HW10

1.

The KKT condition is

$$\exists \mu_1^*, \mu_2^* \in R^2, st.$$
$$\begin{pmatrix} 2x_1^* \\ 2x_2^* \end{pmatrix} + \mu_1^* \begin{pmatrix} 2x_1^* - 2 \\ 2x_2^* - 2 \end{pmatrix} + \mu_2^* \begin{pmatrix} 2x_1^* - 2 \\ 2x_2^* \end{pmatrix} = 0$$
$$\mu_1^* \geq 0$$
$$\mu_2^* \geq 0$$
$$\mu_1^*[(x_1^* - 1)^2 + (x_2^* - 1)^2 - 1] = 0$$
$$\mu_2^*\left[(x_1^* - 1)^2 + x_2^{*2} - 1\right] = 0$$

The optimal point $x^* = \begin{pmatrix} 1-\frac{\sqrt{2}}{2} \\ 1-\frac{\sqrt{2}}{2} \end{pmatrix}$ and $\mu_1^* = \sqrt{2} - 1, \mu_2^* = 0.$

2.

(a)

$$x^* = \begin{pmatrix} 1 \\ 0 \end{pmatrix} and\ f^* = 1$$

(b)

The KKT condition is

$$\exists \mu_1^*, \mu_2^* \in R^2, st.$$
$$\begin{pmatrix} 2x_1^* \\ 2x_2^* \end{pmatrix} + \mu_1^* \begin{pmatrix} 2x_1^* - 2 \\ 2x_2^* - 2 \end{pmatrix} + \mu_2^* \begin{pmatrix} 2x_1^* - 2 \\ 2x_2^* + 2 \end{pmatrix} = 0$$
$$\mu_1^* \geq 0$$
$$\mu_2^* \geq 0$$
$$\mu_1^*[(x_1^* - 1)^2 + (x_2^* - 1)^2 - 1] = 0$$
$$\mu_2^*[(x_1^* - 1)^2 + (x_2^* + 1)^2 - 1] = 0$$

$$\nabla g_1(x^*) = \begin{pmatrix} 2x_1^* - 2 \\ 2x_2^* - 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$
$$\nabla g_2(x^*) = \begin{pmatrix} 2x_1^* - 2 \\ 2x_2^* + 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

Because

$$\nabla g_1(x^*) = -\nabla g_2(x^*)$$

So, x^* is a regular point.

3.

$x^{(1)}$ is not an optimal solution, because $-x_1^{(1)^2} + x_2^{(1)^2} = -\frac{49}{16} < 0.$

$x^{(2)}$ is not an optimal solution, because g_1 and g_2 are both inactive at $x^{(2)}$.

$x^{(3)}$ is an optimal solution.

The KKT multipliers are $\mu_1^* = \frac{1}{2}\ and\ \mu_2^* = 0.$

4.

(a)

The KKT condition is

$$(x^* - z) + \mu^* y = 0$$
$$\mu^* y^T x^* = 0$$

Let $x^* = z - \lambda y$. Namely, $x_i^* = z_i^* - \lambda y_i^*, i = 1,2,3, \dots, n.$

$$(x^* - z) + \mu^* y = (z - \lambda y - z) + \mu^* y = 0$$
$$\lambda = \mu^*$$

So, there exists $\lambda = \mu^* \in R$ which makes $x^* = z - \lambda y$ satisfies the KKT condition.

(b)

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optimal point : [0.33333333333333337, 1.3333333333333335, 1.6666666666666665]
lambda multiplier : 0.6666666666666666
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