Comme Appelé du Néant—
As If Summoned from the Void: The Life of Alexandre Grothendieck

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This is the first part of a two-part article about the life of Alexandre Grothendieck. The second part of the article will appear in the next issue of the *Notices*.

Et toute science, quand nous l'entendons non comme un instrument de pouvoir et de domination, mais comme aventure de connaissance de notre espèce à travers les âges, n'est autre chose que cette harmonie, plus ou moins vaste et plus ou moins riche d'une époque à l'autre, qui se déploie au cours des générations et des siècles, par le délicat contrepoint de tous les thèmes apparus tour à tour, comme appelés du néant.

And every science, when we understand it not as an instrument of power and domination but as an adventure in knowledge pursued by our species across the ages, is nothing but this harmony, more or less vast, more or less rich from one epoch to another, which unfurls over the course of generations and centuries, by the delicate counterpoint of all the themes appearing in turn, as if summoned from the void.

—*Récoltes et Semailles*, page P20

Alexandre Grothendieck is a mathematician of immense sensitivity to things mathematical, of profound perception of the intricate and elegant lines of their architecture. A couple of high points from his biography—he was a founding member of the Institut des Hautes Études Scientifiques (IHÉS) and received the Fields Medal in 1966—suffice to secure his place in the pantheon of twentieth century mathematics. But such details cannot capture the essence of his work, which is rooted in something far more organic and humble. As he wrote in his long memoir, *Récoltes et Semailles* (*Reapings and Sowings*, R&S), "What makes the quality of a researcher's inventiveness and imagination is the *quality of his attention* to hearing the voices of things" (emphasis in the original, page P27). Today Grothendieck's own voice, embodied in his written works, reaches us as if through a void: now seventy-six years old, he has for more than a decade lived in seclusion in a remote hamlet in the south of France.

Grothendieck changed the landscape of mathematics with a viewpoint that is "cosmically general", in the words of Hyman Bass of the University of Michigan. This viewpoint has been so thoroughly absorbed into mathematics that nowadays it is difficult for newcomers to imagine that the field was not always this way. Grothendieck left his deepest mark on algebraic geometry, where he placed emphasis on discovering relationships among mathematical objects as a way of understanding the objects themselves. He had an extremely powerful, almost other-worldly ability of abstraction that allowed him to see problems in a highly general context, and he used this ability with exquisite precision. Indeed, the trend toward increasing generality and abstraction, which can be seen across the whole field since the middle of the twentieth

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century, is due in no small part to Grothendieck's influence. At the same time, generality for its own sake, which can lead to sterile and uninteresting mathematics, is something he never engaged in.

Grothendieck's early life during World War II had a good deal of chaos and trauma, and his educational background was not the best. How he emerged from these deprived beginnings and forged a life for himself as one of the leading mathematicians in the world is a story of high drama—as is his decision in 1970 to abruptly leave the mathematical milieu in which his greatest achievements blossomed and which was so deeply influenced by his extraordinary personality.

Early Life

> Ce qui me satisfaisait le moins, dans nos livres de maths [au lycée], c'était l'absence de toute définition sérieuse de la notion de longueur (d'une courbe), d'aire (d'une surface), de volume (d'un solide). Je me suis promis de combler cette lacune, dès que j'en aurais le loisir.
>
> What was least satisfying to me in our [high school] math books was the absence of any serious definition of the notion of length (of a curve), of area (of a surface), of volume (of a solid). I promised myself I would fill this gap when I had the chance.
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> —*Récoltes et Semailles*, page P3

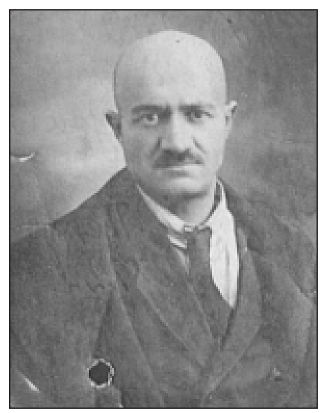
Armand Borel of the Institute for Advanced Study in Princeton, who died in August 2003 at the age of 80, remembered the first time he met Grothendieck, at a Bourbaki seminar in Paris in November 1949. During a break between lectures, Borel, then in his mid-twenties, was chatting with Charles Ehresmann, who at forty-five years of age was a leading figure in French mathematics. As Borel recalled, a young man strode up to Ehresmann and, without any preamble, demanded, "Are you an expert on topological groups?" Ehresmann, not wanting to seem immodest, replied that yes, he knew something about topological groups. The young man insisted, "But I need a *real* expert!" This was Alexandre Grothendieck, age twenty-one—brash, intense, not exactly impolite but having little sense of social niceties. Borel remembered the question Grothendieck asked: Is every local topological group the germ of a global topological group? As it turned out, Borel knew a counterexample. It was a question that showed Grothendieck was already thinking in very general terms.

Grothendieck's time in Paris in the late 1940s was his first real contact with the world of mathematical research. Up to that time, his life story—

Grothendieck's mother, Hanka, 1917.

at least what is known of it—contains few clues that he was destined to become a dominant figure in that world. Many of the details about Grothendieck's family background and early life are sketchy or unknown. Winfried Scharlau of the Universität Münster is writing a biography of Grothendieck and has studied carefully this part of his life. Much of the information in the following biographical sketch comes from an interview with Scharlau and from biographical materials he has assembled about Grothendieck [Scharlau].

Grothendieck's father, whose name may have been Alexander Shapiro, was born into a Jewish family in Novozybkov in Ukraine on October 11, 1889. Shapiro was an anarchist and took part in various uprisings in czarist Russia in the early twentieth century. Arrested at the age of seventeen, he managed to elude a death sentence, but, after escaping and being recaptured a few times, he spent a total of about ten years in prison. Grothendieck's father has sometimes been confused with another more famous activist also named Alexander Shapiro, who participated in some of the same political movements. This other Shapiro, who was portrayed in John Reed's book *Ten Days that Shook the World*, emigrated to New York and died there in 1946, by which time Grothendieck's father had already been dead for four years. Another distinguishing detail is that Grothendieck's father had only one arm. According to Justine Bumby, who lived with Grothendieck for a period in the 1970s and had a son by him, his father lost his arm in a suicide attempt while trying to avoid being captured by the police. Grothendieck himself may unwittingly have contributed to the confusion between the two Shapiros; for example, Pierre Cartier of the Institut des Hautes Études Scientifiques mentioned in [Cartier2] Grothendieck's



Grothendieck's father, Sascha, ca. 1922.

maintaining that one of the figures in Reed's book was his father.

In 1921 Shapiro left Russia and was stateless for the rest of his life. To hide his political past, he obtained identity papers with the name Alexander Tanaroff, and for the rest of his life he lived under this name. He spent time in France, Germany, and Belgium, where he associated with anarchist and other revolutionary groups. In the radical circles of Berlin in the mid-1920s, he met Grothendieck's mother, Johanna (Hanka) Grothendieck. She had been born on August 21, 1900, into a bourgeois family of Lutherans in Hamburg. Rebelling against her traditional upbringing, she was drawn to Berlin, which was then a hotbed of avant-garde culture and revolutionary social movements. Both she and Shapiro yearned to be writers. He never published anything, but she published some newspaper articles; in particular, between 1920 and 1922, she wrote for a leftist weekly newspaper called *Der Pranger*, which had taken up the cause of prostitutes living on the fringe of Hamburg society. Much later, in the late 1940s, she wrote an autobiographical novel called *Eine Frau*, which was never published.

A. Grothendieck as a child.

For most of his life, Tanaroff was a street photographer, an occupation that allowed him to earn an independent living without being in an employer-employee relationship that would have run counter to his anarchist principles. He and Hanka had each been married before, and each had a child from the previous marriage, she a daughter and he a son. Alexandre Grothendieck was born in Berlin on March 28, 1928, into a family consisting of Hanka, Tanaroff, and Hanka's daughter from her first marriage, Maidi, who was four years older than Alexandre. He was known in the family, and to his close friends later on, as Shurik; his father's nickname was Sascha. Although he never met his half-brother, Grothendieck dedicated to him the manuscript *A La Poursuite des Champs* (*Pursuing Stacks*), written in the 1980s.

In 1933, when the Nazis came to power, Shapiro fled Berlin for Paris. In December that year, Hanka decided to follow her husband, so she put her son in the care of a foster family in Blankenese, near Hamburg; Maidi was left in an institution for handicapped children in Berlin, although she was not handicapped (R&S, pages 472–473). The foster family was headed by Wilhelm Heydorn, whose remarkable life is outlined in his biography, *Nur Mensch Sein!* [Heydorn]; the book contains a photograph of Alexandre Grothendieck from 1934, and he is mentioned briefly. Heydorn had been a Lutheran priest and army officer, then left the church and worked as an elementary school teacher and a *Heilpraktiker* (which nowadays might be translated roughly as "practitioner of alternative medicine"). In 1930 he founded an idealistic political party called the "Menschheitspartei" ("Humanity Party"), which was outlawed by the Nazis. Heydorn had four children of his own, and he and his wife Dagmar, following their sense of Christian duty, took in several foster children who were separated from their families in the tumultuous period leading up to World War II.

Grothendieck remained with the Heydorn family for five years, between the ages of five and eleven, and attended school. A memoir by Dagmar Heydorn recalled the young Alexandre as being very free, completely honest, and lacking in inhibitions. During his time with the Heydorns, Grothendieck received only a few letters from his mother and no word at all from his father. Although Hanka still had relatives in Hamburg, no one ever came to visit her son. The sudden separation from his parents was highly traumatic for Grothendieck, as he indicated in *Récoltes et Semailles* (page 473). Scharlau speculated that the young Alexandre was probably not especially happy with the Heydorns. Having started life in a liberal home headed by a couple of anarchists, the stricter atmosphere of the Heydorn household probably chafed. He was actually closer to some other families who lived near the Heydorns, and as an adult he continued to write to them for many years. He also wrote to the Heydorns and visited Hamburg several times, the last time in the mid-1980s.

By 1939, with war imminent, political pressure increased on the Heydorns, and they could no longer keep the foster children. Grothendieck was an especially difficult case, because he looked Jewish. The exact whereabouts of his parents were unknown, but Dagmar Heydorn wrote to the French consulate in Hamburg and managed to get a message to Shapiro in Paris and to Hanka in Nîmes. Once contact with his parents was made, Grothendieck, then 11 years old, was put on a train from Hamburg to Paris. He was reunited with his parents in May 1939, and they spent a brief time together before the war began.

It is not clear exactly what Grothendieck's parents were doing while he was in Hamburg, but they remained politically active. They went to Spain to fight in the Spanish Civil War and were among the many who fled to France when Franco triumphed. Because of their political activities, Hanka and her husband were viewed in France as dangerous foreigners. Some time after Grothendieck joined them there, Shapiro was put into the internment camp Le Vernet, the worst of all the French camps. It is probable that he never again saw his wife and son.

In August 1942 he was deported by the French authorities to Auschwitz, where he was killed. What happened to Maidi at this time is unclear, but eventually she married an American soldier and emigrated to the United States; she passed away a couple of years ago.

In 1940 Hanka and her son were put into an internment camp in Rieucros, near Mende. As internment camps went, the one at Rieucros was one of the better ones, and Grothendieck was permitted to go to the *lycée* (high school) in Mende. Nevertheless, it was a life of deprivation and uncertainty. He told Bumby that he and his mother were sometimes shunned by French people who did not know of Hanka's opposition to the Nazis. Once he ran away from the camp with the intention of assassinating Hitler, but he was quickly caught and returned. "This could easily have cost him his life", Bumby noted. He had always been strong and a good boxer, attributes that were useful at this time, as he was sometimes the target of bullying.

After two years, mother and son were separated; Hanka was sent to another internment camp, and her son ended up in the town of Chambon-sur-Lignon. André Trocmé, a Protestant pastor, had transformed the mountain resort town of Chambon into a stronghold of resistance against the Nazis and a haven for protecting Jews and others endangered during the war [Hallie]. There Grothendieck was taken into a children's home supported by a Swiss organization. He attended the Collège Cévenol, set up in Chambon to provide an education for the young people, and earned a *baccalauréat*. The heroic efforts of the Chambonnais kept the refugees safe, but life was nevertheless precarious. In *Récoltes et Semailles* Grothendieck mentioned the periodic roundups of Jews that would send him and his fellow students scattering to hide in the woods for a few days (page P2).

He also related some of his memories of his schooling in Mende and Chambon. It is clear that, despite the difficulties and dislocation of his youth, he had a strong internal compass from an early age. In his mathematics classes, he did not depend on his teachers to distinguish what was deep from what was inconsequential, what was right from what was wrong. He found the mathematics problems in the texts to be repetitive and presented in isolation from anything that would give them meaning. "These were the book's problems, and not *my* problems," he wrote. When a problem did seize him, he lost himself in it completely, without regard to how much time he spent on it (page P3).

From Montpellier to Paris to Nancy

> Monsieur Soula [mon professeur de calcul] m'assurait…que les derniers problèmes qui s'étaient encore posés en maths avaient été résolus, il y avait vingt ou trente ans, par un dénommé Lebesgue. Il aurait développé justement (drôle de coïncidence, décidement!) une théorie de la mesure et de l'intégration, laquelle mettait un point final à la mathématique.
>
> Mr. Soula [my calculus teacher] assured me that the final problems posed in mathematics had been resolved, twenty or thirty years before, by a certain Lebesgue. He had exactly developed (an amusing coincidence, certainly!) a theory of measure and integration, which was the endpoint of mathematics.
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> —*Récoltes et Semailles*, page P4

By the time the war ended in Europe, in May 1945, Alexandre Grothendieck was seventeen years old. He and his mother went to live in Maisargues, a village in a wine-growing region outside of Montpellier. He enrolled at the Université de Montpellier, and the two survived on his student scholarship and by doing seasonal work in the grape harvest; his mother also worked at housecleaning. Over time he attended the university courses less and less, as he found that the teachers were mostly repeating what was in the textbooks. At the time, Montpellier "was among the most backward of French universities in the teaching of mathematics," wrote Jean Dieudonné [D1].

In this uninspiring environment, Grothendieck devoted most of his three years at Montpellier to filling the gap that he had felt in his high school textbooks about how to provide a satisfactory definition of length, area, and volume. On his own, he essentially rediscovered measure theory and the notion of the Lebesgue integral. This episode is one of several parallels between the life of Grothendieck and that of Albert Einstein; as a young man Einstein developed on his own ideas in statistical physics that he later found out had already been discovered by Josiah Willard Gibbs.

In 1948, having finished his Licenceès Sciences at Montpellier, Grothendieck went to Paris, the main center for mathematics in France. In an article about Grothendieck that appeared in a French magazine in 1995 [Ikonicoff], a French education official, André Magnier, recalled Grothendieck's application for a scholarship to go to Paris. Magnier asked him to describe the project he had been working on at Montpellier. "I was astounded," the article quoted Magnier as saying. "Instead of a meeting of twenty minutes, he went on for two hours explaining to me how he had reconstructed, 'with the tools available', theories that had taken decades to construct. He showed an extraordinary sagacity." Magnier also added: "Grothendieck gave the impression of being an extraordinary young man, but imbalanced by suffering and deprivation." Magnier immediately recommended Grothendieck for the scholarship.

Grothendieck's calculus teacher at Montpellier, Monsieur Soula, recommended he go to Paris and make contact with Cartan, who had been Soula's teacher. Whether the name Cartan referred to the father, Élie Cartan, who was then close to eighty years old, or his son, Henri Cartan, then in his mid-forties, Grothendieck did not know (R&S, page 19). When he arrived in Paris, in the autumn of 1948, he showed to mathematicians there the work he had done in Montpellier. Just as Soula had told him, the results were already known. But Grothendieck was not disappointed. In fact, this early solitary effort was probably critical to his development as a mathematician. In *Récoltes et Semailles,* he said of this time: "Without knowing it, I learned in solitude what is essential to the metier of a mathematician—something that no master can truly teach. Without having been told, I nevertheless knew 'in my gut' that I was a mathematician: someone who 'does' math, in the fullest sense of the word—like one 'makes' love" (page P5).

He began attending the legendary seminar run by Henri Cartan at the École Normale Supérieure. This seminar followed a pattern that Grothendieck was to take up with great vigor later in his career, in which a theme is investigated in lectures over the course of the year and the lectures are systematically written up and published. The theme for the Cartan seminar for 1948–1949 was simplicial algebraic topology and sheaf theory—then cutting-edge topics that were not being taught anywhere else in France [D1]. Indeed, this was not long after the notion of sheaves had been formulated by Jean Leray. In the Cartan seminar, Grothendieck encountered for the first time many of the outstanding mathematicians of the day, including Claude Chevalley, Jean Delsarte, Jean Dieudonné, Roger Godement, Laurent Schwartz, and André Weil. Among Cartan's students at this time was Jean-Pierre Serre. In addition to attending the Cartan seminar, Grothendieck went to a course on the then-new notion of locally convex spaces, given by Leray at the Collège de France.

As the son of the geometer Élie Cartan, as an outstanding mathematician in his own right, and as a professor at the École Normale Supérieure, Henri Cartan was in many ways the center of the Parisian mathematical elite. Also, he was one of the few French mathematicians who made efforts to reach out to German colleagues after the war. This was despite his intimate knowledge of the war's horrors: his brother, who had joined the Résistance, had been captured by the Germans and beheaded. Cartan and many of the top mathematicians of the time—such as Ehresmann, Leray, Chevalley, Delsarte, Dieudonné, and Weil—shared the common background of having been *normaliens*, meaning that they were graduates of the École Normale Supérieure, the most prestigious institution of higher education in France.

When Grothendieck joined Cartan's seminar, he was an outsider: not only was he a German speaker living in postwar France, but his meager educational background contrasted sharply with that of the group he found himself in. And yet in *Récoltes et Semailles*, Grothendieck said he did not feel like a stranger in this milieu and related warm memories of the "benevolent welcome" he received (pages 19–20). His outspokenness drew notice: in a tribute to Cartan for his 100th birthday, Jean Cerf recalled seeing in the Cartan seminar around this time "a stranger (it was Grothendieck) who took the liberty of speaking to Cartan, as if to his equal, from the back of the room" [Cerf]. Grothendieck felt free to ask questions, and yet, he wrote, he also found himself struggling to learn things that those around him seemed to grasp instantly and play with "like they had known them from the cradle." (R&S, page P6). This may have been one reason why, in October 1949, on the advice of Cartan and Weil, he left the rarefied atmosphere of Paris for the slower-paced Nancy. Also, as Dieudonné wrote [D1], Grothendieck was at this time showing more interest in topological vector spaces than in algebraic geometry, so Nancy was the natural place for him to go.

Apprenticeship in Nancy

> …l'affection circulait…depuis ce premier moment où j'ai été reçu avec affection à Nancy, en 1949, dans la maison de Laurent et Hélène Schwartz (où je faisais un peu partie de la famille), celle de Dieudonné, celle de Godement (qu'en un temps je hantais également régulièrement). Cette chaleur affectueuse qui a entouré mes premiers pas dans le monde mathématique, et que j'ai eu tendance un peu à oublier, a été importante pour toute ma vie de mathématicien.
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> …the affection circulated…from that first moment when I was received with affection in Nancy in 1949, in the house of Laurent and Hélène Schwartz (where I was somewhat a member of the family), in that of Dieudonné, in that of Godement (which at that time also became one of my regular haunts). This affectionate warmth that surrounded my first steps in the mathematical

world, and that I have had some tendency to forget, was important in my entire life as a mathematician.

—*Récoltes et Semailles*, page 42

In the late 1940s, Nancy was one of the strongest mathematical centers in France; indeed, the fictitious Nicolas Bourbaki was said to have come from the "University of Nancago", a name that makes reference to Weil's time at the University of Chicago as well as to his fellow Bourbakists in Nancy. The Nancy faculty included Delsarte, Godement, Dieudonné, and Schwartz. Among Grothendieck's fellow students at Nancy were Jacques-Louis Lions and Bernard Malgrange, who like Grothendieck were students of Schwartz, as well as Paulo Ribenboim, a Brazilian who at twenty-two years of age arrived in Nancy about the same time as Grothendieck.

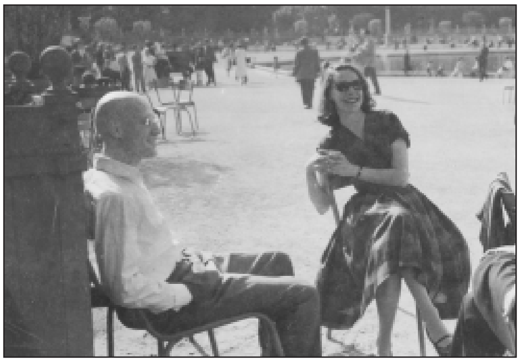
According to Ribenboim, who is today a professor emeritus at Queen's University in Ontario, the pace in Nancy was less hectic than in Paris, and professors had more time for the students. Ribenboim said he had the impression that Grothendieck had come to Nancy because his lack of background had made it hard for him to follow Cartan's high-powered seminar. Not that Grothendieck came out and said this: "He was not the guy who would admit he didn't understand!" Ribenboim remarked. Nevertheless, Grothendieck's extraordinary talents were apparent, and Ribenboim remembered looking up to him as an ideal. Grothendieck could be extremely intense, sometimes expressing himself in a brazen way, Ribenboim recalled: "He was not mean, but very demanding of himself and everyone else." Grothendieck had very few books; rather than learning things by reading, he would try to reconstruct them on his own. And he worked very hard. Ribenboim remembered Schwartz telling him: You seem to be a nice, well-balanced young man; you should make friends with Grothendieck and do something so that he is not only working.

Dieudonné and Schwartz were running a seminar in Nancy on topological vector spaces. As Dieudonné explained in [D1], by this time Banach spaces and their duality were well understood, but locally convex spaces had only recently been introduced, and a general theory for their duality had not yet been worked out. In working in this area, he and Schwartz had run into a series of problems, which they decided to turn over to Grothendieck. They were astonished when, some months later, he had solved every one of them and gone on to work on other questions in functional analysis. "When, in 1953, it was time to grant him a doctor's degree, it was necessary to choose from among six papers he had written, any one of which was at the level of a good dissertation," Dieudonné wrote. The paper chosen for his thesis was "Produits tensoriels topologiques et espaces nucléaires," which shows the first signs of the generality of thinking that would come to characterize Grothendieck's entire oeuvre. The notion of nuclear spaces, which has had wide applications, was first proposed in this paper. Schwartz popularized Grothendieck's results in a Paris seminar, "Les produits tensoriels d'après Grothendieck," published in 1954 [Schwartz]. In addition, Grothendieck's thesis appeared as a monograph in 1955 in the *Memoirs of the AMS* series; it was reprinted for the seventh time in 1990 [Gthesis].

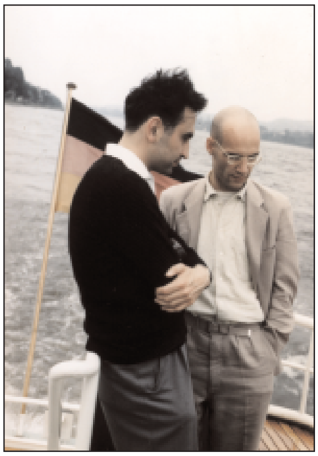
Grothendieck's work in functional analysis "was quite remarkable," commented Edward G. Effros of the University of California at Los Angeles. "He was arguably the first to realize that the algebraic/categorical methods that flourished after the Second World War could be used in this highly analytic branch of functional analysis." In some ways, Grothendieck was ahead of his time. Effros noted that it took at least fifteen years before Grothendieck's work was fully incorporated into mainstream Banach space theory, partly because of a reluctance to adopt his more algebraic perspective. The influence of his work has grown in recent years, Effros said, with the "quantization" of Banach space theory, for which Grothendieck's categorical approach is especially well suited.

Top: Party at Hirzebruch home, 1961 Arbeitstagung (left to right) Dorothea von Viereck, Raoul Bott, Grothendieck.
Center, with Michael Atiyah.
Bottom: Bonn, 1961, excursion during Arbeitstagung, Ioan James, Michael Atiyah, Grothendieck.

Although Grothendieck's mathematical work had gotten off to a promising start, his personal life was unsettled. He lived in Nancy with his mother, who as Ribenboim recalled was occasionally bedridden because of tuberculosis. She had contracted the disease in the internment camps. It was around this time that she was writing her autobiographical novel *Eine Frau*. A liaison between Grothendieck and an older woman who ran the boarding house where



he and his mother rented rooms resulted in the birth of his first child, a son named Serge; Serge was raised mostly by his mother. After he finished his Ph.D., Grothendieck's prospects for permanent employment were bleak: he was stateless, and at that time it was difficult for noncitizens to get permanent jobs in France. Becoming a French citizen would have entailed military service, which Grothendieck refused to do. Since 1950 he had had a position through the Centre National de la Recherche Scientifique (CNRS), but this was more like a fellowship than a permanent job. At some point he considered learning carpentry as a way to earn money (R&S, page 1246(*)).



Top: Paris, with Karin Tate, 1964. Bottom: with E. Luft, an excursion on the Rhine, 1961.

Laurent Schwartz visited Brazil in 1952 and told people there about his brilliant young student who was having trouble finding a job in France. As a result Grothendieck received an offer of a visiting professor position at the Universidade de São Paulo, which he held during 1953 and 1954. According to José Barros-Neto, who was then a student in São Paulo and is now a professor emeritus at Rutgers University, Grothendieck made a special arrangement so that he would be able to return to Paris to attend seminars that took place in the fall. The second language for the Brazilian mathematical community was French, so it was easy for Grothendieck to teach and converse with his colleagues. In going to São Paulo, Grothendieck was carrying on a tradition of scientific exchange between Brazil and France: in addition to Schwartz, Weil, Dieudonné, and Delsarte had all visited Brazil in the 1940s and 1950s. Weil came to São Paulo in January 1945 and stayed until the fall of 1947, when he went to the University of Chicago. The mathematical ties between France and Brazil continue to this day. The Instituto de Matemática Pura e Aplicada in Rio de Janeiro has a Brazil-France cooperative agreement that brings many French mathematicians to IMPA.

In *Récoltes et Semailles*, Grothendieck referred to 1954 as "the wearisome year" (*"l'année pénible"*) (page 163). For the whole year he tried without success to make headway on the problem of approximation in topological vector spaces, a problem that was resolved only some twenty years later by methods different from those Grothendieck was attempting to use. This was "the only time in my life when doing mathematics became burdensome for me!" he wrote. This frustration taught him a lesson: always have several mathematical "irons in the fire," so that if one problem proves too stubborn, there is something else to work on.

Chaim Honig, a professor at the Universidade de São Paulo, was an assistant in the mathematics department when Grothendieck was there, and they became good friends. Honig said Grothendieck led a somewhat spartan and lonely existence, living off of milk and bananas and completely immersing himself in mathematics. Honig once asked Grothendieck why he had gone into mathematics. Grothendieck replied that he had two special passions, mathematics and piano, but he chose mathematics because he thought it would be easier to earn a living that way. His gift for mathematics was so abundantly clear, said Honig, "I was astonished that at any moment he could hesitate between mathematics and music."

Grothendieck planned to write a book on topological vector spaces with Leopoldo Nachbin, who was in Rio de Janeiro, but the book never materialized. However, Grothendieck taught a course in São Paulo on topological vector spaces and wrote up the notes, which were subsequently published by the university. Barros-Neto was a student in the course and wrote an introductory chapter for the notes, giving some basic prerequisites. Barros-Neto recalled that at the time he was in Brazil Grothendieck was talking about changing fields. He was "very, very ambitious," Barros-Neto said. "You could sense that drive—he had to do something fundamental, important, basic."

A Rising Star

> La chose essentielle, c'était que Serre à chaque fois sentait fortement la riche substance derrière un énoncé qui, de but en blanc, ne m'aurait sans doute fait ni chaud ni froid—et qu'il arrivait à "faire passer" cette perception d'une substance riche, tangible, mystérieuse—cette perception qui est en même temps *désir* de connaître cette substance, d'y pénétrer.
>
> The essential thing was that Serre each time strongly sensed the rich meaning behind a statement that, on the page, would doubtless have left me neither hot nor cold—and that he could "transmit" this perception of a rich, tangible, and mysterious substance—this perception that is at the same time the

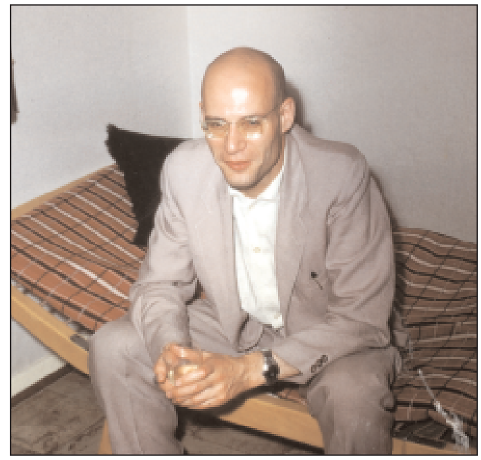
desire to understand this substance, to penetrate it.

—*Récoltes et Semailles*, page 556

Bernard Malgrange of the Université de Grenoble recalled that after Grothendieck wrote his thesis he asserted that he was no longer interested in topological vector spaces. "He told me, 'There is nothing more to do, the subject is dead'," Malgrange recalled. At that time, students were required to prepare a "second thesis", which did not contain original work but which was intended to demonstrate depth of understanding of another area of mathematics far removed from the thesis topic. Grothendieck's second thesis was on sheaf theory, and this work may have planted the seeds for his interest in algebraic geometry, where he was to do his greatest work. After Grothendieck's thesis defense, which took place in Paris, Malgrange recalled that he, Grothendieck, and Henri Cartan piled into a taxicab to go to lunch at the home of Laurent Schwartz. They took a cab because Malgrange had broken his leg skiing. "In the taxi Cartan explained to Grothendieck some wrong things Grothendieck had said about sheaf theory," Malgrange recalled.

After leaving Brazil Grothendieck spent the year of 1955 at the University of Kansas, probably at the invitation of N. Aronszajn [Corr]. There Grothendieck began to immerse himself in homological algebra. It was while he was at Kansas that he wrote "Sur quelques points d'algèbre homologique," which came to be known informally among specialists as the "Tôhoku paper" after the name of the journal in which it appeared, the *Tôhoku Mathematical Journal* [To]. This paper, which became a classic in homological algebra, extended the work of Cartan and Eilenberg on modules. Also while he was in Kansas, Grothendieck wrote "A general theory of fiber spaces with structure sheaf," which appeared as a report of the National Science Foundation. This report developed his initial ideas on nonabelian cohomology, a subject to which he later returned in the context of algebraic geometry.

Around this time, Grothendieck began corresponding with Jean-Pierre Serre of the Collège de France, whom he had met in Paris and later encountered in Nancy; a selection of their letters was published in the original French in 2001 and in a dual French-English version in 2003 [Corr]. This was the beginning of a long and fruitful interaction. The letters display a deep and vibrant mathematical bond between two very different mathematicians. Grothendieck shows a high-flying imagination that is frequently brought back to earth by Serre's incisive understanding and wider knowledge. Sometimes in the letters Grothendieck displays a surprising level of ignorance: for example, at one point, he asks Serre if the Riemann zeta function has infinitely many zeros ([Corr], page 204). "His knowledge of classical algebraic geometry was practically zero," recalled Serre. "My own knowledge of classical algebraic geometry was a little bit better, but not very much, but I tried to help him with that. But…there were so many open questions that it didn't matter." Grothendieck was not one for keeping up on the latest literature, and to a large degree he depended on Serre to tell him what was going on. In *Récoltes et Semailles* Grothendieck wrote that most of what he learned in geometry, apart from what he taught himself, he learned from Serre (pages 555–556). But Serre did not simply teach Grothendieck things; he was able to digest ideas and to discuss them in a way that Grothendieck found especially compelling. Grothendieck called Serre a "detonator," one who provided a spark that set the fuse burning for an explosion of ideas.



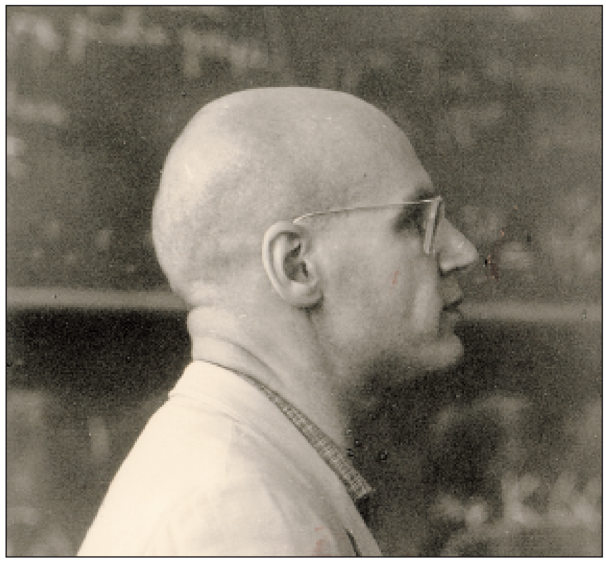
During an Arbeitstagung in 1961, an evening at the Hirzebruch home in Bonn.

Indeed, Grothendieck traced many of the central themes of his work back to Serre. For example, it was Serre who around 1955 described the Weil conjectures to Grothendieck in a cohomological context—a context that was not made explicit in Weil's original formulation of the conjectures and was the one that could hook Grothendieck (R&S, page 840). Through his idea of a "Kählerian" analogue of the Weil conjectures, Serre also inspired Grothendieck's conception of the so-called "standard conjectures," which are more general and would imply the Weil conjectures as a corollary (R&S, page 210).

When Grothendieck returned to France in 1956 after his year in Kansas, he held a CNRS position and spent most of his time in Paris. He and Serre continued to correspond by letter and to talk regularly by telephone. This was when Grothendieck began to work more deeply in topology and algebraic geometry. He "was bubbling with ideas," recalled Armand Borel. "I was sure something first-rate would come out of him. But then what came out was even much higher than I had expected. It was his version of Riemann-Roch, and that's a fantastic theorem. This is really a masterpiece of mathematics."

The Riemann-Roch theorem was proved in its classical form in the mid-nineteenth century. The question it addresses is, What is the dimension of the space of meromorphic functions on a compact Riemann surface having poles of at most given orders at a specified finite set of points? The answer is the Riemann-Roch formula, which expresses the dimension in terms of invariants of the surface—thereby providing a profound link between the analytic and topological properties of the surface. Friedrich Hirzebruch made a big advance in 1953, when he generalized the Riemann-Roch theorem to apply not just to Riemann surfaces but to projective nonsingular varieties over the complex numbers. The mathematical world cheered at this tour de force, which might have seemed to be the final word on the matter.

"Grothendieck came along and said, 'No, the Riemann-Roch theorem is *not* a theorem about varieties, it's a theorem about morphisms between varieties'," said Nicholas Katz of Princeton University. "This was a fundamentally new point of view…the very statement of the theorem completely changed." The basic philosophy of category theory, that one should pay more attention to the arrows between objects than to the objects themselves, was just then beginning to have an influence. "What [Grothendieck] did is he applied this philosophy on a very hard piece of mathematics," Borel said. "This was really in the spirit of categories and functors, but no one had ever thought about doing this in such a hard topic…. If people had been given that statement and had understood it, there might have been others who would have been able to prove it. But the statement itself was ten years ahead of anybody else."



Bonn, around 1965.

This theorem, which was also proved independently by Gerard Washnitzer in 1959 [Washnitzer], applies not just to a complex algebraic variety—the case where the ground field has characteristic zero—but to any proper smooth algebraic variety regardless of the ground field. The Hirzebruch-Riemann-Roch theorem then follows as a special case. A far-reaching generalization of the Riemann-Roch theorem came in 1963, with the proof by Michael Atiyah and Isadore Singer of the Atiyah-Singer Index Theorem. In the course of his proof, Grothendieck introduced what are now called Grothendieck groups, which essentially provide a new kind of topological invariant. Grothendieck himself called them K-groups, and they provided the starting point for the development of topological K-theory by Atiyah and Hirzebruch. Topological K-theory then provided the inspiration for algebraic K-theory, and both have been active fields of research ever since.

The *Arbeitstagung*, which means literally "working meeting," was begun by Hirzebruch at the Universität Bonn and has been a forum for cutting-edge mathematics research for more than forty years. It was at the very first *Arbeitstagung* in July 1957 that Grothendieck spoke about his work on Riemann-Roch. But in a curious twist, the result was not published under his name; it appears in a paper by Borel and Serre [BS] (the proof also appeared later as an *exposé* in volume 6 of *Séminaire de Géometrie Algébrique du Bois Marie* from 1966-67). While visiting the IAS in the fall of 1957, Serre received a letter from Grothendieck containing an outline of the proof (November 1, 1957, letter in [Corr]). He and Borel organized a seminar to try to understand it. As Grothendieck was busy with many other things, he suggested to his colleagues that they write up and publish their seminar notes. But Borel speculated that there may have been another reason Grothendieck was not interested in writing up the result himself. "The main philosophy of Grothendieck was that mathematics should be reduced to a succession of small, natural steps," Borel said. "As long as you have not been able to do this, you have not understood what is going on…. And his proof of Riemann-Roch used a trick, *une astuce*. So he didn't like it, so he didn't want to publish it…. He had many other things to do, and he was not interested in writing up this trick."

This was not the last time Grothendieck would revolutionize the viewpoint on a subject. "This just kept happening over and over again, where he would come upon some problem that people had thought about for, in some cases, a hundred years…and just completely transformed what people thought the subject was about," Katz remarked. Grothendieck was not only solving outstanding problems but reworking the very questions they posed.

A New World Opens

> [J'ai fini] par me rendre compte que cette idéologie du "nous, les grands et nobles esprits…", sous une forme particulièrement extrême et virulente, avait sévi en ma mère depuis son enfance, et dominé sa relation aux autres, qu'elle se plaisait à regarder du haut de sa grandeur avec une commisération souvent dédaigneuse, voire méprisante.
>
> [I eventually] realized that this ideology of "we, the grand and noble spirits…", in a particularly extreme and virulent form, raged in my mother since her childhood and dominated her relations to others, whom she liked to view from the height of her grandeur with a pity that was frequently disdainful, even contemptuous.
>
> —*Récoltes et Semailles*, page 30

According to Honig, Grothendieck's mother was with him at least part of the time that he was in Brazil, though Honig says he never met her. Whether she was with him in Kansas is not clear. When Grothendieck returned to France in 1956, they may not have continued living together. In a letter to Serre written in Paris in November 1957, Grothendieck asked whether he might be able to rent a Paris apartment that Serre was planning to vacate. "I am interested in it for my mother, who is not doing so well in Bois-Colombes, and is terribly isolated," Grothendieck explained [Corr]. In fact, his mother died before the year's end.

Friends and colleagues say that Grothendieck spoke with great admiration, almost adulation, of both of his parents. And in *Récoltes et Semailles* Grothendieck expressed a deep and elemental love for them. For years he had in his office a striking portrait of his father, painted by a fellow detainee in the Le Vernet camp. As Pierre Cartier described it, the portrait showed a man with his head shaved and a "fiery expression" in the eyes [Cartier1]; for many years Grothendieck also shaved his head. According to Ribenboim, Hanka Grothendieck was very proud of her brilliant son, and he in turn had an extremely deep attachment to his mother.

After her death, Grothendieck went through a period of soul-searching, during which he stopped all mathematical activity and thought about becoming a writer. After several months, he decided to return to mathematics, to finish work on some of the ideas he had begun developing. This was 1958, the year that, as Grothendieck put it, was "probably the most fecund of all my mathematical life." (R&S, page P24) By this time he was living with a woman named Mireille, whom he was to marry a few years later and with whom he had three children, Johanna, Mathieu, and Alexandre. Mireille had been close to Grothendieck's mother and, according to several people who knew them, was quite a bit older than he was.

With Mireille and baby Mathieu, Paris, May 1965.

John Tate of the University of Texas at Austin and his wife at the time, Karin Tate, spent the academic year 1957–58 in Paris, where they met Grothendieck for the first time. Grothendieck displayed none of the arrogance he attributed to his mother. "He was just friendly, and at the same time rather naive and childlike," John Tate recalled. "Many mathematicians are rather childlike, unworldly in some sense, but Grothendieck more than most. He just seemed like an innocent—not very sophisticated, no pretense, no sham. He thought very clearly and explained things very patiently, without any sense of superiority. He wasn't contaminated by civilization or power or one-upmanship." Karin Tate recalled that Grothendieck had a great capacity for enjoyment, he was charming, and he loved to laugh. But he could also be extremely intense, seeing things in black-and-white with no shades of gray. And he was honest: "You always knew where you stood with him," she said. "He didn't pretend anything. He was direct." Both she and her brother, Michael Artin of the Massachusetts Institute of Technology, saw similarities between Grothendieck's personality and that of their father, Emil Artin.

Grothendieck had "an incredible idealistic streak," Karin Tate remembered. For example, he refused to have any rugs in his house because he believed that rugs were merely a decorative luxury. She also remembered him wearing sandals made out of tires. "He thought these were fantastic," she said. "They were a symbol of the kind of thing he respected—you take what you have, and you make do." In his idealism, he could also be wildly impractical. Before Grothendieck and Mireille visited Harvard for the first time in 1958, he gave her one of his favorite novels so that she could improve her rather weak knowledge of English. The novel was *Moby Dick*.

The Birth of the New Geometry

> Avec un recul de près de trente ans, je peux dire maintenant que c'est l'année [1958] vraiment où est née la vision de la géometrie nouvelle, dans le sillage

> des deux maître-outils de cette géometrie: les schémas (qui représentent une métamorphose de l'ancienne notion de "variété algébrique"), et les topos (qui représentent une métamorphose, plus profonde encore, de la notion d'espace).
>
> With hindsight of thirty years, I can now say that [1958] is the year where the vision of the new geometry was really born, in the wake of two mastertools of this geometry: schemes (which represent a metamorphosis of the old notion of "algebraic variety"), and toposes (which represent a metamorphosis, yet more profound, of the notion of space).
>
> —*Récoltes et Semailles*, page P23

In August 1958, Grothendieck gave a plenary lecture at the International Congress of Mathematicians in Edinburgh [Edin]. The talk outlined, with a remarkable prescience, many of the main themes that he was to work on for the next dozen years. It was clear by this time that he was aiming to prove the famous conjectures of André Weil, which hinted at a profound unity between the discrete world of algebraic varieties and the continuous world of topology.

At this time, algebraic geometry was evolving rapidly, with many open questions that did not require a great deal of background. Originally the main objects of study were varieties over the complex numbers. During the early part of the twentieth century, this area was a specialty of Italian mathematicians, such as Guido Castelnuovo, Federigo Enriques, and Francesco Severi. Although they developed many ingenious ideas, not all of their results were proved rigorously. In the 1930s and 1940s, other mathematicians, among them B. L. van der Waerden, André Weil, and Oscar Zariski, wanted to work with varieties over arbitrary fields, particularly varieties over fields of characteristic p, which are important in number theory. But, because of the lack of rigor of the Italian school of algebraic geometry, it was necessary to build new foundations for the field. This is what Weil did in his 1946 book *Foundations of Algebraic Geometry* [Weil1].

Weil's conjectures appeared in his 1949 paper [Weil2]. Motivated by problems in number theory, Weil studied a certain zeta function that had been introduced in special cases by Emil Artin; it is called a zeta function because it was defined in analogy to the Riemann zeta function. Given an algebraic variety V defined over a finite field of characteristic p, one can count the number of points of V that are rational over this field, as well as the corresponding numbers for each finite extension field. These numbers are then incorporated into a generating function, which is the zeta function of V. Weil proved for both curves and abelian varieties three facts about this zeta function: it is rational, it satisfies a functional equation, and its zeros and poles have a certain specific form. This form, once a change of variables is made, corresponds exactly to the Riemann hypothesis. Moreover, Weil observed that, if V arose from reduction modulo p of a variety W in characteristic zero, then the Betti numbers of W can be read off the zeta function of V, when the zeta function is expressed as a rational function. The Weil conjectures ask whether these same facts hold true if one defines such a zeta function for a projective nonsingular algebraic variety. In particular, would topological data such as the Betti numbers emerge in the zeta function? This conjectured link between algebraic geometry and topology hinted that some of the new tools, such as cohomology theory, that were then being developed for topological spaces, could be adapted for use with algebraic varieties. Because of its similarity to the classical Riemann hypothesis, the third of the Weil conjectures is sometimes called the "congruence Riemann hypothesis"; this one turned out to be the most difficult of the three to prove.

"As soon as [the Weil] conjectures were made, it was clear that they were somehow going to play a central role," Katz said, "both because they were fabulous just as 'black-box' statements, but also because it seemed obvious that solving them required developing incredible new tools that would somehow have to be incredibly valuable in their own way—which turned out to be completely correct." Pierre Deligne of the Institute for Advanced Study said that it was the conjectured link between algebraic geometry and topology that attracted Grothendieck. He liked the idea of "turning this dream of Weil into powerful machinery," Deligne remarked.

Grothendieck was not interested in the Weil conjectures because they were famous or because other people considered them to be difficult. Indeed, he was not motivated by the challenge of hard problems. What interested him were problems that seemed to point to larger, hidden structures. "He would aim at finding and creating the home which was the problem's natural habitat," Deligne noted. "That was the part that interested him, more than solving the problem." This approach contrasts with that of another great mathematician of the time, John Nash. In his mathematical prime, Nash searched out specific problems considered by his colleagues to be the most important and challenging [Nasar]. "Nash was like an Olympian athlete," remarked Hyman Bass of the University of Michigan. "He was interested in enormous personal challenges." If Nash is an ideal example of a problem-solver, then Grothendieck is an ideal example of a theory-builder. Grothendieck, said Bass, "had a sweeping vision of what mathematics could be."

In the fall of 1958, Grothendieck made the first of his several visits to the mathematics department at Harvard University. Tate was a professor there, and the chairman was Oscar Zariski. By this time Grothendieck had reproved, by recently developed cohomological methods, the connectedness theorem that was one of Zariski's biggest results, proved in the 1940s. According to David Mumford of Brown University, who was Zariski's student at the time, Zariski never took up the new methods himself, but he understood their power and wanted his students to be exposed to them, and this was why he invited Grothendieck to Harvard.

Zariski and Grothendieck got along well, Mumford noted, though as mathematicians they were very different. It was said that Zariski, when he got stuck, would go to the blackboard and draw a picture of a self-intersecting curve, which would allow him to refresh his understanding of various ideas. "The rumor was that he would draw this in the corner of the blackboard, and then he would erase it and then he would do his algebra," explained Mumford. "He had to clear his mind by creating a geometric picture and reconstructing the link from the geometry to the algebra." According to Mumford, this is something Grothendieck would never do; he seemed never to work from examples, except for ones that were extremely simple, almost trivial. He also rarely drew pictures, apart from homological diagrams.

When Grothendieck was first invited to Harvard, Mumford recalled, he had some correspondence with Zariski before the visit. This was not long after the era of the House Un-American Activities Committee, and one requirement for getting a visa was swearing that one would not work to overthrow the government of the United States. Grothendieck told Zariski he would refuse to take such a pledge. When told he might end up in jail, Grothendieck said jail would be acceptable as long as students could visit and he could have as many books as he wanted.

In Grothendieck's lectures at Harvard, Mumford found the leaps into abstraction to be breathtaking. Once he asked Grothendieck how to prove a certain lemma and got in reply a highly abstract argument. Mumford did not at first believe that such an abstract argument could prove so concrete a lemma. "Then I went away and thought about it for a couple of days, and I realized it was exactly right," Mumford recalled. "He had more than anybody else I've ever met this ability to make an absolutely startling leap into something an order of magnitude more abstract…. He would always look for some way of formulating a problem, stripping apparently everything away from it, so you don't think anything is left. And yet something *is* left, and he could find real structure in this seeming vacuum."

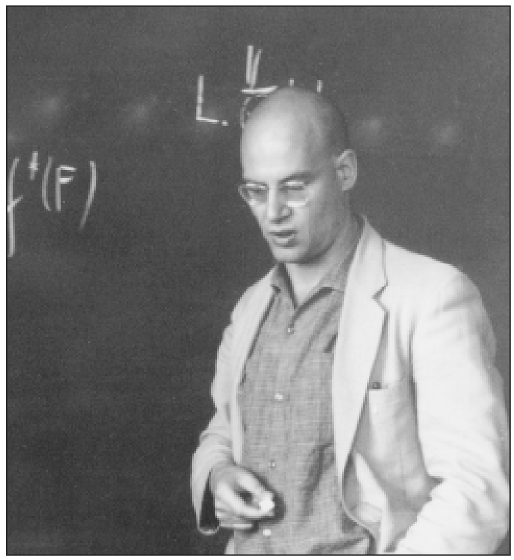
The Heroic Years

> Pendant les années héroiques de l'IHÉS, Dieudonné et moi en avons été les seuls membres, et les seuls aussi à lui donner crédibilité et audience dans le monde scientifique. …Je me sentais un peu comme un cofondateur "scientifique", avec Dieudonné, de mon institution d'attache, et je comptais bien y finir mes jours! J'avais fini par m'identifier fortement à l'IHÉS….
>
> During the heroic years of the IHÉS, Dieudonné and I were the only members, and the only ones also giving it credibility and an audience in the scientific world. …I felt myself a bit like a "scientific" co-founder, with Dieudonné, of the institution where I was on the faculty, and I counted on ending my days there! I ended up strongly identifying with the IHÉS….
>
> —*Récoltes et Semailles*, page 169

In June 1958, the Institut des Hautes Études Scientifiques (IHÉS) was formally established in a meeting of its sponsors at the Sorbonne in Paris. The founder, Léon Motchane, a businessman with a doctoral degree in physics, had a vision of establishing in France an independent research institution akin to the Institute for Advanced Study in Princeton. The original plan for the IHÉS was to focus on fundamental research in three areas: mathematics, theoretical physics, and the methodology of human sciences. While the third area never gained a foothold, within a decade the IHÉS had established itself as one of the world's top centers for mathematics and theoretical physics, with a small but stellar faculty and an active visitor program.

According to the doctoral thesis of historian of science David Aubin [Aubin], it was at the Edinburgh Congress in 1958, or possibly before, that Motchane persuaded Dieudonné and Grothendieck to accept professorships at the newly established IHÉS. Cartier wrote in [Cartier2] that Motchane originally wanted to hire Dieudonné, who made it a condition of his taking the position that an offer also be made to Grothendieck. Because the IHÉS has been from the start independent of the state, there was no problem in hiring Grothendieck despite his being stateless. The two professors formally took up their positions in March 1959, and Grothendieck started his seminar on algebraic geometry in May of that year. René Thom, who had received a Fields Medal at the 1958 Congress, joined the faculty in October 1963, and the IHÉS section on theoretical physics was launched with the appointments of Louis Michel in 1962 and of David Ruelle in 1964. Thus by the mid-1960s, Motchane had assembled an outstanding group of researchers for his new institute.

Up to 1962, the IHÉS had no permanent quarters. Office space was rented from the Fondation Thiers, and seminars were given there or at universities in Paris. Aubin reported that an early visitor to the IHÉS, Arthur Wightman, was expected to work from his hotel room. It is said that, when a visitor noted the inadequate library, Grothendieck replied, "We don't read books, we write them!" Indeed, in the early years, much of the institute's activity centered on the "Publications mathématiques de l'IHÉS," which began with the initial volumes of the foundational work *Éléments de Géométrie Algébrique*, universally known by its acronym EGA. In fact, the writing of EGA had begun half a year before Dieudonné and Grothendieck formally took their positions at the IHÉS; a reference in [Corr] dates the beginning of the writing to the autumn of 1958.



Around 1965.

The authorship of EGA is attributed to Grothendieck, "with the collaboration of Jean Dieudonné." Grothendieck wrote notes and drafts, which were fleshed out and polished by Dieudonné. As Armand Borel explained it, Grothendieck was the one who had the global vision for EGA, whereas Dieudonné had only a line-by-line understanding. "Dieudonné put this in a rather heavy style," Borel remarked. At the same time, "Dieudonné was of course fantastically efficient. No one else could have done it without ruining his own work." For some wanting to enter the field at that time, learning from EGA could be a daunting challenge. Nowadays it is seldom used as an introduction to the field, as there are many other, more approachable texts to choose from. But those texts do not do what EGA aims to do, which is to explain fully and systematically some of the tools needed to investigate schemes. When he was at Princeton University, Gerd Faltings, now at the Max-Planck-Institut für Mathematik in Bonn, encouraged his doctoral students to read EGA. And for many mathematicians today, EGA remains a useful and comprehensive reference. The current IHÉS director, Jean-Pierre Bourguignon, says that the institute still sells over 100 copies of EGA every year.

Grothendieck's plans for what EGA would cover were enormous. In a letter to Serre from August 1959, he gave a brief outline, which included the fundamental group, category theory, residues, duality, intersections, Weil cohomology, and "God willing, a little homotopy." "Unless there are unexpected difficulties or I get bogged down, the multiplodocus should be ready in 3 years' time, or 4 at the outside," Grothendieck optimistically wrote, using his and Serre's joking term "multiplodocus," meaning a very long paper. "We will be able to start doing algebraic geometry!" he crowed. As it turned out, EGA ran out of steam after almost exponential growth: chapters one and two are each one volume, chapter three is two volumes, and the last, chapter four, runs four volumes. Altogether, they comprise 1,800 pages. Despite its falling short of Grothendieck's plans, EGA is a monumental work.

It is no coincidence that the title of EGA echoes the title of the series by Nicolas Bourbaki, *Éléments de Mathématique*, which in turn echoes Euclid's *Elements*: Grothendieck was a member of Bourbaki for several years, starting in the late 1950s and was close to many of the other members. Bourbaki was the pseudonym for a group of mathematicians, most of them French, who collaborated on writing a series of foundational treatises on mathematics. Dieudonné was a founder of the Bourbaki group, together with Henri Cartan, Claude Chevalley, Jean Delsarte, and André Weil. Usually there were about ten members, and the group's composition evolved over the years. The first Bourbaki book appeared in 1939, and the group's influence was at its height during the 1950s and 1960s. The purpose of the books was to provide axiomatic treatments of central areas of mathematics at a level of generality that would make the books useful to the largest number of mathematicians. The books were born in a crucible of animated and sometimes heated discussions among the group's members, many of whom had strong personalities and highly individual points of view. Borel, who was a member of Bourbaki for 25 years, wrote that this collaboration may have been "a unique occurrence in the history of mathematics" [Borel]. Bourbaki pooled the efforts of some of the top mathematicians of the day, who unselfishly and anonymously devoted a good deal of time and energy to writing texts that would make a wide swath of the field accessible. The texts had a large impact, and by the 1970s and 1980s, there were grumblings that Bourbaki had too much influence. Also, some criticized the style of the books as being excessively abstract and general.

The work of Bourbaki and that of Grothendieck bear some similarities in the level of generality and abstraction and also in the aim of being foundational, thorough, and systematic. The main difference is that Bourbaki covered a range of mathematical areas, while Grothendieck focused on developing new ideas in algebraic geometry, with the Weil conjectures as a primary goal. In addition, Grothendieck's work was very much centered on his own internal vision, whereas Bourbaki was a collaborative effort that forged a synthesis of the viewpoints of its members.

Borel described in [Borel] the March 1957 meeting of Bourbaki, dubbed the "Congress of the inflexible functor" because of Grothendieck's proposal that a Bourbaki draft on sheaf theory be redone from a more categorical viewpoint. Bourbaki abandoned this idea, believing it could lead to an endless cycle of foundation-building. Grothendieck "could not really collaborate with Bourbaki because he had his big machine, and Bourbaki was not general enough for him," Serre recalled. In addition, Serre remarked, "I don't think he liked very much the system of Bourbaki, where we would really discuss drafts in detail and criticize them. …That was not his way of doing mathematics. He wanted to do it himself." Grothendieck left Bourbaki in 1960, though he remained close to many of its members.

Stories have circulated that Grothendieck left Bourbaki because of clashes with Weil, but in fact the two had only a brief overlap: following the edict that members must retire at age 50, Weil left the group in 1956. Nevertheless, it is true that Grothendieck and Weil were very different as mathematicians. As Deligne put it, "Weil felt somewhat that Grothendieck was too ignorant of what the Italian geometers had done and what all the classical literature was, and Weil did not like the style of building a big machine. …Their styles were quite different."

Apart from EGA, the other major part of Grothendieck's oeuvre in algebraic geometry is *Séminaire de Géométrie Algébrique du Bois Marie*, known as SGA, which contains written versions of lectures presented in his IHÉS seminar. They were originally distributed by the IHÉS. SGA 2 was copublished by North-Holland and Masson, while the remaining volumes were published by Springer-Verlag. SGA 1 dates from the seminars of 1960–1961, and the last in the series, SGA 7, dates from 1967–1969. In contrast to EGA, which is intended to set foundations, SGA describes ongoing research as it unfolded in Grothendieck's seminar. He presented many of his results in the Bourbaki Seminar in Paris, and they were collected in FGA, *Fondements de la Géométrie Algébrique*, which appeared in 1962. Together, EGA, SGA, and FGA total around 7,500 pages.

The Magic Fan

> [S]'il y a une chose en mathématique qui (depuis toujours sans doute) me fascine plus que toute autre, ce n'est ni "le nombre", ni "la grandeur", mais toujours *la forme*. Et parmi les mille-et-un visages que choisit la forme pour se révéler à nous, celui qui m'a fasciné plus que tout autre et continue à me fasciner, c'est *la structure* cachée dans les choses mathématiques.
>
> [I]f there is one thing in mathematics that fascinates me more than anything else (and doubtless always has), it is neither "number" nor "size", but always *form*. And among the thousand-and-one faces whereby form chooses to reveal itself to us, the one that fascinates me more than any other and continues to fascinate me, is *the structure* hidden in mathematical things.
>
> —*Récoltes et Semailles*, page P27

In the first volume of *Récoltes et Semailles*, Grothendieck presents an expository overview of his work intended to be accessible to nonmathematicians (pages P25–48). There he writes that, at its most fundamental level, this work seeks a unification of two worlds: "the *arithmetic world*, in which live the (so-called) 'spaces' having no notion of continuity, and the *world of continuous size*, in which live the 'spaces' in the proper sense of the term, accessible to the methods of the analyst". The reason the Weil conjectures were so tantalizing is exactly that they provided clues about this unity. Rather than trying to solve the Weil conjectures directly, Grothendieck greatly generalized their entire context. Doing so allowed him to perceive the larger structures in which the conjectures lived and of which they provided only a fleeting glimpse. In this section of *Récoltes et Semailles*, Grothendieck explained some of the key ideas in his work, including *scheme*, *sheaf*, and *topos*.

Basically, a scheme is a generalization of the notion of an algebraic variety. Given the array of finite fields of prime characteristic, a scheme produces in turn an array of varieties, each with its distinct geometry. "The array of these different varieties of different characteristics can be visualized as a sort of 'infinite fan of varieties' (one for each characteristic)," Grothendieck wrote. "The 'scheme' *is* this magic fan, which links, like so many different 'branches', the 'avatars' or 'incarnations' of all the possible characteristics." The generalization to a scheme allows one to study in a unified way all the different "incarnations" of a variety. Before Grothendieck, "I don't think people really believed you could do that," commented Michael Artin. "It was too radical. No one had had the courage to even think this may be the way to work, to work in complete generality. That was very remarkable."

Starting with the insight of the nineteenth-century Italian mathematician Enrico Betti, homology and its dual cohomology were developed as tools for studying topological spaces. Basically, cohomology theories provide invariants, which can be thought of as "yardsticks" for measuring this or that aspect of a space. The great hope, sparked by the insight implicit in the Weil conjectures, was that cohomological methods for topological spaces could be adapted for use with varieties and schemes. This hope was realized to a great extent in the work of Grothendieck and his collaborators. "It was like night and day to [bring] these cohomological techniques" into algebraic geometry, Mumford noted. "It completely turned the field upside down. It's like analysis before and after Fourier analysis. Once you get Fourier techniques, suddenly you have this whole deep insight into a way of looking at a function. It was similar with cohomology."

The notion of a sheaf was conceived by Jean Leray and further developed by Henri Cartan and Jean-Pierre Serre. In his groundbreaking paper known as FAC ("Faisceaux algébriques cohérents", [FAC]), Serre showed how sheaves could be used in algebraic geometry. Without saying exactly what a sheaf is, Grothendieck described in *Récoltes et Semailles* how this notion changed the landscape: When the idea of a sheaf came along, it was as if the good old standard cohomology "yardstick" suddenly multiplied into an infinite array of new "yardsticks", in all sizes and forms, each perfectly suited to its own unique measuring task. What is more, the category of all sheaves over a space carries so much information that one can essentially "forget" what the space is. All the information is in the sheaf—what Grothendieck called the "silent and sure guide" that led him on the path to his discoveries.

The notion of topos, Grothendieck wrote, is "a metamorphosis of the notion of a space." The concept of a sheaf provides a way of translating from the topological setting, where the space lives, to the categorical setting, where the category of sheaves lives. A topos, then, can be described as a category that, without necessarily arising from an ordinary space, nevertheless has all of the "nice" properties of a category of sheaves. The notion of topos, Grothendieck wrote, highlights the fact that "what really counts in a topological space is not at all its 'points' or its subsets of points and their proximity relations and so forth, but rather the *sheaves* on the space and the *category* that they form."

To come up with the idea of topos, Grothendieck "thought very deeply about the notion of space," Deligne commented. "The theory he created to understand those conjectures of Weil was first to create the concept of topos, a generalization of the notion of space, then to define a topos adapted to the problem," he explained. Grothendieck also showed that "one can really work with it, that the intuition we have about ordinary space works [on a topos] also. …This was a very deep idea."

In *Récoltes et Semailles* Grothendieck commented that from a technical point of view much of his work in mathematics consisted in developing the cohomology theories that were lacking. Étale cohomology was one such theory, developed by Grothendieck, Michael Artin, and others, specifically to apply to the Weil conjectures, and indeed it was one of the key ingredients in their proof. But Grothendieck went yet further, developing the concept of a *motive*, which he described as the "ultimate cohomological invariant" of which all others are different realizations or incarnations. A full theory of motives has remained out of grasp, but the concept has generated a good deal of mathematics. For example, in the 1970s Deligne and Robert Langlands of the IAS conjectured precise relationships between motives and automorphic representations. This conjecture, now part of the so-called Langlands Program, first appeared in print in [Langlands]. James Arthur of the University of Toronto said that proving this conjecture in full generality is decades away. But, he pointed out, what Andrew Wiles did in the proof of Fermat's Last Theorem was essentially to prove this conjecture in the case of two-dimensional motives that come from elliptic curves. Another example is the work of Vladimir Voevodsky of the IAS on motivic cohomology, for which he received the Fields Medal in 2002. This work builds on some of Grothendieck's original ideas about motives.

In looking back on this brief retrospective of his mathematical work, Grothendieck wrote that what makes up its essence and power is not results or big theorems, but "ideas, even dreams" (page P51).

The Grothendieck School

> Jusqu'au moment du premier "réveil,"
> en 1970, les relations à mes élèves, tout

> comme ma relation à mon propre travail, était une source de satisfaction et de joie, un des fondements tangibles, irrécusables, d'un sentiment d'harmonie dans ma vie, qui continuait à lui donner un sens….
>
> Until the moment of the first "awakening", in 1970, the relations with my students, just like my relation to my own work, was a source of satisfaction and joy, one of the tangible, unimpeachable bases of a sense of harmony in my life, which continued to give it meaning….
>
> —*Récoltes et Semailles*, page 63

During a visit to Harvard in the fall of 1961, Grothendieck wrote to Serre: "The mathematical atmosphere at Harvard is ravishing, a real breath of fresh air compared with Paris, which is getting more gloomy every year. Here, there are a fair number of intelligent students who are beginning to be familiar with the language of schemes and ask for nothing more than to work on interesting problems, which obviously are not in short supply" [Corr]. Michael Artin was at Harvard at that time as a Benjamin Peirce instructor, after having finished his thesis with Zariski in 1960. Immediately after his thesis, Artin set about learning the new language of schemes, and he also became interested in the idea of étale cohomology. When Grothendieck came to Harvard in 1961, "I asked him to tell me the definition of étale cohomology," Artin recalled with a laugh. The definition had not yet been formulated precisely. Said Artin, "Actually we argued about the definition for the whole fall."

After moving to the Massachusetts Institute of Technology in 1962, Artin gave a seminar on étale cohomology. He spent much of the following two years at the IHÉS working with Grothendieck. Once the definition of étale cohomology was in hand, there was still a lot of work to be done to tame the theory and make it into a tool that could really be used. "The definition looked marvelous, but it came with no guarantees that anything was finite, or could ever be computed, or anything," Mumford commented. This was the work that Artin and Grothendieck plunged into; one product was the Artin representability theorem. Together with Jean-Louis Verdier, they directed the 1963–64 seminar, which focused on étale cohomology. That seminar was written up in the three volumes of SGA 4, which total nearly 1,600 pages.

There might be disagreement with Grothendieck's "gloomy" assessment of the Parisian mathematical scene of the early 1960s, but there is no question that it got an enormous boost when he returned to the IHÉS in 1961 and restarted his seminar. The atmosphere was "fantastic", Artin recalled. The seminar was well populated by the leading lights of Parisian mathematics, as well as mathematicians visiting from other places. A group of brilliant and eager students began to collect around Grothendieck and to write their theses under his direction (the IHÉS does not give degrees, so formally they were students at universities in and around Paris). By 1962 the IHÉS had moved to its permanent home in the middle of a serene, tree-filled park called the Bois-Marie, in the Paris suburb of Bures-sur-Yvette. The gazebo-like building where the seminar was held, with its large picture windows and open, airy feel, provided an unusual and dramatic setting. Grothendieck was the dynamic center of the activities. "The seminars were highly interactive," recalled Hyman Bass, who visited the IHÉS in the 1960s, "but Grothendieck dominated whether he was the speaker or not." He was extremely rigorous and could be rather tough on people. "He was not unkind, but also not coddling," Bass said.

Grothendieck developed a certain pattern of working with students. A typical example is that of Luc Illusie of the Université de Paris-Sud, who became a student of Grothendieck's in 1964. Illusie had been participating in the Paris seminar of Henri Cartan and Laurent Schwartz, and it was Cartan who suggested that Illusie might do a thesis with Grothendieck. Illusie, who had until that time worked only in topology, was apprehensive about meeting this "god" of algebraic geometry. As it turned out, Grothendieck was quite kind and friendly and asked Illusie to explain what he had been working on. After Illusie had spoken for a short time, Grothendieck went to the board and launched into a discussion of sheaves, finiteness conditions, pseudo-coherence, and the like. "It was like a sea, like a continuous stream of mathematics on the board," Illusie recalled. At the end of it Grothendieck said that the next year he would devote his seminar to L-functions and l-adic cohomology and that Illusie should help to write up the notes. When Illusie protested that he knew nothing about algebraic geometry, Grothendieck said it didn't matter: "You will learn quickly."

And Illusie did. "His lectures were so clear, and he made so many efforts to recall what was necessary, all the prerequisites," Illusie remarked. Grothendieck was an excellent teacher, very patient and adept at explaining things clearly. "He took time to explain very simple examples showing how the machinery works," Illusie said. Grothendieck discussed formal properties that are often glossed over as being "trivial" and therefore too obvious to require explanation. Usually "you don't specify them, you don't spend time," Illusie said, but such things are pedagogically very useful. "Sometimes it was a bit lengthy, but it was very good for understanding."

Grothendieck gave Illusie the assignment of writing up notes for some *exposés* of the seminars—namely, *exposés* I, II, and III of SGA 5. The notes done, "I was shivering when I handed them to him," Illusie recalled. A few weeks later, Grothendieck asked Illusie to come to his home to discuss the notes; he often worked at home with colleagues and students. When Grothendieck took the notes out and set them on the table, Illusie saw that they were blackened with penciled comments. The two sat there for several hours as Grothendieck went over each comment. "He could criticize for a comma, for a period, he could criticize for an accent, he could criticize also on the substance of the thing very deeply and propose another organization—it was all kinds of comments," Illusie said. "But all his comments were very up to the point." This kind of line-by-line critique of written notes was typical of Grothendieck's way of working with students. Illusie recalled that a couple of students could not bear this kind of close criticism and ended up writing their theses with someone else. One was nearly reduced to tears after an encounter with Grothendieck. Illusie said, "Some people I remember didn't like it so much. You had to comply with that. …[But] they were not petty criticisms."

Nicholas Katz was also given an assignment when he visited the IHÉS as a postdoc in 1968. Grothendieck suggested that Katz could give a lecture in the seminar about Lefschetz pencils. "I had heard of Lefschetz pencils but really knew as little as is possible to know about them except for having heard of them," Katz recalled. "But by the end of the year I had given a few talks in the seminar, which now exist as part of SGA 7. I learned a tremendous amount from it, and it had a big effect on my future." Katz said that Grothendieck would come to the IHÉS perhaps one day a week to talk to the visitors. "What was completely amazing is he would then somehow get them interested in something, give them something to do," Katz explained. "But with, it seemed to me, a kind of amazing insight into what was a good problem to give to that particular person to think about. And he was somehow mathematically incredibly charismatic, so that it seemed like people felt it was almost a privilege to be asked to do something that was part of Grothendieck's long range vision of the future."

Barry Mazur of Harvard University still remembers today the question that Grothendieck posed to him in one of their first conversations at the IHÉS in the early 1960s, a question that Gerard Washnitzer had originally asked Grothendieck. The question: Can an algebraic variety defined over a field give topologically different manifolds by two different embeddings of the field into the complex numbers? Serre had given some early examples showing that the two manifolds could be different, and Mazur went on to do some work in homotopy theory with Artin that was inspired by this question. But at the time Grothendieck posed it, Mazur was a dedicated differential topologist, and such a question would not have occurred to him. "For [Grothendieck], it was a natural question," Mazur said. "But for me, it was precisely the right kind of motivation to get me to begin to think about algebra." Grothendieck had a real talent for "matching people with open problems. He would size you up and pose a problem that would be just the thing to illuminate the world for you. It's a mode of perceptiveness that's quite wonderful, and rare."

In addition to work with students and colleagues at the IHÉS, Grothendieck maintained correspondence with a large number of mathematicians outside Paris, some of whom were working on parts of his program in other places. For example, Robin Hartshorne of the University of California at Berkeley was at Harvard in 1961 and got the idea for his thesis topic, concerning Hilbert schemes, from Grothendieck's lectures there. Once the thesis was done, Hartshorne sent a copy to Grothendieck, who was by then back in Paris. In a reply dated September 17, 1962, Grothendieck made some brief positive remarks about the thesis. "The next three or four pages [of the letter] are all of his ideas about further theorems that I might be able to develop and other things that one might like to know about the subject," Hartshorne said. Some of the things the letter suggested are "impossibly difficult," he noted; others show a remarkable prescience. After this outpouring of ideas, Grothendieck returned to the thesis and offered three pages of detailed comments.

In his 1958 talk at the Edinburgh Congress, Grothendieck had outlined his ideas for a theory of duality, but because he was busy with other topics in his IHÉS seminar, it did not get treated there. So Hartshorne offered to give a seminar on duality at Harvard and write up the notes. Over the summer of 1963, Grothendieck fed Hartshorne about 250 pages of "pre-notes" that formed the basis for the seminar, which Hartshorne began in the fall of 1963. Questions from the audience helped Hartshorne to develop and refine the theory, which he began to write up in a systematic fashion. He would send each chapter to Grothendieck to critique. "It would come back covered with red ink all over," Hartshorne recalled. "So I'd fix everything he said, and then I would send him the new version. And it would come back again covered with more red ink." After realizing that this was a potentially endless process, Hartshorne decided one day to send the manuscript off to be published; it appeared in the Springer Lecture Notes series in 1966 [Hartshorne].

Grothendieck "had so many ideas that he kept basically all the serious people working in algebraic geometry in the world busy during that time," Hartshorne observed. How did he keep such an enterprise going? "I don't think there is a simple answer," Artin replied. But certainly Grothendieck's energy and breadth were factors. "He was very dynamic, and he did cover a lot of territory," Artin said. "One thing that was remarkable was that he had complete control of the field, which was not inhabited by slouches, for a period of 12 years or so."

During his IHÉS years, Grothendieck's devotion to mathematics was total. His tremendous energy and capacity for work, combined with a tenacious fidelity to his internal vision, produced a flood of ideas that swept many into its currents. He did not shrink from the daunting program he had set for himself, but plunged straight in, taking on tasks great and small. "His mathematical agenda was much more than any single human being could do," Bass commented. He parceled out much of the work to his students and collaborators, while also taking on a great deal himself. What motivated him, as he explained in *Récoltes et Semailles*, was simply the desire to understand, and indeed those who knew him then confirm that he was not propelled by any sense of competition. "At the time, there was never a thought of proving something before somebody else," Serre explained. And in any case, "he could not be in competition with anybody, in a sense, because he wanted to do things his own way, and essentially nobody else wanted to do the same. It was too much work."

The dominance of the Grothendieck school had some detrimental effects. Even Grothendieck's distinguished IHÉS colleague, René Thom, felt the pressure. In [Fields], Thom wrote that his relations with Grothendieck were "less agreeable" than with his other IHÉS colleagues. "His technical superiority was crushing," Thom wrote. "His seminar attracted the whole of Parisian mathematics, whereas I had nothing new to offer. That made me leave the strictly mathematical world and tackle more general notions, like morphogenesis, a subject which interested me more and led me towards a very general form of 'philosophical' biology."

In the historical remarks that appear at the end of his 1988 textbook *Undergraduate Algebraic Geometry*, Miles Reid wrote: "[T]he Grothendieck personality cult had serious side effects: many people who had devoted a large part of their lives to mastering Weil foundations suffered rejection and humiliation. …[A] whole generation of students (mainly French) got themselves brainwashed into the foolish belief that a problem that can't be dressed up in high-powered abstract formalism is unworthy of study." Such "brainwashing" was perhaps an inevitable by-product of the fashions of the times, though Grothendieck himself never pursued abstraction for abstraction's sake. Reid also noted that, apart from the small number of students of Grothendieck who could "take the pace and survive," the people who benefited most from his ideas were those influenced at a distance, particularly American, Japanese, and Russian mathematicians. Pierre Cartier sees Grothendieck's heritage in the work of such Russian mathematicians as Vladimir Drinfeld, Maxim Kontsevich, Yuri Manin, and Vladimir Voevodsky. Said Cartier, "They capture the true spirit of Grothendieck, but they are able to combine it with other things."

Photographs used in this article are courtesy of Friedrich Hirzebruch, Karin Tate, and the website of the Grothendieck Circle (`http://www.grothendieck-circle.org`).

The second part of this article will appear in the next issue of the Notices.

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Comme Appelé du Néant—As If Summoned from the Void: The Life of Alexandre Grothendieck

Allyn Jackson

This is the second part of a two-part article about the life of Alexandre Grothendieck. The first part of the article appeared in the October 2004 issue of the *Notices*.

A Different Way of Thinking

> Dans le travail de découverte, cette attention intense, cette sollicitude ardente sont une force essentielle, tout comme la chaleur du soleil pour l'obscure gestation des semences enfouies dans la terre nourricière, et pour leur humble et miraculeuse éclosion à la lumière du jour.
>
> In the work of discovery, this intense attention, this ardent solicitude, are an essential force, just like the warmth of the sun for the obscure gestation of seeds covered in nourishing soil, and for their humble and miraculous blossoming in the light of day.
>
> —*Récoltes et Semailles*, page P49

Grothendieck had a mathematical style all his own. As Michael Artin of the Massachusetts Institute of Technology commented, in the late 1950s and 1960s "the world needed to get used to him, to his power of abstraction." Nowadays Grothendieck's point of view has been so thoroughly absorbed into algebraic geometry that it is standard fare for graduate students starting in the field, many of whom do not realize that things were once quite different. Nicholas Katz of Princeton University said that when as a young mathematician he first encountered Grothendieck's way of thinking, it seemed completely different and new. But it is hard to articulate what the difference was. As Katz put it, the change in point of view was so fundamental and profound and, once adopted, so completely natural "that it's sort of hard to imagine the time before you thought that way."

Although Grothendieck approached problems from a very general point of view, he did so not for generality's sake but because he was able to use generality in a very fruitful way. "It's a kind of approach that in less gifted hands just leads to what most people would say are sterile generalities," Katz commented. "He somehow knew what general things to think about." Grothendieck always sought the precise level of generality that would provide precisely the right leverage to gain insight into a problem. "He seemed to have the knack, time after time, of stripping away just enough so that it wasn't a special case, but it wasn't a vacuum either," commented John Tate of the University of Texas at Austin. "It's streamlined; there is no baggage. It's just right."

One striking characteristic of Grothendieck's mode of thinking is that it seemed to rely so little on examples. This can be seen in the legend of the so-called "Grothendieck prime". In a mathematical conversation, someone suggested to Grothendieck that they should consider a particular prime number. "You mean an actual number?" Grothendieck asked. The other person replied, yes, an actual prime number. Grothendieck suggested, "All right, take 57."

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But Grothendieck must have known that 57 is not prime, right? Absolutely not, said David Mumford of Brown University. "He doesn't think concretely." Consider by contrast the Indian mathematician Ramanujan, who was intimately familiar with properties of many numbers, some of them huge. That way of thinking represents a world antipodal to that of Grothendieck. "He really never worked on examples," Mumford observed. "I only understand things through examples and then gradually make them more abstract. I don't think it helped Grothendieck in the least to look at an example. He really got control of the situation by thinking of it in absolutely the most abstract possible way. It's just very strange. That's the way his mind worked." Norbert A'Campo of the University of Basel once asked Grothendieck about something related to the Platonic solids. Grothendieck advised caution. The Platonic solids are so beautiful and so exceptional, he said, that one cannot assume such exceptional beauty will hold in more general situations.

One thing Grothendieck said was that one should never try to prove anything that is not almost obvious. This does not mean that one should not be ambitious in choosing things to work on. Rather, "if you don't see that what you are working on is almost obvious, then you are not ready to work on that yet," explained Arthur Ogus of the University of California at Berkeley. "Prepare the way. And that was his approach to mathematics, that everything should be so natural that it just seems completely straightforward." Many mathematicians will choose a well-formulated problem and knock away at it, an approach that Grothendieck disliked. In a well-known passage of *Récoltes et Semailles*, he describes this approach as being comparable to cracking a nut with a hammer and chisel. What he prefers to do is to soften the shell slowly in water, or to leave it in the sun and the rain, and wait for the right moment when the nut opens naturally (pages 552–553). "So a lot of what Grothendieck did looks like the natural landscape of things, because it looks like it grew, as if on its own," Ogus noted.

Grothendieck had a flair for choosing striking, evocative names for new concepts; indeed, he saw the act of naming mathematical objects as an integral part of their discovery, as a way to grasp them even before they have been entirely understood (*R&S*, page P24). One such term is *étale*, which in French is used to describe the sea at slack tide, that is, when the tide is neither going in nor out. At slack tide, the surface of the sea looks like a sheet, which evokes the notion of a covering space. As Grothendieck explained in *Récoltes et Semailles*, he chose the word *topos*, which means "place" in Greek, to suggest the idea of "the 'object *par excellence*' to which topological intuition applies" (pages 40–41). Matching the concept, the word *topos* suggests the most fundamental, primordial notion of space. The term *motif* ("motive" in English) is intended to evoke both meanings of the word: a recurrent theme and something that causes action.

Grothendieck lecturing at the IHÉS.

Grothendieck's attention to choosing names meant that he loathed terminology that seemed unsuitable: In *Récoltes et Semailles*, he said he felt an "internal recoiling" upon hearing for the first time the term *perverse sheaf*. "What an idea to give such a name to a mathematical thing!" he wrote. "Or to any other thing or living being, except in sternness towards a person—for it is evident that of all the 'things' in the universe, we humans are the only ones to whom this term could ever apply" (page 293).

Although Grothendieck possessed great technical power, it was always secondary; it was a means for carrying out his larger vision. He is known for certain results and for developing certain tools, but it is his creation of a new viewpoint on mathematics that is his greatest legacy. In this regard, Grothendieck resembles Evariste Galois; indeed, in various places in *Récoltes et Semailles* Grothendieck wrote that he strongly identified with Galois. He also mentioned that as a young man he read a biography of Galois by Leopold Infeld [Infeld] (page P63).

Ultimately, the wellspring of Grothendieck's achievement in mathematics is something quite humble: his love for the mathematical objects he studied.

A Spirit in Stagnation

> [P]endant vingt-cinq ans, entre 1945 (quand j'avais dix-sept ans) et 1969 (quand j'allais sur les quarante-deux), j'ai investi pratiquement la totalité de

> mon énergie dans la recherche mathématique. Investissement démesuré, certes. Je l'ai payé par une longue stagnation spirituelle, par un "épaississement" progressif, que j'aurai plus d'une fois l'occasion d'évoquer dans les pages de *Récoltes et Semailles*.
>
> [F]or twenty-five years, between 1945 (when I was seventeen years old) and 1969 (when I reached forty-two), I invested practically my entire energy into mathematical research. An excessive investment, certainly. I paid for it with a long spiritual stagnation, with a progressive "dulling", that I have more than once found occasion to evoke in the pages of *Récoltes et Semailles*.
>
> —*Récoltes et Semailles*, page P17

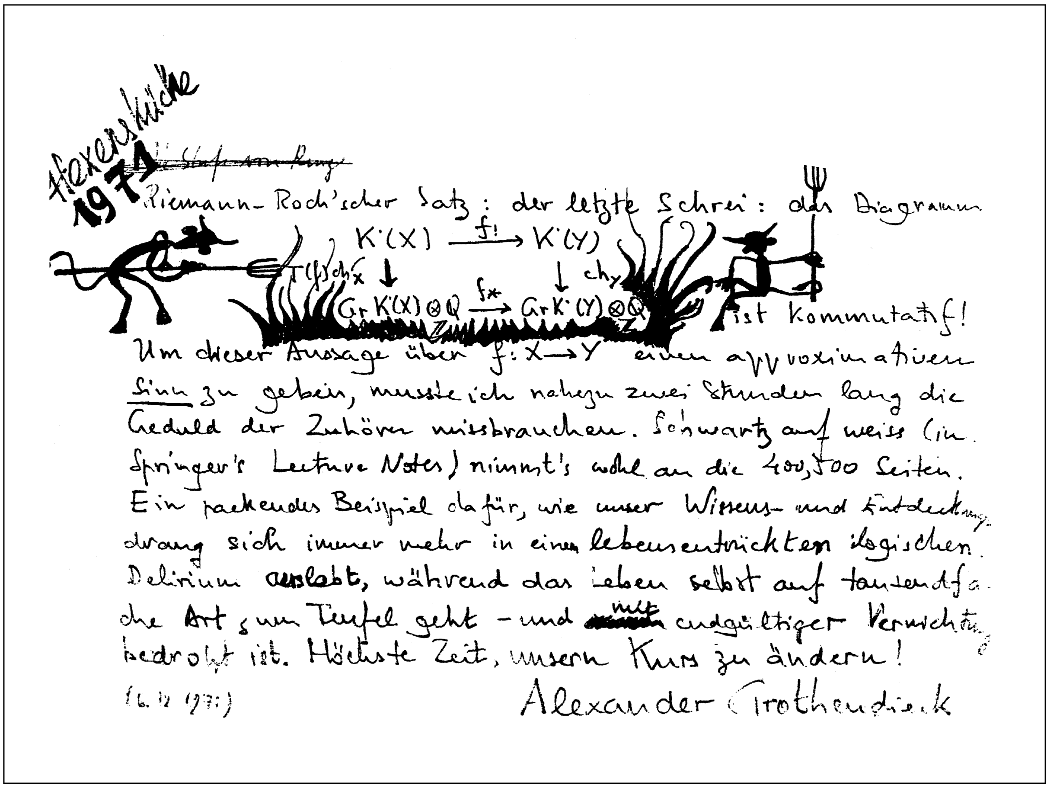
During the 1960s, Barry Mazur of Harvard University visited the Institut des Hautes Études Scientifiques (IHÉS) with his wife. Although by that time Grothendieck had a family and a house of his own, he also kept an apartment in the same building where the Mazurs were living and frequently worked there late into the night. Because the apartment keys did not open the outside doors, which were locked at 11:00 p.m., one might have trouble getting into the building after an evening in Paris. But "I remember we never had any problems," Mazur recalled. "We would take the last train back, absolutely certain that there would be Grothendieck working, his desk by the window. We would throw some gravel at his window and he would open the outside door for us." Grothendieck's apartment was sparely furnished; Mazur remembered a wire sculpture in the outline of a goat and an urn filled with Spanish olives.

This somewhat lonely image of Grothendieck working away into the night in a spartan apartment captures one aspect of his life during the 1960s. At this time he did mathematics nonstop. He was talking to colleagues, advising students, lecturing, carrying on extensive correspondence with mathematicians outside of France, and writing the seemingly endless volumes of *EGA* and *SGA*. It is no exaggeration to say that he was single-handedly leading a large and thriving segment of worldwide research in algebraic geometry. He seemed to have few interests outside of mathematics; colleagues have said that he never read a newspaper. Even among mathematicians, who tend to be single-minded and highly devoted to their work, Grothendieck was an extreme case. "Grothendieck was working on the foundations of algebraic geometry seven days a week, twelve hours a day, for ten years," noted his IHÉS colleague David Ruelle. "He had achieved level -1 and was working on level 0 of something that must be 10 levels high.…At a certain age it becomes clear you will never be able to finish the building."

The extremity of Grothendieck's focus on mathematics is one reason for the "spiritual stagnation" he referred to in *Récoltes et Semailles*, which in turn is one of the reasons behind his departure, in 1970, from the world of mathematics in which he had been a leading figure. One step toward that departure was a crisis within the IHÉS, which led to his resignation. Starting in late 1969, Grothendieck became embroiled in a conflict with the founder and director of the IHÉS, Léon Motchane, over military funding for the institute. As historian of science David Aubin explained [Aubin], during the 1960s, the IHÉS finances were rather precarious, and in some years the institute received a small portion of its budget, never more than about 5 percent, from sources within the French military. All of the permanent IHÉS professors had misgivings about military funding, and in late 1969 they insisted that Motchane quit accepting such funding. Motchane agreed, but, as Aubin noted, he went back on his word just a few months later, when the IHÉS budget was stretched thin and he accepted a grant from the minister of the army. Outraged, Grothendieck tried in vain to persuade the other professors to resign along with him, but none did. Less than a year earlier, Pierre Deligne had joined the IHÉS faculty as a permanent professor, largely on the recommendation of Grothendieck, who now pressed his newly appointed colleague to join him in resigning. Deligne too refused. "Because I was very close to him mathematically, Grothendieck was surprised and deeply disappointed that this closeness of ideas did not extend outside of mathematics," Deligne recalled. Grothendieck's letter of resignation was dated May 25, 1970.

His rupture with the IHÉS was the most visible sign of a profound shift taking place in Grothendieck's life. Toward the end of the 1960s there were other signs as well. Some were small. Mazur recalled that when he was visiting the IHÉS in 1968, Grothendieck told him he had gone to the movies—for the first time in perhaps a decade. Other signs were larger. In 1966, when he was to receive the Fields Medal at the International Congress of Mathematicians (ICM) in Moscow, Grothendieck refused to attend as a protest against the Soviet government. In 1967 Grothendieck made a three-week trip to Vietnam, which clearly left an impression on him. His written account of the trip [Vietnam] described the many air raid alerts and a bombing that left two mathematics teachers dead, as well as the valiant efforts of the Vietnamese to cultivate a mathematical life in their country. A friendship with a Romanian physician named Mircea Dumitrescu led Grothendieck to make in the late 1960s a fairly serious foray into learning some biology. He also discussed physics with Ruelle.

The events of the extraordinary year of 1968 must also have had an impact on Grothendieck. That year saw student protests and social upheavals all over the world, as well as the Soviet Union's brutal crushing of the "Prague Spring". In France the boiling point came in May 1968, when students objecting to university and government policies carried out massive protests that soon turned into riots. In Paris hundreds of thousands of students, teachers, and workers took to the streets to protest police brutality, and the French government, fearing revolution, stationed tanks around the perimeter of the city. Millions of workers went on strike, paralyzing the nation for about two weeks. Karin Tate, who was living in Paris with her husband at the time, John Tate, recalled the chaos that reigned. "Paving stones, batons, and any other missiles that were handy flew through the air," she said. "Soon the entire country was at a standstill. There was no gasoline (truckers were on strike), there were no trains (train workers were on strike), garbage was piling up in Paris (sanitation workers were on strike), there was very little food on the shelves." She and John fled to Bures-sur-Yvette, where her brother, Michael Artin, was visiting the IHÉS. Many Parisian mathematicians took the side of the students in the conflict. Karin Tate said the protests dominated conversations among the mathematicians she knew, though she did not remember discussing the topic with Grothendieck.



Grothendieck wrote this abstract into the colloquium book at the Universität Bielefeld when he spoke there in 1971. The abstract says: "Witch's Kitchen 1971. Riemann-Roch Theorem: The 'dernier cri': The diagram [displayed] is commutative! To give an approximate sense to the statement about $f : X \to Y$, I had to abuse the listeners' patience for almost two hours. Black on white (in Springer Lecture Notes) it probably takes about 400, 500 pages. A gripping example of how our thirst for knowledge and discovery indulges itself more and more in a logical delirium far removed from life, while life itself is going to Hell in a thousand ways—and is under the threat of final extermination. High time to change our course!"

Shortly after his resignation from the IHÉS, Grothendieck plunged into a world completely new to him, the world of protest politics. In a June 26, 1970, lecture at the Université de Paris in Orsay, he spoke not about mathematics but about the threat of nuclear proliferation to the survival of humankind and called upon scientists and mathematicians not to collaborate in any way with the military. Nicholas Katz, who had recently arrived for a visit at the IHÉS and was surprised to hear of Grothendieck's resignation, attended the lecture, which he said drew an audience of hundreds in a very crowded lecture hall. Katz remembered that in the lecture Grothendieck went so far as to say that doing mathematical research was actually "harmful" (*"nuisible"*), given the impending threats to the human race.

A written version of the lecture, "Responsabilité du savant dans le monde d'aujourd'hui: Le savant et l'appareil militaire" ("The responsibility of the scholar in today's world: The scholar and the military apparatus"), circulated as an unpublished manuscript. An appendix described the hostile reactions of the students who attended the lecture and who handed out flyers mocking Grothendieck. One of the flyers is reproduced in the appendix; a typical slogan: "Réussissez, ossifiez-vous, détruisez-vous vous-mêmes: devenez un petit schéma télécommandé par Grothendieck" ("Succeed, ossify, self-destruct: become a little scheme remote-controlled by Grothendieck"). He was clearly seen as a detested member of the establishment.

In another appendix in this manuscript, Grothendieck called for the founding of a group to fight

Tata Institute International Colloquium in 1968. Grothendieck (standing, left) and Armand Borel (seated, facing camera).

for the survival of the human race against environmental degradation and the dangers of military conflict. This group, called "Survival" ("Survivre et Vivre" in French) came into being in July 1970 when Grothendieck delivered his Orsay lecture a second time, at a summer school on algebraic geometry at the University of Montreal. The main activity of Survival was the publication of a newsletter by the same name, the first issue of which was written in English by Grothendieck and is dated August 1970. The newsletter describes an ambitious agenda of publication of books on science, organization of public courses on science aimed at nonexperts, and boycotts of scientific institutions that accept military funds.

That first issue carried a list of the names, professions, and addresses of the group's members, who numbered twenty-five at the time. On the list were several mathematicians, Grothendieck's mother-in-law, and his son Serge. The directors of the group were Grothendieck and three other mathematicians: Claude Chevalley, Denis Guedj, and Pierre Samuel (*R&S*, page 758). Survival was one of many leftist groups that emerged in the wake of the tumultuous 1960s; a similar organization in the United States was the Mathematics Action Group. Too small and diffuse to accumulate much influence, Survival was more active in Paris than in the United States and Canada, due mostly to Grothendieck's presence. When he moved out of Paris in 1973, the group petered out.

At the ICM in Nice in the summer of 1970, Grothendieck tried to recruit members for Survival. He wrote, "I expected massive enrollments—there were (if I remember correctly) two or three" (*R&S*, page 758). Nevertheless, his proselytizing drew a good deal of attention. "First of all, he was one of the world stars in mathematics at that time," said Pierre Cartier of the IHÉS, who attended the congress. "Also, you have to remember the political climate at the time." Many mathematicians opposed the Vietnam War and sympathized with Survival's antimilitary stance. During the congress, Cartier said, Grothendieck sneaked a table in between two publishers' booths in the exhibit area and, assisted by his son Serge, began to hand out the Survival newsletter. This caused a heated row between him and his old colleague and friend, Jean Dieudonné, who had become the first dean of the science faculty at the Université de Nice when it was founded in 1964 and who was responsible for the ICM being held there. Cartier said that he and others tried unsuccessfully to persuade Dieudonné to permit this "unofficial booth". Eventually Grothendieck took the table out to the street in front of the hall in which the congress was being held. But another problem loomed: in delicate negotiations with the mayor of Nice, the congress organizers had promised there would be no street demonstrations. Police officers began to question Grothendieck, and finally the chief of police showed up. Grothendieck was asked to move his table just a few yards back so that it was off the sidewalk. "But he refused," Cartier recalled. "He wanted to be put in jail. He really wanted to be put in jail!" Finally, Cartier said, he and some others moved the table back sufficiently to satisfy the police.

Although Grothendieck's plunge into politics was sudden, he was by no means alone. His good friend Cartier has a long history of political activism. For example, he was among the mathematicians who used the holding of the ICM in Warsaw in 1983 to negotiate the release of one hundred fifty political prisoners in Poland. Cartier traces his activism to the example set by his teacher and mentor, Laurent Schwartz, who was one of the most politically vocal and active academics in France. Schwartz was the thesis adviser of Grothendieck. Another mathematician Grothendieck knew well, Pierre Samuel, is one of the founders of the French Green Party. Outside of France, many mathematicians were politically active. Among the best-known examples in North America are Chandler Davis and Stephen Smale, who were deeply involved in protests against the Vietnam War.

But despite his strong convictions, Grothendieck was never effective in the real world of politics. "He was always an anarchist at heart," Cartier observed. "On many issues, my basic positions are not very far from his positions. But he was so naive that it was totally impossible to do anything with him politically." He was also rather ignorant. Cartier recalled that, after an inconclusive presidential election in France in 1965, the newspapers carried headlines saying that de Gaulle had not been elected. Grothendieck asked if this meant that France would no longer have a president. Cartier had to explain to him what a runoff election is. "Grothendieck was politically illiterate," Cartier said. But he did want to help people: it was not unusual for Grothendieck to give shelter for a few weeks to homeless people or others in need. "He was very generous, he has always been very generous," Cartier said. "He remembered his youth, his difficult youth, when his mother had nothing, and he was always ready to help—but in a nonpolitical way."

The Wild '70s

> [In 1970 J]'ai alors quitté un milieu pour entrer dans un autre—le milieu des gens "des premiers rangs" pour le "marais"; soudain, la plupart de mes nouveaux amis étaient de ceux justement qu'un an avant encore j'aurais tacitement situés dans cette contrée sans nom et sans contours. Le soi-disant marais soudain s'animait et prenait vie par les visages d'amis liés à moi par une aventure commune—une autre aventure!
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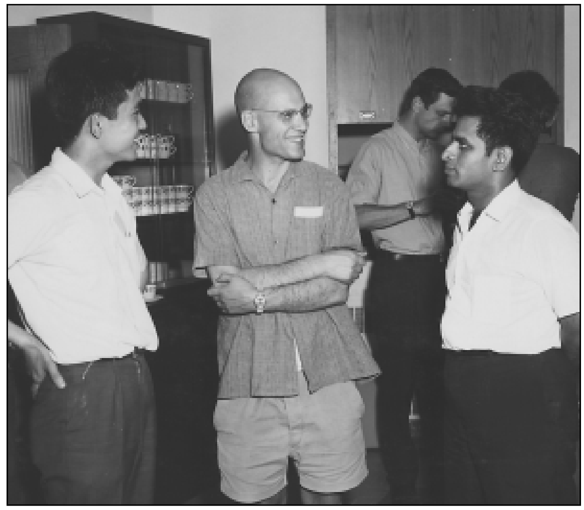
> [In 1970] I left one milieu to enter another—the milieu of people "of the first rank" for the "swamp"; suddenly, the majority of my new friends were those who just a year before I had tacitly situated in this region without name and without shape. The so-called swamp suddenly moved around and took on life through the faces of friends tied to me by a common adventure—another adventure!
>
> —*Récoltes et Semailles*, page 38

"Légion d'Honneur! Légion d'Honneur!" Grothendieck was shouting from the back of the auditorium, waving a paper facsimile of the Légion d'Honneur cross, a distinction conferred by the French government. The scene was the opening day of a summer school on modular functions, held in Antwerp in the summer of 1972 and supported by the North Atlantic Treaty Organization (NATO). Grothendieck's longtime friend Jean-Pierre Serre of the Collège de France, who had recently received the Légion d'Honneur, was presenting the opening speech. Grothendieck approached Serre and asked, "Do you mind if I go to the blackboard and say something?" Serre replied, "Yes, I mind" and left the room. Grothendieck then mounted the podium and began speaking against NATO support for the conference. Other mathematicians sympathized with this view: One example was Roger Godement, who in April 1971 wrote an open letter explaining his reasons for refusing to attend the conference.



Grothendieck, center, University of Montreal, around 1970.

Unbeknownst to Grothendieck, Cartier and some other mathematicians who were uncomfortable about the NATO support had conducted extensive negotiations to have a NATO representative come to the conference for a public debate. Cartier and others eased Grothendieck off the podium, but the damage had been done: Cartier soon received an angry phone call from the NATO representative, who had heard about the outburst and refused to come, believing that conditions for an orderly debate had been ruined. "To me, it was sad, because from what I remember, I think that the audience was mostly on Grothendieck's political side," Cartier noted. "Even people who were close to his political views or his social views were antagonized by his behavior.…He behaved like a wild teenager."

By the time of the Antwerp meeting, Grothendieck had cut many of the ties that had bound him to an orderly life focused on mathematics. For one thing, he no longer had a permanent position. After he left the IHÉS in 1970, Serre arranged for him to have a visiting position at the Collège de France for two years. This elite institution operates differently from other universities in France (or anywhere else for that matter). Each professor at the Collège must submit for approval by the assembly of all the professors a program of the lectures he or she plans to deliver during the year. Serre recalled that Grothendieck offered two possible programs: one on mathematics and one on the political themes that occupied the Survival group. The committee approved the mathematical program and rejected the other one. So Grothendieck presented mathematical lectures prefaced by long discourses about politics. After two years he applied for a permanent position at the Collège de France, a position that had become vacant with the retirement of Szolem Mandelbrojt. The curriculum vitae

A. Grothendieck with children Serge (left) and Johanna in 1960.

Grothendieck submitted plainly showed that he intended to give up mathematics to focus on tasks he believed to be far more urgent: “the imperatives of survival and the promotion of a stable and humane order on our planet.” How could the Collège appoint to a position in mathematics someone who had declared that he would no longer do any mathematics? “He was rightly refused,” Serre said.

It was also during the period just after he left the IHÉS that Grothendieck’s family life crumbled and he and his wife separated. In the two years after he left the IHÉS, Grothendieck spent a fair amount of time lecturing in mathematics departments in North America. He spread the gospel of Survival by insisting he would give a mathematics lecture only if arrangements were made for him also to give a political lecture. On one such trip in May 1972, he visited Rutgers University and met Justine Bumby (Skalba), then a graduate student of Daniel Gorenstein. Captivated by Grothendieck’s charismatic personality, Bumby left behind her life as a graduate student to follow him, first on the remainder of his trip in the United States, and then on to France, where she lived with him for two years. “He’s the most intelligent person I’ve ever met,” she said. “I was very much in awe of him.”

Their life together was in some ways emblematic of the counterculture years of the 1970s. Once, at a peaceful demonstration in Avignon, the police intervened, harassing and pushing away the demonstrators. Grothendieck got angry when they started pestering him, Bumby recalled. “He was a good boxer, so he was very fast,” she said. “We see the policemen approaching us, and we are all scared, and then the next thing we know, the two policemen are on the ground.” Grothendieck had single-handedly decked two police officers. After some other officers had subdued him, Bumby and Grothendieck were bundled into a wagon and taken to the police station. When his identification papers revealed that he was a professor at the Collège de France, the two were taken in to see the chief of police, who spoke to them in English, as Bumby spoke no French. After a short conversation, in which the police chief expressed his desire to avoid trouble between police and professors, the two were released and no charges were brought.

Shortly after Bumby came to France with Grothendieck, he started a commune in a large house he had rented just south of Paris in Chatenay-Malabry, and they lived there together. She said he sold organically grown vegetables and sea salt out of the basement of the house. The commune was a bustling place: Bumby said that Grothendieck held meetings, which might attract up to a hundred people, about the issues raised in the Survival group, and these attracted considerable media attention. However, the commune dissolved fairly rapidly as a result of complicated personal relationships among the members. It was around this time that Grothendieck’s position ended at the Collège de France, and in the fall of 1972 he took a temporary position teaching for one year at the Université de Paris in Orsay. After that, Grothendieck obtained a position called *professeur à titre personnel*, which is attached to a single individual and can be taken to any university in France. Grothendieck took his to the Université de Montpellier, where he was to remain until his retirement in 1988.

In early 1973 he and Bumby moved to Olmet-le-sec, a rural village in the south of France. This area was at the time a magnet for hippies and others in the counterculture movement who wanted to return to a simpler lifestyle close to the land. Here Grothendieck again attempted to start up a commune, but personality conflicts led to its collapse. At various times three of Grothendieck’s children came to live in the Paris commune and in the one in Olmet. After the latter commune dissolved, he moved with Bumby and his children to Villecun, a short distance away. Bumby noted that Grothendieck had a hard time adjusting to the ways of the people attracted to the counterculture movement. “His students in mathematics had been very serious, and they were very disciplined, very hardworking people,” she said. “In the counterculture he was meeting people who would loaf around all day listening to music.” Having been an undisputed leader in mathematics, Grothendieck now found himself in a very different milieu, in which his views were not always taken seriously. “He was used to people agreeing with his opinions when he was doing algebraic geometry,” Bumby remarked. “When he switched to politics all the people who would have agreed with him before suddenly disagreed with him.... It was something he wasn’t used to.”

Although most of the time Grothendieck was very warm and affectionate, Bumby said, he sometimes had violent outbursts followed by periods of silent withdrawal. There were also disturbing episodes in which he would launch into a monologue in German, even though she understood no German. "He would just go on as if I wasn't there," she said. "It was kind of scary." He was frugal, sometimes compulsively so: one time, to avoid throwing away three quarts of leftover coffee, he drank it—with the predictable result that he got quite sick afterward. Bumby said she believes that his speaking German and his extreme frugality may have been connected psychologically to the hardships he endured as a child, especially the time when he lived with his mother in the internment camps.

Grothendieck may have been experiencing some kind of psychological breakdown, and Bumby today wonders whether she should have sought treatment for him. Whether he would have submitted to such treatment is unclear. They parted ways not long after their son, John, was born in the fall of 1973. After spending some time in Paris, Bumby moved back to the United States. She married a mathematician who was a widower, Richard Bumby of Rutgers University, and they raised John and Richard's two daughters. John exhibited a good deal of mathematical talent and was a mathematics major at Harvard University. He recently finished his Ph.D. in statistics at Rutgers. Grothendieck has had no contact with this son.

During the early 1970s, Grothendieck's interests were very far from those of the mathematical world he had left behind. But that world intruded in a major way in the summer of 1973, when, at a conference in honor of W. V. D. Hodge in Cambridge, England, Pierre Deligne presented a series of lectures about his proof of the last and most stubborn of the Weil conjectures. Grothendieck's former student Luc Illusie was at the conference and wrote to him with the news. Wanting to know more, Grothendieck, accompanied by Bumby, visited the IHÉS in July 1973.

In 1959 Bernard Dwork proved by p-adic methods the first Weil conjecture (which says that the zeta function of a variety over a finite field is a rational function). Grothendieck's 1964 l-adic proof of this conjecture was more general and introduced his "formalism of the six operations." In the 1960s Grothendieck also proved the second Weil conjecture (which says that the zeta function of a variety satisfies a functional equation). Finding a way to prove the last Weil conjecture (sometimes called the "congruence Riemann Hypothesis") was a major inspiration for much of his work. He formulated what he called the "standard conjectures," which, if they could be proved, would imply all of the Weil conjectures. The standard conjectures were also formulated independently around the same time by Enrico Bombieri. To this day, the standard conjectures remain inaccessible. Deligne found a clever way to circumvent them when he proved the last Weil conjecture. One of the key ideas he used came from a paper by R. A. Rankin [Rankin], which is about the classical theory of modular forms and of which Grothendieck was unaware. As John Tate put it, "For the proof of the last Weil conjecture, you needed another ingredient that was more classical. That was Grothendieck's blind spot."

When Bumby and Grothendieck turned up at the IHÉS that summer, among the visitors was William Messing of the University of Minnesota. Messing first met Grothendieck in 1966, when as a graduate student at Princeton he attended a series of lectures Grothendieck gave at Haverford College. These lectures made a deep impression on Messing, and Grothendieck became his informal thesis adviser. In 1970 Messing joined the Survival group at the Montreal meeting at which it was founded. The following year, while Grothendieck was visiting Kingston University in Ontario, he and Messing made a car trip to visit Alex Jameson, an Indian activist living on a reservation near Buffalo, New York. Grothendieck was pursuing a quixotic hope of helping the Indians resolve a dispute over a land treaty.

In the summer of 1973 Messing was living in a small studio in the Ormaille, the housing complex for IHÉS visitors. Excitement was bubbling among the mathematicians over Deligne's breakthrough. "Grothendieck was with Justine," Messing recalled. "They came for dinner, and Katz and I spent the evening explaining to Grothendieck the main new and different things in Deligne's proof of the last of the Weil conjectures. He was pretty excited." At the same time, Grothendieck expressed disappointment that the proof bypassed the question of whether or not the standard conjectures were true. "I think he certainly would have been very happy to have proven [all the Weil conjectures] himself," Katz remarked. "But in his mind, the Weil conjectures were important because they were the tip of the iceberg reflecting some fundamental structures in mathematics that he wanted to discover and develop." A proof of the standard conjectures would reveal that structure in a much deeper way.

Later during that visit Grothendieck also met with Deligne to discuss the proof. Deligne recalled that Grothendieck was not as interested in the proof as he would have been had it used the theory of motives. "If I had done it using motives, he would have been very interested, because it would have meant the theory of motives had been developed," Deligne remarked. "Since the proof used a trick, he did not care." In trying to develop the theory of motives, Grothendieck had run into a major technical difficulty. "The most serious problem was that, for his idea of motives to work, one had to be able to construct enough algebraic cycles," Deligne explained. "I think he tried very hard and he failed. And since then nobody has been able to succeed." According to Deligne, this technical obstacle to developing the theory of motives was probably far more frustrating to Grothendieck than his inability to prove the last Weil conjecture.

A Distant Voice

> [J]'ai quitté "le grand monde" mathématique en 1970.…Après quelques années de militantisme anti-militariste et écologique, style "révolution culturelle", dont tu as sans doute eu quelque écho ici et là, je disparais pratiquement de la circulation, perdu dans une université de province Dieu sait où. La rumeur dit que je passe mon temps à garder des moutons et à forer des puits. La vérité est qu'à part beaucoup d'autres occupations, j'allais bravement, comme tout le monde, faire mes cours à la Fac (c'était là mon peu original gagne-pain, et ça l'est encore aujourd'hui).
>
> I left "the great world" of mathematics in 1970.…After several years of anti-military and ecological militancy, "cultural revolution"-style, of which you have no doubt heard an echo here and there, I just about disappeared from circulation, lost in a university in some province, God knows where. Rumor has it that I pass my time tending sheep and drilling wells. The truth is that apart from many other occupations, I bravely went, like everyone else, to teach my courses in the Department (this was the way I originally earned my bread, and it's the same today).
>
> —*Récoltes et Semailles*, page L3

When Grothendieck came to the Université de Montpellier in 1973, Yves Ladegaillerie, then twenty-five years old, was a *maître des conferences* there, having finished his doctorate at the Institut Henri Poincaré in Paris three years earlier. Grothendieck proposed that Ladegaillerie do a *thèse d'état* with him in topology and spent a great deal of time initiating the younger mathematician into his vision and methods. In a brief memoir about Grothendieck, Ladegaillerie wrote: "I had had as professors in Paris some of the great mathematicians of the day, from Schwartz to Cartan, but Grothendieck was completely different, an extra-terrestrial. Rather than translating things into another language, he thought and spoke directly in the language of modern structural mathematics, to whose creation he had contributed greatly" [Ladegaillerie]. Once, in order to verify a certain algebraic computation involving braids, Ladegaillerie made a little model using some string and a small plank with holes. This made Grothendieck laugh out of sheer delight: "At that moment, he was like a child before a wizard who performed a trick, and he told me: 'I would never have thought of doing that'."

Grothendieck lived an ascetic, unconventional life in an old house without electricity in Villecun, about thirty-five miles outside of Montpellier. Ladegaillerie remembered seeing Justine Bumby and her baby there, though she soon was gone. Many friends, acquaintances, and students went to visit Grothendieck, including people from the ecology movement. In 1974 the leader of a group of Buddhist missionaries from Japan came to visit Grothendieck, and after that many other adherents of Buddhism passed through his home (*R&S*, page 759). Once, after being host to a Buddhist monk whose travel documents were not in order, Grothendieck became the first person in France ever to be charged under an obscure 1949 law against "gratuitously lodging and feeding a stranger in an irregular situation" (*R&S*, page 53). As someone who had been stateless all his life, Grothendieck was outraged at the charge and tried to launch a campaign against it. He even traveled to Paris to speak about it at a Bourbaki seminar. His campaign made headlines in French national newspapers. Ultimately he paid a fine and received a suspended sentence.

It was around this time that Grothendieck learned to drive. He had an ancient Citroën of a model called 2CV and known informally as a *deux chevaux*. One of his students, Jean Malgoire, now a *maître des conferences* at Montpellier, recalled a terrifying journey through a torrential rainstorm with Grothendieck at the wheel. In addition to being a poor driver, Grothendieck was far more occupied with the discourse he was presenting to his passengers than with the condition of the road. "I was sure we would never get there alive!" Malgoire said. "I understood then that Alexandre had a very special relationship with reality.… Rather than adapting to what was real, he believed that reality would adapt itself to him." One time, while driving a moped, Grothendieck collided head-on with an automobile. According to Ladegaillerie, he had turned his eyes from the road to get an apricot out of a bag that was behind him. Although he had a leg fracture serious enough to require surgery, he requested acupuncture as the only anesthetic. He agreed to take antibiotics only when the surgeon told him that the alternative was to amputate the broken leg.

At the Université de Montpellier, Grothendieck had a regular faculty position and taught at all levels. Although the students were not as strong as the ones he had had in Paris, he nevertheless poured a great deal of energy, enthusiasm, and patience into his teaching. He had an unconventional teaching style. For an examination involving polyhedra, he had students submit paper-and-glue models, much to the dismay of those who had to shepherd the exam papers through the grading process. One person who took undergraduate courses from him at Montpellier is Susan Holmes, now a statistician at Stanford University. "I found him very inspiring, as he was both unconventional and kind to the students, who really didn't understand at all that he was a great mathematician," she recalled. He showed up in the worn-out attire of a hippie and distributed his homegrown organic apples in class. "He definitely did not explain in a linear fashion suited to undergraduates, but his teaching was very inspiring, and one got the impression of some wonderful mysterious 'big picture'," Holmes said.

Grothendieck was never one for reading as a way to learn about and understand mathematics. Talking to others had always been his primary way of finding out what was going on in the field. His departure from the intense, stimulating atmosphere of the IHÉS, where oral exchanges were his primary mode of communication about mathematics, was an enormous change for him. Compared with the pace he kept during the 1960s, Grothendieck's later mathematical work was sporadic. Although he had several Ph.D. students at Montpellier, he did not establish anything like the thriving school he had headed at the IHÉS. Some of Grothendieck's former students and colleagues from his Paris days traveled to Montpellier to visit him. The most frequent of these visitors was Deligne, who during the 1970s was the main person keeping Grothendieck aware of new developments.

At Montpellier, Grothendieck did not have a seminar that met consistently. He formed a small working group with Ladegaillerie, Malgoire, and some of his other students, but according to Ladegaillerie it never really got off the ground. During 1980–81, he ran a seminar, whose sole attendee was Malgoire, on relations between Galois groups and fundamental groups. This is the subject of his 1,300-page manuscript *La Longue Marche à Travers la Théorie de Galois* (*The Long March through Galois Theory*), completed in 1981. Grothendieck did not publish *La Longue Marche*, but through Malgoire's efforts part of it was published in 1995 by the Université de Montpellier [Marche]. There was also a small working seminar in which Ladegaillerie gave some lectures on William Thurston's work on Teichmüller spaces, which stimulated Grothendieck's interest in this subject.

By the 1980s Grothendieck felt he had done all he could in trying to motivate the less-than-enthusiastic students at Montpellier and decided to apply for a position as a researcher in the Centre National de la Recherche Scientifique (CNRS). The CNRS, an agency of the French government, employs mathematicians and scientists to do research. Based at universities or research institutions, CNRS positions usually entail no teaching. In the 1950s, before he went to the IHÉS, Grothendieck had held a CNRS position. In the 1970s he applied to reenter the CNRS but was turned down. At that time, Michel Raynaud of the Université de Paris-Orsay was on the committee of mathematicians that reviewed CNRS applications. Raynaud said the CNRS administration had been hesitant to take Grothendieck on, arguing that it was unclear whether he would continue doing mathematics. The committee could not contradict this argument, and the application was turned down.

When Grothendieck reapplied to the CNRS in 1984, his application was once again controversial. Jean-Pierre Bourguignon, now director of the IHÉS, chaired the committee in charge of reviewing applications in mathematics, among which was Grothendieck's. According to Bourguignon, in the handwritten letter required for the application, Grothendieck listed several tasks he would not perform, such as supervising research students. Because CNRS contracts obligate researchers to perform some of these tasks, this letter was viewed by the CNRS administration as proof of Grothendieck's ineligibility. Bourguignon said he tried to get Grothendieck to amend his application so that it did not state explicitly all the tasks he refused to carry out, but Grothendieck would not budge. After considerable effort on the part of several people, Grothendieck was eventually put on a special kind of position, called a *position asterisquée*, that was acceptable to him and to the CNRS. The CNRS did not actually hire him but was in charge only of paying his salary, and he retained his university affiliation. So for his last few years at Montpellier before his retirement in 1988, Grothendieck did not teach and spent less and less time at the university.

The mathematical part of Grothendieck's 1984 application to the CNRS was the now-famous manuscript *Esquisse d'un Programme*. In it he outlines, in a somewhat mysterious but nevertheless penetrating and visionary fashion, a new area that he called "anabelian algebraic geometry". He also muses on the inadequacy of general topology and presents ideas for a renewal in the form of what he called "tame topology". The *Esquisse* also contains his ideas about *dessins d'enfants*, which he originally developed in order to have a simple way of explaining to students some notions in algebraic geometry and which have since spawned a good deal of research. Grothendieck sent the *Esquisse* to mathematicians who he thought might take an interest in it, and the manuscript circulated unpublished for several years.

Leila Schneps of the Université de Paris VI read the *Esquisse* in 1991. Before that she had identified

Grothendieck with the foundational works of *EGA* and *SGA*, and she found that the *Esquisse* was completely different. "It was a wild expression of mathematical imagination," she recalled. "I loved it. I was bowled over, and I wanted to start working on it right away." She became an enthusiastic evangelist for the research program described in the *Esquisse*, and she and others have made a good deal of progress on it. She said, "Some of it doesn't even seem to make sense at first, but then you work for two years, and you go back and look at it, and you say, 'He *knew* this'." She edited a book on *dessins d'enfants*, which appeared in 1994 [Schneps1], and in 1995 she and Pierre Lochak, also of the Université de Paris VI, organized a conference around the *Esquisse*. The *Esquisse* appeared for the first time in print in the proceedings of that conference [Schneps2].

Aside from the *Esquisse* and *La Longue Marche*, Grothendieck wrote at least one other mathematical work during the 1980s. *À la Poursuite des Champs* (*Pursuing Stacks*), which runs 1,500 pages, began as a letter to Daniel Quillen of the University of Oxford. Completed in 1983, it sketches Grothendieck's vision of a synthesis of homotopical algebra, homological algebra, and topos theory. *À la Poursuite des Champs* circulated widely among mathematicians but was never published. Although its topic is mathematics, the style of *À la Poursuite des Champs* is completely different from the style of his earlier mathematical writings. It was written as a sort of "log book" on a mathematical voyage of discovery, which includes all the false starts, wrong turns, and sudden inspirations that characterize mathematical discovery but that are typically omitted from written mathematical works. When nonmathematical matters drew his attention, they become part of the log book too: for example, *À la Poursuite des Champs* contains a digression about the birth of one of his grandchildren. During the 1990s he wrote a 2,000-page mathematical work on the foundations of homotopy theory called *Les Dérivateurs*, which he gave to Malgoire in 1995 and which is now being made available on the Web [Deriv].

While he was at Montpellier, Grothendieck's uncompromising, "anti-establishment" bent seems to have become more pronounced. After Ladegaillerie's thesis was finished, Grothendieck wrote to Springer-Verlag to suggest that it be published in the Lecture Notes series. He was outraged when he received the reply that the series no longer published theses. The thesis was submitted for publication anyway, with the predictable result that it was rejected. According to Ladegaillerie, Grothendieck wrote letters about this to colleagues, in an effort to build a campaign against Springer. Ladegaillerie decided to publish his thesis in the form of several papers rather than as a whole, and the main part appeared in *Topology*. Grothendieck reproached him for having cut the work into publishable pieces. As Ladegaillerie put it, Grothendieck tried to enlist him in his "fight against the establishment," but Ladegaillerie resisted, believing that such a fight was unreasonable and unjustified.

"Despite such disagreements, we have stayed friends, with highs and lows," Ladegaillerie said. Of his work with Grothendieck, Ladegaillerie said, "It was fascinating to work with a genius. I don't like this word, but for Grothendieck there is no other word possible.…It was fascinating, but it was also frightening, because the man was not ordinary." Memories of working on mathematics with Grothendieck long into the night, by the light of a kerosene lamp, are "the greatest memories of my life as a mathematician."

Reaping and Sowing

> Il y a beaucoup de choses dans *Récoltes et Semailles*, et les uns et les autres y verront sans doute beaucoup de choses différentes: un *voyage* à la découverte d'un passé; une *méditation* sur l'existence; un *tableau de moeurs* d'un milieu et d'une époque (ou le tableau du glissement insidieux et implacable d'une époque à une autre…); une *enquête* (quasiment policière par moments, et en d'autres frisant le roman de cape et d'épée dans les bas-fonds de la mégapolis mathématique…); une vaste *divagation mathématique* (qui sèmera plus d'un…); un traité pratique de psychanalyse appliquée (ou, au choix, un livre de *"psychanalyse-fiction"*); une panégyrique de la *connaissance de soi*; "Mes *confessions*"; un *journal* intime; une psychologie de la *découverte et de la création*; un *réquisitoire* (impitoyable, comme il se doit…), voire un *règlement de comptes* dans "le beau monde mathématique" (et sans faire de cadeaux).
>
> There are many things in *Récoltes et Semailles*, and different people will no doubt see in it many different things: a *voyage* to the discovery of a past; a *meditation* on existence; a *portrait of the morals* of a milieu and of an era (or the portrait of an insidious and relentless sliding of one era into another…); an *inquest* (almost detective-style at times, and at others bordering on cloak-and-dagger fiction set in the underbelly of the mathematical megapolis); a vast *mathematical ramble* (which will leave more than one reader in the dust…); a practical treatise on applied psychology (or, if you like, a book of

> *"psychoanalytic-fiction"*); a panegyric on *self-knowledge*; "My *confessions*"; a private *diary*; a psychology of *discovery and creation*; an indictment (pitiless, as is fitting), even a *settling of scores* in "the world of elite mathematics" (and without any gifts).
>
> —*Récoltes et Semailles*, page L2

Between June 1983 and February 1986, Grothendieck wrote *Récoltes et Semailles: Réflexions et témoignage sur un passé de mathématicien* (*Reapings and Sowings: Reflections and testimony about the past of a mathematician*). It is a work that defies categorization. The title suggests a memoir, but *Récoltes et Semailles* is something more and less than a memoir. It is more, in that it contains not only memories of events in his life but also analyses, often quite deep and minute, of the moral and psychological significance of those events and his attempts to reconcile their meaning with his view of himself and the world. These analyses lead him into philosophical musings about the role of discovery and creativity in mathematics and in life more generally. At the same time, *Récoltes et Semailles* is something less than a memoir, in that it does not attempt a systematic and comprehensive account of events in Grothendieck's life. He is not writing for future biographers or historians, but primarily for himself. *Récoltes et Semailles* is a probing examination of matters closest to his heart. He brings to this work the searching curiosity, the same drive to get to the very bottom of things, that he brought to his mathematics. The result is a dense, multilayered work that reveals a great and sometimes terrifying mind carrying out the difficult work of trying to understand itself and the world.

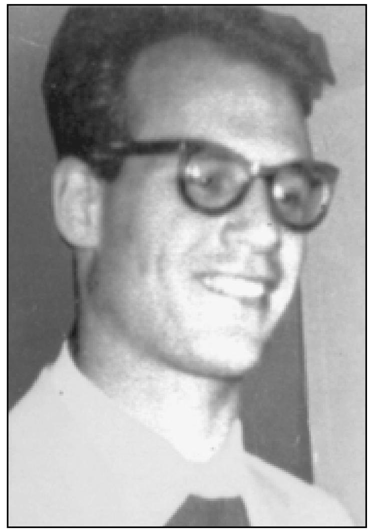
Needless to say, *Récoltes et Semailles* is not an easy read, and Grothendieck makes a lot of demands on his readers. Much of it has a quotidian feel, and in some parts he is obviously setting down his thoughts as they evolve from one day to the next. As a result, within the space of a page there can be sudden and sometimes disconcerting changes in mood and topic. The organization is complex. The main text is divided into numbered sections, each with its own carefully chosen and evocative title. Within each section there are cross-references to other sections, as well as numerous footnotes, some quite long and substantial, and sometimes even footnotes to the footnotes. The wide-ranging vocabulary presents special challenges for those whose native language is not French, as does his penchant for using colloquialisms, some of them rather vulgar. Through it all Grothendieck writes with great care, insight, and clarity, in a pungent and arresting style. He often succeeds at describing things that at first glance would seem quite ineffable.

One of the reasons for the complexity of the structure of *Récoltes et Semailles*, and for its spontaneity, is that Grothendieck wrote it without a definite plan in mind. He started writing it as an introduction to *À la Poursuite des Champs*, which was to mark his return to making a serious investment of time and energy in doing and publishing mathematics. The introduction was intended to explain the new spirit of his research, which would not focus on the precise and exhaustive foundation-building of his earlier work, but would take readers on a "voyage of discovery" of new mathematical worlds. Grothendieck envisioned *Récoltes et Semailles* as the first volume of a series called *Réflexions*, which would contain his thoughts and reflections on things mathematical and otherwise. The second volume was to have been *À la Poursuite des Champs*, and *La Longue Marche à Travers la Théorie de Galois* and *Esquisse d'un Programme* were also to have been included.

In the first part of *Récoltes et Semailles*, which he called "Fatuité et Renouvellement" ("Complacency and Renewal"), Grothendieck does a lot of soul-searching about the mathematical community in which he worked. The welcoming atmosphere he encountered upon joining that community as a newcomer in 1948 began to disappear, he says, as mathematicians came to use their reputations to set themselves in a superior position. Mathematics became a way to gain power, and the elite mathematicians of the day became smug, feared figures who used that power to discourage and disdain when it served their interests. He ruefully recounts some instances in which he himself displayed attitudes of conceit and haughtiness and realizes that these attitudes had coalesced into a "sportive" or competitive approach to mathematics that had begun to hamper his ability to open himself to the beauty of mathematical things.

It was after writing "Fatuité et Renouvellement" that he was suddenly struck by "the insidious reality of a *Burial* of my oeuvre and at the same time of my person, which suddenly imposed itself on me, with an irresistible force and with this very name, 'The Burial', on [April 19, 1984]." (*R&S*, page L8). On that date he began writing what eventually became a three-part series called "L'Enterrement" ("The Burial"), comprising more than one thousand pages. In it he strongly attacks some of his former students and colleagues, whom he believes tried to "bury" his work and his style of mathematics by pilfering his ideas and not according proper credit to him. He also champions the work of Zoghman Mebkhout, who during the 1970s developed some of Grothendieck's ideas and whose work Grothendieck believes was unfairly marginalized and ignored. "L'Enterrement" presents six mathematical areas, or "construction sites" (*"chantiers"*), that he says were abandoned when he left the IHÉS in 1970 and that he believes his students should have developed. Throughout "L'Enterrement" he closely analyzes his relationship with Pierre Deligne, the most brilliant of all of his students and the one with whom he had the closest mathematical affinity.

"L'Enterrement (II) ou La Clef du Yin et du Yang" ("The Burial (II) or the Key to Yin and Yang") is rather different from the other two parts of "L'Enterrement" in being less directly concerned with the investigation of the "burial". This second part, which Grothendieck notes is the most personal and deepest part of *Récoltes et Semailles*, constitutes a wide-ranging meditation on diverse themes such as creativity, intuition, violence, conflict, and the self. He uses the "yin-yang" dialectic to analyze different styles of doing mathematics, concluding that his own style is fundamentally "yin", or feminine. This style is captured in one especially evocative section called "La mer qui monte…" ("The rising sea…"). He likens his approach to mathematics to a sea: "The sea advances imperceptibly and without sound, nothing seems to happen and nothing is disturbed, the water is so far off one hardly hears it. But it ends up surrounding the stubborn substance, which little by little becomes a peninsula, then an island, then an islet, which itself is submerged, as if dissolved by the ocean stretching away as far as the eye can see" (*R&S*, page 553).


Grothendieck in a photograph from the 1950s.

In "L'Enterrement" he pursues some of the themes established in "Fatuité et Renouvellement" concerning the competitive, snobbish attitudes of the upper crust of the mathematical world. For example, he notes that much of his work in mathematics was marked by an "attitude of service": service to the mathematical community of writing clear and complete expositions that make fundamental and foundational ideas widely accessible. Although he candidly admits that his own conceit sometimes led him into elitist attitudes, he says that he never lost this spontaneous sense of service, "service to all those who leaped with me into a common adventure" (*R&S*, page 630, (*)). He believes that the mathematical community lost this sense of service as personal aggrandizement and the development of an exclusionary elite became the order of the day. He also decries the devaluation of vision and intuition in favor of technical mastery.

Apart from "Fatuité et Renouvellement" and the three parts that make up "L'Enterrement", *Récoltes et Semailles* has two introductory volumes, as well as an appendix to "La Clef du Yin et du Yang". About two hundred copies were sent out to his mathematical colleagues. Despite Grothendieck's intention to publish it, the original French-language version of *Récoltes et Semailles* has never appeared in print, as the strong attacks it contains could be deemed libelous. Nevertheless, it has circulated widely. Copies can be found on bookshelves in mathematicians' offices all over the world, especially in France, and in some libraries in universities and mathematics institutes. Historian of science Alain Herreman of the Université de Rennes has undertaken an effort to post on the Web html files containing the entire French original, and partial translations into English, Russian, and Spanish have appeared there too [*R&S*]. A Japanese translation of a large portion of *Récoltes et Semailles* was prepared by Yūichi Tsuji, who knew Grothendieck through the Survival group, and was published in the 1990s by Gendaisūgakusha, a mathematics publisher. According to Michel Waldschmidt of the Université de Paris VI, who was president of the Société Mathématique de France (SMF) during 2001–04, the society considered, during his presidency, the question of whether to publish *Récoltes et Semailles*. The question raised strong opinions both for and against, Waldschmidt said, and ultimately the SMF decided against publication.

Many mathematicians, especially some of Grothendieck's former students, were shocked and hurt by the accusations in *Récoltes et Semailles*. One of them, Luc Illusie of the Université de Paris-Orsay, recalled that he talked to another former student, Jean-Louis Verdier, about whether they should try to discuss the accusations with Grothendieck. According to Illusie, Verdier, who died in 1989, felt that Grothendieck's state of mind was such that there was no sound basis for discussion. But, Illusie said, "I thought, 'It is not possible that Grothendieck has become like that. I will try to reason and to discuss with him. Maybe I will agree with him on some points that he is right and on others he is not right.' Eventually, we settled the material points, but nothing really emerged, and he remained convinced that everyone was against him."

In *Récoltes et Semailles* Grothendieck says that, after he left the mathematical world in 1970, his style of doing mathematics was held in contempt and that many of the paths he had broken went undeveloped. It is true that after that time, research in algebraic geometry began to shift, mixing the highly general approach that characterized his work with investigation of specific problems. Deligne's proof of the Weil conjecture, which was very much in the spirit of Grothendieck but which also incorporated many new ideas, was one of the great advances of the 1970s. Along with developments in the theory of D-modules and Deligne's mixed Hodge theory, greater attention began to be paid to more specific problems, such as the classification theory of varieties and questions about low-dimensional varieties. Also, after the Antwerp meeting of 1972, collaborations grew between algebraic geometry and representation theory, leading to advances in the theory of automorphic forms and the Langlands program. As Illusie put it, all these developments show that there has been "quite a natural balance between general theory and the study of specific examples at great length, to enrich the theory itself."

Récoltes et Semailles also contains the accusation that Grothendieck's work was not always properly credited. Indeed, his work was so well known and fundamental that credit was not always specifically accorded to him. "It is true that everybody knew he had invented motives, for instance, or l-adic cohomology, and so there was no need to quote his name every time one used them," remarked Jean-Pierre Serre. "His name was rarely mentioned because of that. On the other hand, it was well known that it was due to him. Nobody was saying that it was due to someone else." Serre noted that Grothendieck's complaining about lack of credit is in sharp contrast to his behavior during the 1960s, when he shared his ideas with great generosity and in some cases attached other people's names to ideas he himself had come up with. "It was sad to read *Récoltes et Semailles* because of that," Serre said.

Even granting that there was a shift away from Grothendieck's style of mathematics and that credit was not always specifically accorded to him, it is a long leap from there to the deliberate "burial" that he asserts took place. "In retrospect, very few mathematical ideas have been as widely used as Grothendieck's," said Illusie. "Everybody who is doing algebraic or arithmetic geometry now uses Grothendieck's language, ideas, theorems, and so on. So when you think one second, it is completely ridiculous that he suggested that he could have been buried." There is no question that mathematics suffered a great loss when Grothendieck halted his research career in 1970. But mathematics did not stop; others continued to work, following their own ideas and interests. In February 1986, after receiving a copy of *Récoltes et Semailles*, Serre wrote to Grothendieck: "You are surprised and indignant that your former students did not continue the work that you had undertaken and largely completed. But you do not ask the most obvious question, the one every reader expects you to answer: and you, why did you abandon the work in question?" [Corr].

Contents of *Récoltes et Semailles*

Although the accusations of a "burial" have generated a good deal of notoriety, there is much more to *Récoltes et Semailles*. Those who have read beyond those parts have been deeply touched by the work's beauty and insights. Grothendieck's critique of how the highly competitive atmosphere of the mathematical world stifles creativity and renewal of the field resonated with many. In *Récoltes et Semailles* Grothendieck puts the highest value on the innocent, childlike curiosity that gives birth to the creative impulse, and he mourns the way it is trampled on by competitiveness and the desire for power and prestige.

"I am one of quite probably a minority who think that *Récoltes et Semailles* is a miraculous document," said William Messing. "That is not to say that there are not parts that are excessive and have aspects of what might be referred to as paranoia. But it's very striking that the person who created *EGA* and *SGA* would write in such a style. The systematic and soul-searching aspect is of a piece with his approach to mathematics. Those who have really read it—as opposed to looking at five pages of negative comments—tend to think of it as an extraordinary document."

Lightness Descending

> [A]ujourd'hui je ne suis plus, comme naguère, le prisonnier de tâches

> interminables, qui si souvent m'avaient interdit de m'élancer dans l'inconnu, mathématique ou non. Le temps des *tâches* pour moi est révolu. Si l'âge m'a apporté quelque chose, c'est d'être plus léger.
>
> Today I am no longer, as I once was, the prisoner of interminable tasks, which so often prevented me from leaping into the unknown, mathematical or otherwise. The time of *tasks* for me is over. If age has brought me anything, it is lightness.
>
> —*Esquisse d'un Programme*

"[T]he ethics of the scientific profession (especially among mathematicians) have degraded to such a degree that pure and simple theft between colleagues (especially at the expense of those who have no position of power to defend themselves) has almost become the general rule and is in any case tolerated by all, even in the most flagrant and iniquitous cases." So wrote Grothendieck in an April 19, 1988, letter to the Royal Swedish Academy of Sciences in which he declined the 1988 Crafoord Prize. He also sent to the academy the introductory volumes of *Récoltes et Semailles*. The academy had awarded the prize of around US$200,000 to him and Pierre Deligne. Grothendieck's letter became widely known when it was published in *Le Monde* on May 4, 1988 [LeMonde]. To play into the game of accepting prizes and honors, Grothendieck wrote, would be to validate "a spirit and an evolution in the scientific world that I see as profoundly unhealthy, and condemned to disappear soon, so suicidal is it, spiritually as well as intellectually and materially." Evidently these sentiments resonated with many readers of *Le Monde*. One of the newspaper's editors told Jean-Pierre Bourguignon that the paper had received more reactions to Grothendieck's letter than to any other preceding it and that most of the letters registered approval that finally a scientist had recognized how corrupt the scientific milieu had become. News of the letter appeared in other magazines and newspapers, and it was avidly discussed within the mathematical community. An English translation was published in the *Mathematical Intelligencer* [Intell], and a short item appeared in the *Notices* [Notices].

The same year in which he turned down the Crafoord Prize, Grothendieck retired from the Université de Montpellier at the age of sixty. Also that year, six mathematicians decided to assemble a collection of articles as a "Festschrift" on the occasion of Grothendieck's sixtieth birthday [Festschrift] (there was also a special issue of the journal *K-Theory* dedicated to Grothendieck). The Festschrift seems to have been an attempt to make amends with Grothendieck and to show that he had not been "buried", as he asserted in *Récoltes et Semailles*. Some of the people contributing papers were among those he had most heavily criticized. When the Festschrift appeared in 1990, Illusie, who was one of the editors, sent a copy to Grothendieck, whose reaction was extremely bitter. In a letter to Illusie, he objected strongly to the brief foreword of the volume and also to the fact that he had not been told that the volume would appear. He said his work had been used like "confetti," like bright, worthless bits one throws into the air to give the pretense of happiness and celebration while ignoring the malaise underneath. Grothendieck submitted this letter for publication in the *Bulletin de la Société Mathématique de France*. When the SMF told him that the *Bulletin* carries only mathematics articles but that the letter could appear instead in the *SMF Gazette*, Grothendieck refused. The letter was never published.

After he retired, Grothendieck spent little time at the Université de Montpellier, though he continued to live in the area, in a village called Les Aumettes. At this time, Ladegaillerie said, Grothendieck seemed to be going through a deep spiritual crisis and wrote strange letters "that made us fear the worst about his condition." During 1987–88, Grothendieck wrote *La Clef des Songes ou Dialogue avec le Bon Dieu* (*The Key to Dreams or Dialogue with the Good Lord*), which expresses his conviction that God exists and that He speaks to people through their dreams. It also contains a good deal of material about Grothendieck's early life. *La Clef des Songes* runs about three hundred pages and is accompanied by another five hundred pages of notes. According to a lecture given in the summer of 2004 by Winfried Scharlau of the Universität Münster, Grothendieck subsumed *La Clef des Songes* under a collection of works that he called *Méditations* and that included the material making up *Réflexions*, as well as a poetical work called "Eloge de l'Inceste" ("The Eulogy to Incest"). Neither that work nor *La Clef des Songes* was ever widely distributed.

Many of Grothendieck's friends and colleagues became aware of his increasing preoccupation with spiritual matters when they received "La Lettre de la Bonne Nouvelle" ("The Letter of Good News"), which is dated January 26, 1990, and which he sent to about two hundred fifty people. The letter states: "You are part of a group of two to three thousand people, personally known to me, whom God destines for a great mission: That of announcing and preparing the *'New Age'* (or *Age of Liberation…*), which will commence on the *'Day of Truth'*, 14 October 1996." He says that God manifested Himself to him for the first time in 1986 and communicated to him through dreams. He also describes encounters with a deity named Flora, who imparts revelations but also cruelly tests his faith. Although the content of the letter is baffling, the way it is written is perfectly lucid. Three months later Grothendieck sent a "correction", stating that he was no longer certain of the truth of the revelations described in "La Lettre de la Bonne Nouvelle". He writes: "That I was the victim of a mystification by one of more 'spirits' (among which my limited capacity could not distinguish), invested with prodigious powers over my body and in my psyche, I no longer have the least doubt." Together, the two letters impart an impression of deep disturbance and suffering.

In July 1990 Grothendieck asked Malgoire to take possession of all of his mathematical papers, including books, preprints, correspondence, and manuscripts in various states of preparation. Grothendieck wanted to "lighten" himself of many things, as Malgoire put it. He burned a huge amount of material, most of it nonmathematical, including letters that his parents had exchanged in the 1930s. He showed Malgoire a 200-liter oil drum filled with cinders and estimated he had destroyed a total of 25,000 pages. Grothendieck also left many papers and other items, including his mother's death mask, with a friend named Yolande Levine, to whom he had been very close for the preceding decade. He then disappeared into the Pyrenées to live in complete isolation. A small number of people knew where he was, and he instructed them not to forward any mail that arrived for him at the university. Malgoire said that even today, close to fifteen years after Grothendieck went into seclusion, the university still gets a great deal of correspondence addressed to him. In 1995 Grothendieck formally conferred the legal rights to his mathematical papers to Malgoire.

Grothendieck has had very little contact with mathematicians in the past fifteen years. Among the few who have seen him are Leila Schneps and Pierre Lochak, who met him in the mid-1990s. They told him about the progress made on the program he had outlined in the *Esquisse d'un Programme*, and he was surprised to learn that people were still interested in his work. He had developed a strong interest in physics but expressed frustration with what he felt was a lack of rigor in that field. Lochak and Schneps exchanged some letters with him and also sent him some books on physics that he had asked for. In one letter he asked a disarmingly simple question: What is a meter? His letters began to swing between warm friendliness and cold suspicion, and eventually he severed all contact with them. Although the friendship with Grothendieck could not be sustained, Lochak and Schneps retain a fervent admiration and a deep attachment to the man and his work. Together they painstakingly typed into TeX a large chunk of the handwritten *La Longue Marche à Travers la Théorie de Galois*. They have also started a website, the Grothendieck Circle, which contains a wealth of material about Grothendieck, his life, and his work [Circle].

The Dancing Star

> Ich sage euch: man muß noch Chaos in sich haben, um einen tanzenden Stern gebären zu können. Ich sage euch: ihr habt noch Chaos in euch.
>
> I tell you: one must have chaos inside, to give birth to a dancing star. I tell you: you have yet chaos in you.
>
> —Friedrich Nietzsche, *Also sprach Zarathustra*

The work of Alexandre Grothendieck has had a profound impact on modern mathematics and, more broadly, ranks among the most important advances in human knowledge during the twentieth century. The stature of Grothendieck can be compared to that of, for example, Albert Einstein. Each of them opened revolutionary new perspectives that transformed the terrain of exploration, and each sought fundamental, unifying connections among phenomena. Grothendieck's propensity for investigating how mathematical objects behave relative to one another echoes the relativistic viewpoint proposed by Einstein. Grothendieck's work also has parallels with another great twentieth-century advance, that of quantum mechanics, which turned conventional notions upside down by replacing point particles by "probability clouds". "[T]hese 'probability clouds', replacing the reassuring material particles of before, remind me strangely of the elusive 'open neighborhoods' that populate the toposes, like evanescent phantoms, to surround the imaginary 'points'," he wrote (*R&S*, page P60).

Yet, as extraordinary as Grothendieck's achievements are, he traced his creative capacity to something rather humble: the naive, avid curiosity of a child. "Discovery is the privilege of the child," he wrote in *Récoltes et Semailles* (page 1), "the child who has no fear of being once again wrong, of looking like an idiot, of not being serious, of not doing things like everyone else." For the work of discovery and creation, Grothendieck saw intellectual endowment and technical power as secondary to the child's simple thirst to know and understand. This child is inside each of us, though it may be marginalized, neglected, or drowned out. "Each of us can rediscover what discovery and creation are, and no one can invent them" (*R&S*, page 2).

One aspect of this childlike curiosity is a scrupulous fidelity to truth. Grothendieck taught his students an important discipline when writing about mathematics: never say anything false. Statements that were almost or essentially true were not permitted. It was acceptable to be vague, but when one gives precise details, one must say only things that are true. Indeed, Grothendieck's life has been a constant search for truth. From his mathematical work through *Récoltes et Semailles* and even "La Lettre de la Bonne Nouvelle", Grothendieck wrote with the unblinking honesty of a child. He spoke the truth—*his* truth, as he perceived it. Even when he made factual mistakes or was misled by incorrect assumptions, he presented candidly what was in his mind. He has never tried to hide who he is and what he thinks.

Grothendieck's search for truth took him to the very roots of mathematical ideas and to the far reaches of human psychological perception. He has had a long journey. "In his solitary retirement in the Pyrenées, Alexandre Grothendieck has the right to rest after all he has been through," wrote Yves Ladegaillerie [Ladegaillerie]. "He deserves our admiration and our respect but, above all, in thinking of what we owe him, we must leave him in peace."

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