MODULES OVER MONADS, MONADIC SYNTAX AND THE CATEGORY OF UNTYPED LAMBDA-CALCULI

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Abstract. We define a notion of module over a monad and use it to propose a new definition (or semantics) for abstract syntax

(still limited) experience, non emerge as clear winners." For one such solution and a fair account of earlier ones, see e.g. [MM02]. For some more recent solutions, see ([Hof99, FPT99, GP99]). Based on modules over monads, we propose what we hope to become the "winning" abstract notion of syntax, which we call monadic syntax. It goes as follows.

a computer-checked proof of our theorem. Indeed, while the ideas of

Example 2.12. The final module is the product of the empty family.

Example 2.13. Given a R-module M, we have a natural "evaluation" morphism

$$\text{app} : M \times R \longrightarrow M.$$

Proposition 2.14. *Derivation yields a cartesian functor from R-modules to R-modules.*

Definition 2.15 (Pull-back)**.** Given a morphism $f : A \to B$ of monads and a B-module M, we define its pull-back $f M$ as follows: we set $f M(X) := M(X$

Definition 4.1. A monad R is said to be a monadic group when its tautological module is equipped with a structure of group object in the category of R-modules.

Example

Theorem 5.4. *The lambda-calculus* LC *is an initial object in the category of lambda-calculi.*

6. Proofs

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