Review for Test 3

Math 1552, Integral Calculus Sections 10.1-10.9

1. Terminology review: complete the following statements.
(a) A geometric series has the general form
The series converges when and diverges when
(b) A p-series has the general form and and
diverges when test. To show these results, we can use the test.
(c) The harmonic series and telescoping series
(d) If you want to show a series converges, compare it to a series that also
converges. If you want to show a series diverges, compare it to a series that also diverges.
(e) If the direct comparison test does not have the correct inequality, you can instead use
the test. In this test, if the limit is a number (not equal
to), then both series converge or both series diverge.
(f) In the ratio and root tests, the series will if the limit is less than 1 and if the limit is greater than 1. If the limit equals 1, then the test is
(g) If $\lim_{n\to\infty} a_n = 0$, then what do we know about the series $\sum_k a_k$?
(h) A sequence is an infinite of terms.
A sequence $\{a_n\}$ converges if:
(i) The smallest value that is greater than or equal to every term in a sequence is called
the The largest value that is less than or equal to every term in the sequence
is called the If both of these values are finite, then we say the sequence is
(·) A
(j) A sequence is called monotonic if the terms are,
or If a sequence is both monotonic and bounded, then we know it must

(k) A power series has the general form: To find the radius of convergence
R, use either the or test. The series converges when
x-c < R. To find the interval of convergence, don't forget to check the If
we differentiate or integrate a power series, the radius of convergence of the new series is
 .
(l) A Taylor Polynomial has the general form: The error in using P_n to approximate a value of x is given by the Lagrange Remainder term:
(m) A Taylor series has general form: The Taylor series can be used to approximate $f(x)$ wherever the series
(n) A MacLaurin series is a Taylor series centered at Some standard MacLaurin series are:
$e^x =$
$\sin x =$
$\cos x =$
$\frac{1}{1-x} =$

 $\ln(1+x) =$

2. Sum the series

$$\sum_{k=2}^{\infty} \frac{4^{2k} - 1}{17^{k-1}}.$$

3. Find the sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)(2k+3)}.$$

4. Determine whether the following series converge or diverge. Justify your answers using the tests we discussed in class.

(a)
$$\sum_{k=1}^{\infty} \frac{e^k}{(1+4e^k)^{3\cdot 2}}$$

(b)
$$\sum_{k=2}^{\infty} \left(\frac{k-5}{k}\right)^{k^2}$$

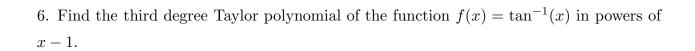
(c)
$$\sum_{k=1}^{\infty} \frac{k^2 \cdot 2^{k+1}}{k!}$$

(d)
$$\sum_{k=1}^{\infty} \frac{1}{1+2+3+...+k}$$

5. For each sequence, determine: (i) the l.u.b. and g.l.b.; (ii) whether the sequence is monotonic; (iii) whether the sequence converges or diverges, and the limit if it is convergent. (a) $\left\{ \left(\frac{n}{n+2} \right)^{3n} \right\}$

(b)
$$\left\{\frac{\cos(n\pi)}{4^n}\right\}$$

(c) $\left\{ (-1)^n \frac{n+2}{n+4} \right\}$



7. Use a Taylor polynomial to estimate the value of \sqrt{e} with an error of at most 0.01. HINT: Choose a=0 and use the fact that e<3.

8. Use the MacLaurin series for $f(x) = \frac{1}{1-x}$ to find a power series representation of the function

$$g(x) = \frac{x}{(1-x)^3}.$$

HINT: You will need to differentiate.

9. Find the radius and interval of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{5^k}{\sqrt{k}} (3 - 2x)^k.$$

10. Determine whether each of the alternating series below converge absolutely, converge conditionally, or diverge. Use the convergence tests from class to justify your answer.

(a)
$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\ln k}{k^4}$$

(b)
$$\sum_{k=2}^{\infty} (-1)^k \frac{4k^2}{k^3 + 1}$$