HW7

1. (a) $\nabla f(\boldsymbol{x}) = \begin{pmatrix} e^{x_1+3x_2-0.1} + e^{x_1-3x_2-0.1} - e^{-x_1-0.1} & 3e^{x_1+3x_2-0.1} - 3e^{x_1-3x_2-0.1} \end{pmatrix}$

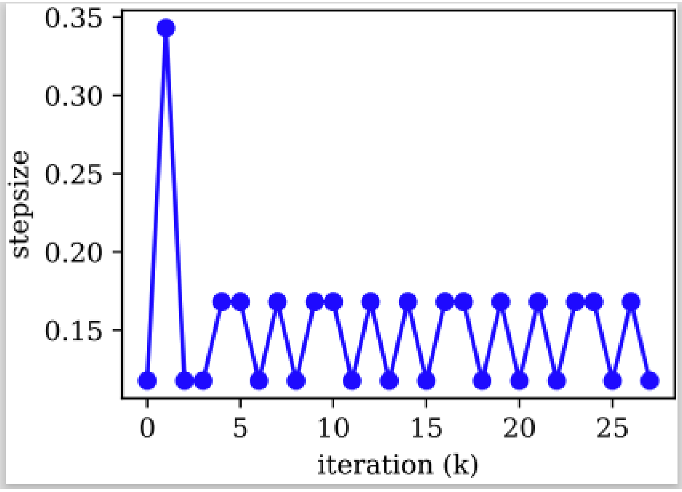
$$\begin{cases} e^{x_1+3x_2-0.1} + e^{x_1-3x_2-0.1} - e^{-x_1-0.1} = 0 \\ 3e^{x_1+3x_2-0.1} - 3e^{x_1-3x_2-0.1} = 0 \end{cases}$$

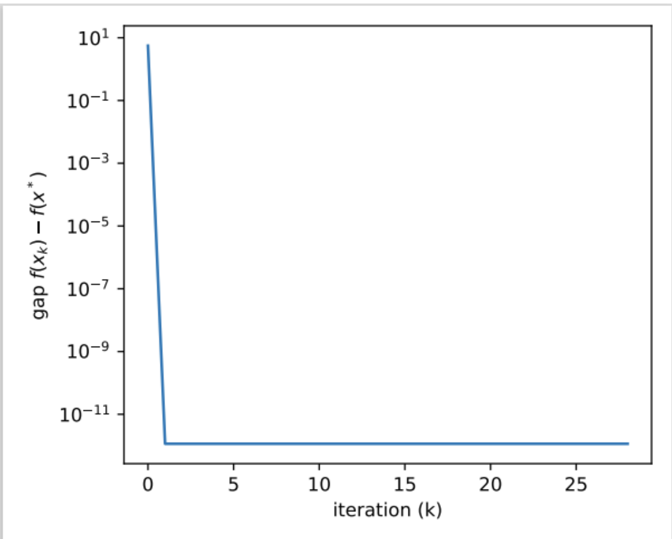
The solution is $\boldsymbol{x}^* = \begin{pmatrix} -\frac{1}{2}ln2 \\ 0 \end{pmatrix}$.

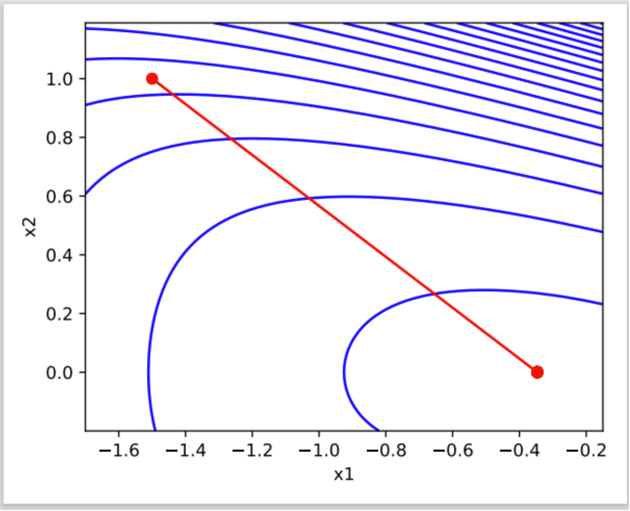
The optimal value is $2e^{\frac{1}{2}ln2-0.1}$.

(b)

```
gradient descent with Armijo
  number of iterations in outer loop: 28
  total number of iterations in inner loop: 151
  solution: [-3.46574284e-01  3.04072749e-07]
  value: 2.5592666966593645
```







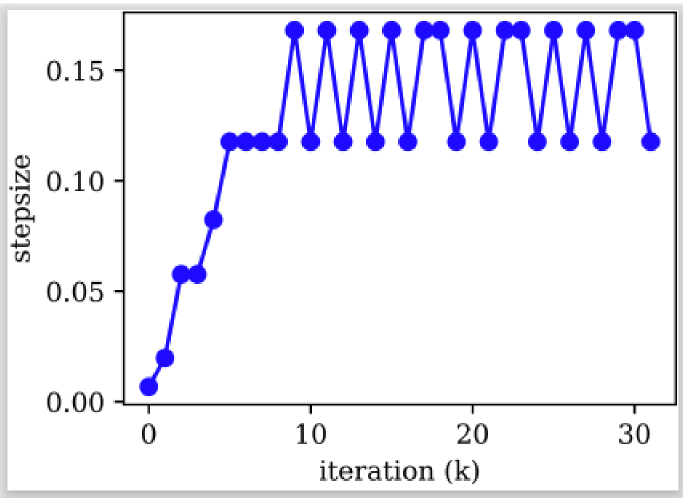
(c)when step size = 0.1.

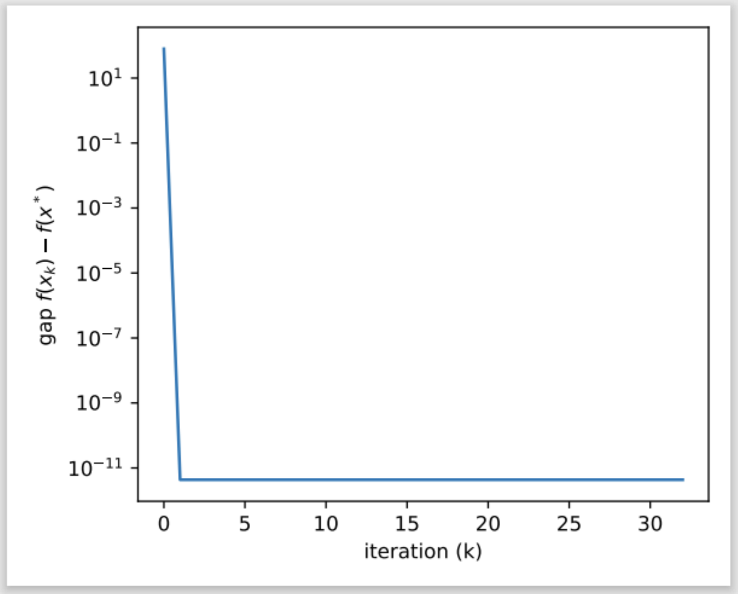
```
gradient descent with constant stepsize  0.1
  number of iterations: 44
  solution: [-3.46576607e-01  3.21465960e-18]
  value: 2.559266696669859
```

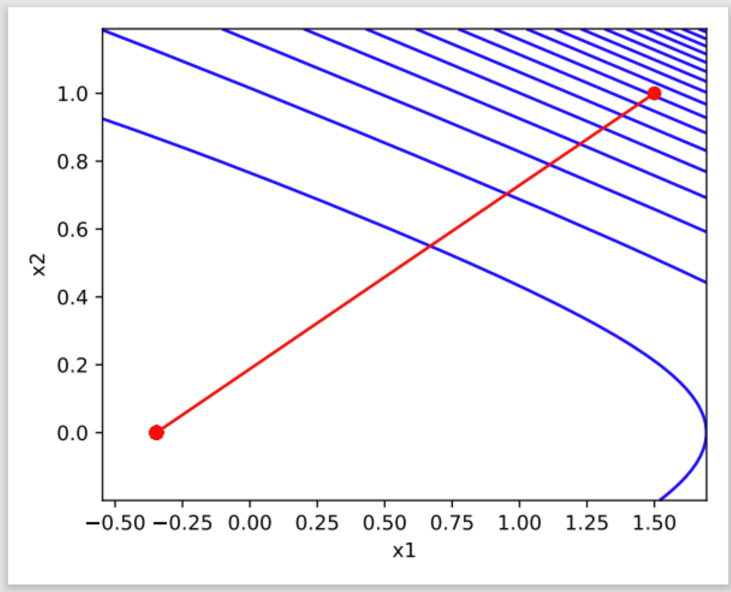
```
gradient descent with constant stepsize  0.01
  number of iterations: 489
  solution: [-3.46577419e-01  8.65140907e-18]
  value: 2.559266696676969
```

(d)

```
gradient descent with Armijo
  number of iterations in outer loop: 32
  total number of iterations in inner loop: 197
  solution: [-3.4657238e-01  6.5447655e-07]
  value: 2.5592666966625575
```







(e)

```
gradient descent with constant stepsize  0.005
  number of iterations: 984
  solution: [-3.46569713e-01 -7.62280416e-18]
  value: 2.559266696677449
```

2. (a) $||x_{k+1} - x^*|| = ||x_k - t\nabla f(x_k) - t\varepsilon_k - x^*||$

$$\leq ||x_k - t\nabla f(x_k) - x^*|| + ||-t\varepsilon_k||$$

$$\leq ||x_k - t\nabla f(x_k) - x^*|| + tE = ||\widetilde{x_{k+1}} - x^*|| + tE$$

(b) By the convergence analysis of noiseless case,

$$||\widetilde{x_{k+1}} - x^*|| \leq q||\widetilde{x_k} - x^*||.$$

In the noiseless case, $\widetilde{x_k} = x_k$.

By the conclusion of (a),

$$||x_{k+1} - x^*|| \leq ||\widetilde{x_{k+1}} - x^*|| + tE$$

$$\leq q\|\widetilde{x_k} - x^*\| + tE$$
$$\leq q\|x_k - x^*\| + tE$$

(c)By the conclusion of (b),

$$\begin{aligned}\|x_k - x^*\| &\leq q\|x_{k-1} - x^*\| + tE\\ &\leq q(q\|x_{k-2} - x^*\| + tE) + tE\\ &\leq q\big(q\big(q\|x_{k-3} - x^*\| + tE\big) + tE\big) + tE\\ &\leq \cdots\end{aligned}$$

Iteriate the steps for k times.

So, $\|x_k - x^*\| \leq q^k\|x_0 - x^*\| + \frac{1-q^k}{1-q}tE$

(d)Apparently, $q = \sqrt{1-mt} < 1$.

So, when $k \to +\infty$, $q^k\|x_0 - x^*\| \to 0$ and $\frac{1-q^k}{1-q}tE \to \frac{tE}{1-q}$.

Namely, $sup \lim\limits_{k\to+\infty}\|x_k - x^*\| < \frac{tE}{1-q}$.

The taylor expansion of $\sqrt{1+x}$ at 0 is

$$\sqrt{1+x} = 1 + \frac{1}{2}x + \frac{1}{2}x^2\left(-\frac{1}{4}(1+\gamma)^{-\frac{3}{2}}\right),$$ where γ is between 0 and x.

So, $\sqrt{1-mt} \geq 1 - \frac{1}{2}mt$.

So, $\frac{tE}{1-q} \leq \frac{tE}{1-1-\frac{1}{2}mt} = \frac{E}{2m}$.

So, $sup \lim\limits_{k\to+\infty}\|x_k - x^*\| \leq \frac{E}{2m}$.