Brandvain and Coop. Sperm dependent female meiotic drive

In[80]:=
```
ClearAll["Global`*"]
```

Model 1. Female drive depends on sperm haplotype

The B allele is transmited with probability, d, in heterozygous females when fertilized by B-bearing sperm.
x represents the deviation from Hardy - Weinberg Equilibrium

Setup

In[89]:=
```
(*Allele and Genotype frequencies*)
fA = 1 - fB;
fAA  =  fA^2 + fA fB x;
fAB  =  2 fA fB (1 - x);
fBB =  fB^2 + fA fB x;
```

Drive

In[93]:=
```
(*Genotype frequencies after drive*)
fAA_Drive = FullSimplify[fA (fAA + fAB / 2)];
fAB_Drive  = FullSimplify[fB (fAA + fAB * (1 - d)) + fA (fAB / 2 + fBB)];
fBB_Drive  = FullSimplify[fB (fAB d + fBB )];
```

Selection

In[113]:=
```
wAA = 1; wAB = 1 - hs; wBB = 1 - s; (*genotypic fitnesses*)
W̅ = FullSimplify[fAA_Drive wAA + fAB_Drive wAB + fBB_Drive wBB]; (*mean fitness*)
fAA_Sel = FullSimplify[(fAA_Drive * wAA) / W̅];
fAB_Sel = FullSimplify[(fAB_Drive * wAB) / W̅];
fBB_Sel = FullSimplify[(fBB_Drive wBB) / W̅];
fA_Sel = FullSimplify[fAA_Sel + fAB_Sel / 2];
fB_Sel = FullSimplify[fBB_Sel + fAB_Sel / 2];
ΔfA = FullSimplify[fA_Sel - fA];
ΔfB = FullSimplify[fB_Sel - fB];
```

Analysis

Note, we assume no deviation from Hardy-Weinberg [i.e. x=0] for all analytical results, and therefore these answers are approximations. In the supplamentary material we show thats results of exact recursions are remarkably consistant from these approximate analystical solutions.

Assuming the cost of drive is fully recessive [i.e. hs is zero]

Invasion

In[147]:=
```
ΔfBinvade = (FullSimplify[ΔfB /. hs → 0 /. x → 0] / fB^2 /. fB → 0)
```

Out[147]=
$$\frac{1}{2}\,(-1 + d\,(2 - 4\,s))$$

In[167]:=
```
spermDepReceesiveInvade = Solve[ΔfBinvade == 0, s]
```

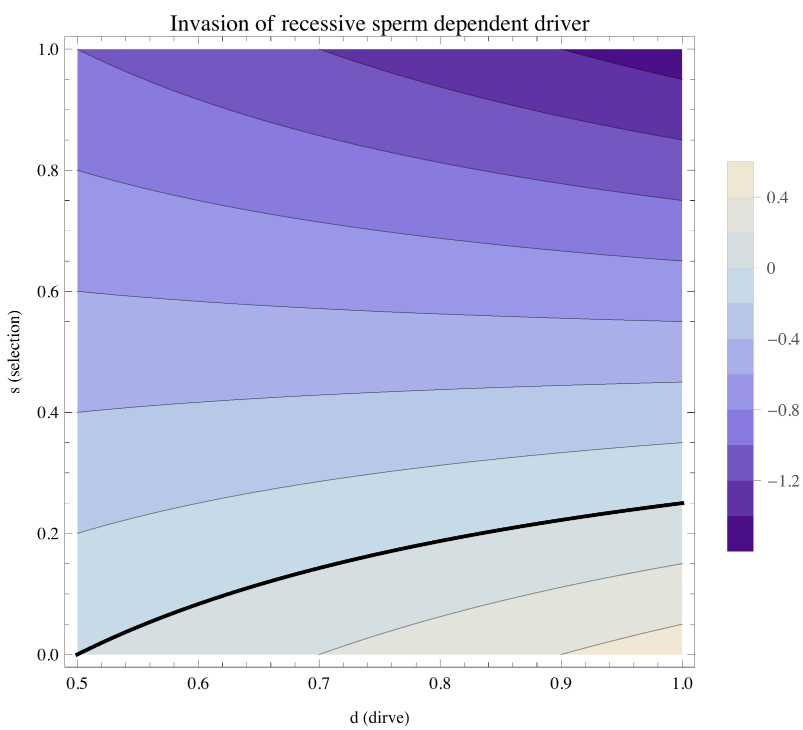
Out[167]=
$$\left\{\left\{s \to \frac{-1 + 2\,d}{4\,d}\right\}\right\}$$

In[252]:=
```
plotInvasion4spermDepRecessive =
  Plot[s /. spermDepReceesiveInvade[[1]], {d, .5, 1}, PlotStyle → {Black, Thick}];
```

In[253]:=
```
plotRelChange4RarespermDepRecessive = ContourPlot[{ΔfBinvade}, {d, 0.5, 1}, {s, 0, 1},
   PlotLegends → Automatic, FrameLabel → {"d (dirve)", "s (selection)"},
   PlotLabel → "Invasion of recessive sperm dependent driver"];
```

In[254]:=
```
Show[plotRelChange4RarespermDepRecessive, plotInvasion4spermDepRecessive]
```

Out[254]=



Fixation

In[222]:=
```
ΔfBfix = FullSimplify[FullSimplify[ΔfB /. hs → 0 /. x → 0] / fA] /. fB → 1
```

Out[222]=
$$\frac{-1 + 2\,d - 2\,s}{2 - 2\,s}$$

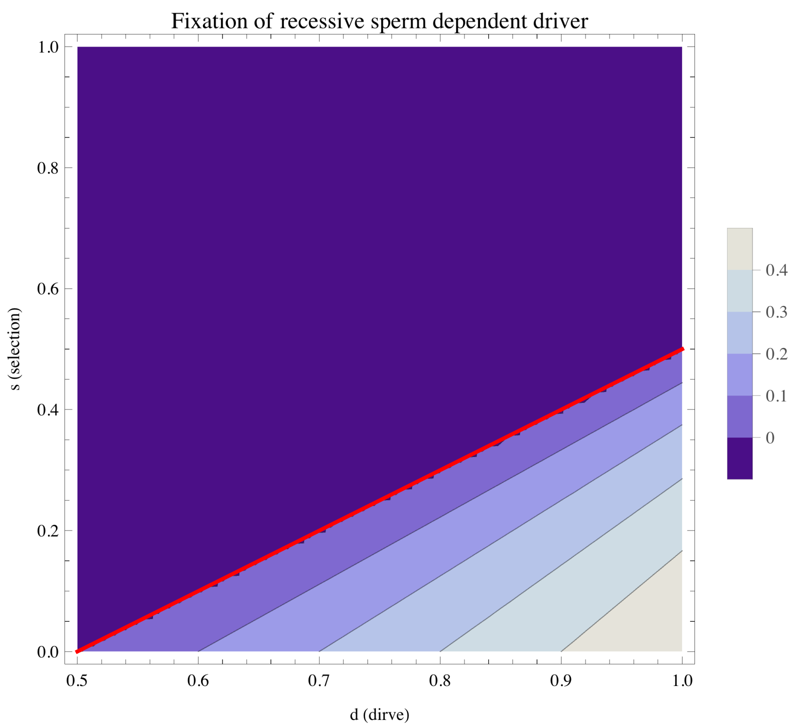
In[223]:=
```
spermDepReceesiveFix = Solve[ΔfBfix == 0, s]
(s /. spermDepReceesiveFix [[1]])
```

Out[246]=
$$\frac{1}{2}(-1+2\,d)$$

In[255]:=
```
plotFixation4spermDepRecessive =
  Plot[s /. spermDepReceesiveFix [[1]], {d, .5, 1}, PlotStyle → {Red, Thick}];
```

In[256]:=
```
(*Note we artificially rescaled z to be -.1 for all negative values*)
plotRelChange4CommonSpermDepRecessive =
  ContourPlot[If[s > (s /. spermDepReceesiveFix [[1]]), -.1, ΔfBfix], {d, 0.5, 1},
   {s, 0, 1}, PlotLegends → Automatic, FrameLabel → {"d (dirve)", "s (selection)"},
   PlotLabel → "Fixation of recessive sperm dependent driver"];
```

In[257]:=
```
Show[plotRelChange4CommonSpermDepRecessive, plotFixation4spermDepRecessive]
```

Out[257]=


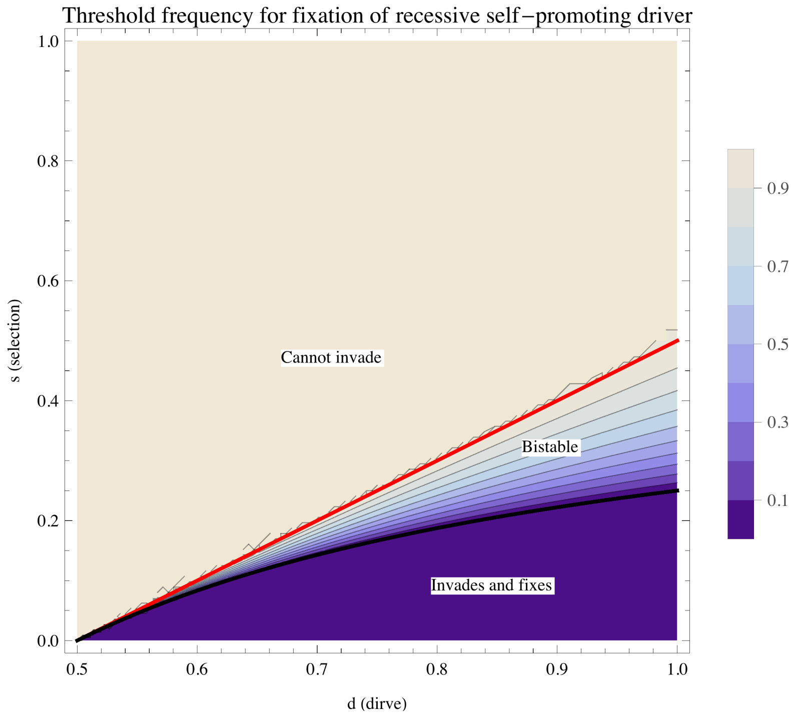
Bistability Point

In[274]:=
```
FBbistabSpermDepReceesive = Solve[FullSimplify[ΔfB /. hs → 0 /. x → 0] == 0, fB][[4]]
```

Out[274]=
$$\left\{fB \to \frac{1-2\,d+4\,d\,s}{-2\,s+4\,d\,s}\right\}$$

In[284]:=
```
bistab = ContourPlot[(If[fB < 0, 0, If[fB > 1, 1, fB]]) /. FBbistabSpermDepReceesive,
  {d, .5, 1}, {s, 0, 1}, PlotLegends → Automatic,
  FrameLabel → {"d (dirve)", "s (selection)"}, PlotLabel →
   "Threshold frequency for fixation of recessive self-promoting driver"]
```

In[285]:=
```
Show[bistab, plotFixation4spermDepRecessive, plotInvasion4spermDepRecessive]
```



Assuming the cost of drive is not fully recessive [i.e. hs is nonzero]

Invasion

Note with any heterozgous cost (i.e. hs > 0) a self - promoting driver cannot invade

In[293]:= `FullSimplify[FullSimplify[ΔfB /. x → 0] / fB] /. fB → 0`

Out[293]= $-\text{hs}$

Fixation

In[295]:= `ΔfBfix = FullSimplify[FullSimplify[FullSimplify[ΔfB /. x → 0] / fA] /. fB → 1]`

Out[295]= $$\frac{1 + 2\,d\,(-1 + \text{hs}) - 3\,\text{hs} + 2\,s}{2\,(-1 + s)}$$

In[303]:= `spermDepNotReceesiveFix = Solve[ΔfBfix == 0, s]`

Out[303]= $$\left\{\left\{s \to \frac{1}{2}\,(-1 + 2\,d + 3\,\text{hs} - 2\,d\,\text{hs})\right\}\right\}$$

In[317]:= `spermDepAddFix = Solve[ΔfBfix == 0 /. hs → s / 2, s]`

Out[317]= $$\left\{\left\{s \to \frac{2\,(-1 + 2\,d)}{1 + 2\,d}\right\}\right\}$$

In[318]:=
```
plotspermDepAddFix =
  Plot[s /. spermDepAddReceesiveFix , {d, .5, 1}, PlotStyle → {Red, Thick}];
```

Bistability Point

In[319]:=
```
FBbistabSpermDepNotReceesive = Solve[FullSimplify[ΔfB /. x → 0] == 0, fB][[3]]
```

Out[319]=

$$\left\{fB \to \left(-1 + 2\,d + 3\,hs + 2\,d\,hs - 4\,d\,s - \sqrt{-8\,hs\,(-2\,hs + 4\,d\,hs + 2\,s - 4\,d\,s) + (1 - 2\,d - 3\,hs - 2\,d\,hs + 4\,d\,s)^2}\right) \Big/ (2\,(-2\,hs + 4\,d\,hs + 2\,s - 4\,d\,s))\right\}$$

An Example of a non - recessive driver [Assuming additivity]

In[332]:=
```
bistab = ContourPlot[
   (If[fB < 0, 0, If[fB > 1, 1, fB]]) /. FBbistabSpermDepNotReceesive /. hs → (s / 2),
   {d, .5, 1}, {s, 0, 1}, PlotLegends → Automatic,
   FrameLabel → {"d (dirve)", "s (selection)"}, PlotLabel →
    "Threshold frequency for fixation of recessive self-promoting driver"];
```

In[333]:=
```
Show[bistab, plotspermDepAddFix]
```