# Multiagent Transition Systems: Protocol-Stack Mathematics for Distributed Computing

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#### Abstract

Presently, the practice of distributed computing is such that problems exist in a mathematical realm different from their solutions: a problem is presented as a set of requirements on possible process or system behaviors, and the solution is presented as algorithmic pseudocode satisfying the requirements. Here, we present a novel mathematical realm, termed multiagent transition systems, that aims to accommodate both distributed computing problems and their solutions. A problem is presented as a specification—a multiagent transition system—and a solution as an implementation of the specification by another, lower-level multiagent transition systems.

This duality of roles of a multiagent transition system can be exploited all the way from a high-level distributed computing problem description down to an agreed-upon base layer, say TCP/IP, resulting in a mathematical protocol stack where each protocol is implemented by the one below it. Correct implementations are compositional and thus provide also an implementation of the protocol stack as a whole. The framework also offers a formal, yet natural, notion of faults and their resilience.

We present two illustrations of the power of the approach: First, we provide multiagent transition systems specifying a centralized single-chain protocol and a distributed longest-chain protocol, show that the single-chain protocol is universal in that it can implement any centralized multiagent transition system, show an implementation of this protocol by the longest-chain protocol, and conclude—via the compositionality of correct implementations—that the distributed longest-chain protocol is universal for centralized multiagent transition systems. Second, we describe a DAG-based blockchain consensus protocol stack that addresses each of the key tasks of a blockchain protocol—dissemination, equivocation-exclusion, and ordering—by a different layer of the stack. Additional applications of this mathematical framework are underway.

#### 1 Introduction

The standard approach in distributed computing is to define a problem, typically by specifying requirements and constraints on possible process or system

behavior, and then to present a solution, typically in the form of algorithmic pseudocode, and argue that it satisfies the requirements and constrained specified by the problem. The problem and the solution exist in different mathematical realms: The problem is described mathematically but informally, and the solution is a high-level—formal [12, 11] or informal—description of a putative workable program code. As such, different problems, and hence their solutions, cannot be composed.

In our approach, problem and solution are best understood via the standard computer science notions of 'specification' and 'implementation'. Moreover, in our approach, specifications and implementations all live in the same mathematical realm: They are both multiagent transition systems, where the 'solution' implements the 'problem'. But the implementing transition system can also serve as a specification awaiting still a lower-level implementation, iterating this dual role of specification/implementation all the way down to an agreed-upon bottom layer. Hence, problems can be solved piecemeal, layer by layer, and solutions can be composed. That's the logical/mathematical value of the mathematical protocol stack approach resulting from the use of multiagent transition systems. Unlike I/O Automata [12] or TLA+ [11], multiagent transition systems are not a formal specification language; as presented here, they are a rather-abstract mathematical domain, without an associated formal syntax. Also, note that specification languages and formalisms for distributed and concurrent systems necessarily include methods for composing concurrent processes.

Here, we are not concerned with this 'horizontal', modular notion of compositionality [2]. Rather, we focus on the 'vertical' composition of implementations among concurrent or distributed systems, a notion of composition that is akin to function composition. Our approach is similar in spirit to that of universal composability [4], devised for the analysis of cryptographic protocols, but is different in at least two respects: First, it is more abstract as it does not assume, from the outset, a specific notion of communication. Second, its notion of composition is different: Whereas universal composability, like the notion of protocol composition is common in the practice of protocol design (e.g. [10, 7]), is a 'run-time' notion that composes protocols by replacing a call to one subroutine by a call to another, our notion of composition is 'compile-time' – we do not compose protocols, but compose implementations (of one protocol by another), resulting in a new protocol that realizes the functionality of the high-level protocol using the primitives of the low-level protocol. In particular, it seems that the universality results of Section 3 cannot be expressed in the model of universal composability.

In the following we develop a mathematical theory of multiagent transition systems—their specification, implementation, composition, and fault tolerance—and provide examples of its application.

# 2 Multiagent Transition Systems: Implementation, Composition & Resilience

Here we present multiagent transition systems.

**Definition 1** (Transition System with Faults). Given a set S, referred to as states, the transitions over S are all pairs  $(s,s') \in S^2$ , also written  $s \to s'$ . A transition system  $TS = (S, s_0, T)$  consists of a set of states S, an initial state  $s_0 \in S$ , and set of (correct) transitions  $T \subseteq S^2$ . A computation of TS is a sequence of transitions  $s \to s' \to \cdots$ , and a run of TS is a computation that starts from  $s_0$ . A transition  $s \to s' \in T$  is correct, and a computation of correct transitions is correct. A transition in  $S^2 \setminus T$  is faulty, and a computation is faulty if it includes a faulty transition. We denote by  $s \xrightarrow{*} s' \in T$  the existence of a correct computation (empty if s = s') from s to s'.

Note that any fault can be modelled with the notion thus defined, by enlarging S. Often, a limited adversary is considered (e.g. limited in computational power), in which case a restricted set of faulty transition is considered. Transitions are unlabeled for simplicity and the transition systems do not specify final states as we focus on distributed multiagent transition systems (defined below) that are typically non-terminating.

With this, we can define the notion of a correct implementation:

**Definition 2** (Implementation, Safety, Completeness, Liveness, Correctness). Given two transition systems  $TS = (S, s_0, T)$  and  $TS' = (S', s'_0, T')$  an *implementation of* TS by TS' is a function  $\sigma : S' \to S$  where  $\sigma(s'_0) = s_0$ . The implementation  $\sigma$  is:

- 1. Safe if  $s_0' \stackrel{*}{\to} y \to y' \in T'$  implies that  $s_0 \stackrel{*}{\to} x \stackrel{*}{\to} x' \in T$  for some  $x = \sigma(y)$  and  $x' = \sigma(y')$  in S. If x = x' then the T' transition  $y \to y'$  stutters, else it activates T.
- 2. Complete if  $s_0 \stackrel{*}{\to} x \to x' \in T$ , implies that  $s_0' \stackrel{*}{\to} y \stackrel{*}{\to} y' \in T'$  for some  $y, y' \in S'$  such that  $x = \sigma(y)$  and  $x' = \sigma(y')$ .
- 3. Live if  $s_0' \xrightarrow{*} y \in T'$  implies that there is  $y' \in S'$ ,  $y' \neq y$ , such that  $y \xrightarrow{*} y' \in T'$  and  $s_0 \xrightarrow{*} \sigma(y) \xrightarrow{*} \sigma(y') \in T$ ,  $\sigma(y) \neq \sigma(y')$ .

An implementation is *correct* if it is safe, complete and live.

Our definition is not unlike standard definitions (e.g. [9, 1]), but is refined to support the composition of correct implementations thus defined, which to the best of our knowledge has not been previously explored within a formal framework. Safety ensures that the implementation performs only correct computations. Completeness ensures that every correct computation of the implemented transition system (henceforth - specification), can be realized by a correct computation of the implementation. Liveness ensures that the implementation can

always activate the specification. This implies that any correct run of the implementation that stutters indefinitely has infinitely many opportunities to activate the specification. Under the standard assumption that an opportunity that is presented infinitely often is eventually seized, a live implementation does not deadlock as it eventually activates the specification.

A key property of correct implementations is their compositionality:

**Lemma 1** (Composing Implementations). The composition of correct implementations is in turn correct.

*Proof.* Assume transition systems TS1 = (S1, s1, T1), TC2 = (C2, s2, T2), TC3 = (C3, s3, T3) and correct implementations  $\sigma_{21} : C2 \to S1$  and  $\sigma_3 : C3 \to C3$ , and let  $\sigma_{31} := \sigma_{21} \circ \sigma_3$ .

- 1. Safety:  $s3 \stackrel{*}{\to} z \to z' \in T3$  implies that  $s2 \stackrel{*}{\to} y \to y' \in T2$  for  $y = \sigma_3(z)$ ,  $y' = \sigma_3(z')$  in C2 by safety of  $\sigma_3$ , which implies that  $s1 \stackrel{*}{\to} x \stackrel{*}{\to} x' \in T1$  for  $x = \sigma_{21}(y)$  and  $x' = \sigma_{21}(y')$  in S1 by safety of  $\sigma_{21}$ . Hence  $\sigma_{31}$  is safe.
- 2. Completeness: Assume  $s1 \stackrel{*}{\to} x \to x' \in T1$ . By completeness of  $\sigma_{21}$ ,  $s2 \stackrel{*}{\to} y \stackrel{*}{\to} y' \in T2$  for some  $y, y' \in C2$  such that  $x = \sigma_{21}(y)$  and  $x' = \sigma_{21}(y')$ . By completeness of  $\sigma_3$ ,  $s3 \stackrel{*}{\to} z \stackrel{*}{\to} z' \in T3$  for some  $z, z' \in C3$  such that  $y = \sigma_3(z)$  and  $y' = \sigma_3(z')$ . Hence  $\sigma_{31}$  is complete.
- 3. Liveness: Assume  $s3 \stackrel{*}{\to} z \in T3$ . By safety of  $\sigma_3$ ,  $s2 \stackrel{*}{\to} \sigma_3(z) \in T2$ . Let  $y = \sigma_3(z)$ . By liveness of  $\sigma_{21}$  there is  $y' \in C2$ ,  $y' \neq y$ , such that  $y \stackrel{*}{\to} y' \in T2$  and  $s1 \stackrel{*}{\to} \sigma_{21}(y) \stackrel{*}{\to} \sigma_{21}(y') \in T1$ ,  $\sigma_{21}(y) \neq \sigma_{21}(y')$ . By completeness of  $\sigma_3$ ,  $s3 \stackrel{*}{\to} z \stackrel{*}{\to} z' \in T3$  for some  $z' \in C3$  such that  $\sigma_3(z') = y'$ . Hence the liveness condition for  $\sigma_{31}$  is satisfied: There is a  $z' \in C3$ ,  $z' \neq z$ , such that  $z \stackrel{*}{\to} z' \in T3$  and  $s1 \stackrel{*}{\to} \sigma_{31}(z) \stackrel{*}{\to} \sigma_{31}(z') \in T1$ ,  $\sigma_{31}(z) \neq \sigma_{31}(z')$ .

This completes the proof.

Unlike shared-memory systems, distributed systems have a state that increases in some natural sense as the computation progresses, e.g. through the accumulation of messages and the maintenance of history of local states.

**Definition 3** (Partial Order). A reflexive partial order on a set S is denoted by  $\preceq_S$  (with S omitted if clear from the context),  $s \prec s'$  stands for  $s \preceq s' \& s' \not \preceq s$ , and  $s \simeq s'$  for  $s \preceq s' \& s' \preceq s$ . The partial order is *strict* if  $s \simeq s'$  implies s = s' and *unbounded* if for every  $s \in S$  there is an  $s' \in S$  such that  $s \prec s'$ . We say that  $s, s' \in S$  are *consistent* wrt  $\preceq$  if  $s \preceq s'$  or  $s' \preceq s$  (or both).

It is often possible to associate a partial order with a distributed system, wrt which the local state of each agent only increases. Therefore we focus on the following type of transition systems:

**Definition 4** (Monotonic & Monotonically-Closed Transition System). Given a partial order  $\leq$  on S, a transition system  $(S, s_0, T)$  is *monotonic* with respect to  $\leq$  if  $s \rightarrow s' \in T$  implies  $s \leq s'$ , and *monotonically-closed* wrt  $\leq$  if, in addition,  $s_0 \xrightarrow{*} s \in T$  and  $s \leq s'$  implies that  $s \xrightarrow{*} s' \in T$ .

When distributed systems are monotonically-closed wrt a (strict) partial order, the following Definition 5 and Lemma 2 can be a powerful tool in proving that one protocol can correctly implement another.

**Definition 5** (Order-Preserving Implementation). Let transition systems  $TS = (S, s_0, T)$  and  $TS' = (S', s'_0, T')$  be monotonic wrt the partial orders  $\leq$  and  $\leq$ ', respectively. Then an implementation  $\sigma: S' \to S$  of TS by TS' is order-preserving wrt  $\leq$  and  $\leq$ ' if:

- 1. Up condition:  $y_1 \leq' y_2$  implies that  $\sigma(y_1) \leq \sigma(y_2)$
- 2. **Down condition:**  $s_0 \stackrel{*}{\to} x_1 \in T, x_1 \preceq x_2$  implies that there are  $y_1, y_2 \in S'$  such that  $x_1 = \sigma(y_1), x_2 = \sigma(y_2), s_0' \stackrel{*}{\to} y_1 \in T'$  and  $y_1 \preceq' y_2$ .

Note that if  $\leq'$  is induced by  $\sigma$  and  $\leq$ , namely defined by  $y_1 \leq' y_2$  if  $\sigma(y_1) \leq \sigma(y_2)$ , then the Up condition holds trivially. Furthermore, if  $\leq$  is strict, then  $y_1 \simeq' y_2$  implies that  $\sigma(y_1) = \sigma(y_2)$ . The following Lemma is the linchpin of the proofs of protocol stack theorems here and in forthcoming applications of the framework.

**Lemma 2** (Correct Implementations Among Monotonically-Closed Transition Systems). Assume two transition systems  $TS = (S, s_0, T)$  and  $TS' = (S', s'_0, T')$ , each monotonically-closed wrt the unbounded partial orders  $\leq$  and  $\leq'$ , respectively, and an implementation  $\sigma: S' \to S$  among them. If  $\sigma$  is order-preserving then it is correct.

*Proof.* We have to show that:

1. Safety:  $s'_0 \stackrel{*}{\to} y \to y' \in T'$  implies that  $s_0 \stackrel{*}{\to} x \stackrel{*}{\to} x' \in T$  for  $x = \sigma(y)$  and  $x' = \sigma(y')$  in S.

By monotonicity of TS' it follows that  $s_0' \leq y \prec' y'$ ; by the Up condition on  $\sigma$ , it follows that  $s_0 \leq \sigma(y) \leq \sigma(y')$ ; by assumption that TS is monotonically-closed it follows that  $s_0 \stackrel{*}{\to} x \stackrel{*}{\to} x' \in T$  for  $x = \sigma(y)$  and  $x' = \sigma(y')$  in S. Hence  $\sigma$  is safe.

2. Completeness:  $s_0 \xrightarrow{*} x \to x' \in T$  implies  $s_0' \xrightarrow{*} y \xrightarrow{*} \in T'$  for some  $y, y' \in S'$  such that  $x = \sigma(y)$  and  $x' = \sigma(y')$ .

Let  $s_0 \stackrel{*}{\to} x \to x' \in T$ . By monotonicity of TS,  $s_0 \leq x \leq x'$ ; by the Down condition on  $\sigma$ , there are  $y, y' \in S'$  such that  $x = \sigma(y)$ ,  $x' = \sigma(y')$ , and  $y \leq y'$ ; by assumption that TS' is monotonically-closed,  $s_0' \stackrel{*}{\to} y \stackrel{*}{\to} y' \in T'$ . Hence  $\sigma$  is complete.

3. Liveness: If  $s_0' \xrightarrow{*} y \in T'$  then there is some  $y' \in S'$ ,  $y \neq y'$ , such that  $y \xrightarrow{*} y' \in T'$  and  $s_0 \xrightarrow{*} \sigma(y) \xrightarrow{*} \sigma(y') \in T$ ,  $\sigma(y) \neq \sigma(y')$ .

Let  $s_0' \stackrel{*}{\to} y \in T'$  and  $x = \sigma(y)$ . By monotonicity of TS,  $s_0 \leq x$ ; since  $\leq$  is unbounded there is an  $x' \in S$  such that x < x'; since TS is monotonicallyclosed,  $s_0 \stackrel{*}{\to} x \stackrel{*}{\to} x' \in T$ . By the Down condition on  $\sigma$ ,  $y \leq' y'$ ,  $x' = \sigma(y')$  for some  $y' \in S'$ . Since TS' is monotonically-closed,  $y \stackrel{*}{\to} y' \in T'$ . Since  $x \neq x'$  then by their definition so are  $y \neq y'$ . Hence  $\sigma$  is live.

This completes the proof of correctness of  $\sigma$ .

If all transition systems in a protocol stack are monotonically-closed, then Lemma 2 makes it is sufficient to establish that an implementation of one protocol by the next is order-preserving to prove it correct. A key challenge in showing that Lemma 2 applies is proving that the implementation satisfies the Down condition (Def. 5), which can be addressed by finding an 'inverse' to  $\sigma$  as follows:

**Observation 1** (Representative Implementation State). Assume TS and TS' as in Lemma 2 and an implementation  $\sigma: S' \to S$  that satisfies the Up condition of Definition 5. If there is a function  $\hat{\sigma}: S \to S'$  such that  $x \simeq \sigma(\hat{\sigma}(x))$  for every  $x \in S$ , and  $x_1 \preceq x_2$  implies that  $\hat{\sigma}(x_1) \preceq' \hat{\sigma}(x_2)$ , then  $\sigma$  also satisfies the Down condition.

*Proof.* As TS' is monotonically-closed, it has a computation  $\hat{\sigma}(x) \stackrel{*}{\to} \hat{\sigma}(x') \in T'$  that satisfies the Down condition.

Given a specification, typically the implementing transition system has to be 'programmed', with its states restricted to states that encode specification states and its transitions restricted to behave according to the specification. In the absence of a programming language, such 'programming' is captured as follows:

**Definition 6** (Transition System Instance). Given transition systems  $TS = (S, s_0, T), TS' = (S', s'_0, T')$ , then TS' is an instance of TS if  $s_0 = s'_0, S' \subseteq S$  and  $T' \subseteq T$ .

The definition suggests at least two specific ways to construct an instance: Via choosing the states and restricting the transitions to be only among these states; or via choosing transitions. Specifically, (i) Choose some  $S' \subset S$  and define T' := T/S', namely  $T' := \{(s \to s' \in T : s, s' \in S'\}$ . (ii) Choose some  $T' \subset T$ .

When a protocol employs an instance to implement the full protocol above it, this results in the need to compose implementations among three transition systems where the bottom protocol implements the full middle protocol, and an instance of the middle protocol implements the top protocol. The following lemma enables that.

**Lemma 3** (Restricting a Correct Implementation to an Instance). Let  $\sigma: C2 \to S1$  be an order-preserving implementation of TS1 = (S1, s1, T1) by TC2 = (C2, s2, T2), monotonically-closed respectively with  $\preceq_1$  and  $\preceq_2$ . Let TS1' = (S1', s1, T1') be an instance of TS1 and TC2' = (C2', s2, T2') be the instance of TC2 defined by  $C2' := \{s \in C2 : \sigma(s) \in S1'\}$ , with T2' := T2/C2', and assume that both instances are also monotonically-closed wrt  $\preceq_1$  and  $\preceq_2$ , respectively. If  $y_1 \to y_2 \in T2$  &  $\sigma(y_1) \in S1'$  implies that  $\sigma(y_2) \in S1'$  then the restriction of  $\sigma$  to C2' is a correct implementation of TS1' by TC2'.

*Proof.* Assume TS1, TC2, TS1', TC2' and  $\sigma$  as in the Lemma and that  $y \to y' \in T2 \& \sigma(y) \in S1'$  implies that  $\sigma(y') \in S1'$ . Define  $\sigma' : C2' \to S1'$  to be the restriction of  $\sigma$  to C2'. We have to show that  $\sigma'$  is correct, namely, we have to show:

- 1. Safety:  $s2 \xrightarrow{*} y \rightarrow y' \in T2'$  implies that  $s1 \xrightarrow{*} x \xrightarrow{*} x' \in T1'$  for  $x = \sigma'(y)$  and  $x' = \sigma'(y')$  in S1.
  - This follows from the safety of  $\sigma$ ,  $S1' \subseteq S1$  and the assumption that  $y \to y' \in T2 \& \sigma(y) \in S1'$  implies that  $\sigma(y') \in S1'$ .
- 2. Completeness:  $s1 \xrightarrow{*} x \to x' \in T1'$ , implies that there are  $y, y' \in C2'$  such that  $x = \sigma'(y), x' = \sigma'(y')$ , and  $s2 \xrightarrow{*} y \xrightarrow{*} y' \in T2'$ .
  - By completeness of  $\sigma$ , there are  $y, y' \in C2$  such that  $x = \sigma(y)$ ,  $x' = \sigma(y')$ , and  $s2 \stackrel{*}{\to} y \stackrel{*}{\to} y' \in T2$ . By definition of C2' as the domain of  $\sigma$ ,  $y, y' \in C2'$ . As  $y \stackrel{*}{\to} y' \in T2$ , then  $y \leq_2 y'$ . By assumption that TC2' is monotonically-closed, there is a computation  $s2 \stackrel{*}{\to} y \stackrel{*}{\to} y' \in T2'$ .
- 3. Liveness:  $s2 \xrightarrow{*} y \in T2'$  implies that there is for some  $y' \in C2'$ ,  $y \neq y'$ , for which  $y \xrightarrow{*} y' \in T2'$  and  $s1 \xrightarrow{*} \sigma'(y) \xrightarrow{*} \sigma'(y') \in T1'$ .
  - Assume  $s2 \stackrel{*}{\to} y \in T2'$ . By liveness of  $\sigma$ ,  $\sigma(y) \stackrel{*}{\to} \sigma(y') \in T1$  for some  $y' \in C2$ ,  $y \stackrel{*}{\to} y' \in T2$ . By definition of C2', also  $y' \in C2'$ . As  $\sigma(y)$ ,  $\sigma(y') \in S1'$ , then by definition of T1' as T1/S1',  $\sigma'(y) \stackrel{*}{\to} \sigma'(y') \in T1'$ . By the assumption that  $z \stackrel{*}{\to} z' \in T2$  &  $\sigma(z) \in S1'$  implies that  $\sigma(z') \in S1'$ , we conclude that the entire computation  $y \stackrel{*}{\to} y' \in T2'$ . Hence  $\sigma'$  is live.

This completes the proof.

In the following we assume a set  $\Pi = \{p_1, \ldots, p_n\}$  of  $n \geq 3$  agents, or miners, each equipped with a single and unique cryptographic key pair (aka trusted PKI). We overload an agent  $p \in \Pi$  to sometimes mean its public key and sometimes mean its index in [n]; the use should be clear from the context. Next we define multiagent transition systems, in which each transition is effected by a particular agent, as realized, for example, by each agent signing the new state, or the state increment (e.g. message sent or block created), resulting from its transition.

**Definition 7** (Multiagent, Centralized, Distributed, Synchronous & Asynchronous Transition Systems). Given agents  $\Pi$ , a transition system  $TS = (C, c_0, T)$  is:

- 1. multiagent over  $\Pi$  if there is a partition  $C^2 = \bigcup_{p \in P} C_p^2$  such that each transition  $c \to c' \in C_p^2$ ,  $p \in \Pi$ , referred to as a *p-transition*, is effected by p.
- 2. distributed if, in addition,
  - (a) there is a set S for which  $C = S^{\Pi}$ , in which case S is referred to as local states, C as configurations over  $\Pi$  and S, and  $c_p \in S$  as the local state of p in c, for any configuration  $c \in C$  and agent  $p \in \Pi$ ; and
  - (b) every p-transition  $c \to c' \in C_p^2$  only affects p's local state, namely  $c_p \neq c_p'$  and  $c_q' = c_q$  for all  $q \neq p \in \Pi$ .

Else TS is centralized.

A partial order  $\leq$  on the local states S naturally extends to configurations:  $c \leq c'$  if  $c_p \leq c'_p$  for every  $p \in \Pi$ .

- 3. asynchronous, or Einsteinian, if, in addition,
  - (a) there is a partial order  $\leq$  on S wrt which TS is monotonic, and
  - (b) for every p-transition  $c \to c' \in T$  and for every  $d, d' \in C$  that satisfies the following asynchrony condition, T also includes the p-transition  $d \to d'$

**Asynchrony condition**:  $c \leq d$ ,  $c_p = d_p$ ,  $c'_p = d'_p$ , and  $d_q = d'_q$  for every  $q \neq p \in \Pi$ .

Else TS is synchronous, or Newtonian.

Note that the definition of a distributed multiagent transition systems refers to  $C^2$  (all transitions, including faulty ones) rather than to T (correct transitions), so that even a malicious faulty agent cannot affect another agent's local state. Also note that in both synchronous and asynchronous transition systems, a transition of an agent may depend on the the local states of other agents. However, in a synchronous/Newtonian transition system a transition may depend on the present local state of all other agents, and may be disabled as soon as some other agent changes it local state; hence the synchronous attribute. On the other hand, in an asynchronous/Einsteinian transition system agents cannot tell whether they observe the present local state of another agent or its past state. Formally, if agent p can make a local p-transition in a configuration, it can still make it in any subsequent configuration in which other agents have updated their local states; hence the asynchronous attribute. Also note that any asynchronous transition system has a synchronous counterpart as an instance, obtained by retaining the minimal transition from every equivalence

class of transitions related by the Asynchrony condition. More generally, an instance of an asynchronous transition system is not necessarily asynchronous.

Next, we formalize fault resilience by an implementation. In the following, we

**Definition 8** (Fault Resilience). Given multiagent transition systems  $TS = (C, c_0, T), TS' = (C', c'_0, T')$  over  $\Pi$ , and a set of faulty transitions  $F \subset C'^2 \setminus T'$ , a correct implementation  $\sigma : C' \to C$  is F-resilient if:

- 1. **Safety**: For any TS' run  $r' = c'_0 \to c'_1 \to \ldots \in T' \cup F$ , there is a correct TS run  $\sigma(r') := \sigma(c'_0) \xrightarrow{*} \sigma(c'_1) \xrightarrow{*} \ldots$ , namely  $\sigma(r') \in T$ , and
- 2. **Liveness**: For any TS' run  $c'_0 \stackrel{*}{\to} c'_1 \in T' \cup F$  there is a correct TS' computation that does not include F-transitions  $c'_1 \stackrel{*}{\to} c'_2 \in T'$  that activates TS, namely such that  $\sigma(c'_1) \neq \sigma(c'_2)$ .

**Definition 9** (Can Implement). Given multiagent transition systems  $TS = (C, c_0, T), TS' = (C', c'_0, T')$  over  $\Pi, TS'$  can implement TS correctly if there is an instance  $TS'' = (S'', c'_0, T'')$  of TS' and a correct implementation  $\sigma: S'' \to S$  of TS by TS''; and given  $F \subseteq C'^2 \setminus T'$ , it can do so with F-resilience if  $\sigma$  is F-resilient.

We note that the above captures an adversary that corrupts agents, thus affecting safety, but not an adversary that controls the network, and can thus affect liveness. A multiagent transition system can model such an adversary by granting it control over the scheduler, namely the order of transitions. With such control, the adversary can model process failure and recovery, and arbitrary message delays (if the transition system has a notion of message sending and receipt), thus affecting liveness. With such control, the adversary is typically limited so that it cannot delay an enabled transition (or message) indefinitely. To capture such a restriction, we make the *fairness assumption* that in any infinite computation, a transition that is enabled infinitely often is eventually taken. In other words, an infinite computation of which every suffix has a configuration from which the transition can be taken, but it is never taken, violates the fairness assumption.

The following Lemma is useful in designing fault-resilient implementations among monotonically-closed transition systems, and in proving their resilience. It states that if the non-faulty agents can ensure that computations of the implemented transition system are monotonically increasing, the implementation is resilient to the faulty agents.

**Lemma 4** (Fault-Resilience of Order-Preserving Implementations). Assume multiagent transition systems  $TS = (S, c_0, T)$  and  $TS' = (S', c'_0, T')$ , each monotonically-closed wrt the unbounded partial orders  $\leq$  and  $\leq$ ', and an order-preserving correct implementation  $\sigma: S' \to S$  of TS by TS', and let  $F \subseteq S'^2 \setminus T'$ . If for every TS' run  $r' = c'_0 \stackrel{*}{\to} c'_1 \stackrel{*}{\to} c'_2 \in T' \cup F$ :

1. **Safety**:  $\sigma(c'_1) \leq \sigma(c'_2)$ , and

2. **Liveness**: there is a correct TS' computation  $c_2' \xrightarrow{*} c_3' \in T'$  for which  $\sigma(c_2') \prec \sigma(c_3')$ ,

then  $\sigma$  is fault resilient.

*Proof.* For safety: Let  $r'=c'_0\to c'_1\to\ldots$  be a TS' run in which less than f of the agents are faulty, and consider any two consecutive configurations  $c'_i\to c'_{i+1}\in r',\ i\geq 0$ . If  $\sigma(c'_i)=\sigma(c'_{i+1})$  then  $\sigma(c'_i)\overset{*}{\to}\sigma(c'_{i+1})\in T$  via the empty computation. Else  $\sigma(c'_i)\neq\sigma(c'_{i+1})$ . Since  $\sigma$  is order-preserving, then  $\sigma(c'_i)\prec\sigma(c'_{i+1})$ . Since TS is monotonically-closed wrt  $\preceq$ , then  $\sigma(c'_i)\overset{*}{\to}\sigma(c'_{i+1})\in T$ . Hence  $\sigma(r')=c_0\overset{*}{\to}\sigma(c'_1)\overset{*}{\to}\ldots\in T$ , with  $\sigma$  satisfying the safety requirement for fault resilience.

For liveness: Let  $c_0' \stackrel{*}{\to} c_1' \in T' \cup F$  be a TS' run. By assumption, there is a correct TS' computation  $c_1' \stackrel{*}{\to} c_2' \in T'$  that activates TS, namely for which  $\sigma(c_1') \prec \sigma(c_2')$ , with  $\sigma$  satisfying the liveness requirement for fault resilience. Hence  $\sigma$  is fault resilient.

Note that the Lemma is resilient to an adaptive adversary that chooses faulty

Note that the Lemma is resilient to an adaptive adversary that chooses faulty transitions from F as it pleases.

# 3 Example Application: Universality of the Longest-Chain Protocol

Next, we provide examples of the application of the approach by showing various protocols and their implementations, resulting in the mathematical protocol stack depicted in Figure 1. First, we show:

- 1. Multiagent transition systems specifying:
  - (P0) a centralized single-chain protocol, and
  - (P1) a distributed synchronous longest-chain protocol.
- 2. Universality of the single-chain protocol P0, in that it can implement any centralized multiagent transition system.
- 3. An implementation of P0 by the longest-chain protocol P1.
- 4. Concluding—via the compositionality of correct implementations—that the distributed longest-chain protocol P1 is universal for centralized multiagent transition systems.

Protocol P0 maintains a shared global sequence of signed acts and lets any agent extend it with a signed act. As such, it is a centralized multiagent transition system: Each transition is effected by one agent, but it affects a shared global state. The protocol can be viewed as an abstract specification of Byzantine Atomic Broadcast [6, 17].

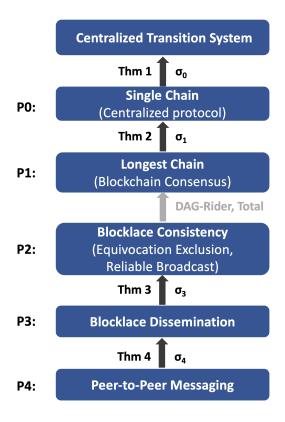


Figure 1: A Multiagent Transition System Protocol Stack: Blocks depict protocols, arrows implementations. Arrows are labeled with the mapping and the related theorem. DAG-Rider [10] and Total [13] are algorithms for ordering a DAG that can be used to implement P1 by P2.

**Definition 10** (P0: Centralized Ordering Protocol). Assume a set of acts  $\mathcal{A}$ . Let  $(p, a) \in \Pi \times \mathcal{A}$  denote the act a signed by agent p, referred to as a signed act by p, and let  $S0 := (\Pi \times \mathcal{A})^*$  denote all sequences of signed acts, each referred to as a chain. The P1 ordering protocol is a transition system P0 =  $(S0, \Lambda, T0)$ , with the initial state being the empty chain  $\Lambda$ , and transitions T0 being all pairs  $s \to s'$  of chains where  $s' = s \cdot (p, a)$  for some signed act  $(p, a) \in \Pi \times \mathcal{A}$ .

Note that in P0,  $T0 = S0^2$ , hence P0 does not specify any faulty transitions; its instances, however, may do so.

**Definition 11** (Prefix,  $\leq$ ). A sequence x is a *prefix* of a sequence x' if x' can be obtained from x by appending to it zero or more elements. In the following, we reserve  $x \leq y$  (without a subscript) to denote that sequence x is a prefix of sequence y. Note that  $\leq$  is strict and unbounded.

**Observation 2.** P0 is monotonically-closed wrt  $\leq$ .

*Proof.* P0 is monotonic wrt  $\leq$  since every transition increases its sequence. Given two sequences  $x, x' \in S^*$  such that  $x \prec x'$ , let  $x' = x \cdot s_1 \cdot \ldots \cdot s_k$ , for some  $k \geq 1$ . Then  $x \stackrel{*}{\to} x'$  via the sequence of transitions  $x \to x \cdot s_1 \to \ldots \to x \cdot s_1 \ldots s_k$ . Hence P0 is monotonically-closed.

Note that P0 can realize any blockchain application that requires total ordering of signed acts, e.g. smart contracts. More generally, our first theorem shows that P0 is a *universal centralized multiagent transition system*, in that it can implement correctly any centralized multiagent transition system. While the theorem's proof is quite straightforward—the states of the implementation are histories of the states of the specification—it provides a simple illustration of the methodology of multiagent transition systems and their implementation.

**Theorem 1** (P0 Universality). Let TS = (S, s0, T) be a centralized multiagent transition system (Def. 7). Then P0 can implement TS correctly.

*Proof Outline.* We define an instance (Def. 6) PC of P0 (Def 12), an implementation  $\sigma_0$  of TS by PC (Def. 13), and prove the implementation to be correct (Prop. 1).

**Definition 12** (PC). Given a centralized multiagent transition system TS = (S, s0, T), the transition system PC=  $(S^+, s0, TC)$  has non-empty sequences over S as states and for every transition  $s \to s' \in T0$  and every  $x \in S^*$ , TC has the transition  $x \cdot s \to x \cdot s \cdot s' \in TC$ .

**Observation 3.** PC is an instance of P0.

*Proof.* The states  $S^+$  of PC are a subset of the states  $S^*$  of P0. Every transition  $x \cdot s \to x \cdot s' \in TC$  of PC is also a P0 transition by definition.

The implementation  $\sigma_0: S^+ \mapsto S$  simply extract the last element of its non-empty input sequence:

**Definition 13**  $(\sigma_0)$ . Given a nonempty sequence  $x \cdot s \in S^+$ ,  $\sigma_0(x \cdot s) := s$ .

Centralized transition systems in general are not amenable to partial orders, hence the tools developed above for monotonic transition systems do not apply in this case, and a direct proof of the correctness of  $\sigma_0$  is needed.

**Proposition 1.** The implementation  $\sigma_0$  is correct.

*Proof.* To prove that  $\sigma_0$  is correct we have to show:

1. Safety:  $s0 \xrightarrow{*} y \to y' \in TC$  implies that  $s0 \xrightarrow{*} x \xrightarrow{*} x' \in T$  for  $x = \sigma_0(y)$  and  $x' = \sigma_0(y')$  in S.

Let  $y = s0 \cdot s_1 \cdot \ldots \cdot s_k$ ,  $y' = y \cdot s_{k+1}$ , for  $k \geq 1$ . For each transition  $s0 \cdot s_1 \cdot \ldots \cdot s_i \rightarrow s0 \cdot s_1 \cdot \ldots \cdot s_i \cdot s_{i+1}$ ,  $i \leq k$ , the transition  $s_i \rightarrow s_{i+1} \in T$  by definition of TC. Hence  $s0 \stackrel{*}{\to} x \stackrel{*}{\to} x' \in T$ , satisfying the safety condition.

- 2. Completeness:  $s0 \stackrel{*}{\to} x \to x' \in T$  implies that there are  $y, y' \in SC$  such that  $x = \sigma_0(y), x' = \sigma_0(y'),$  and  $s0 \stackrel{*}{\to} y \stackrel{*}{\to} y' \in TC.$ 
  - Let  $x = s_k$ ,  $x' = s_{k+1}$ ,  $k \ge 1$ , and  $s0 \to s_1 \to \ldots \to s_k \to s_{k+1} \in T$ . Then  $y = s0 \cdot s_1 \cdot \ldots \cdot s_k$  and  $y' = y \cdot s_{k+1}$  satisfy the completeness condition.
- 3. Liveness:  $s0 \xrightarrow{*} y \in TC$  implies that there is for some  $y' \in SC$ ,  $y \neq y'$ , for which  $y \xrightarrow{*} y' \in TC$  and  $s0 \xrightarrow{*} \sigma_0(y) \xrightarrow{*} \sigma_0(y') \in T$ .

Let 
$$y = y0 \cdot s \in S^+$$
 and  $y' = y \cdot s' \in S^+$ . Then  $s0 \xrightarrow{*} \sigma_0(y) \xrightarrow{*} \sigma_0(y') = s0 \xrightarrow{*} s \rightarrow s' \in T$ , satisfying the liveness condition.

This completes the proof.

The single-chain protocol P0 is a centralized ordering protocol. As a first step towards its implementation via an asynchronous distributed protocol, we show here a distributed synchronous ordering protocol – the longest-chain protocol P1. Recall that two sequences x, x' are consistent if  $x \leq x'$  or  $x' \leq x$ , inconsistent otherwise (Def. 3).

**Definition 14** (P1: Distributed Synchronous Longest-Chain Protocol). The P1 distributed synchronous longest-chain protocol is a distributed multiagent transition system P1 = (C1, c0, T1), where C1 are all configurations over agents  $\Pi$  and local states S0, initial configuration  $c0 = \{\Lambda\}^{\Pi}$  having empty sequences as local states, and transitions T1 being all p-transitions  $c \to c'$ , for every  $p \in \Pi$ , where  $c'_p := c_p \cdot x$  for any  $x \in S1$  such that  $c'_p$  and  $c_q$  are consistent for every  $q \in \Pi$ .

Note that the dynamics of P1 can be viewed as an abstraction of longestchain consensus protocols (e.g. Nakamoto [14]), since the consistency requirement entails that only an agent with the longest chain may extend it; others are bound to copy blocks into their local chain till they catch up, and only then may contribute. Also note that we do not restrict the block added by an agent to be signed by that agent; this flexibility is needed as in subsequent protocols an agent may finalize blocks issued by others before the issuers of these blocks finalize them.

**Observation 4.** P1 is monotonically-closed wrt  $\leq$ .

The proof is similar to the proof of Observation 2.

**Observation 5** (Consistent Configurations). A configuration  $c \in C1$  is consistent if  $c_p$  and  $c_q$  are consistent for every  $p, q \in \Pi$ . Let r a run of P1 and  $c \in r$  a configuration. Then c is consistent.

*Proof.* The proof is by induction on the length of r. All empty sequences of the initial configuration of r are pairwise consistent. Assume the  $n^{th}$  configuration c of r is consistent and consider the next r transition  $c \to c' \in T1$ . The transition modifies only one local state in c and preserves its consistency with the other local states in c' by its definition. Hence all the local states of c' are pairwise consistent and hence c' is consistent.

Hence the following implementation of P0 by P1 is well-defined.

**Definition 15**  $(\sigma_1)$ . The implementation  $\sigma_1: C1 \mapsto S0$  maps every configuration  $c \in C1$  to  $c_p \in S0$ , where p satisfies that  $c_q \leq c_p$  for any  $q \in \Pi$ .

**Proposition 2.**  $\sigma_1$  is order-preserving wrt the prefix relation  $\leq$ .

*Proof.* We have to show that:

- 1. Up condition:  $y \leq y'$  for  $y, y' \in S1$  implies that  $\sigma_1(y) \leq \sigma_1(y')$
- 2. **Down condition:**  $s_0 \stackrel{*}{\to} x \in T0$ ,  $x \stackrel{\prec}{=} x'$  implies that there are  $y, y' \in S1$  such that  $x = \sigma_1(y)$ ,  $x' = \sigma_1(y')$ ,  $c_0 \stackrel{\prec}{\to} y \in T1$  and  $y \stackrel{\prec}{=} y'$ .

Regarding the Up condition, assume that  $y \leq y'$  and that  $y'_p$  is the longest chain

in y'. Then  $\sigma_1(y)=y_p\preceq y'_p=\sigma_1(y')$ . Regarding the Down condition, define  $y_p:=x,\ y'_p:=x'$ , and  $y_q:=y'_q:=\Lambda$ for every  $q \neq p \in \Pi$ . Then  $x = y_p = \sigma_1(y), x' = y'_p = \sigma_1(y'), c0 \stackrel{*}{\to} y \in T1$  by the same transitions that lead from s0 to x, and  $y \leq y'$  by construction.

**Theorem 2.** P1 can implement P0 correctly.

*Proof.* Both P0 and P1 are monotonically-closed wrt the prefix relation  $\prec$  (Observations 2, 4). The implementation  $\sigma_1$  of P0 by P1 is order preserving (Proposition 2). Hence, according to Lemma 2,  $\sigma_1$  is correct.

Corollary 1. The distributed synchronous longest-chain protocol P1 is universal for centralized multiagent transition systems.

*Proof.* Given a centralized transition system TS, a correct implementation  $\sigma_0$ of TS by P0 exists according to Theorem 1. The implementation  $\sigma_1$  of P0 by P1 is correct according to Theorem 2. Then, Lemmas 1 and 3 ensure that even though an instance PC of P0 was used in implementing TS, the result of the composition  $\sigma_{10} := \sigma_1 \circ \sigma_0$  is a correct implementation of TS by P1.

#### 4 Example Application: A DAG-Based Blockchain **Protocol Stack**

As another example, we present a protocol stack that addresses the three key tasks of a blockchain protocol (whether it is Nakamoto Consensus [14], State-Machine Replication [18] or Byzantine Atomic Broadcast [10]): Dissemination [7], equivocation exclusion [8], and ordering. The desired behavior of processes in a distributed blockchain consensus protocol has been specified by the longest-chain protocol P1 above.

The protocol stack presented here (Figure 1) employs a partially-ordered generalization of the totally-ordered blockchain data structure, termed blocklace, which represents the partial order via a DAG. The use of a DAG for blockchain consensus [3, 15, 16], and in particular algorithms for the total-ordering of an equivocation-free DAG are known [13, 10]. Hence we illustrate here only the bottom-part of the stack, utilizing the blocklace to address equivocation-exclusion and dissemination. While the protocol stack is presented mainly as an illustration of the mathematical approach, it has independent value. In particular, its approach to DAG-based equivocation-exclusion is novel, and the use of a partially-ordered data structure is geared for applications that need to record causality.

# 4.1 The Blocklace: A Partially-Ordered Generalization of the Totally-Ordered Blockchain

In the following we assume a given set of payloads A.

**Definition 16** (Block). A block is a triple b = (p, a, H), referred to as a p-block,  $p \in \Pi$ , with  $a \in \mathcal{A}$  being the payload of b, and H is a (possibly empty) finite set of hash pointers to blocks, namely for each  $h \in H$ , h = hash(b') for some block b'. Such a hash pointer h is a q-pointer if b' is a q-block. The set H may have at most one q-pointer for any miner  $q \in \Pi$ , and if H has no self-edge then b is called *initial*. The depth of b, depth(b), is the maximal length of any path emanating from b.

Note that hash being cryptographic implies that a circle of pointers cannot be effectively computed.

**Definition 17** (Dangling Pointer, Grounded). A hash pointer h = hash(b) for some block b is dangling in B if  $b \notin B$ . A set of blocks B is grounded if no block  $b \in B$  has a pointer dangling in B.

The non-dangling pointers of a set of blocks B induce finite-degree directed graph (B, E),  $E \subset B \times B$ , with blocks B as vertices and directed edges  $(b, b') \in E$  if  $b, b' \in B$  and b includes a hash pointer to b'. We overload B to also mean its induced graph (B, E).

**Definition 18** (Blocklace). Let  $\mathcal{B}$  be the maximal set of blocks over  $\mathcal{A}$  and hash for which the induced directed graph  $(\mathcal{B}, \mathcal{E})$  is acyclic. A blocklace over  $\mathcal{A}$  is a set of blocks  $\mathcal{B} \subset \mathcal{B}$ .

#### Lemma 5. $\mathcal{B}$ is well-defined.

Proof. Given  $\mathcal{A}$ , we enumerate recursively all elements of  $\mathcal{B}$  and prove that the result is unique. First, let  $\mathcal{B}$  include all initial blocks of the form  $(p, a, \emptyset)$ , for any  $p \in \Pi$  and  $a \in \mathcal{A}$ . Next, iterate adding to  $\mathcal{B}$  all blocks of the form (p, a, H), with  $p \in \Pi$ ,  $a \in \mathcal{A}$ , and H a set of hash pointers with at most one q-pointer h = hash(b) for any  $q \in \Pi$  and some q-block  $b \in \mathcal{B}$ . Note that  $\mathcal{B}$  is acyclic and maximal by construction. To prove that it is unique, assume that there are two sets  $\mathcal{B} \neq \mathcal{B}'$  that satisfy our construction, and that wlog  $\mathcal{B} \not\subset \mathcal{B}'$ . Let b = (p, a, H) be a first block such that  $b \in \mathcal{B} \setminus \mathcal{B}'$ , namely a block for which every block b' pointed to by some  $h \in H$  is in  $\mathcal{B} \cap \mathcal{B}'$ . As by assumption all blocks pointed to by H are in  $\mathcal{B}'$ , then by construction  $\mathcal{B}'$  includes b, a contradiction.

The two key blocklace notions used in our protocols are *acknowledgement* and *approval*, defined next.

**Definition 19** ( $\succ$ , Acknowledge). Given a blocklace  $B \subseteq \mathcal{B}$ , the strict partial order  $\succ_B$  is defined by  $b' \succ_B b$  if B has a non-empty path of directed edges from b' to b (B is omitted if  $B = \mathcal{B}$ ). Given a blocklace B, b' acknowledges b in B if  $b' \succ_B b$ . Miner p acknowledges b via B if there is a p-block  $b' \in B$  that acknowledges b, and a group of miners  $Q \subseteq \Pi$  acknowledge b via B if for every miner  $p \in Q$  there is a p-block  $b' \in B$  that acknowledges b.

**Definition 20** (Closure, Tip). The closure of  $b \in \mathcal{B}$  wrt  $\succ$  is the set  $[b] := \{b' \in \mathcal{B} : b \succeq b'\}$ . The closure of  $B \subset \mathcal{B}$  wrt  $\succ$  is the set  $[B] := \bigcup_{b \in B} [b]$ . A block  $b \in \mathcal{B}$  is a tip of B if  $[b] = [B] \cup \{b\}$ .

Note that a set of blocks is grounded iff it includes its closure (and thus is identical to it):

**Observation 6.**  $B \subset \mathcal{B}$  is grounded iff  $[B] \subseteq B$ .

With this, we can define the basic notion of equivocation (called double-spend if the block's payload is a transfer of funds).

**Definition 21** (Equivocation, Equivocator, Consistent Blocklace). Two p-blocks  $b \neq b' \in \mathcal{B}$ ,  $p \in \Pi$ , are an equivocation of p if they are not consistent wrt  $\succ$ , namely  $b' \not\succ b$  and  $b \not\succ b'$ . A miner p is an equivocator in B if [B] has an equivocation of p. A blocklace B is consistent if it is grounded and does not include an equivocation.

Namely, two p-blocks are an equivocation of p if they do not acknowledge each other in  $\mathcal{B}$ . In particular, two initial p-blocks constitute an equivocation by p. As p-blocks are cryptographically signed by p, an equivocation of p is a volitional fault of p. Also, note that an inspection of a blocklace p can conclude that p is not an equivocator in p only if p is grounded, lest a dangling pointer in p points to a yet-to-be-uncovered equivocating p-block.

**Definition 22** (Approval). Given a blocklace B, a block b approves b' in B if b acknowledges b' in B and does not acknowledge any block b'' in B that together with b' forms an equivocation. A miner p approves b' in B if there is a p-block b that approves b' in B, in which case we also say that p approves b' in B via b. A set of miners  $Q \subseteq \Pi$  approve b' via B' in B if every miner  $p \in Q$  approves b' in B via some p-block  $b \in B'$ ,  $B' \subseteq B$ .

**Observation 7.** Approval is monotonic wrt  $\supset$ .

Proof. TBC 
$$\Box$$

Namely, if b or p approve b' in B they also approve b' in  $B' \supset B$ .

A key observation is that a miner cannot approve an equivocation of another miner without being an equivocator itself (Fig. 2):



Figure 2: Observing an Equivocation: Initial blocks are at the bottom. Assume  $b_1, b_2$  are an equivocation (Def. 21) by the red miner. According to the figure, b'' approves  $b_2$  (Def. 22). However, since b' acknowledges b'' (Def. 19) it also acknowledges  $b_2$  and hence does not approve the equivocating  $b_1$  (nor  $b_2$ ).

**Observation 8.** [Approving an Equivocation] If miner  $p \in \Pi$  approves an equivocation  $b_1, b_2$  in a blocklace  $B \subseteq \mathcal{B}$ , then p is an equivocator in B.

*Proof.* Assume that p approves  $b_1$  and  $b_2$  via two blocks, b', b'', respectively, that do not constitute an equivocation, so wlog assume that  $b' \succ b''$  (See Figure 2). However, since  $b' \succ b''$  and  $b'' \succ b_2$ , then  $b' \succ b_2$ , in contradiction to the assumption that b' approves  $b_1$ .

We note that once a miner p observes an equivocation b, b' by another miner q, it would not approve any subsequent block b'' by q that acknowledges say b but not b', as such a block would constitute an equivocation with the block b' of the equivocation it does not acknowledge.

We define an equivocation to be a fault, and hence by assumption at most f miners may equivocate in any run.

**Lemma 6** (No Supermajority Approval for Equivocation). If there are at most f equivocators in a blocklace  $B \subset \mathcal{B}$  with an equivocation  $b, b' \in B$ , then not both b, b' have supermajority approval in B.

*Proof.* Assume that there are two supermajorities  $S, S' \subseteq P$ , where S approves b and S' approves b'. A counting argument shows that two supermajorities must have in common at least one correct miner. Let  $p \in S \cap S'$  be such a correct miner. Observation 8 shows that a miner that approves an equivocation must be an equivocator, hence p is an equivocator. A contradiction.

**Definition 23** (Finality by Supermajority). A block  $b \in B$  is *final* in B if the set of blocks that approve b in B is a supermajority. The *final subset* of B is the set  $\phi(B) := \{b \in B : b \text{ is final in } B\}$ .

From the definition of finality and Lemma 6 we conclude:

**Observation 9** (Finality Excludes Equivocation).  $\phi(B)$  is consistent.

And conclude that finality is indeed final:

**Observation 10** (Finality is Monotonic). Let  $B \subseteq B'$  be two blocklaces. Then  $\phi(B) \subseteq \phi(B')$ .

## 4.2 Blocklace Consistency Protocol P2

Next we specify the blocklace consistency protocol P2. As an equivocation-free DAG protocol, it can support a correct implementation of the longest-chain protocol P1, along the lines of DAG-Rider [10] and its Total predecessor [13]. These protocols assume equivocation-free dissemination (i.e., reliable broadcast) of blocks, and then have each agent compute the totally-ordered chain from its local DAG.

Recall that a blocklace is consistent if its closure does not include an equivocation (Def. 21), and the notion of a multiagent configuration (Def. 7).

**Definition 24** (Blocklace Configuration). A blocklace configuration over  $\Pi$  and  $\mathcal{A}$  is a configuration over  $\Pi$  and  $\mathcal{B}(\mathcal{A})$ . The initial blocklace configuration is empty,  $c0 := \{\emptyset\}^{\Pi}$ .

We may overload a blocklace configuration c to also denote the union of its local blocklaces  $\bigcup_{p \in P} c_p$ ; in particular, we say that c is consistent if the union of its local blocklaces is.

**Definition 25** (P2: Blocklace Consistency Protocol). The P2 blocklace consistency protocol is a transition system P2 = (C2, c0, T2), with states C2 being all consistent blocklace configurations over  $\Pi$  and  $\mathcal{A}$ , and transitions T2 being all p-transitions  $c \to c'$ ,  $p \in \Pi$ , where  $c'_p := c_p \cup \{b\}$  with a p-block  $b \in \mathcal{B}(\mathcal{A})$  such that:

- 1. **Grounded**:  $[b] \subseteq c'$ , and
- 2. Non-equivocating: b acknowledges every p-block in  $c_p$ .

The definition ensures that:

**Observation 11.** A correct P2 run only produces consistent blocklace configurations.

*Proof.* By induction on the length of the run. Initially, an empty configuration is grounded and consistent by Definitions 17 and 21. Assume a consistent configuration c and a P2 p-transition  $c \to c'$ . The p-block added to c has no dangling pointers in c since its closure is in c, hence c' is grounded. And it is not an equivocation since it acknowledges all p-blocks in c, hence c' is consistent.  $\square$ 

**Observation 12.** Define  $c \leq_2 c'$  if  $c, c' \in C2$  are consistent and  $c \subseteq c'$ . Then P2 is monotonically-closed wrt  $\leq_2$  (Definition 4).

Proof. Assume a P2 p-transition  $c \to c' \in T2$  that adds the p-block b to  $c_p$ . Then  $c_p \subseteq c_p \cup \{b\} = c'_p$ , and  $\forall q \neq p : c_q = c'_q \ c \preceq_2 c'$ , hence  $c \preceq_2 c'$ . Consider a P2 computation  $c0 \xrightarrow{*} c \in T2$  and some consistent c' for which  $c \preceq_2 c'$ . Since c is consistent, there is an ordering  $b_1, b_2, \ldots b_k$  of  $c' \setminus c$  satisfying  $b_i \succ b_{i+1}$ ,  $i \in [k-1]$ . Let  $c^11 := c$ , and for every  $i \in [k-1]$  and p-block  $b_i$ , define the p-transition  $c^i \to c^{i+1}$  to be the p-transition  $c^i \to x^i_p \cup \{b_i\} = c^{i+1}_p$ . By construction,  $b_i$  has no dangling pointers in c and acknowledges all p-blocks in c. Hence  $c = c_1 \to c_2 \ldots \to c_k = c' \in T2$ , concluding that P2 is monotonically-closed wrt  $\preceq_2$ .

The following observation has implications on the ease of a correct distributed implementation of P2.

Observation 13. P2 is asynchronous (Def. 7).

Proof. P2 is monotonic wrt  $\leq_2$  according to Observation 12. Consider a P2 p-transition  $c \to c'$ , namely  $c'_p = c_p \cup \{b\}$  and  $\forall q \neq p : c_q = c'_q$ . The asynchrony condition requires that for every  $d, d' \in C2$  such that  $c \leq d$ ,  $c_p = d_p$ ,  $c'_p = d'_p$ , and  $d_q = d'_q$  for every  $q \neq p \in \Pi$ , T2 also includes the p-transition  $d \to d'$ . Consider any  $d \in C2$  such that  $c \leq d$ ,  $c_p = d_p$ . Since  $c \leq d$ , the grounded conditions for a T2 p-transition that adds b to  $d_p$  is met, and since  $c_p = d_p$ , also the non-equivocation condition is met. Since the transition only adds b to  $d_p$ , it follows that  $c'_p = c_p \cup \{b\} = d_p \cup \{b\} = d'_p$ , and  $d_q = d'_q$  for every  $q \neq p \in \Pi$ . Hence P2 is asynchronous.

#### 4.3 Blocklace Dissemination Protocol P3

In the blocklace consistency protocol P2 each agent trusts the other agent's blocks to be consistent. In the blocklace dissemination protocol P3 each agent maintains its own copy of the blocklace. The protocol P3 ensures that if an agent is an equivocator then eventually all correct agents will know that. This by itself is not sufficient for the correct agents to exclude equivocations, as different agents may observe different acts of an equivocator p and acknowledge them before detecting that p is an equivocator. Next, we define the protocol P3 and show its fault-resilient implementation of P2.

**Definition 26** (P3: Blocklace Dissemination Protocol). The P3 blocklace dissemination protocol is a transition system P3 = (C3, c0, T3), with states C3 all blocklace configurations over  $\Pi$  and  $\mathcal{A}$ , transitions T3 all p-transitions,  $p \in \Pi$ , where  $c \to c'$ ,  $c'_p := c_p \cup \{b\}$ , and:

- 1. p-Acts:  $b \in \mathcal{B}(\mathcal{A})$  is a p-block such that  $[b] = c_p \cup \{b\}$ , or
- 2. p-Delivers:  $b \in c_q \setminus c_p$ ,  $[b] \subset c_p \cup \{b\}$

We also consider specifically the following faulty transitions:

3. p-Equivocates:  $b \in \mathcal{B}(\mathcal{A})$  is a p-block such that  $[b] \subset c_p \cup \{b\}$ 

and refer to an agent taking such a transition as equivocator.

The p-Acts transition requirements ensure that in any run of P3, the local blocklace of a correct agent p does not include equivocation by p; it may include equivocations of other faulty miners, delivered by p, but not both approved by p as long as p is not an equivocator (Ob. 2). The closure requirement  $[b] = c_p \cup \{b\}$  ensures that a block by a correct miner acknowledges all the blocks in its local blocklace. The p-Delivers transition allows an agent to obtain a block from another agent, provided that it has not pointers dangling in  $c_p$ . Relating to standard notions of communication protocols, non-grounded blocks can be thought of as out-of-order messages, which are buffered and ignored until missing in-order messages that precede them are delivered.

Note that correct miners do not equivocate in P3, so in a correct computation all configurations are consistent.

The partial order for P3 is based on the subset relation  $\subseteq$  as in P2, but is more refined since local blocklaces in P3 are not disjoint as in P2.

**Observation 14.** Define  $c \preceq_3 c'$  for  $c, c' \in C3$  if  $c_p \subseteq c'_p$  and  $c_p, c'_p$  are grounded for every  $p \in \Pi$ . Then P3 is monotonically-closed wrt  $\preceq_3$ .

*Proof.* Essentially the same as the proof of Observation 12; note that the proof does not depends on P2 configurations being consistent, only on them on being grounded, which holds also for P3.

Under the fairness assumptions, in a correct run r of a computation of P3, for any block b in the local blocklace  $c_p$  of some agent p in a configuration c of r and for any other agent  $q \neq p$ , there is a subsequent configuration c' in r in which the block b is included in  $c'_q$ . We observe that this holds among correct miners, when faulty miners may repetitively delete all or part of their state arbitrarily during the computation. In particular, if holds even if a faulty agent p deletes a p-block from its state before some other agents have delivered it, achieving message dissemination among correct agents. Let F be the set of all p-transitions  $c_p \to c'_p \in C3^2$  where  $c'_p \subset c_p$ .

**Observation 15.** In a P3 run where incorrect miners take only  $F \cup T3$  transitions, any block known to a correct miner will eventually be known to every correct miner.

*Proof.* If there is a block  $b \in c_q \setminus c_p$  in some configuration c of a computation of P3, where p and q are correct in r, then p-Delivers of b from q is enabled in r from c onwards, and therefore will eventually be taken.

Recall that Definition 8 is wrt to a set of faulty transitions F. Here, we consider any such set F of p-Equivocates transitions by less than f agents.

The theorem we aim to prove here is:

**Theorem 3.** Protocol P3 can implement P2 correctly with resilience to less than f equivocators.

Proof Outline. We define an implementation  $\sigma_3$  of P2 by P3 (Def. 27), and prove that  $\sigma_3$  is order-preserving wrt  $\leq_3$  and  $\leq_2$  (Prop. 3) and resilient to less than f equivocators (Prop. 4). As P2 is monotonically-closed wrt  $\leq_2$  (Ob. 12 above), the conditions for Lemma 2 are fulfilled, thus completing the proof.

The key challenge in implementing P2 is excluding equivocations. In a distributed context, an equivocation, and more specifically a double-spend, can be initiated by a malicious agent in order to perform two independent transactions with different agents but with the same digital asset. The standard blockchain solution for equivocations (by Nakamoto consensus [14] or PBFT [5]) employs ordering: order all acts, and in case of equivocation simply ignore the second act of the two. However, ordering is more difficult than excluding equivocations [8] and, indeed, the implementation we present of P2 by P3 excludes equivocations without ordering. In particular, it has the freedom to exclude one or both acts of an equivocation.

Note that finality  $\phi$  is a global property defined for the blocklace configuration c as a whole, not for each local blocklace.

**Definition 27**  $(\sigma_3)$ . The implementation  $\sigma_3: C3 \to C2$  maps every C3 configuration y into the C2 configuration  $x = \sigma_3(y)$ , defined by  $x_p := \{b : b \text{ is a } p\text{-block in } \phi(y)\}$  for every  $p \in \Pi$ .

Namely, the implementation extracts from a blocklace configuration its final subset and assigns each resulting p-block to the local blocklace of its creator p. If a block b is final, then under the fairness assumption and given the bound on faulty miners, the blocklace dissemination protocol ensures that eventually b will be final in every agent's blocklace.

#### **Proposition 3.** $\sigma_3$ is order-preserving wrt $\leq_3$ and $\leq_2$ .

*Proof.* As  $\leq_2$  is a strict partial order, the equivalence relation it induces on P2 is equality, and hence the Up condition is trivially satisfied. For the Down condition, we have to show that  $c0 \stackrel{*}{\to} x \in T2$ ,  $x \leq_2 x'$  implies that there are  $y, y' \in C3$  such that  $x = \sigma_3(y)$ ,  $x' = \sigma_3(y')$ ,  $c0 \stackrel{*}{\to} y \in T3$  and  $y \leq_3 y'$ .

Consider such  $x \leq_3 x' \in C2$  and  $y \in C3$ , and construct for x, x' representative implementation states (Observation 1)  $y, y' \in C3$  as follows. Let  $y_p := x \cup ACK$ ,  $y'_p := x' \cup ACK'$ , where ACK is a set of blocks in which every agent  $p \in \Pi$  has a p-block that acknowledges every block  $b \in x$ , presumably by pointing to the tip of x, but does not acknowledge any other blocks in ACK. The set of blocks  $ACK' \supseteq ACK$  is constructed similarly for every  $b \in x'$ , and x', and includes also ACK. By construction  $x_p := \{b : b \text{ is a } p\text{-block in } \phi(y)\}$  and similarly for x' and y'. Hence  $x = \sigma_3(y)$ ,  $x' = \sigma_3(y')$ .

**Proposition 4.**  $\sigma_3$  is resilient to less than a third of the miners equivocating.

*Proof.* Given the correct implementation  $\sigma_3: C3 \to C2$  and a P3 run r with less than f equivocators, we have to show that:

1. safety: the P2 run  $\sigma_3(r)$  is correct.

The final subset computed by  $\sigma_3$  for each r configuration is consistent in the face of less than f equivocators according to Observation 9. Hence the transitions in  $\sigma_3(r)$  are correct.

2. liveness: for every  $c \in r$  there is a P3 computation  $c \stackrel{*}{\to} c' \in T3$  of transitions by agents correct in r that activates P2.

Given a configuration  $c \in r$ , by assumption less than f of the agents are equivocators. As the remaining correct agents constitute a supermajority, one of them, say p, can make a p-Acts transition, with all the correct agents acknowledging it, resulting in the act being final and P2 activated.

This completes the proof of Theorem 3.

## 4.4 Messaging Protocol P4

The messaging protocol P4 provides reliable but unordered peer-to-peer messaging. It can be readily implemented by TCP/IP.

**Definition 28** (Message). We assume a base set  $\mathcal{X}$  of payloads. A message is a triple m = (q, p, x), referred to as a p-message to  $q, p, q \in \Pi$ , with  $x \in \mathcal{X}$  being its payload; such a message is signed by p's private key, and may be encrypted by q's public key. Let  $\mathcal{M}(\mathcal{X})$  denote the set of all messages with payloads in  $\mathcal{X}$ .

Note that 'self-messages' with p=q are possible and can be used for local state update.

**Definition 29** (Messaging Protocol P4). The messaging protocol P4 = (C4, c0, T4) is a distributed multiagent transition system with states being configurations over  $\Pi$  and message sequences  $\mathcal{M}(\mathcal{X})^*$ . T4 has all p-transitions  $c \to c'$ ,  $p \in \Pi$ , where  $c'_p := c_p \cdot M$  and  $M \neq \Lambda$  is finite set of message, each  $m \in M$  is either a q-message for  $p, m = (p, q, x) \in c_q \setminus c_p, q \in \Pi$ , or a p-messages.

Namely, a p-transition may deliver messages for p and respond with p-messages.

Next we show a simple correct implementation of P3 by P4. Then, we extend the implementation so that it achieve dissemination among correct agents in the face of agents deleting their local states, in the sense of Observation 17.

**Theorem 4.** P4 can implement correctly P3.

Proof Outline. We define an instance of P4, termed P4B (Def. 30), with a partial order  $\leq_4$ , and observe that P4B is monotonically-closed wrt  $\leq_4$  (Prop. 16). We define an implementation  $\sigma_4$  of P3 by P4B (Def. 31), and prove that  $\sigma_4$  is order-preserving wrt  $\leq_4$  and  $\leq_3$  (Prop. 5). As P3 is monotonically-closed wrt  $\leq_3$  (Ob. 14 above), the conditions for Lemma 2 are fulfilled, thus completing the proof.

The transition system P4B mimics the intended behavior of protocol P3 using message passing. Namely, when a miner p creates a block b as in a p-Acts transition of protocol P3, then it sends b to all agents, as specified by the p-Delivers transition of P3. It uses the following function  $\beta$  maps a sequence of  $\mathcal{M}(\mathcal{B}(\mathcal{A}))$  messages into the blocklace they carry, provided the incrementally constructed set of blocks is always grounded.

$$\beta(M) = \begin{cases} B \cup \beta(M') & \text{if } M = (p, q, B) \cdot M' \text{ and the result is grounded} \\ \bot & \text{otherwise} \end{cases}$$

**Definition 30** (P4B). The transition system instance P4B= (C4B, c0, T4B) of P4 is defined as follows. By definition,  $\mathcal{M}(\mathcal{B}(\mathcal{A}))$  has messages of the form (p,q,B), where  $B \subset \mathcal{B}(\mathcal{A})$ . We define  $C4B \subset C4$  to be the set of configurations over message sequences  $\mathcal{M}(\mathcal{B}(\mathcal{A}))^*$ , and T4B having all p-transitions  $c \to c' \in T4_p$  for each  $p \in \Pi$ , that satisfy  $c'_p = c_p \cdot M$ , where M is:

- 1. **p-Acts**: A sequence of p-messages (q, p, b) for every  $q \in \Pi$  and every p-block  $b \notin \beta(c_p)$  such that  $[b] = c_p \cup \{b\}$ .
- 2. **p-Delivers**:  $M = \{(p, q, b)\} \subseteq c_q \setminus c_p \text{ and } [b] \subset \beta(c'_p).$

Note that the p-Delivers transition ignores (aka buffers) messages that include out-of-order, non-grounded blocks.

We observe that:

**Observation 16.** Define  $c \leq_4 c'$  for  $c, c' \in C4B$  if  $\beta(c_p) \subseteq \beta(c'_p)$  for every  $p \in \Pi$ . Then P4B is monotonically-closed wrt  $\leq_4$ 

*Proof.* Essentially the same as the proof of Observation 14.  $\Box$ 

We define the implementation and prove necessary propositions.

**Definition 31**  $(\sigma_4)$ . Define  $\sigma_4: C4B \to S_3$  by  $\sigma(c)_p := \beta(c_p)$  for all  $p \in \Pi$ .

**Proposition 5.**  $\sigma_4$  order-preserving wrt  $\leq_4$  and  $\leq_3$ 

*Proof.* As  $\leq_3$  is a strict partial order, the equivalence relation it induces on P3 is equality, and hence the Up condition is trivially satisfied. For the Down condition, we have to show that  $c0 \stackrel{*}{\to} x \in T3$ ,  $x \leq_3 x'$  implies that there are  $y, y' \in C4$  such that  $x = \sigma_4(y)$ ,  $x' = \sigma_4(y')$ ,  $c0 \stackrel{*}{\to} y \in T4$  and  $y \leq_4 y'$ .

Consider such  $x \leq_4 x' \in C3$  and  $y \in C4B$ , and construct for x, x' representative implementation states (Observation 1)  $y, y' \in C4B$  as follows. Since P3 is monotonically-closed, there is a computation  $x = x^1 \to x^2 \ldots \to x^k = x' \in T3$ . In particular, we choose a computation in which each p-Acts transition of some  $p \in \Pi$  with block p is followed by p-Delivers transition of p from p by each  $p \neq p \in \Pi$ . We construct a computation p = p = p = p. We construct a computation p = p = p = p and  $p \in p$  is grounded, as follows. Note that  $p \in p$  for each  $p \in p$ . For each  $p \in p$  for each  $p \in p$  for each  $p \in p$ . For each  $p \in p$  for each  $p \in p$ . For each  $p \in p$  for each  $p \in p$ 

- 1. p-Acts, (which only adds the block  $b_i$  to  $x_p^i$ ), let  $y^i \to y^{i+1}$  be the corresponding P3 p-Acts transition (which adds messages to all  $q \neq p \in \Pi$ , each with a cordial package of that contains  $b_i$ ). Note that since  $\beta(y_p^i) = x_p^i$  the transition is enabled, and also that  $\beta(y_p^{i+1}) = x_p^{i+1}$ .
- 2. p-Delivers, which delivers b from q, let  $y^i \to y^{i+1}$  be a corresponding P3 p-Delivers transition that delivers a message from q with b. Such a transition is enabled since the message was sent by q when taking the p-Acts transition. Note that since  $\beta(y_p^i) = x_p^i$  then  $\beta(y_p^{i+1}) = x_p^{i+1}$ .

By construction  $x_p := \beta(y_p)$  for every  $p \in \Pi$ , and similarly for x' and y'. Hence  $x = \sigma_4(y)$ ,  $x' = \sigma_4(y')$ .

This completes the proof of Theorem 4.

Next, we extend P4B to achieve message dissemination among correct agents in the face of faulty agents that delete their local state, in effect preventing delivery of as-yet undelivered messages. To do so, when an agent p creates a block, it sends it to every agent q with a so-called *cordial package* of all blocks known to p and not known to q, to the best of p's knowledge.

**Definition 32** (P4D). The transition system instance P4D= (C4D, c0, T4D) of P4 is defined by C4D = C4B, and T4B having all p-transitions  $c \to c' \in T4_p$  for each  $p \in \Pi$ , that satisfy  $c'_p = c_p \cdot M$ , where M is:

- 1. p-Acts: A sequence of p-messages (q, p, b') for every  $b' \in [b] \setminus B'$  and every  $q \neq p \in \Pi$ , where  $b \notin \beta(c_p)$  is a p-block such that  $[b] = c_p \cup \{b\}$ , and B' = [b''] if there is a q-block  $b'' \in c_p$  such that  $b \succ b''$ , else  $B' = \emptyset$ .
- 2. **p-Delivers**:  $M = \{(p, q, b)\} \subseteq c_q \setminus c_p \text{ and } [b] \subseteq \beta(c'_p).$

As the p-Acts in P4B and P4D affect  $\beta$  in the same way (the additional blocks sent by p in P4D are already known to p),  $\sigma_4$  and Theorem 4 apply to P4D. However, due to the cordial package sent by p-Acts, it is also resilient to agents that delete their state, in the sense of Observation 17, with the same notion of faulty transitions F, with which agents may delete some or all of their local state.

**Observation 17.** In a P4D run where incorrect miners take only  $F \cup T4D$  transitions, any block known to a correct miner will eventually be known to every correct miner.

Proof. If there is a block  $b \in \beta(c_q) \setminus \beta(c_p)$  in some configuration c of a computation of P4D, where p and q are correct in r, then eventually p will make a p-Acts transition resulting in configuration c', and if by that time it has not delivered a message with a q-block b' such that  $b \in [b']$ , indicating that q already knows b, then it will add a message (q, p, b) to its local state  $c'_p$ . Then p-Delivers of the message (q, p, b) is enabled in r from c' onwards, and therefore will eventually be taken.

## 5 Conclusions

Multiagent transition systems provide a powerful tool for proving the correctness of implementations among transition systems, if the transitions systems are monotonically-closed wrt a partial order, and if the implementation is order-preserving wrt the partial orders of the implementation and the specification. As shown by the examples presented as well as by forthcoming applications of this framework, this is often the case in multiagent transition systems specifying distributed protocols.

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