1. $\forall\, y_1 = f(x_1), y_2 = f(x_2)$, where $y_i \in C, i = 1,2$ and $\theta \epsilon [0,1]$, need to show that $\theta x_1 + \bar{\theta} x_2 \in f^{-1}(C)$, namely $f(\theta x_1 + \bar{\theta} x_2) \in C$.
   Since f is an affine function, $\theta y_1 + \bar{\theta} y_2 = \theta f(x_1) + \bar{\theta} f(x_2) = A(\theta x_1 + \bar{\theta} x_2) + b$.
   Since C is convex set, $\theta y_1 + \bar{\theta} y_2 = A(\theta x_1 + \bar{\theta} x_2) + b \in C$.
   So, $\theta x_1 + \bar{\theta} x_2 \in f^{-1}(C)$, $f^{-1}(C)$ is convex set.
2. $\forall\, \theta \in [0,1]$, $x_{11}, x_{12} \in C_1$, $x_{21}, x_{22} \in C_2$, then $x_{11} - x_{21}$ and $x_{12} - x_{22} \in C$.
   Since C_1 and C_2 are convex set, $\theta x_{11} + \bar{\theta} x_{12} \in C_1$ and $\theta x_{21} + \bar{\theta} x_{22} \in C_2$.
   Then $\theta(x_{11} - x_{21}) + \bar{\theta}(x_{12} - x_{22}) = (\theta x_{11} + \bar{\theta} x_{12}) - (\theta x_{21} + \bar{\theta} x_{22})$.
   Thus, $\theta(x_{11} - x_{21}) + \bar{\theta}(x_{12} - x_{22}) \in C$.
   If $0 \in C$, $\exists\, x_1 \in C_1$ and $x_2 \in C_2$ such that $x_1 = x_2$.
   While $C_1 \cap C_2 = \emptyset$.
   Therefore, $0 \notin C$.
3. a) $\forall\ x_i \in int\ C, i = 1,2$ and $\theta \in [0,1]$.
      Since C is convex set, $\theta x_1 + \bar{\theta} x_2 \in C$.
      Since $x_i \in int\ C, i = 1,2$, there exists r such that $B(x_i, r) \subset C, i = 1,2$.
      Select a ρ which $\|\rho\| < r$, $(x_i + \rho) \in B(x_i, r) \subset C, i = 1,2$.
      Therefore, $\theta(x_1 + \rho) + \bar{\theta}(x_2 + \rho) \in C$.
      Thus, $\theta x_1 + \bar{\theta} x_2 \in int\ C$, namely $int\ C$ is convex set.
   b) Let $x_i \in \bar{C}$, $i = 1,2$, need to show that $\forall\, \theta \in [0,1], \theta x_1 + \bar{\theta} x_2 \in \bar{C}$.
      $\exists\, r\ such\ that\ B(x_i, r) \cap C \neq \emptyset, i = 1,2$. Let $y_i \in B(x_i, r) \cap C \neq \emptyset, i = 1,2$.
      Since C is convex set, $\forall\, \theta \in [0,1], \theta y_1 + \bar{\theta} y_2 \in C \subset \bar{C}$.
      Because $\left\|(\theta y_1 + \bar{\theta} y_2) - (\theta x_1 + \bar{\theta} x_2)\right\| \le r$, $\theta x_1 + \bar{\theta} x_2 \in \bar{C}$.
      Namely, $\bar{C}$ is convex set.
4. a) $\forall\, y_i = \sum_{k=1}^{m} \theta_{ik} x_k$, where $\sum_{k=1}^{m} \theta_{ik} = 1$ and $i = 1,2$
      $\forall\, \alpha \in [0,1]$, $\alpha y_1 + (1-\alpha) y_2 = \sum_{k=1}^{m} (\alpha\theta_{1k} + (1-\alpha)\theta_{2k}) x_k$,
      Since $\sum_{k=1}^{m} \theta_{ik} = 1$, $i = 1,2$, $\sum_{k=1}^{m} \alpha\theta_{1k} = \alpha$ and $\sum_{k=1}^{m} (1-\alpha)\theta_{2k} = 1-\alpha$,
      $\sum_{k=1}^{m} \alpha\theta_{1k} + (1-\alpha)\theta_{2k} = 1$.
      So $\alpha y_1 + (1-\alpha) y_2 \in C$. Namely, C is convex set.
   b) $\forall\, x_k \in S$ and $\alpha \in [0,1]$, let $k = i$,j, $\theta_k = \begin{cases} \alpha, k = j \\ (1-\alpha),\ k = i, \\ 0,\ k \neq i,j \end{cases}$ so $C \subset conv\ S$.

      So $C = conv\ S$.
5. $\|x_0 - x\|_2 < \|x_i - x\|_2, i = 1,2,\dots,k$
   $({x_0}^T - x^T)(x_0 - x) \le ({x_i}^T - x^T)(x_i - x)$
   $2({x_i}^T - {x_0}^T)x \le {x_i}^T x_i - {x_0}^T x_0$
   Let $A = 2({x_1}^T - {x_0}^T, {x_2}^T - {x_0}^T, \dots, {x_k}^T - {x_0}^T)^T$ and $b = ({x_2}^T x_1 - {x_0}^T x_0,\ {x_2}^T x_2 - {x_0}^T x_0, \dots, {x_k}^T x_k - {x_0}^T x_0)^T$.
   Then, $V = \{x : Ax \le b\}$.