dot{\theta}^2$$

$$m_B\ddot{r} + (k - m\dot{\theta}^2)r = \frac{k}{2} + mg\cos\theta$$

$$\text{if } \ddot{r} = 0 \implies r = \frac{k}{2}\cdot\frac{1}{k - m\dot{\theta}^2} = \frac{k}{2}\cdot\frac{1}{k - m\cdot\frac{2g}{L}}$$