HW5

1. Apparently, $\widehat{\boldsymbol{x}_0} = \frac{\boldsymbol{x}_0}{\|\boldsymbol{x}_0\|} \in \boldsymbol{X}$.

   $\nabla f(x) = \boldsymbol{x} - \boldsymbol{x}_0$.

   $$\nabla f(\widehat{\boldsymbol{x}}_0)(\widehat{\boldsymbol{x}}_0 - \boldsymbol{x}) = (\widehat{\boldsymbol{x}}_0 - \boldsymbol{x}_0)^T(\widehat{\boldsymbol{x}}_0 - \boldsymbol{x})$$
   $$= \left(\widehat{\boldsymbol{x}}_0{}^T - \boldsymbol{x}_0{}^T\right)(\widehat{\boldsymbol{x}}_0 - \boldsymbol{x})$$

   Because $\widehat{\boldsymbol{x}_0} = \frac{\boldsymbol{x}_0}{\|\boldsymbol{x}_0\|}$, $\widehat{\boldsymbol{x}}_0{}^T - \boldsymbol{x}_0{}^T$ is parallel to $\widehat{\boldsymbol{x}}_0$.

   Because $\boldsymbol{x} \in \overline{\boldsymbol{B}}$, $\|\boldsymbol{x}\| \le \|\widehat{\boldsymbol{x}_0}\| = 1$.

   Therefore, $\left(\widehat{\boldsymbol{x}}_0{}^T - \boldsymbol{x}_0{}^T\right)\widehat{\boldsymbol{x}}_0 \ge \left(\widehat{\boldsymbol{x}}_0{}^T - \boldsymbol{x}_0{}^T\right)\boldsymbol{x}$.

   Namely, $\nabla f(\widehat{\boldsymbol{x}}_0)(\widehat{\boldsymbol{x}}_0 - \boldsymbol{x}) \ge 0$.

   By the first-order optimality condition, $\widehat{\boldsymbol{x}}_0$ is the solution to the problem.

   Thus, $\widehat{\boldsymbol{x}}_0$ is the projection of $\boldsymbol{x}_0$ onto $\overline{B}$.

2.

```
a)
Status optimal
Optimal value 0.5999999999116253
Optimal var x1 : 0.3999999999724491 x2 : 0.1999999999391762
b)
Status unbounded
Optimal value -inf
Optimal var x1 : None x2 : None
c)
Status optimal
Optimal value -2.2491441767693299e-10
Optimal var x1 : -2.2491441767693299e-10 x2 : 1.5537158969947242
d)
Status optimal
Optimal value 0.3333333334080862
Optimal var x1 : 0.3333333334080862 x2 : 0.3333333286259564
e)
Status optimal
Optimal value 0.5000000000000002
Optimal var x1 : 0.5000000000000001 x2 : 0.1666666666666667
```

3. (a) $min(\boldsymbol{1}^T\boldsymbol{s})$

   s.t $-\boldsymbol{1} \le \boldsymbol{x} \le \boldsymbol{1}$

   $-\boldsymbol{s} \le \boldsymbol{A}\boldsymbol{x} - \boldsymbol{b} \le \boldsymbol{s}$

   (b)

```
b)
Status optimal
Optimal value 13.999999990735517
Optimal var [ 1. -1.]
```

(c)

```
c)
Status optimal
Optimal value 13.999999998611607
Optimal var x : [ 1. -1.] s : [4. 6. 4.]
```

4. a) The problem can be converted to:

$$min(2\omega_1 - 3)^2 + (\omega_2 - 2)^2 + 4$$

Apparently, $(2\omega_1 - 3)^2 + (\omega_2 - 2)^2 \geq 0$.

Thus, the optimal value is 4.

The optimal point $\omega_0 = (1.5,\ 2)$.

b)

```
when t = 1
Status optimal
Optimal value 9.0000000633334
Optimal var [9.99962136e-01 3.78500362e-05]

when t = 100
Status optimal
Optimal value 4.000000000000032
Optimal var [1.49999997 1.99999984]
```

When $t = 1$, the solution is not the same as (a).

Neither of them has zero components.

(c)

```
when t = 1
Status optimal
Optimal value 7.857489685034338
Optimal var [0.86270479 0.50570788]
```

```
when t = 100
Status optimal
Optimal value 4.0000000000000195
Optimal var [1.50000002 2.00000013]
```

When $t = 1$, the solution is not the same as (a).

Neither of them has zero components.