# Key Exposure Problem of Chameleon Hashing

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12-4-2024

### Literature Review

Title	Date	Author(s)	Strategy
Chameleon Hashing Without Key Exposure	2004	Xiaofeng Chen, Fangguo Zhang, Kwangjo Kim	Alter private key with signature
On the Key Exposure Problem in Chameleon Hashes	2004	Giuseppe Ateniese Breno de Medeiros	Multi
Chameleon Hashes Without Key Exposure Based on Factoring	2007	Wei Gao,Xueli Wang,Dongqing Xie HNU	Factoring
Key-Exposure Free Chameleon Hashing and Signatures Based on Discrete Logarithm Systems	2009	Xiaofeng Chen, Fangguo Zhang, Haibo Tian, Baodian Wei, Kwangjo Kim	Discrete Logarithm
Identity-based chameleon hashing and signatures without key exposure	2014	Xiaofeng Chen, Fangguo Zhang, Willy Susilo, Haibo Tian, Jin Li, Kwangjo Kim	Identity-Based
Quantum resistant key-exposure free chameleon hash and applications in redactable blockchain	2021	Chunhui Wua, Lishan Keb, Yusong Duc	-

#### The Initial Scheme

- The secret key is easy to obtain
- Weak non-transferability
- Weak non-repudiation

——Chameleon Hashing and Signatures
Hugo Krawczyk, Tal Rabin 1997

- System Parameters Generation  $\mathcal{PG}$ : Let G be a Gap Diffie-Hellman group generated by g, whose order is a prime q. The system parameters are  $SP = \{G, q, g\}$ .
- **Key Generation**  $\mathcal{KG}$ : Each user randomly chooses an integer  $x \in Z_q^*$  as his private key, and publishes his public key  $y = g^x$ . The validity of y can be ensured by a certificate issued by a trusted third party.
- Hashing Computation  $\mathcal{H}$ : On input the public key y of a certain user. Randomly chooses an integer  $a \in \mathbb{Z}_q^*$ , and computes  $(g^a, y^a)$ . Our novel hash function is defined as

$$h = \operatorname{Hash}(m, g^a, y^a) = g^m y^a$$

- Collision Computation  $\mathcal{F}$ : For any valid hash value h, the algorithm  $\mathcal{F}$  can be used to compute a hash collision with the trapdoor information x

$$\mathcal{F}(x, h, m, g^a, y^a, m') = (g^{a'}, y^{a'}),$$

where  $g^{a'} = g^a g^{x^{-1}(m-m')}$  and  $y^{a'} = y^a g^{m-m'}$ . Note that

$$\operatorname{Hash}(m', g^{a'}, y^{a'}) = g^{m'} y^{a'}$$

$$= g^{m'} y^a g^{m-m'}$$

$$= g^m y^a$$

$$= \operatorname{Hash}(m, g^a, y^a)$$

and  $\langle g, y, g^{a'}, y^{a'} \rangle$  is a valid Diffie-Hellman tuple. Therefore, the forgery is successful.

#### Motivation

One disadvantage of the initial chameleon signature scheme is that signature forgery results in the signer recovering the recipient's trapdoor information, *i.e.*, private key. Therefore, the signer can use this informa-

——Chameleon Hashing without Key Exposure Xiaofeng Chen

Notice that if the recipient forges the signature, and two pairs (m,r) and (m',r') become known to the signer (during a dispute), the signer can recover the secret key x of the recipient from  $h = g^m y^r = g^{m'} y^{r'}$ , giving  $x = \frac{m' - m}{r - r'}$ .

——On the Key Exposure Problem in Chameleon Hashes Giuseppe Ateniese

已知 
$$g^m y^r = g^{m'} y^{r'}$$
  
所以有  $g^m g^{x \cdot r} = g^{m'} y^{x \cdot r'}$   
则有  $m + x \cdot r = m' + x \cdot r'$   
移项得到  $x = \frac{m - m'}{r' - r}$ 

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                                                                                           Chameleon_Hash.py 3 X
                                                                                           D ~ III ...
     C: > Users > coper > Desktop > key exposure > codes > 比襄代码 > 🍨 Chameleon_Hash.py > ...
8
            def Forge(SK, m1, r1, m2): # 求r'
                x, y, gcd = exgcd(SK, q)
      103
                h1, h2 = treatMSG(SM3(m1)), treatMSG(SM3(m2))
      104
                result = x * (h1 - h2 + SK * r1) % q
      105
      106
                return result
      107
            if name == " main ":
      108
                length = 256 # 随机大素数长度
      109
                result,p,q = primeFactorization(length)
      110
      111
                g = getGenerator(result)
      112
                SK = getSecretKey()
      113
                PK = getPublicKey(g, SK)
      114
                msg1='Hello, World~'
      115
                msg2='How are you ?'
      116
                r1 = random.randint(1, q) # r
      117
                print(ChameleonHash(PK,g,msg1,r1))
      118
                rr = Forge( SK , msg1 , r1 , msg2 )
      119
                print( rr
      120
      121
      122
                print()
      123
                print( quickPower(g,treatMSG(SM3(msg1)),p)*quickPower(PK,r1,p)%p )
                print( quickPower(g,treatMSG(SM3(msg2)),p)*quickPower(PK,rr,p)%p )
      124
      125
      126
                x = (treatMSG(SM3(msg1))-treatMSG(SM3(msg2)))*invert(mpz(rr-r1),mpz(q))%q
      127
      128
      129
                print( SK )
      130
      186836427970183293468810081641282336982686264660903135620618489860834630497512907
      19518220593625345666328571108925766583165149240508231519372296134900796447779
      186836427970183293468810081641282336982686264660903135620618489860834630497512907
      186836427970183293468810081641282336982686264660903135620618489860834630497512907
      23674972315893148621850345711374397473062325260277113744971857947044325067922
      23674972315893148621850345711374397473062325260277113744971857947044325067922
      [Done] exited with code=0 in 0.585 seconds
× ⊗ 0 △ 3 (A) 0
                                                              ○ 行 128, 列 15 空格: 4 UTF-8 CRLF {} Python ○
```

# 1. Alter private key with signature

- Without key exposure
- Provide non-repudiation and tamper-proofing

cryptographic hash function, define  $I = H(ID_S||ID_R||ID_T)$ , where  $ID_S$ ,  $ID_R$ , and  $ID_T$  denote the identity of signer, recipient, and transaction, respectively.

- Signature Generation SG: Suppose the signed message is m. The signer S randomly chooses an integer  $a \in Z_q^*$ , and computes the chameleon hash function value  $h = (g*I)^m y_R^a$ , here  $y_R$  denotes the public key of the recipient R. Assume SIGN is any secure signature scheme based on the assumption that CDHP in G is intractable. The signature  $\sigma$  for the message m consists of

$$(m, I, g^a, y_R^a, \overline{SIGN}_{x_S}(h)).$$

Where  $x_S$  denotes the private key of the singer S.

- Signature Verification SV: Given a signature  $\sigma$ , the recipient first verifies whether  $\langle g, y_R, g^a, y_R^a \rangle$  is a valid Diffie-Hellman tuple. If tuple is invalid, he rejects the signature; else, he then computes the chameleon hash value  $h = (g * I)^m y_R^a$  and verifies the validity of  $SIGN_{x_S}(h)$  with the public key  $y_S$  of the signer.

Source: Xiaofeng Chen, Fangguo Zhang, Kwangjo Kim: Chameleon Hashing Without Key Exposure. Information Security. 2004. 87-98

## 2. Two Strategies

- 2.1. Single Trapdoor(Without Message Hiding)
  - Security Dependency:
     whether it is safe to sign the same message twice without redundancy
  - Different:
     place r in the exponent of y with e=H(m,r).

**Key Generation:** The scheme specifies a *safe* prime p of bitlength  $\kappa$ . This means that p=2q+1, where q is also prime, and a generator g of the subgroup of quadratic residues  $\mathbf{Q}_p$  of  $\mathbf{Z}_p^*$ , i.e, g has order q. The recipient chooses as secret key x at random in [1,q-1], and his public key is computed as  $(g,y=g^x)$ . Let  $\mathcal{H}$  be a collision-resistant hash function, mapping arbitrary-length bitstrings to strings of fixed length  $\tau \colon \mathcal{H} : \{0,1\}^* \to \{0,1\}^\tau$ .

**The Hash Scheme:** To commit to a message m, it is sufficient to choose random values  $(r, s) \in \mathbf{Z}_q \times \mathbf{Z}_q$ , and compute:

$$e = \mathcal{H}(m, r)$$
; and  $\operatorname{Hash}(m, r, s) = r - (y^e g^s \bmod p) \bmod q$ .

**Collision Finding:** Let C denote the output of the chameleon hash on input the triple (m, r, s). A collision (m', r', s') can be found by computing (m', r', s') such that:

$$e' = \mathcal{H}(m', r')$$
; and  $C = r' - (y^{e'}g^{s'} \bmod p) \bmod q$ .

First, the recipient chooses a random message m', a random value  $k' \in [1, q-1]$ , and computes  $r' = C + (g^{k'} \mod p) \mod q$ ,  $e' = \mathcal{H}(m', r')$ , and  $s' = k' - e'x \mod q$ . Notice that indeed:

$$r'-(y^{e'}g^{s'} \bmod p) \bmod q = C+(g^{k'} \bmod p)-(g^{xe'}g^{s'} \bmod p) \bmod q) = C.$$

**Key Exposure Freeness and Collision-Resistance:** The security of the scheme depends on whethe twice signing a message (without redundancy), using the above variant of Nyberg-Rueppel, is secure. This was proven in appendix A to [14], where the concept of twinning signature schemes is considered. The only difference from the scheme above is that we have substituted  $e = \mathcal{H}(m,r)$  for r in the exponent of y. The only modification to the proof, which is formally the same, is that the probability of collisions is changed from finding collisions in the whole ring  $\mathbf{Z}_q$  to finding them over the image of  $\mathcal{H}(\cdot)$ . Therefore, provided that this hash is collision-resistant, the conclusion of security is unchanged. Notice that we do not need to model the function  $\mathcal{H}(\cdot)$  as a random oracle. Instead, the proof of security for the twin Nyberg-Rueppel works in the generic model of computation.

Source: Giuseppe Ateniese, Breno de Medeiros: On the Key Exposure Problem in Chameleon Hashes. Security in Communication Networks.2004.165-197

## 2. Two Strategies

#### 2.2. Double Trapdoors (With Message Hiding)-Based on RSA

Hash Function: 
$$H(\mathcal{L}, m, r) = J^{\mathcal{H}(m)} r^e \mod n$$

Where  ${\cal L}$  is a label,  $B=J^d$ , J depend on  ${\cal L}$ 

已知 
$$J^{\mathcal{H}(m)}r^e=J^{\mathcal{H}(m')}r'^e$$
  $\dfrac{r'}{r}=B^{\mathcal{H}(m)-\mathcal{H}(m')}$  所以有  $J^{\mathcal{H}(m)\cdot -e}r^{e\cdot -e}=J^{\mathcal{H}(m')\cdot -e}r'^{e\cdot -e}$   $B$  is easy to be calculate if adversary have a pair of  $(m,r)$  则有  $J^{\mathcal{H}(m)\cdot d}r=J^{\mathcal{H}(m')\cdot d}r'$  But the secret key that  $d$  remains safe In this way everybody can forge a pair of new message,  $r'=rB^{\mathcal{H}(m)-\mathcal{H}(m')}$  which so called **Message Hiding**

Source: Giuseppe Ateniese, Breno de Medeiros: On the Key Exposure Problem in Chameleon Hashes. Security in Communication Networks.2004.165-197

#### 3. Based on Factoring

#### 3.1. Signature Scheme

ullet Sign. On input the message  $m \in \{0,1\}^*$ , set the signature as:

$$\sigma \stackrel{R}{\leftarrow} |H(m)|^{\frac{1}{2}} \mod N$$

where if  $H(m) \in QR_N$ , |H(m)| = H(m), else |H(m)| = -H(m).

• Ver. On input  $\sigma, m$ , verify the following equality  $\sigma^2 \equiv \pm H(m) \mod N$ . In other words, if  $\sigma^2 \equiv H(m) \mod N$  or  $\sigma^2 \equiv -H(m) \mod N$ , the signature is valid.

If one know the secret key (p,q), Quadratic and non-quadratic remainders is computable by CRT, or it's a hard problem.

```
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                                                                                                                                                                                                                                                                                                                           ON THE PROPERTY OF THE PROPERT
                                    p , q = getPrime( len ) , getPrime( len )
   12
   13
                                  return SK , PK
                     def GenHash( SK ):
                                  return all quadratic_residues_mod_n( p , q )
                    def Sign( m , SK , Hashed_list ):
                                   hashed = bytes_to_long(m) % (p*q)
                                  if not is_quadratic_residue_mod_n( hashed , SK ):
   23
                                                S = Gennon quadratic remainders(Hashed list)
                                   return hashed
   27
                     def Verf( hashed . PK . Hashed list ):
                                  if (hashed*hashed)%N in Hashed list:
   31
   32
   33
                                                return False
                                 __name__ == '__main__':
                                 SK , PK = GenKey(5)
                                  Hashed list = GenHash( SK )
                                 # print( Hashed list )
                                  random.shuffle(Hashed list)#forge
                                  m = "痞老板会偷走美味蟹黄包秘方".encode()
                                  h = Sign( m , SK , Hashed_list
                                    print( Verf( h , PK , Hashed_list )
[Running] python -u "c:\Users\coper\Desktop\key exposure\codes\paper3\main.py"
[Done] exited with code=0 in 0.137 seconds
                                                                                                                                                                                         ○ 行49,列5 空格:4 UTF-8 CRLF () Python 3.8.10 64-bit ○
```

Source: Wei Gao, Xue-Li Wang, Dong-Qing Xie: Chameleon Hashes Without Key Exposure Based on Factoring. Journal of Computer Science and Technology.2007.109-113

## 3. Based on Factoring

#### 3.2. Chameleon Hash

暂未跑通实验代码 有待后续继续研究

```
★ 文件(F) 编辑(E) 选择(S) 查看(V) 转到(G) 运行(R) 终端(T) …
                                                                                                                                                                                                                          C: > Users > coper > Desktop > key exposure > codes > paper3.2 > # main.py > @ Collision_Extension
                                                                                                                                                                                                                              33 def Hash( PK , L , m , r , k ):
                         from Crypto.Util.number import *
                                                                                                                                                                                                                                            N = PK
                         import random
                                                                                                                                                                                                                                              J = int(sm3_hash(L),16)
                         from gmssl import sm3, func
                                                                                                                                                                                                                                              b = int(random.choice('02'))-1
                                                                                                                                                                                                                                              h = b*pow(J,m,N)*pow(r,2**log_star(k),N)%N
                        def log star(x):
                               # 先检查 x 是否大于 1. 因为 log_star(x) 只对 x > 1 有意义
                                                                                                                                                                                                                                      def Collision_Extension( m2 , r2 , k , PK , L ):
                                       raise ValueError("x must be greater than 1")
                                                                                                                                                                                                                                               m2 = int(sm3_hash(m2), 16)
                                                                                                                                                                                                                                              print( m2 )
                                       x = math.log(x) # 计算 x 的对数
                                                                                                                                                                                                                                              b = int(random.choice('02'))-1
                                         count += 1 # 迭代次数加 1
                                                                                                                                                                                                                                              if m2 > 2**(log_star(k)-1):
                                                                                                                                                                                                                                                       m3 = m2 - 2**(log_star(k)-1)
                                                                                                                                                                                                                                                      r3 = r2*b % N
                         def generate_binary_string(length):
                                                                                                                                                                                                                                                       m3 = 2**(log star(k)-1) + m2
                               return ''.join(random.choice('01') for in range(length))
                                                                                                                                                                                                                                                       r3 = r2//b % N
                                                                                                                                                                                                                                                judge = (Hash(PK,L,m2,r2,k)==Hash(PK,L,m3,r3,k))
                                                                                                                                                                                                                                               print( Hash(PK,L,m2,r2,k) , Hash(PK,L,m3,r3,k) )
                             p , q = getPrime( len ) , getPrime( len )
                                                                                                                                                                                                                                              return m3 , r3 , judge
                                while p==q:#确保pq不相等
                                       q = getPrime( len )
                                                                                                                                                                                                                                      if __name__ == '__main__'
                               # print( p , q )
                                                                                                                                                                                                                                             k = 10
                               SK = p, q
                                                                                                                                                                                                                                              m1 = generate_binary_string( log_star(k) )
                               return SK , PK
                                                                                                                                                                                                                                              r1 = random.randint( 1, PK )
               30 def sm3_hash(data):
                                                                                                                                                                                                                                               m2 = "痞老板会偷走美味蟹黄包秘方"
                               return sm3.sm3_hash(func.bytes_to_list(data.encode('utf-8')))
                                                                                                                                                                                                                                              r2 = random.randint( 1, PK )
               33 def Hash( PK , L , m , r , k ):
                                                                                                                                                                                                                                                print( Collision_Extension(m2,r2,k,PK,L) )
            (655 (4) Mill Widselfs 1504 MI 1245
                                                                                                                                                                                                                                                                                                                                                                                                     ∨ ≣ A ... ^ ×
            [Running] python -u "c:\Users\coper\Desktop\key exposure\codes\paper3.2\main.py"
            28744761583631866400251521235628005744903349723876654061928182567863605234049
            (28744761583631866400251521235628005744903349723876654061928182567863605234047, 328, False)
            [Done] exited with code=0 in 0.169 seconds
× ⊗0∆4 ₩0
                                                                                                                                                                                                                                                                                                                                    ● 行50,列23 空性4 UTF-8 CRLF () Python 3.8.10 64-bit 8 □
                                                                                                                                                                                                                                                                                                                                                            ^ $\Phi$ $\phi}
```

Source: Wei Gao, Xue-Li Wang, Dong-Qing Xie: Chameleon Hashes Without Key Exposure Based on Factoring. Journal of Computer Science and Technology.2007.109-113

# 4. Discrete Logarithm

 $r^\prime$  is not a direct random number, but is generated through the random number a and a pair of message, and can be calculated by the equation

$$calculate\ H: H=g^ah^m$$
  $calculate\ r': r'=(g^{a'},y^{a'})=(g^ah^{m-m'},g^ah^{x(m-m')})$ 

- System Parameters Generation  $\mathcal{PG}$ : Let  $\mathbb{G}$  be a GDH group generated by g, whose order is a prime q. Let  $H:\{0,1\}^* \to \mathbb{G}^*$  be a full-domain collision-resistant hash function. The system parameters are  $SP = \{\mathbb{G}, q, g, H\}$ .
- **Key Generation**  $\mathcal{KG}$ : Any user randomly chooses an integer  $x \in_R \mathbb{Z}_q^*$  as his trapdoor key, and publishes his hash key  $y = g^x$ . The validity of y can be ensured by a certificate issued by a trusted certification authority.
- Hashing Computation  $\mathcal{H}$ : On input the hash key y, a customized identity I, let h = H(y, I). Chooses a random integer  $a \in_R \mathbb{Z}_q^*$ , and computes  $r = (g^a, y^a)$ . Our proposed chameleon hash function is defined as

$$\mathcal{H} = \operatorname{Hash}(I, m, r) = g^a h^m.$$

- Collision Computation  $\mathcal{F}$ : For any valid hash value  $\mathcal{H}$ , the algorithm  $\mathcal{F}$  can be used to compute a hash collision with the trapdoor key x as follows:

$$\mathcal{F}(\mathcal{H}, x, I, m, r, m') = r' = (g^{a'}, y^{a'}),$$

where  $g^{a'} = g^a h^{m-m'}$  and  $y^{a'} = y^a h^{x(m-m')}$ .

Note that

$$\operatorname{Hash}(I, m', r') = g^{a'}h^{m'} = g^{a}h^{m-m'}h^{m'} = g^{a}h^{m} = \operatorname{Hash}(I, m, r)$$

Source: Xiaofeng Chen, Fangguo Zhang, Haibo Tian, Baodian Wei, Kwangjo Kim: Key-Exposure Free Chameleon Hashing and Signatures Based on Discrete Logarithm Systems. 2009

## 4. Discrete Logarithm

#### ----Simulation

#### 最一开始按照论文逻辑写代码,一直跑不通

```
75 def calc c( m1 , a1 , m2 , SK , PK , g , q ):
        r2 = (pow(g,a1,q)*pow(h,SM3(m1)-SM3(m2),q)%q, pow(y,a1,q)*pow(h,x*(SM3(m1)-SM3(m2)),q)%q)
    if name == ' main ':
       q , g , I = init( 512 )
        SK = getSecretKey(q)
       PK = getPublicKey(g, SK, q)
       h = SM3(str(PK)+I)
        a1 = random.randint(1,q)
       r1 = (pow(g,a1,q), pow(PK,a1,q))
        m2 = "ggggg'
        a2 = random.randint(1,q)
       r2 = (pow(g,a2,q), pow(PK,a2,q))
       # r2 = calc_c( m1 , a1 , m2 , SK , PK , g , q )
        print( Hash(h,m1,q,g,a1) )
       print( Hash(h,m2,q,g,a2) )
        print(Hash(h,m1,q,g,a1) == Hash(h,m2,q,g,a2)
546795102425672888993141676329418087539
False
```

后来注意到a'是不可计算的,换了种写法就通了

```
    Chameleon Hash,pv 3 ● OnlineOffline.pv 1

                                                                                              \triangleright \checkmark \square
C: > Users > coper > Desktop > key exposure > codes > paper4 > ♥ paper4.py > .
      def init( len ):
           q = getPrime( len )
           g = fast_find_generator(q)
          I = "小明小绿小白"
          return q , g , I
  70
       def Hash( h , m , q , ga ):
           return ga*pow(h,SM3(m),q)%q
  73
  74
      def calc_c( m1 , a1 , m2 , SK , PK , g , q , h ):
           r2 = (pow(g,a1,q)*pow(h,SM3(m1)-SM3(m2),q)%q, pow(y,a1,q)*pow(h,x*(SM3(m1)-SM3(m2),q)%q)
  79
          __name__ == '__main__':
          q , g , I = init( 512
           SK = getSecretKev(q)
           PK = getPublicKey(g, SK, q)
           h = SM3(str(PK)+I)
           a1 = random.randint(1,q)
           r1 = (pow(g,a1,q), pow(PK,a1,q))
           m2 = "ggggg'
           a2 = random.randint(1,q)
          \# r2 = (pow(g,a2,q), pow(PK,a2,q))
           r2 = calc c( m1 , a1 , m2 , SK , PK , g , q , h )
           ga2 , ya2 = r2
  93
           ga1 , ya1 = r1
           print( Hash(h,m1,q,ga1) )
           print( Hash(h,m2,q,ga2) )
           print( Hash(h,m1,q,ga1) == Hash(h,m2,q,ga2)
                                                                                      ∨ <u>≡</u> 6 ··· ^ ×
 [Running] python -u "c:\Users\coper\Desktop\key exposure\codes\paper4\paper4.py"
 90332040621991358746553863389066344902025477726861662952958073999591810339472969748823286775637
903320406219913587465538633890663449020254777268616629529580739995918103394729
26110199421137377585273610222285098985586140972159454772927
True
[Done] exited with code=0 in 0.2 seconds
```

Source: Xiaofeng Chen, Fangguo Zhang, Haibo Tian, Baodian Wei, Kwangjo Kim: Key-Exposure Free Chameleon Hashing and Signatures Based on Discrete Logarithm Systems. 2009

#### 5. ID-Based

```
SetUp: (SK, PK) \leftarrow k 生成公钥私钥
```

 $Extract: TK \leftarrow (SK, ID)$  生成与哈希密钥ID相关联的陷门密钥

 $Hash: Hash(ID, L, m, r) \leftarrow (PK, ID, L, m, r)$ 

L为定制身份,m为消息,r为随机数,哈希结果不依赖于TK

Forge: r' = F(TK, ID, L, h, m, r, m')

Even if the hash function construction is not strong enough, it is only possible to leak ID. The hash result does not depend on TK, so there is no key leakage.

- Setup: PKG runs this probabilistic polynomial-time algorithm to generate a pair of secret/public keys (SK, PK) defining the scheme. PKG publishes the system parameters SP including the public key PK, and keeps the secret key SK as the master key. The input to this algorithm is a security parameter k.
- Extract: A deterministic polynomial-time algorithm that, on input the master key SK and an identity string ID, outputs the trapdoor key TK associated to the hash key ID.
- Hash: A probabilistic polynomial-time algorithm that, on input the master public key PK, an identity string ID, a customized identity L, a message m, and a random string r, outputs the hash value  $h = \mathsf{Hash}(PK, ID, L, m, r)$  Note that h does not depend on TK and we denote  $h = \mathsf{Hash}(ID, L, m, r)$  for simplicity throughout this paper.
- Forge: A deterministic polynomial-time algorithm  $\mathcal{F}$  that, on input the trapdoor key TK associated to the identity string ID, a customized identity L, a hash value h of a message m, a random string r, and another message

 $m' \neq m$ , outputs a string r' that satisfies

$$h = Hash(ID, L, m, r) = Hash(ID, L, m', r').$$

More precisely,

$$r' = \mathcal{F}(TK, ID, L, h, m, r, m').$$

Moreover, if r is uniformly distributed in a finite space  $\mathcal{R}$ , then the distribution of r' is computationally indistinguishable from uniform in  $\mathcal{R}$ .

Source: Xiaofeng Chen, Fangguo Zhang, Willy Susilo, Haibo Tian, Jin Li, Kwangjo Kim: Identity-based chameleon hashing and signatures without key exposure. Information Security and Privacy.2009.200–215

### 6. Quantum Resistant

• 后量子的,比较难懂,还没看完(悲)

Source: Chunhui Wu, Lishan Ke, Yusong Du: Quantum resistant key-exposure free chameleon hash and applications in redactable blockchain. Information Sciences. 2021. 438-449

## 7. Summary and Outlook

- 3.2的实验代码
- 第六篇再花时间读一下,实在读不懂就搁置一下
- Survey的密钥泄露部分可以开写了
- 第二篇的第一个理论是否存在连分数攻击的安全风险