HW11

1\. (a) $h(t) = \mathcal{F}^{-1}(H(j\omega))$

$$= \frac{1}{2\pi}\int_0^{+\infty} -je^{j\omega t}\,d\omega + \frac{1}{2\pi}\int_{-\infty}^{0} je^{j\omega t}\,d\omega$$

$$= \frac{-j}{2\pi}\cdot\frac{1}{jt}\int_0^{+\infty} e^{j\omega t}\,dj\omega t + \frac{1}{2\pi t}\int_{-\infty}^{0} e^{j\omega t}\,dj\omega t.$$

$$= -\frac{1}{2\pi t}\, e^{j\omega t}\Big|_0^{+\infty} + \frac{1}{2\pi t}\, e^{j\omega t}\Big|_{-\infty}^{0}$$

$$= \frac{1}{\pi t}$$

(b) Take the Fourier transform of the both sides of the equation

$$X_a(j\omega) = X(j\omega) + j\,X_p(j\omega)$$

$$X_a(j\omega) = X(j\omega) + j\,X(j\omega)\cdot H(j\omega)$$

$$= X(j\omega) + j\,X(j\omega)\cdot(-j\,\mathrm{sgn}(\omega))$$

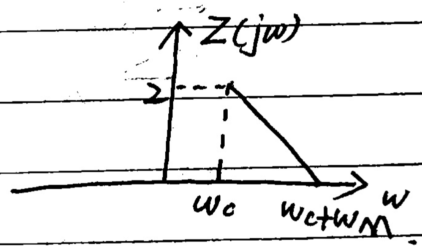
$$= 2X(j\omega)\cdot u(\omega)$$

$$\therefore\ X_a(j\omega) = 2X(j\omega)\cdot u(\omega)$$

(c) By the frequency shifting property

$$Z(j\omega) = X_a(j(\omega-\omega_c))$$

$$= 2X(j(\omega-\omega_c))\cdot u(\omega-\omega_c)$$



(d) $y_u(t) = \mathrm{Re}\,Z(t)$

$$= \frac{1}{2}(Z(t) + Z^*(t))$$

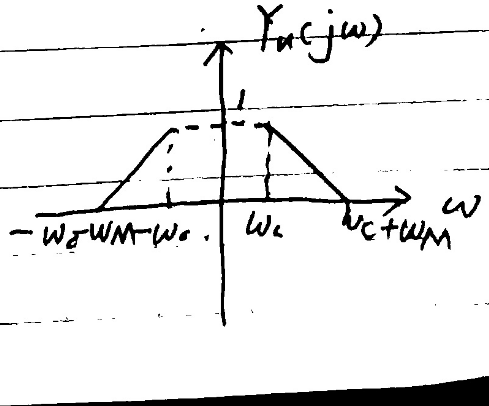
$$\therefore\ Y_u(j\omega) = \frac{1}{2}[Z(j\omega) + Z^*(j\omega)]$$

$$= \frac{1}{2}[Z(j\omega) + Z(-j\omega)]$$

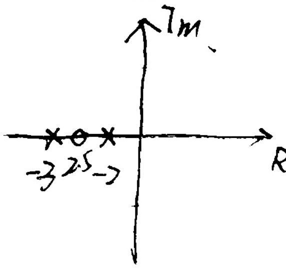
$$= X(j(\omega-\omega_c))\,u(\omega-\omega_c) + X(+j(-\omega-\omega_c))\,u(-\omega-\omega_c)$$

Because $X(j\omega)$ is even

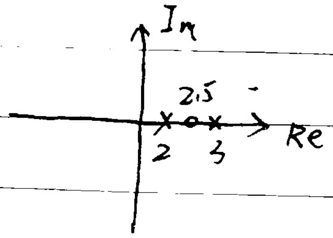
$$Y_u(j\omega) = X(j(\omega-\omega_c))\,u(\omega-\omega_c) + X(j(\omega+\omega_c))\,u(-\omega-\omega_c)$$



2. (a) $X(s) = \frac{1}{s+2} + \frac{1}{s+3}$, $Re\, s > -2$



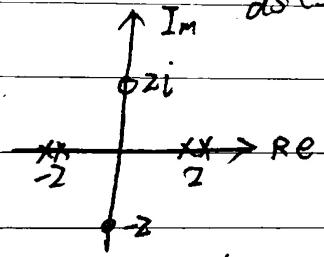
(b) $X(s) = \frac{1}{s-2} + \frac{1}{s-3} = \frac{2s-5}{s^2-5s+6}$, $Re\, s < 2$



(c) $e^{-2|t|} = e^{-2t}u(t) + e^{2t}u(-t)$

$e^{-2|t|} \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2} + \frac{1}{s-2}$

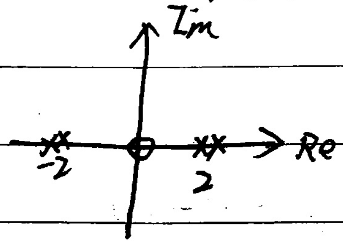
$\therefore te^{-2|t|} \overset{\mathcal{L}}{\longleftrightarrow} \frac{d}{ds}\left(\frac{1}{s+2} + \frac{1}{s-2}\right) = \frac{2s^2+8}{(s^2-4)^2}$.



$\therefore X(s) = \frac{2s^2+8}{(s^2-4)^2}$ $\quad -2 < Re(s) < 2$

(d) For the similar reason to (c),

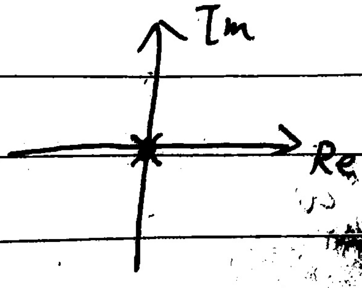
$X(s) = \frac{-4s}{(s+2)^2(s-2)^2} = \frac{-4s}{(s^2-4)^2}$.



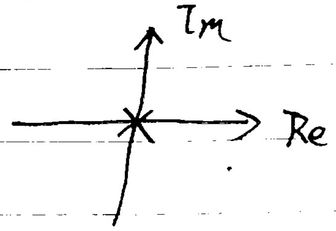
(e) $x(t) = u(t) - u(t-1)$

$\therefore X(s) = \frac{1}{s} - e^{-s}\frac{1}{s}$

$= (1-e^{-s})\frac{1}{s}$ $\quad s \in \mathbb{C}$



f) $X(s) = 1 + \frac{1}{s} \quad \mathrm{Re}\, s > 0$



3. (a) $\cos 3t\, u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{s}{s^2+9}$

$e^{-t}\cos 3t\, u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{(s+1)}{(s+1)^2+9}$

$\therefore x(t) = e^{-t}\cos 3t\, u(t)$

(b) $X(s) = \frac{2}{s+4} - \frac{1}{s+3}$

$\therefore x(t) = 2e^{-4t}u(t) - e^{-3t}u(t)$

(c) $X(s) = 1 + \frac{3s}{s^2-s+1}$

$= 1 + \frac{3s}{(s-\frac{1}{2})^2+\frac{3}{4}}$

$= 1 + 3\frac{s-\frac{1}{2}}{(s-\frac{1}{2})^2+\frac{3}{4}} + \frac{\frac{3}{2}}{(s-\frac{1}{2})^2+\frac{3}{4}}$

$\therefore x(t) = \delta(t) + 3\cos\frac{\sqrt{3}}{2}t\, u(t)\cdot e^{\frac{1}{2}t} + \frac{3}{2}\cancel{\cos}\sqrt{3}e^{\frac{1}{2}}\sin(\sqrt{3}t/2)\, u(t)$

(d) $X(s) = \frac{s^2-s+1}{(s+1)^2} = 1 - \frac{3s}{(s^2+1)^2}$

$\therefore x(t) = \delta(t) - 3\frac{d}{dt}(e^{-t}t\, u(t))$

$= \delta(t) - 3(-e^{-t}t + e^{-t})u(t)$

$= \delta(t) + (3e^{-t}t - 3e^{-t})\cdot u(t)$