Review for Test 3

Math 1552, Integral Calculus

Sections 10.1-10.9

1. Terminology review: complete the following statements.

(a) A geometric series has the general form
____________. The series converges when ____________ and diverges when ____________.

(b) A p-series has the general form ____________. The series converges when ____________ and diverges when ____________. To show these results, we can use the ____________ test.

(c) The harmonic series ____________ and telescoping series ____________.

(d) If you want to show a series converges, compare it to a ____________ series that also converges. If you want to show a series diverges, compare it to a ____________ series that also diverges.

(e) If the direct comparison test does not have the correct inequality, you can instead use the ____________ ____________ test. In this test, if the limit is a ____________ number (not equal to ____________), then both series converge or both series diverge.

(f) In the ratio and root tests, the series will ____________ if the limit is less than 1 and ____________ if the limit is greater than 1. If the limit equals 1, then the test is ____________.

(g) If $\lim_{n\to\infty} a_n = 0$, then what do we know about the series $\sum_k a_k$? ____________

(h) A **sequence** is an infinite ____________ of terms.
A sequence $\{a_n\}$ converges if: ____________.

(i) The smallest value that is greater than or equal to every term in a sequence is called the ____________. The largest value that is less than or equal to every term in the sequence is called the ____________. If both of these values are finite, then we say the sequence is ____________.

(j) A sequence is called monotonic if the terms are ____________, ____________, ____________, or ____________. If a sequence is both monotonic and bounded, then we know it must ____________.

(k) A power series has the general form: _____________. To find the radius of convergence R, use either the _____________ or _____________ test. The series converges _____________ when $|x - c| < R$. To find the interval of convergence, don't forget to check the _____________. If we differentiate or integrate a power series, the radius of convergence of the new series is _____________.

(l) A Taylor Polynomial has the general form: _____________. The error in using P_n to approximate a value of x is given by the Lagrange Remainder term: _____________.

(m) A Taylor series has general form: _____________. The Taylor series can be used to approximate $f(x)$ wherever the series _____________.

(n) A MacLaurin series is a Taylor series centered at _____________. Some standard MacLaurin series are:

$$e^x =$$

$$\sin x =$$

$$\cos x =$$

$$\frac{1}{1-x} =$$

$$\ln(1+x) =$$

2. Sum the series

$$\sum_{k=2}^{\infty} \frac{4^{2k} - 1}{17^{k-1}}.$$

3. Find the sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)(2k+3)}.$$

4. Determine whether the following series converge or diverge. Justify your answers using the tests we discussed in class.

(a) $\sum_{k=1}^{\infty} \frac{e^k}{(1+4e^k)^{3.2}}$

(b) $\sum_{k=2}^{\infty} \left(\frac{k-5}{k}\right)^{k^2}$

(c) $\sum_{k=1}^{\infty} \frac{k^2 \cdot 2^{k+1}}{k!}$

(d) $\sum_{k=1}^{\infty} \frac{1}{1+2+3+\ldots+k}$

5. For each sequence, determine: (i) the l.u.b. and g.l.b.; (ii) whether the sequence is monotonic; (iii) whether the sequence converges or diverges, and the limit if it is convergent.

(a) $\left\{\left(\frac{n}{n+2}\right)^{3n}\right\}$

(b) $\left\{\frac{\cos(n\pi)}{4^n}\right\}$

(c) $\left\{(-1)^n \frac{n+2}{n+4}\right\}$

6. Find the third degree Taylor polynomial of the function $f(x) = \tan^{-1}(x)$ in powers of $x - 1$.

7. Use a Taylor polynomial to estimate the value of $\sqrt{e}$ with an error of at most 0.01. HINT: Choose $a = 0$ and use the fact that $e < 3$.

8. Use the MacLaurin series for $f(x) = \frac{1}{1-x}$ to find a power series representation of the function

$$g(x) = \frac{x}{(1-x)^3}.$$

HINT: You will need to differentiate.

9. Find the radius and interval of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{5^k}{\sqrt{k}}(3-2x)^k.$$

10. Determine whether each of the alternating series below converge absolutely, converge conditionally, or diverge. Use the convergence tests from class to justify your answer.

(a)

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\ln k}{k^4}$$

(b)

$$\sum_{k=2}^{\infty}(-1)^{k}\frac{4k^2}{k^3+1}$$