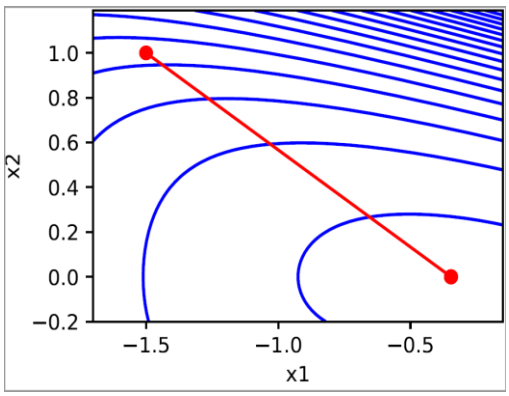
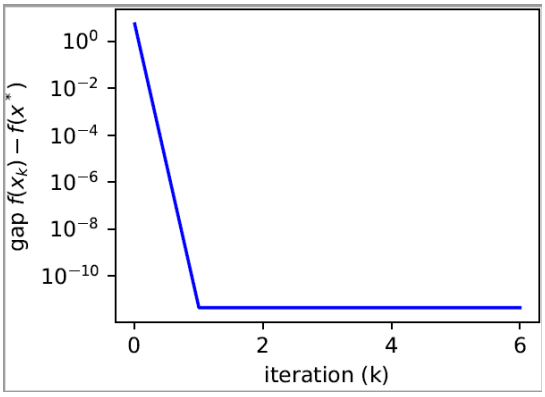
HW8

1.

a)

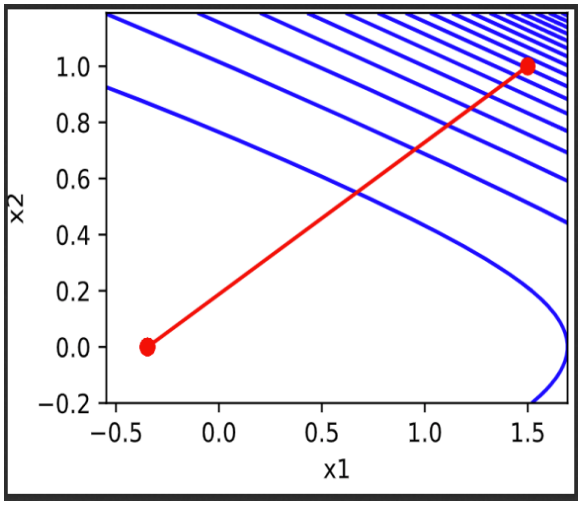
```
Newton's method
  number of iterations: 6
  solution: [-3.46575460e-01  4.43431294e-15]
  value: 2.5592666966626867
```

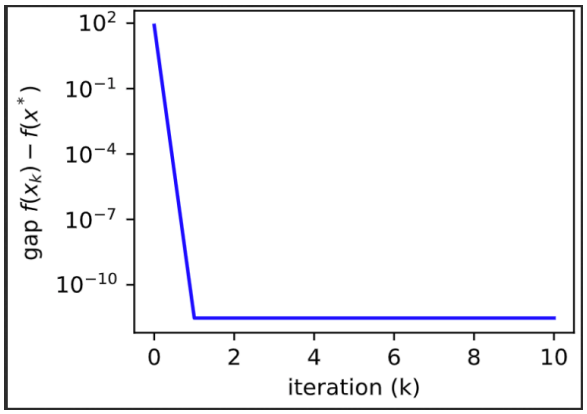




b)

```
Newton's method
  number of iterations: 10
  solution: [-3.46575112e-01 -1.41769540e-16]
  value: 2.5592666966611786
```





2.

a)

$$\nabla f(\omega) = -\sum_{i=0}^{m}\left(1-\sigma(y_i {x_i}^T \omega)\right) y_i x_i = -\sum_{i=0}^{m} y_i x_i + \sum_{i=0}^{m}(\sigma(y_i {x_i}^T \omega) y_i x_i)$$

$$\nabla^2 f(x) = 0 + \nabla \sum_{i=0}^{m}(\sigma(y_i {x_i}^T \omega) y_i x_i)$$

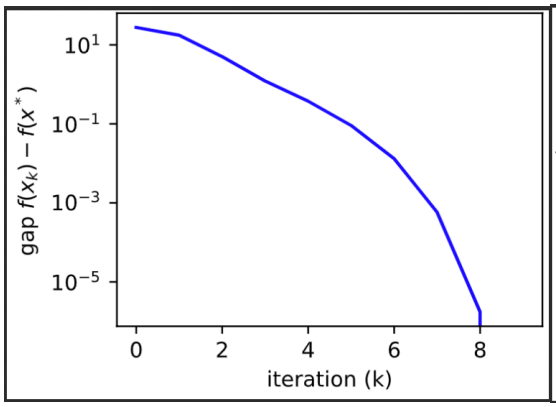
$$\nabla^2 f(x) = \sum_{i=0}^{m} \sigma'(y_i {x_i}^T \omega) y_i {x_i}^T y_i x_i$$

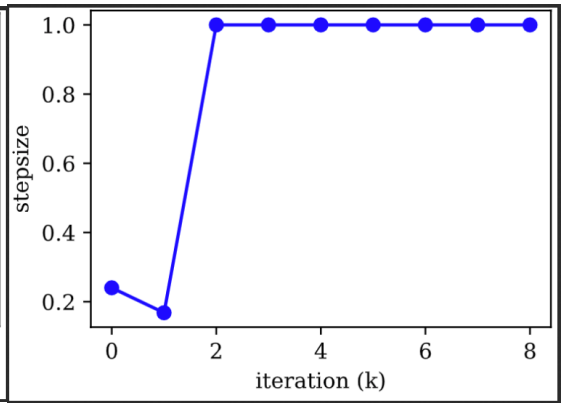
Because $y_i \in \{+1, -1\}$,

$$\nabla^2 f(x) = \sum_{i=0}^{m} \sigma'(y_i {x_i}^T \omega) {x_i}^T x_i$$

b)

```
Damped Newton's method
  number of iterations in outer loop: 9
  total number of iterations in inner loop: 9
  solution: [-1.47021306  4.44400878 -4.3758784 ]
  value: 2.876681099986131
```





c)

An exception occurred.

```
numpy.linalg.LinAlgError: Singular matrix
```

Because in one iteration, a $\nabla^2 f(x)$ becomes a singular matrix, which make it impossible to produce the next x_k.

3.

a) $x_{k+1} = x_k - [\nabla^2 f(x_k)]^{-1} \cdot \nabla f(x_k) = x_k - \frac{4(x_k-a)^3}{12(x_k-a)^2} = x_k - \frac{1}{3}(x_k - a) = \frac{2}{3}x_k + \frac{1}{3}a$

b) $y_{k+1} = |x_{k+1} - a| = \left|\frac{2}{3}x_k + \frac{1}{3}a - a\right| = \frac{2}{3}|x_k - a| = \frac{2}{3}y_k$

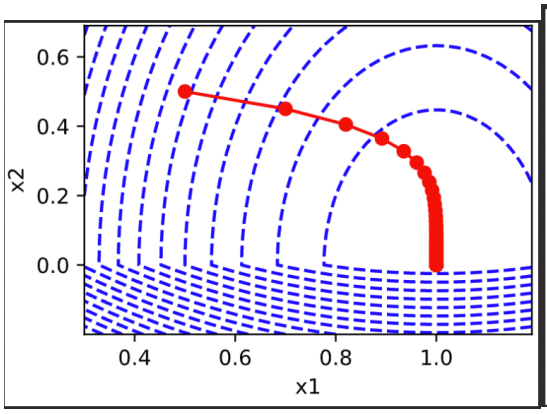
c) Therefore, y_k decays exponentially.

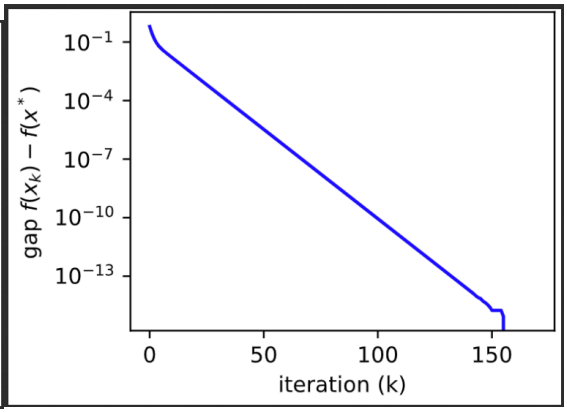
Namely, x_k decays to 0 exponentially.

4.

a)

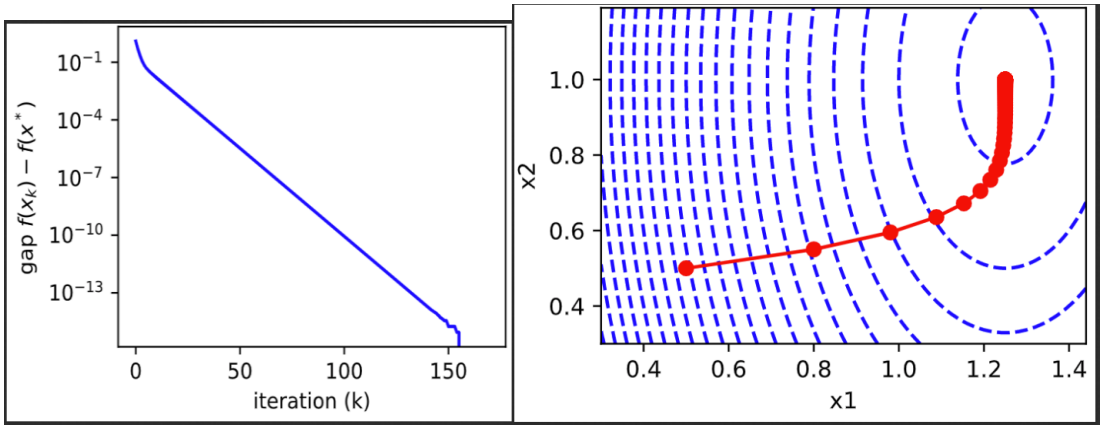
```
lambda =  2
number of iterations: 169
solution: [1.00000000e+00 9.24600449e-09]
value: 6.5
```





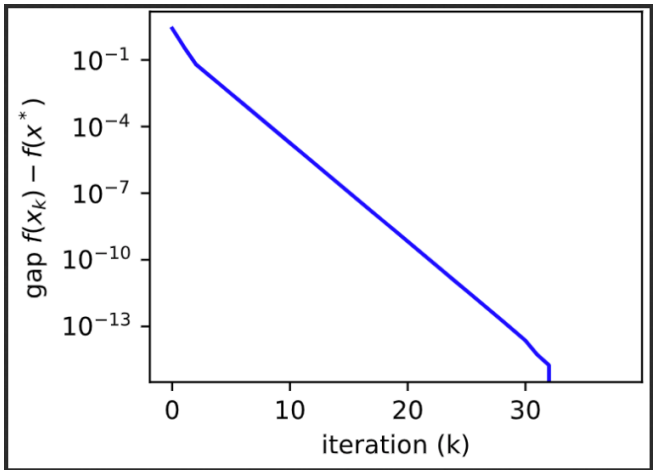
b)

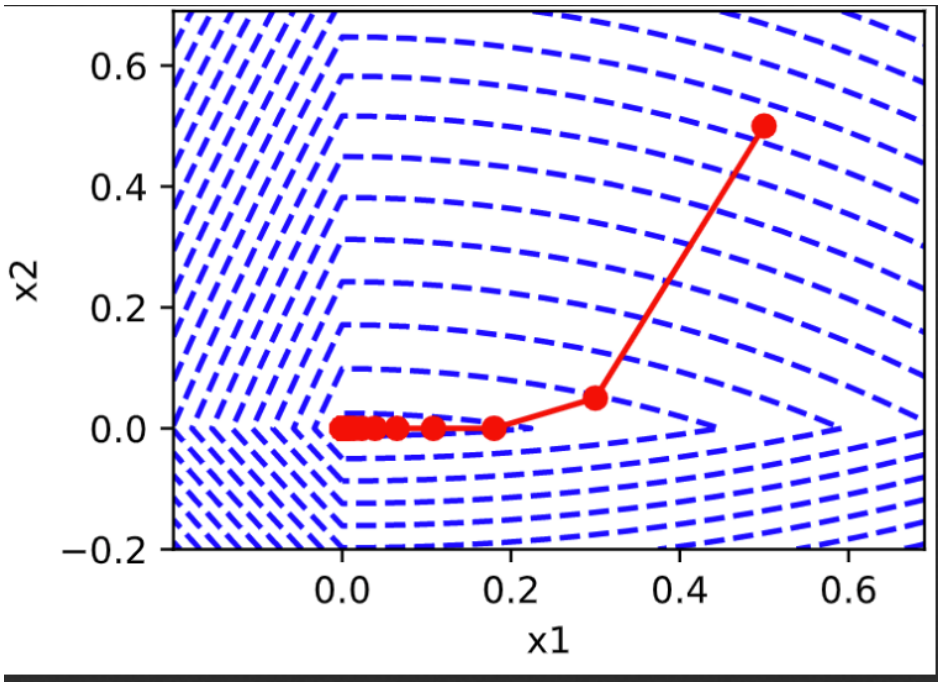
```
lambda =  1
number of iterations: 169
solution: [1.25       0.99999999]
value: 4.875
```



c)

```
lambda =  6
number of iterations: 38
solution: [1.85659632e-09 0.00000000e+00]
value: 8.500000000000002
```




0.6
0.4
0.2
0.0
−0.2
x2
0.0
0.2
0.4
0.6
x1