1. a) $2{x_1}^2 + {x_2}^2 + x_1x_2 - 3x_1 - 5x_2 \geq \frac{3}{2}(x_1-1)^2 + \frac{1}{2}(x_2-5)^2 - 14$

$Aparently, when\ \|x\| \to +\infty, f(x) \to +\infty, therefore\ f(x)\ is\ coerive.$

b) $f(x)\ doesn't\ have\ maximum.$

$The\ minimum\ of\ f(x)\ is\ \ -14, when\ x = \binom{1}{5}$.

2. a) *Because there exists a* ω_0 *such that* $y_i{x_i}^T\omega_0 > 0\ \forall\ i = 0,1,2, \ldots \ldots$,

there exist $\omega_1 = -\omega_0$ *such that* $-y_i{x_i}^T\omega_1 > 0\ \forall\ i = 0,1,2, \ldots \ldots$

So f > mlog2. Thus, f has a minimum.

b) $\forall\ \omega,\ \log(1+e^{-y_ix_i\omega}) > \log(e^{-y_ix_i\omega}) = -y_ix_i\omega.$

Let h(ω) = $-y_{i_0}x_{i_0}\omega$. $\mathrm{Log}(1+e^{-y_{i0}x_{i0}\omega}) > h(\omega)$.

$\forall\ i \neq i_0$, $\log(1+e^{-y_ix_i\omega})$ *> 0, so f(ω) > h(ω).*

Let S = $\{\omega |\ \|\omega\| = 1\}$ *be the unit sphere. Because h(ω) is continuous and bounded on S, h(ω) has minimum h(*ω_1*) on S. because there exists* ω_0 *such that* $-y_{i_0}x_{i_0}\omega > 0$*, so h(*$\omega_1$*) > 0. Let h(*$\omega_1$*) = C.*

Straightforward $\forall\ \omega, h(\omega) \geq C\|\omega\| \geq 0.$

So f (ω) has a minimum.

c) $\nabla f(\omega) = \sum_{i=0}^{m} \frac{e^{-y_i{x_i}^T\omega}}{1+e^{-y_i{x_i}^T\omega}}(-y_i{x_i}^T)$

3. a) $Let\ g(t) = f(td+x)$.

$g(1) = g(0) + g'(0) + \frac{1}{2}g''(0)\,t^2$ t∈ $(0,1)$

$g'(0) = d \cdot \nabla f(x+td)$

$g''(0) = d \cdot \nabla^2 f(x+td) \cdot d^T$

So, $f(x+td) = f(x) + \nabla f(x+td)\,d^T + \frac{1}{2}d^T \cdot \nabla^2 f(x+td)\,d^T$

b) *Let g(t) = f(x + td)*

so, $g''(t)dt^2 = \nabla^2 f(x+td)ddt^2$

g''(t)dt = $\nabla^2 f(x+td)d^2dt$

$\int_0^1 \nabla^2 f(x+td)d^2dt = \int_0^1 g''(t)dt = g'(1) - g'(0) = \nabla f(x+d)d - \nabla f(x)d$

So, $\nabla f(x+d) = \nabla f(x) + \int_0^1 \nabla^2 f(x+td)ddt$

4. A *is positive definite. D1 = 6, D2 = 7, D3 = 72.*

B is indefinite. D1 = 1, D2 = -2, D3 = 9.

C is positive semidefinite. D3 = 0.