SOCIAL NETWORK ANALYTICS

Strategic Networks

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Strategic Networks Formation

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Games on Networks

- The interaction of individuals who are connected via a network and whose behaviors are influenced by those around them.
  Such interactions are natural ones to model using game theory, as the payoffs that an individual receives from various choices depends on the behaviors of his or her neighbors.

- A given player's payoffs depends on other players' actions, but only on those to whom the player is linked in the network.

Games on Networks

- **Individuals are located on nodes of a network.**
  They choose actions and their rewards depend on these actions along with the actions of others on the network.
- **The key point:**
  - **The effect of player 1's action on player 2's payoff depends on where the two players are located in a network.**
- **Examples:**
  1. **Value of learning a language** depends on how many friends and colleagues learn the same language.
  2. **Value of acquiring information on market prices** depend on how much information friends acquire.
  3. **Firms collaborate but also compete in markets**.

Networks in Economics

- **Social and economic networks**: nodes are individuals or firms, links are relationships.
- **Many activities shaped by networks**
  - exchange of information and opinions
  - trade of goods and services
  - job opportunities
  - friendships, business partnerships, political/trade alliances
  - risk-sharing, favors, cooperation
  - innovation, technology adoption
  - credit and financial flows
  - peer effects: education, crime, voting

Networks in Economics: Questions

1. How do networks form?
2. Which network structures are likely to emerge?
3. What is the impact of network architecture on economic outcomes?
4. **How does an individual's position in the network affect his actions and welfare?**
5. **Which networks are socially optimal?**

Players and Networks

- **Consider a finite set of players N = {1, . . . , n} who are connected in a network.**
- **A network (or graph) is a pair (N, g), where g is a network on the set of nodes N.** These represent the interaction structure in the game, indicating the other players whose actions impact a given player's payoff.
- Let g denote the two standard ways in which networks are represented: by their **adjacency matrices** as well as by **listing the pairs of nodes that are connected**.
- The network g will sometimes be an $n \times n$ adjacency matrix, with entry g_{ij} denoting whether i is linked to j and can also include the intensity of that relationship.
- At other times the network g denotes the set of all relationships that are present, and so we use notation $ij \in g$ to indicate that i is linked to j.

Players and Networks

- A network
  - is undirected if g is required to be symmetric so that relationships are necessarily reciprocal and $g_{ij} = g_{ji}$ for all i and j and
  - is directed if relationships can be unidirectional.
- **A relationship between two nodes i and j, represented by ij ∈ g, is referred to as a link.**
- **Shorthand notations for the network obtained by**
  - **adding a link ij to an existing network g is g + ij.**
  - **deleting a link ij from an existing network g is g − ij.**
- A given player's payoff depends on other players' actions, but only on those to whom the player is linked in the network.

Key features of linking activity

- **Linking is a decision: Individuals choose on forming links.**
  - **Examples:** Scientists decide on whether or not to collaborate,
    Firms can choose whether or not to form an alliance;
    I can decide whether or not to form a hyperlink with your homepage.

- **Externality:** A link between A and B affects the payoffs of C as well as the rewards for C from linking with A and B.
  - **Examples:** Firm A and Firm B collaboration affects firm C in same market.

Players and Networks

➢ Key issues in modelling:

1. **Payoffs**: linking generates payoffs; how to allocate the payoffs.
2. **Decision power**: who decides on the link, one person, two persons, all players etc.
3. **Information**: what do I know -- about other players and about the network -- when I form a link?

Strategic Network Formation

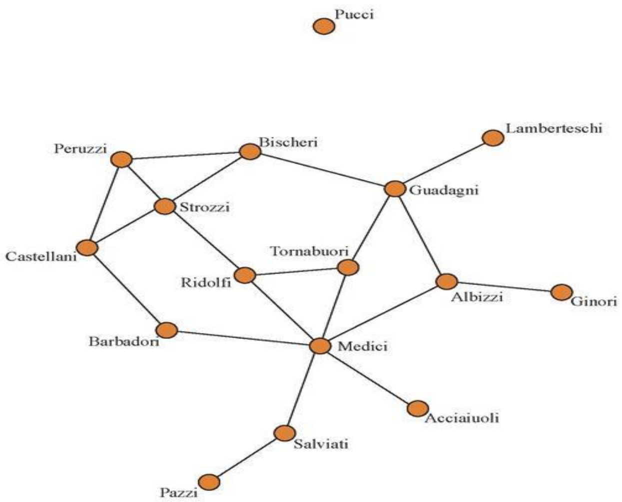
- **Strategic Network Formation** defines how and why networks take particular forms.

- In many networks, **the relation between nodes is determined by the choice of the participating players involved**, not by an arbitrary rule.

- **A "strategic" modeling of network** requires **defining a network's costs and benefits and predicts how individual preferences become outcomes.**

Strategic Network Formation

- **A strategic network formation requires that individuals create relations that are beneficial and drop those that are not.**
- One of the most well-known examples in this context is **the marriage network of sixteen families in Florence**, which **showed how the Medici family gained power and took control of Florence by creating a high number of inter-marriages with the other families.**

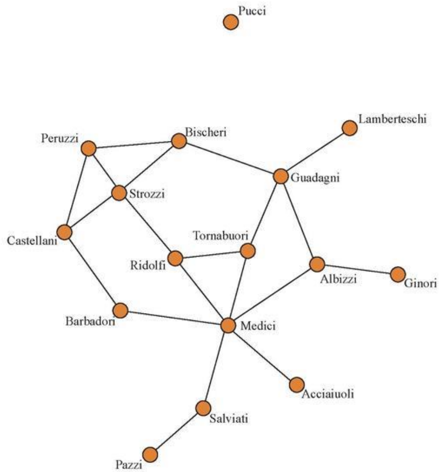


Florence

Strategic Network Formation

- **A strategic network formation requires that individuals create relations that are beneficial and drop those that are not.**
- **“The decisions about profitable relations are not a situation of choice, but a situation of strategic interaction** – an aspect that is best covered by Game Theory”

Florence



Strategic Network Formation

- In these kinds of settings, the nodes are usually called players, where **N={1, 2,…,n} is a set of players that have formed links in a network**.
- **Social Networks** have diverse settings, however the simplest ones can be described by an **undirected graph** whereas more complicated situations are represented by **directed graphs**.
- There are fundamental differences in the way these games are modeled depending on their graph structure.
  If a link exists between player i and player j it is noted as ij.
  In cases of undirected networks, ij is considered equal to ji.
  A network g represents a list of all the links between players.

Strategic Network Formation

- **A network g represents a list of all the links between players.**
- In a more formal setting, **a network g is defined as a set of unordered pairs {i,j},** with i,j element of N . The set of all possible graphs on the set of players N is denoted with G.
- **The benefits that they receive from the network are represented by utility functions.**
  That is, **the payoff to a player i** is represented by a function $\mathbf{u_i: G(N) \rightarrow R}$, where **$u_i$(g) represents the net benefit that i receives if network *g* is in place.**
- **To model strategic network formation the notion of network games is used.**
- **A network game is a set of linked players and their utility functions.**

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Modeling Network Formation

- Network games can be modeled in different ways.
- Some of **the modeling methods that separate the utility allocation from the network formation process** are

1. **Extensive form games,**
2. **Simultaneous move games,**
3. **Pairwise stability concept etc.**

Modeling Network Formation

1. Extensive form game modeling

- If a network is modeled according to the extensive form game concept then the players of the network first propose to create links one after the other and afterwards, they make decisions to create a link or not.
- In such settings, a couple of players decide to either form a link or not by being aware of all the previous players' decisions and by making predictions for the decisions of the following players.

➢ A complete extensive-form representation specifies:

1. the players of a game
2. for every player every opportunity they have to move
3. what each player can do at each of their moves
4. what each player knows for every move
5. the payoffs received by every player for every possible combination of moves

Modeling Network Formation

2. Simultaneous move game modeling

- In simultaneous move game settings, all the players declare at the same time to whom they want to link.
  Even though these sorts of games are easy to understand and analyze, their drawback is that they have multiple Nash Equilibria.

Modeling Network Formation

- [Strategic Network Formation – Matthew O Jackson]

References

1. “Social and Economic Networks”, Mathew O Jackson, Princeton University Press, 2010.

2. https://en.wikipedia.org/wiki/Strategic_Network_Formation

3. Games on Networks - Matthew O. Jackson, Yves Zenou

SOCIAL NETWORK ANALYTICS

Pairwise Stability

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Pairwise Stability

- In social networks, a link between two players is formed only if both of them decide to do so, however either of them can make the decision to delete a link without the other player's approval.
  - Example: Facebook Friendship network

- The concept of Nash equilibrium has a drawback in this case since it does not take into consideration the fact that the players can discuss their decisions.
  To model such a situation a stability concept that takes this fact into account is required.
  A useful stability concept in this case is Pairwise Stability, which accounts for the mutual approval of both players.

Pairwise Stability

- **A network g is pairwise stable if**

(i) for all $ij \in g$, $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$, and

(ii) for all $ij \notin g$, if $u_i(g + ij) > u_i(g)$ then $u_j(g + ij) < u_j(g)$.

Note:

- **g + ij** → The network obtained by **adding a link ij to an existing network g.**
- **g - ij** → The network obtained by **deleting a link ij from an existing network g**.
- **u_i(g)** → Utility of player i with network g.
- **u_i(g-ij)** → Utility of player i with the network obtained by deleting a link ij from an existing network g.
- **u_i(g+ij)** → Utility of player i with the network obtained by adding a link ij to an existing network g.

Pairwise Stability

- **A network g is pairwise stable if**

  (i) for all $ij \in g$, $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$, and

  (ii) for all $ij \notin g$, if $u_i(g + ij) > u_i(g)$ then $u_j(g + ij) < u_j(g)$.

- **A network is pairwise stable if no player wants to sever/remove a link and no two players both want to add a link.**

- **The first part of the definition can be stated as.**
  - The requirement that no player wishes to delete a link that he or she is involved in implies that a player has the discretion to unilaterally terminate relationships that he or she is involved in.

- **Definition (i): no player wants to delete a link unilaterally**
- **Definition (i): no player gains from severing a link – Players should maintain beneficial relationships.**

Pairwise Stability

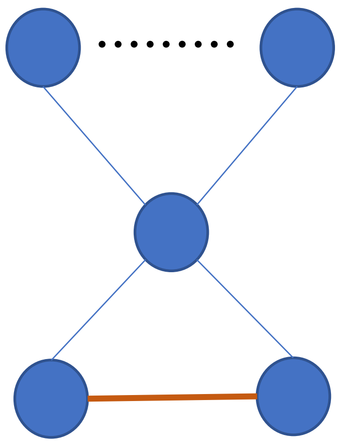
- **A network g is pairwise stable if**

  (i) for all $ij \in g$, $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$, and

  (ii) for all $ij \notin g$, if $u_i(g + ij) > u_i(g)$ then $u_j(g + ij) < u_j(g)$.

- **The second part of the definition can be stated in various ways.**
  - **In order for a network to be pairwise stable, it is required that if some link is not in the network and one of the involved players would benefit from adding it, then the other player would suffer from the addition of the link.**
  - Another way to state this is that **if a network g is such that the creation of some link would benefit both players involved** (with at least one of them strictly benefitting), **then g is not stable.**

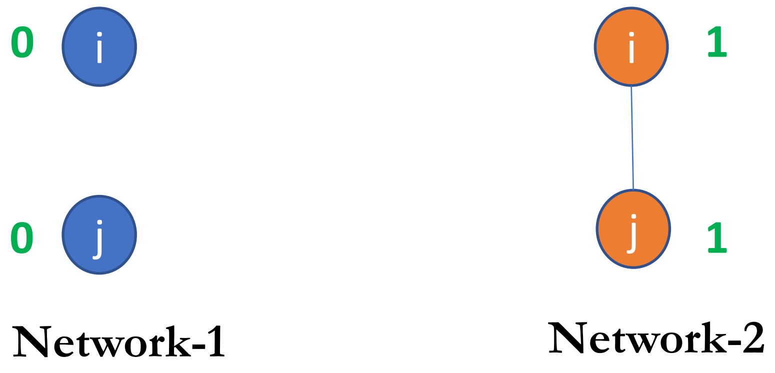
- **Definition(ii): No two agents both gain from adding a link.**
- **Definition(ii): No pair of unlinked players finds it mutually beneficial to form a new link.**



Network g

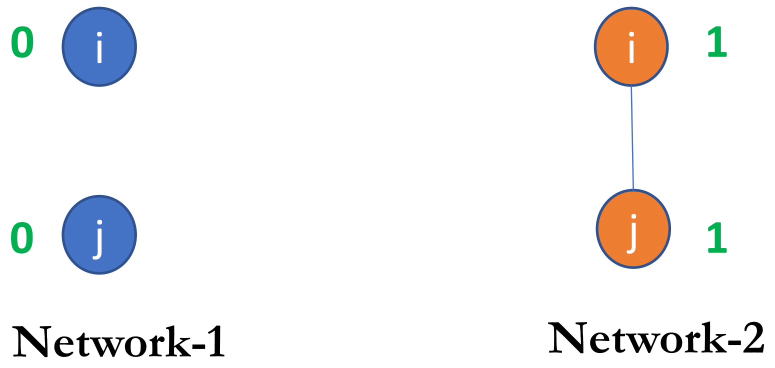
Pairwise Stability

Example 1:



Pairwise Stability

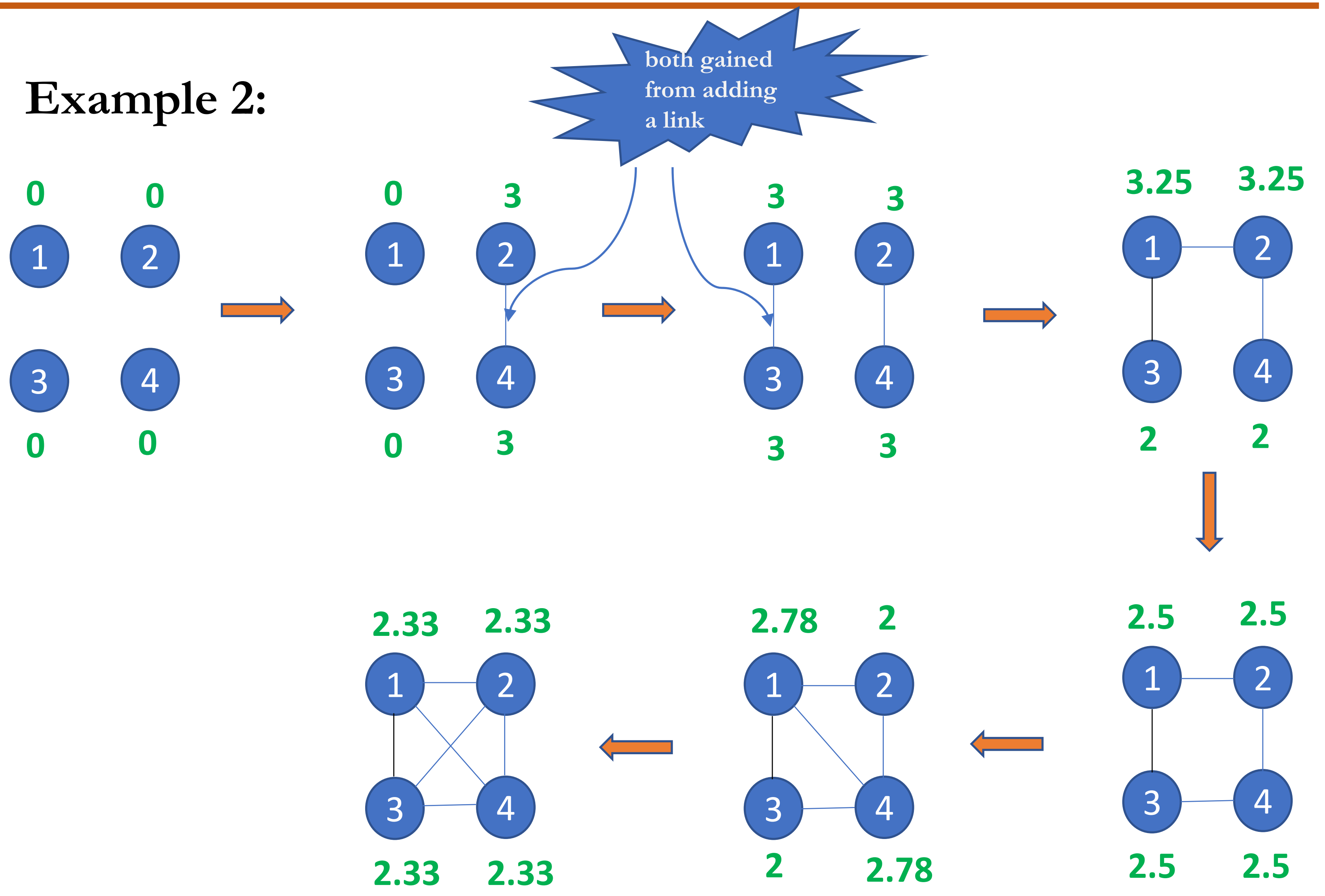
Example 1:



- Only the dyad is pairwise stable.

Pairwise Stability

Example 2:

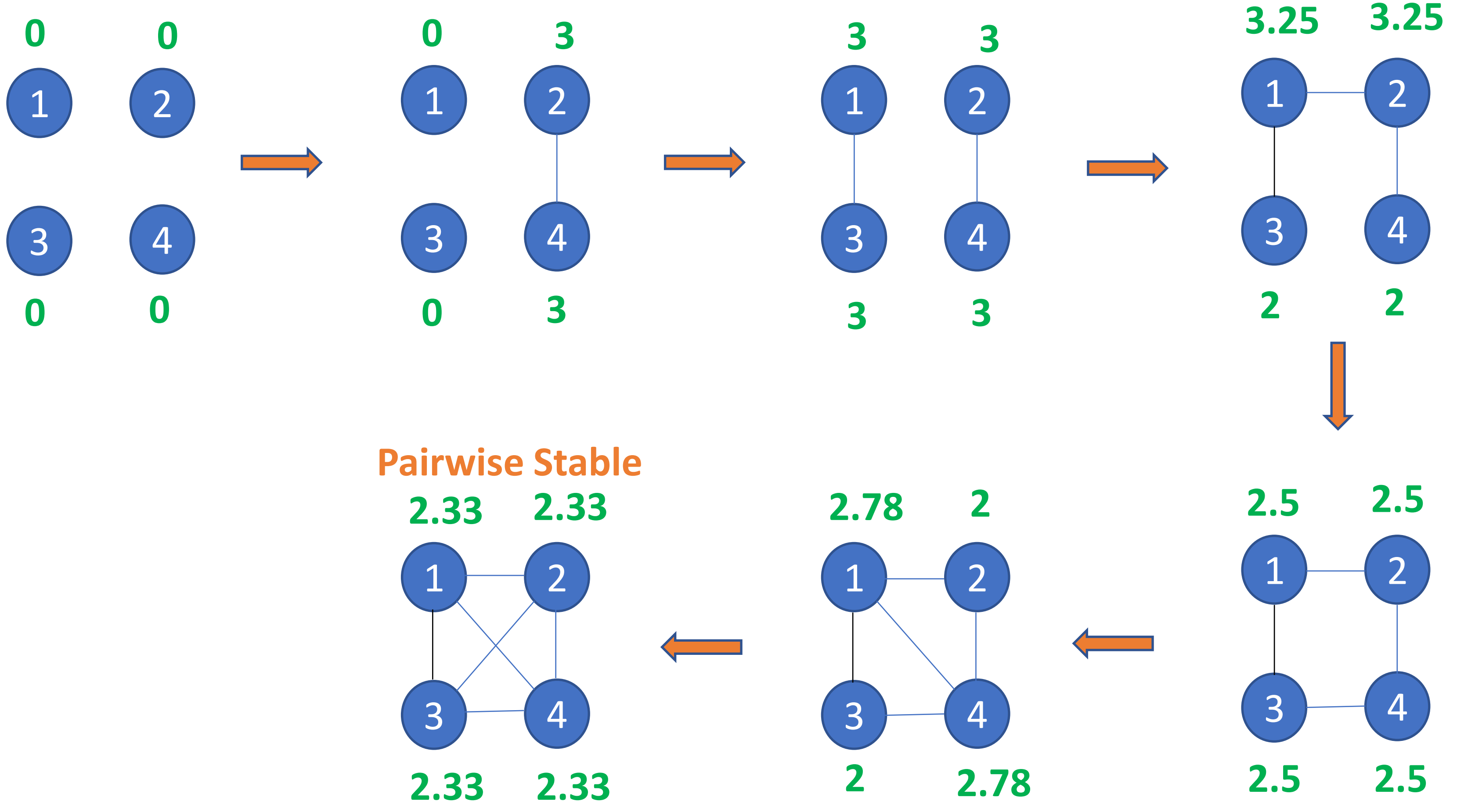


Pairwise Stable:
No player gains from severing/adding a link

Pairwise Stability

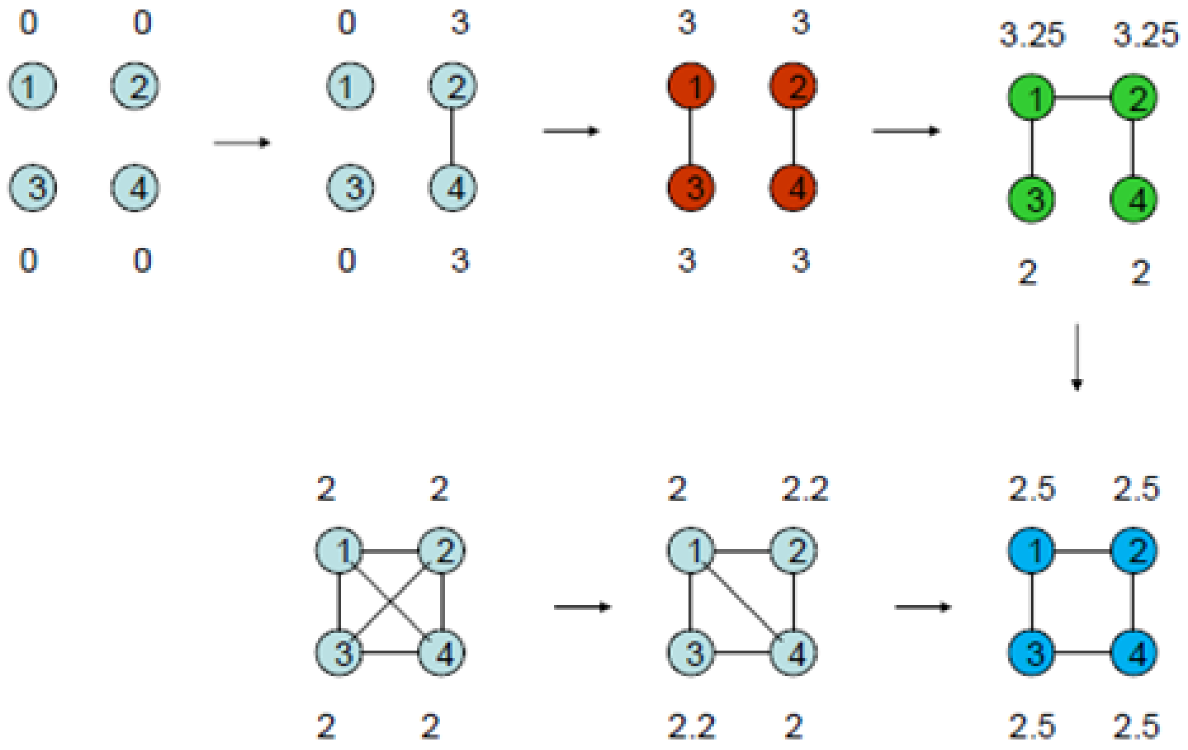
Example 2:

If an arrow is pointing away from a network, it means that the network is not stable, since there would be benefit from deleting a link from a player or by creating a new link from two players of the network.



Pairwise Stable:
No player gains from severing/adding a link

Example 3:



Pairwise Stable:
No player gains from severing/adding a link

Fig: An example of Pairwise Stable Network in a Four Person Society

Pairwise Stability

Example 3:

If an arrow is pointing away from a network, it means that the network is not stable, since there would be benefit from deleting a link from a player or by creating a new link from two players of the network.

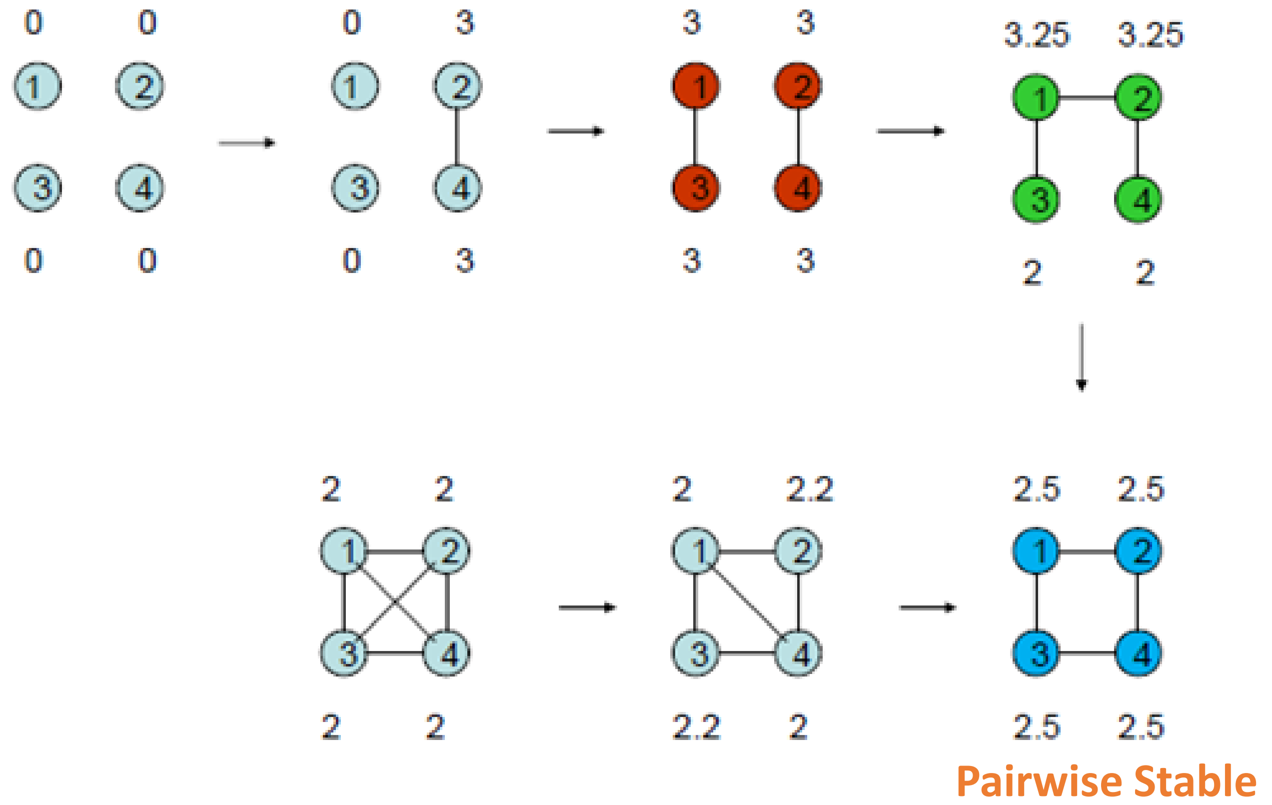


Fig: An example of Pairwise Stable Network in a Four Person Society

Pairwise Stable:
No player gains from severing/adding a link

How “Stable” are Pairwise Stable Networks?

- Pairwise stability is a necessary, but not sufficient, condition for a network to be stable over time.
  1. deviations by only one or two players
  2. changing a single link at a time
  3. myopic incentives; no foresight, no dynamics

Limitations of Pairwise Stability

- Pairwise stability is a weak notion in that it only considers deviations on a single link at a time.
- For instance, it could be that a player would not benefit from severing any single link but would benefit from severing several links simultaneously, and yet the network could still be pairwise stable.
- Second, pairwise stability considers only deviations by at most a pair of players at a time. It might be that some group of players could all be made better off by some more complicated reorganization of their links, which is not accounted for under pairwise stability.

Pairwise Stability

- Pairwise Stability – Matthew O Jackson

References

1. “Social and Economic Networks”, Mathew O Jackson, Princeton University Press, 2010.

2. https://en.wikipedia.org/wiki/Strategic_Network_Formation

SOCIAL NETWORK ANALYTICS

Efficient Networks, Pareto Efficient Networks and Externalities

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Efficient Network:

- **An efficient network is one in which the overall benefits that society sees from a given network is very high.**
  Overall societal benefits play an important role in determining the efficiency of networks.

Best Network:

- **The best network is the one which maximizes the total utility of the society**. This notion was referred to as strong efficiency by Jackson and Wolinsky.

Efficient Networks

- There is a difference between
  - **networks that maximize the social welfare** and
  - **networks that are based on personal incentives**.

- In strategic network formation it is important
  - to look at **the overall social benefit** and
  - to see if networks that players create manage to be efficient for the society in general.

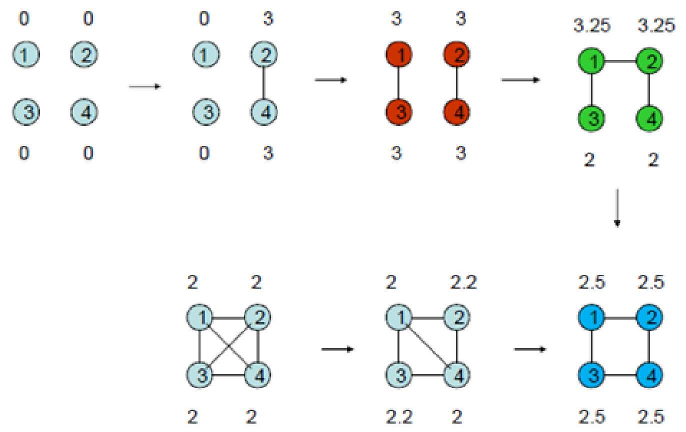
Efficient Networks

- A network g is efficient relative to a profile of utility functions $(u_1, \ldots, u_n)$ if

$$\sum_i u_i(g) \geq \sum_i u_i(g') \text{ for all } g' \in G(N).$$

- It is clear that there will always exist at least one efficient network, given that there are only finitely many networks.

The network in red is Efficient, since all the other link combinations offer lower payoffs to some of the players.



- (3+3+3+3) ≥ 0 i.e., 12 ≥ 0
- (3+3+3+3) ≥ (3+3) i.e., 12 ≥ 6
- (3+3+3+3) ≥ (3.25+3.25+2+2) i.e., 12 ≥ 10.5
- (3+3+3+3) ≥ (2.5+2.5+2.5+2.5) i.e., 12 ≥ 10
- (3+3+3+3) ≥ (2+2.2+2.2+2) i.e., 12 ≥ 8.4
- (3+3+3+3) ≥ (2+2+2+2) i.e., 12 ≥ 8

Efficient Networks: Pareto Efficiency

Pareto Efficiency

- **Pareto Efficiency** is another efficiency concept used by economists to study the overall social welfare.

“A network g is Pareto efficient relative to (u_1,u_2,….u_n) if there does not exist any $g' \in G$ such that $u_i(g') \geq u_i(g)$ for all i with strict inequality for some i”

- The Pareto efficiency notion is more reasonable in settings in which allocation rules are fixed.
- A network can Pareto dominate another network if it has strictly larger benefits for one individual and weakly larger benefits for all individuals.
- **If there exists a network which is not Pareto dominated by another network then it is a Pareto efficient network.**

Efficient Networks: Pareto Efficiency

Example:

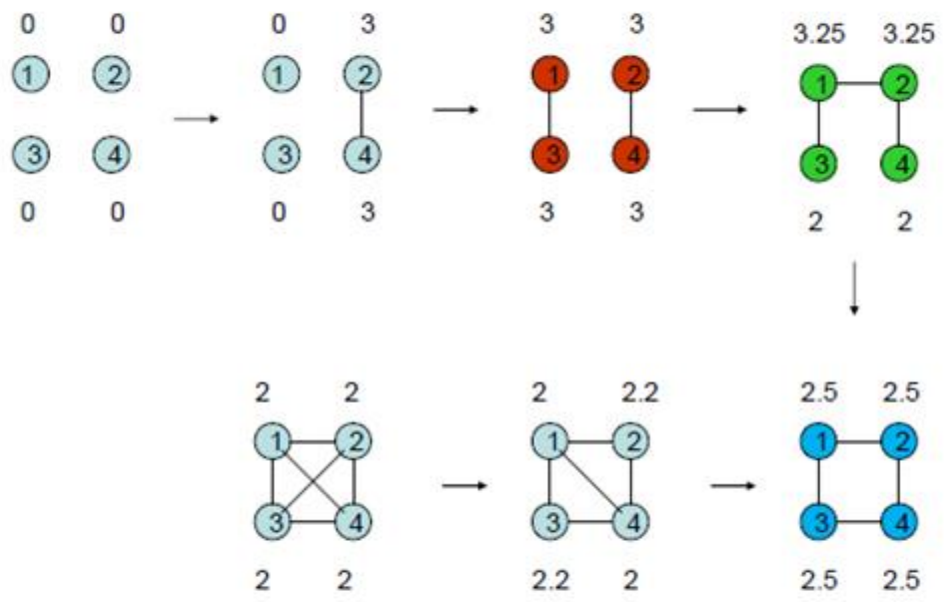
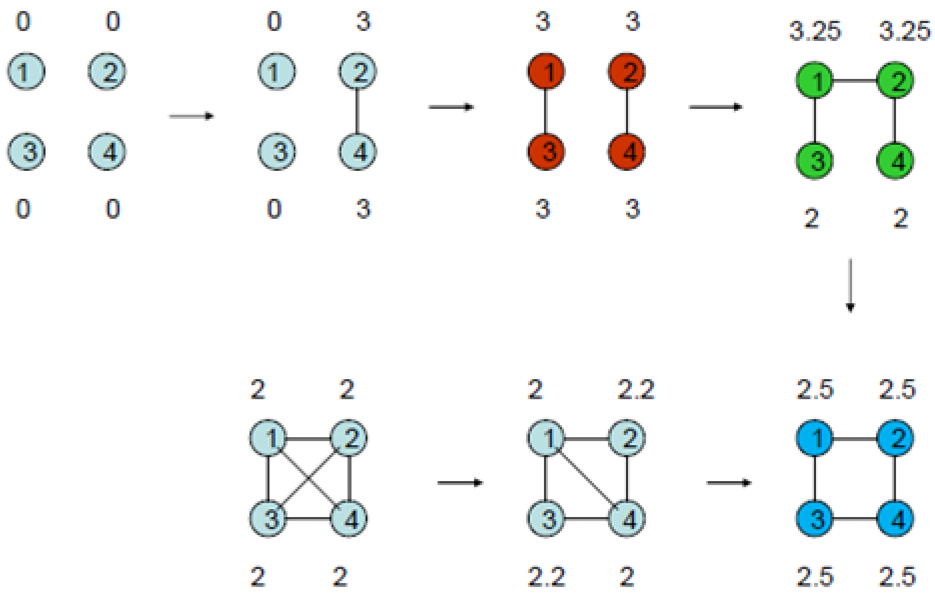


Figure 6.2.2. An Example of Efficient, Pareto Efficient, and Pairwise Stable Networks in a Four Person Society

Pareto efficient network: A network that is not Pareto dominated by any other network.

In the figure "An Example of Effcient, Pareto Effcient, and Pairwise Stable Networks in a Four Person Society" an example with four players is given, where the payoffs of the players are noted by the numbers next to the nodes.

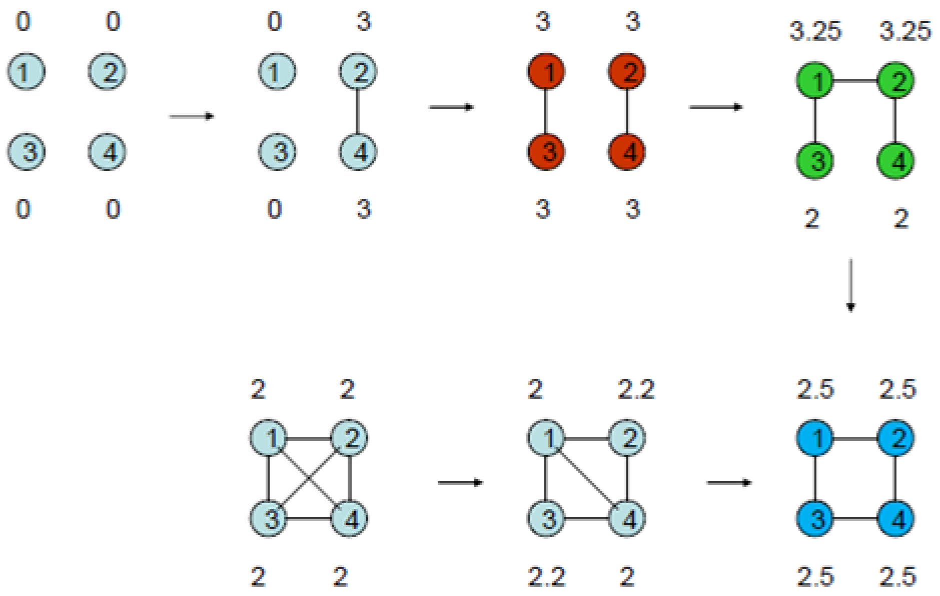
Efficient Networks: Pareto Efficiency



Pareto efficient network: A network that is not Pareto dominated by any other network.

➢ **If an arrow is pointing away from a network, it means that the network is not stable**, since **there would be benefit from deleting a link from a player or by creating a new link from two players of the network**.

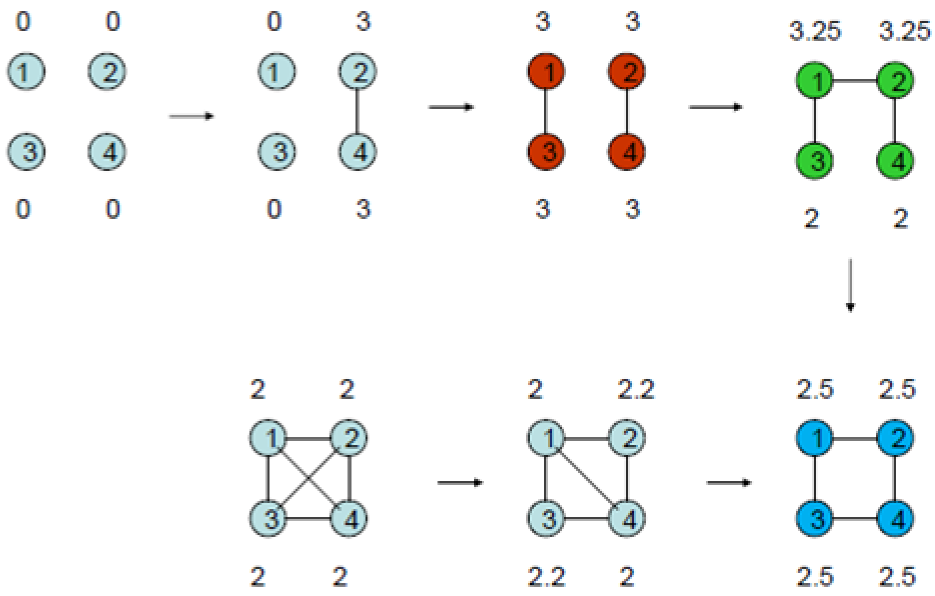
Efficient Networks: Pareto Efficiency



Pareto efficient network:
A network that is not Pareto dominated by any other network.

- **The network in red is Efficient and Pareto efficient**, since all the other link combinations offer lower payoffs to some of the players (and no network is pareto dominating this).
- **The network in red is Pareto efficient -** No network gives all players a higher payoff

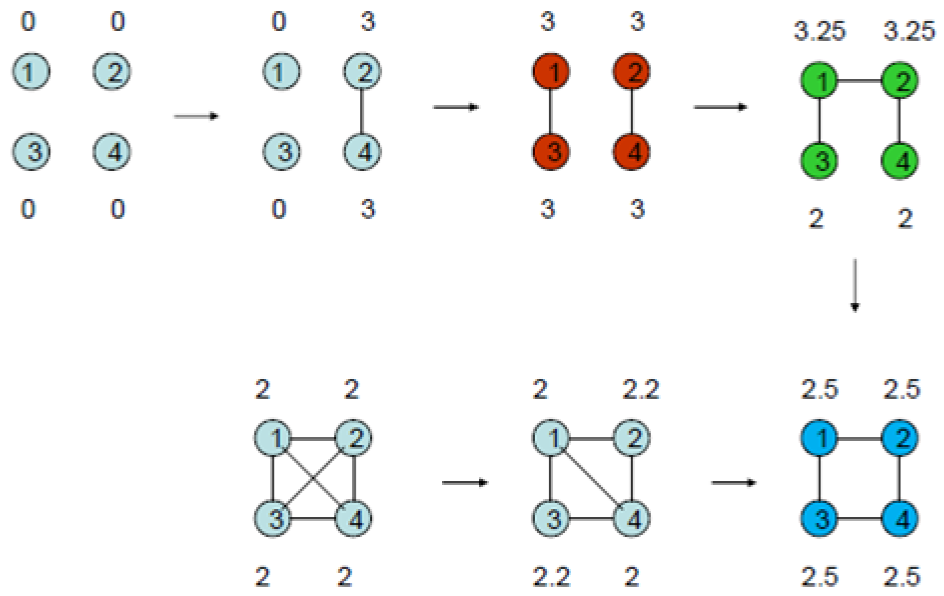
Efficient Networks: Pareto Efficiency



Pareto efficient network:
A network that is not Pareto dominated by any other network.

- **The network in green is Pareto efficient since the payoffs are higher (and not pareto dominated by any another network)** but it is not Pairwise Stable because the players that have created only one link would also benefit by adding links to one another.
- **The network in green is Pareto efficient -** No network gives player-1 and player-2 a higher payoff

Efficient Networks: Pareto Efficiency



Pairwise Stable:
No player gains from severing/adding a link

- **The only Pairwise Stable network in the figure is the dark blue colored one** since none of the players involved would benefit by deleting or creating a link.

Efficient Networks: Pareto Efficiency

Example:

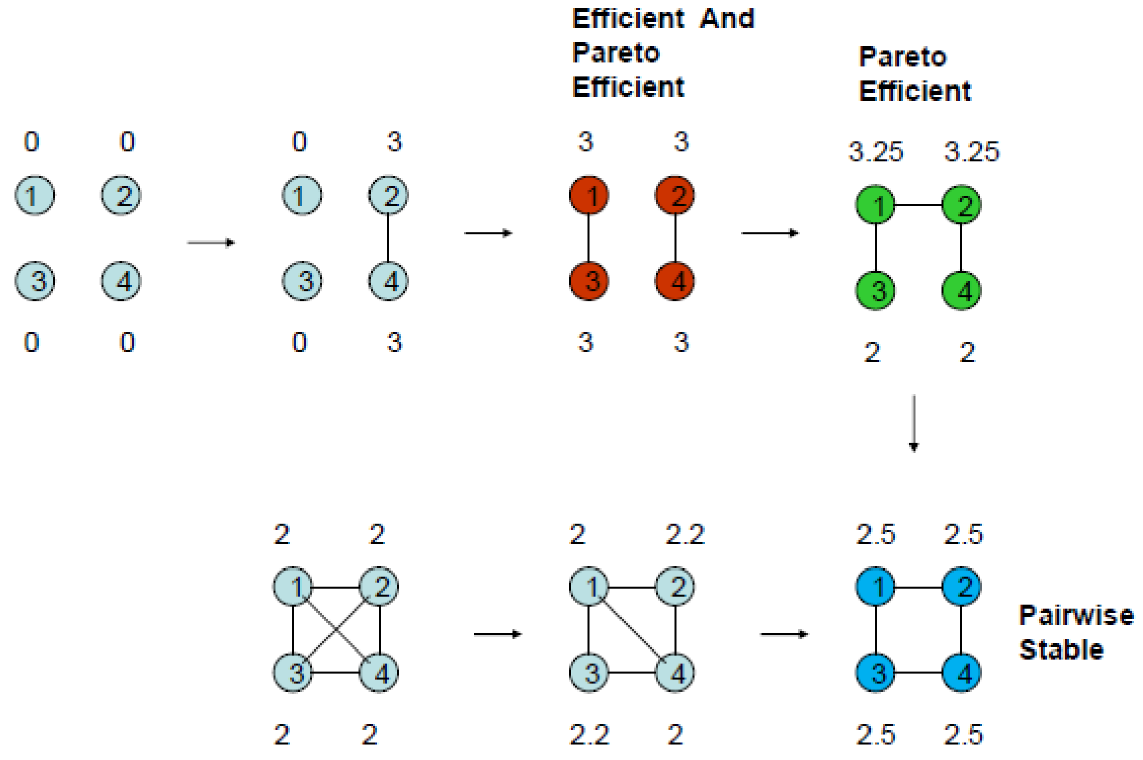


Figure 6.2.2. An Example of Efficient, Pareto Efficient, and Pairwise Stable Networks in a Four Person Society

Distance-Based Utility

- **The utility the players receive does not just come from the direct links that they form with each other, but also from their indirect relations.**
- **The benefit function measures the indirect benefit that players obtain from being close to other players in the network.**
- The distance-based utility assumes that all players' utility functions are alike and it takes into account only the benefits from indirect links that depend on minimum path length. These two features are considered as drawbacks of the distance-based utility.

Externalities

- **Externalities show that players' benefits depend heavily on other players' commitment decisions**.
- The distance-based utility showed that the payoffs of players do not just depend on the direct links that they form, but also on the links that other players have created in the network.

Externalities

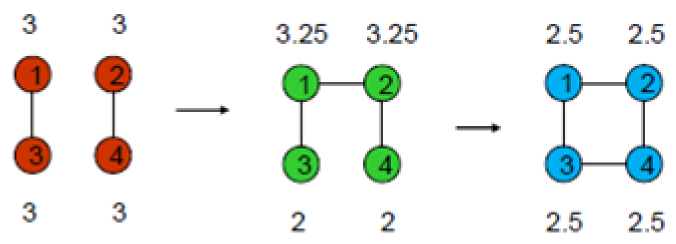
➢ Players may confront positive or negative externalities in networks.

1. Positive externality:

$$u_k(g+ij) \geq u_k(g) \quad \text{if } ij \text{ not in } g \text{ for every } k \neq i,j$$

2. Negative externality:

$$u_k(g+ij) \leq u_k(g) \quad \text{if } ij \text{ not in } g \text{ for every } k \neq i,j$$



Negative externality

➢ The distance-based utility model is an example of positive externalities, since players can only get more benefits when other players increase their number of connections.

➢ On the other hand, a model that faces players with negative externalities is the so-called "Co-Author model" presented by Jackson and Wolinsky in the paper of 1996.

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Externalities

Co-Author model

- Given that working on a research paper requires time and devotion, two researchers can benefit more if are only working with each other at a given period of time and not with many other people.
  Therefore, in the "Co-Author model" researchers benefit more if their other colleagues have fewer links.
- In this model, **if a player's neighbors have many links**, it will bring negative externalities to them.
- In different models, positive or negative externalities lead to inefficiency.

References

1. “Social and Economic Networks”, Mathew O Jackson, Princeton University Press, 2010.

2. https://en.wikipedia.org/wiki/Strategic_Network_Formation

SOCIAL NETWORK ANALYTICS

Pairwise Nash Stability

An Extensive Form Game of Network Formation,
A Simultaneous Link-Announcement Game and Nash Stable

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An Extensive Form Game of Network Formation

- **Players move sequentially and propose links which are then accepted or rejected.**

- The extensive form game is based on an ordering over all possible links, denoted $(i_1j_1\ldots\ldots i_Kj_K)$.
- When the link i_kj_k appears in the ordering, the pair of players i_kj_k decide on whether or not to form that link, knowing the decisions of all pairs coming before them and forecasting the play that will follow them.
- Player i_k moves first and says yes or no and then player j_k says yes or no and the link forms if both say yes.
- A decision to form a link is binding and cannot be undone.

An Extensive Form Game of Network Formation

- If a pair $i_k j_k$ decide not to form a link, but some other pair coming after them forms a link, then $i_k j_k$ are later allowed to reconsider their decision.
  This feature allows a player 1 to make a threat to 2 of the form "I will not form a link with 3 if you do not. But if you do form a link with 3, then I will also do so".
- The way in which this is captured is that the game moves through all the links a first time.
  If at least one link forms, then the game starts again with the same ordering, moving this time only through the links that have not yet been formed.
- **The game continues to move through the remaining unformed links in order, until either all links are formed or there is a round such that all of the links that have not yet formed have been considered and no new links have formed.**

Pairwise Nash Stability

An Extensive Form Game of Network Formation

- Its main drawback are
  - the game can be very difficult to solve, even in very simple settings with only a few players.
  - the ordering of links can have a substantial impact on which networks emerge, and it is not so clear what a natural ordering is.

A Simultaneous Link-Announcement Game

- **Each player simultaneously announce their preferred set of players** (i.e., $S_1=\{2,4,7\}$ $S_2=\{1,4,8\}$) **with whom he or she wishes to be linked.**
- **The links that are formed are those such that both of the players involved in the link named each other.**
- The strategy space of player i is S_i.
- If $\mathbf{S} \in S_1 \times . \times S_n$ is **the profile of strategies played**, then link ij forms if and only if both $j \in S_i$ and $i \in S_j$. The network that forms is

$$g(S) = \{ij \mid i \in S_j \text{ and } j \in S_i\}$$

- In modeling the networks that emerge from the link announcement game, we can use any of a variety of game theoretic solutions, such as Nash equilibrium.

A Simultaneous Link-Announcement Game

- **A network g ∈ G(N) is Nash stable if it results from a pure strategy Nash equilibrium of the link-announcement game**, where player i's payoff as a function of the profile of strategies is $u_i(g(S))$.

Figure 5. Both Networks are Nash Equilibria of the Link Announcement Game

A Simultaneous Link-Announcement Game

Figure 5. Both Networks are Nash Equilibria of the Link Announcement Game

- The main drawback of the game that it has too many Nash equilibria.
- In particular, **$S_i=\emptyset$; for all i is always a Nash equilibrium**, regardless of the payoffs. **Each player refuses to link with any other player, because he or she correctly forecasts that the other players will do the same** (i.e., Coordination failure - nobody manages to name anybody else).
  This is seen most starkly in the dyadic case, as pictured in Figure 5. Here both networks are equilibria, although clearly the network where the link forms is the only reasonable one.

A Simultaneous Link-Announcement Game

- The link formation game may at first seem to be a natural way to model network formation, **it is not reasonable when using Nash equilibrium alone as a solution concept.**

- Basically, **Nash equilibrium allows players to refuse to form links** and thus, effectively to delete links, **Nash equilibrium does not capture the fact that it may be mutually advantageous for two players to form a new relationship**. We need to move beyond Nash equilibrium to capture this.

A Simultaneous Link-Announcement Game

- In the example pictured in Figure 6, it is **a dominant strategy for each player to propose to link with the other player.**

- Consider a triad such that
  - the empty network leads to a payoff of 0 for all players,
  - a single link leads to a payoff of 1 for each of the linked players (and 0 for the other),
  - a two-link network leads to a payoff of -1 for all players, and
  - the complete network leads to a payoff of 1 for all players.

  This pictured in Figure 6.

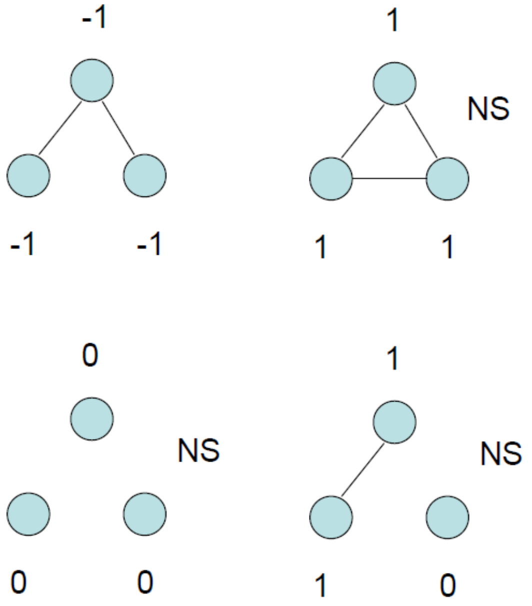


Figure 6.

Pairwise Nash Stability

A Simultaneous Link-Announcement Game

- In this example, all strategies in the link-announcement game are undominated. This means that **the empty network is an outcome of a Nash equilibrium that only uses undominated strategies, where every player announces the empty set of players.**

- **In order to address the fact that it takes the consent of both players to form a link in an undirected network, one has to explicitly consider coordinated actions on the part of pairs of players.** This forces one to move beyond Nash equilibrium, and standard refinements of it, and **somehow coalitional considerations (at least for pairs of players) have to be considered**. That is the reasoning behind **pairwise stability**.

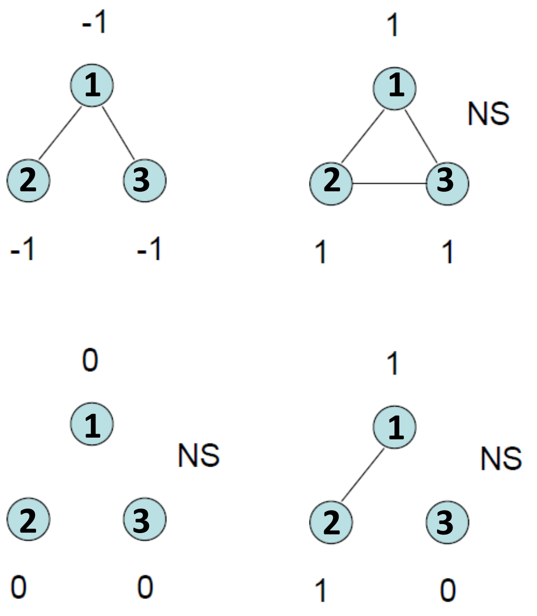


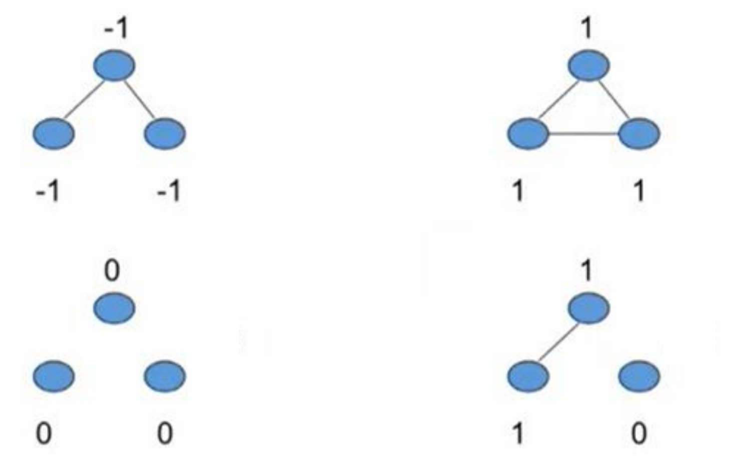
Figure 6.

Pairwise Nash Stability

Nash Stability

- **Nash stable:** $u_i(g(S)) \geq u_i(g(S'_i, S_{-i}))$ for all i S'_i

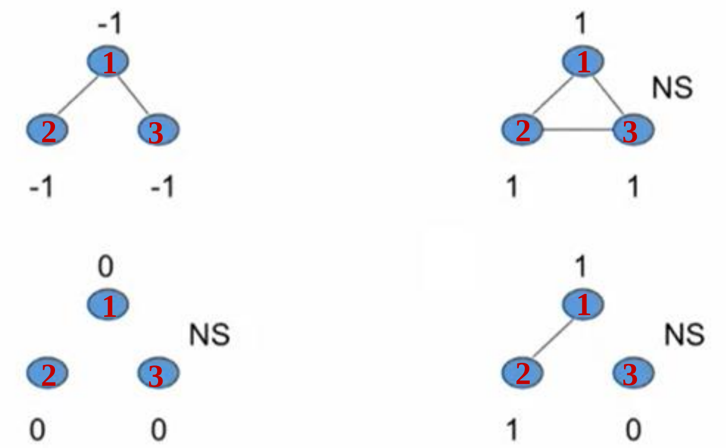
- **So, network g is nash stable if and only if no player wants to delete some set of his or her links.**



Note: If **S** $\in S_1 \times . \times S_n$ is **the profile of strategies played**, then link ij forms if and only if both $j \in S_i$ and $i \in S_j$. The network that forms is **$g(S) = \{ij \mid i \in S_j$ and $j \in S_i\}$**

Pairwise Nash Stability

Nash Stability



Nash Stable: Network g is nash stable if and only if no player wants to delete some set of his or her links.

Pairwise Stable: No player gains from severing/adding a link

Nash Stable:

Network with payoffs (-1,-1,-1) is not Nash Stable because palyer1 and player3 are better off by deleting a link (i.e., incentive to deviate).

Network with payoffs (1,1,0) is Nash Stable because no incentive to deviate for palyer1 and player 2 and player3 $S_3=\emptyset$

Network with payoffs (1,1,1) is Nash Stable because no incentive to deviate for players.

Network with payoffs (0,0,0) is Nash Stable, regardless of the payoffs. Each player refuses to link with any other player, because he or she correctly forecasts that the other players will do the same (i.e., Coordination failure - nobody manages to name anybody else).

Pairwise Stable:

Network with payoffs (1,1,0) is Pairwise Stable because **no gain by creating a new link from two players of the network**.

Network with payoffs (1,1,1) is Pairwise stable because **no gain by removing a link between two players of the network**.

Pairwise Nash Stability

- **Pairwise stability overcomes the difficulties inherent in examining Nash equilibria of the link-announcement game, it restricts attention to changes of one link at a time.** This can lead to over-connected networks being pairwise stable, even when some player would bene.t from deleting multiple links at once, as is pictured in the example in Figure 7.

- In the example in Figure 7, **the reasonable network is the one that is both Nash stable and pairwise stable.** This has led to a concept of pairwise Nash stable networks.

- **A network is pairwise Nash stable if it is Nash stable and pairwise stable.**

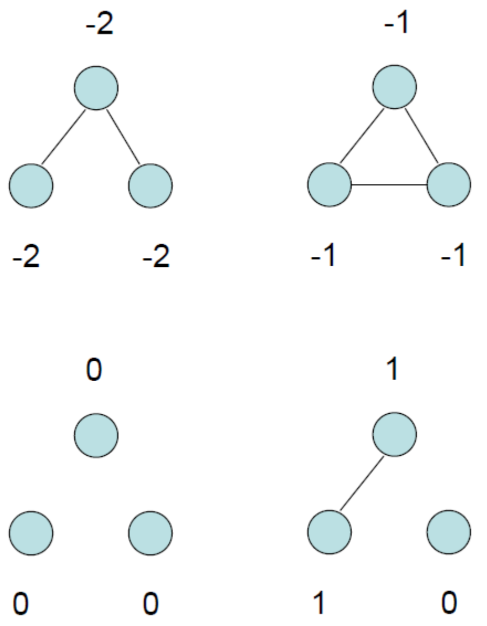
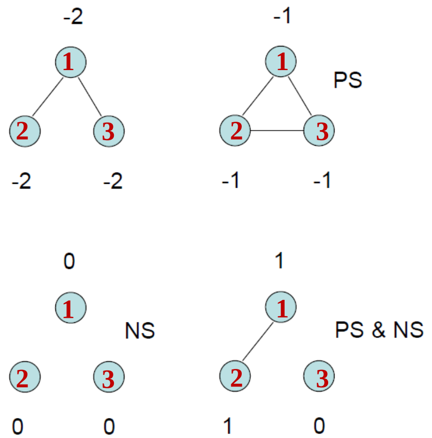


Figure 7.

Pairwise Nash Stability

- A network is pairwise Nash stable if it is Nash stable and pairwise stable.



Nash Stable: Network g is nash stable if and only if no player wants to delete some set of his or her links.

Pairwise Stable: No player gains from severing/adding a link

Nash Stable:

Network with payoffs (-2,-2,-2) is not Nash Stable because palyer1 and player3 are better off by deleting a link (i.e., incentive to deviate).

Network with payoffs (1,1,0) is Nash Stable because no incentive to deviate for palyer1 and player 2 and player3 $S_3=\emptyset$

Network with payoffs (-1,-1,-1) is not Nash Stable because **incentive to deviate for players by** deleting some set of his links.

Network with payoffs (0,0,0) is Nash Stable, regardless of the payoffs. Each player refuses to link with any other player, because he or she correctly forecasts that the other players will do the same (i.e., Coordination failure - nobody manages to name anybody else).

Pairwise Stable:

Network with payoffs (1,1,0) is Pairwise Stable because **no gain by creating/deleting a link between two players of the network**.

Network with payoffs (-1,-1,-1) is Pairwise stable because **no gain by removing a link between two players of the network**.

References

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Pareto Efficiency

- We say that one network Pareto dominates another if it leads to a weakly higher payoffs for all individuals, and a strictly higher payoffs for at least one.
- A network is then Pareto efficient if it is not Pareto dominated by any other network.
- Pareto domination indicates unanimity in the ordering between two networks, and

  thus is a quite compelling argument in favour of the dominating network compared to the dominated network, at least from a purely welfaristic perspective.
- The difficulty, is of course, that such a unanimous ordering can be quite rare, and so while Pareto domination can help us rule out some networks, we are often faced with a very large set of Pareto efficient networks, and so it may not be very prescriptive or discriminating.

Efficient Networks

- There are many networks that are not pictured in Figure 6.2.2.
- Let any permutation of the pictured networks have correspondingly permuted payoffs to the players, and any networks that are not permutations of the pictured ones lead to payoffs of 0 for all players.
- The arrows in the figure indicate that the network that the arrow points away from is unstable in that some player would benefit by deleting a link, or two players would each benefit by adding a link.
- The network the arrow points to indicates which network would result if the player(s) who benefit from the action take the action.

Efficient Networks

- In this figure there is just one efficient network, marked in red, which is to have match the players into two pairs and have each player have one link. This is also a Pareto efficient network, as any other network leads to lower payoffs for some player.
- The efficient network here is not pairwise stable, as two disconnected players would benefit from adding a link. If such an action is taken, then the new network is the green one.
- The players who have formed the link have increased their payoffs from 3 to 3.25, while this has led to a lowering of the payoffs of the other two players.
- The green network is also Pareto efficient, as there is no other network that gives all players a weakly higher payoff with some a strictly higher payoff.

Efficient Networks

- However, the green network is not pairwise stable. Here, the two players who have only one link would benefit by adding a link to each other. This then leads to the dark blue network, which is the only **pairwise stable network** out of those pictured. **No player would gain by adding or severing a link here.**
- We already see a conflict between stability and efficiency here, as the only pairwise stable network (or networks, if we count the permutations) is Pareto dominated by the efficient (red) network.

Distance-Based Utility

Distance-Based Utility

- **The utility the players receive does not just come from the direct links that they form with each other, but also from their indirect relations.**

- **The benefit function measures the indirect benefit that players obtain from being close to other players in the network.** When we consider distance, the utility function takes the form

$$u_i(g) = \sum_{j \neq i : j \in N^{n-1}(g)} b(l_{ij}(g)) - d_i(g)c,$$

where $l_{ij}(g)$ represents the shortest path length between player i and player j

- The distance-based utility assumes that all players' utility functions are alike and it takes into account only the benefits from indirect links that depend on minimum path length. These two features are considered as drawbacks of the distance-based utility.