Calc 2 midterm 2 Study Guide

Sequence, Series, & Infinite Series

{10.1, 10.2, 10.3}

$\{1, \underbrace{4}_{\text{term of a sequence}}, 7, 10, 13, 16, 19, 22, \ldots\}$ → sequence

- ordered list $\{a_n\}$
- recurrence relation $a_{n+1} = f(a_n)$
- explicit formula $a_n = f(n)$

$\{a_1, a_2, a_3, \ldots, a_n, \ldots\}$

to make explicit formula for...
- Arithmetic: $a_n = a_1 + d(n-1)$
- Geometric: $a_n = a_1 (r)^{n-1}$

To find the convergence of a sequence we can take the limit* or use squeeze theorem

$1 + 4 + 7 + 10 + 13 + 16 + 19 + 22 + \ldots$ → infinite series

partial sums: S_1, S_2, S_3, … S_n

$$a_1 + a_2 + a_3 + \ldots = \sum_{k=1}^{\infty} a_k$$

$$S_n = a_1 + a_2 + a_3 + \ldots a_n = \sum_{k=1}^{n} a_k$$

Finding convergence of a series is a little harder

- if $\lim_{k\to\infty} \{S_n\}$ converges to L the infinite series converges to L as well. $\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} \underbrace{\sum_{k=1}^{n} a_k}_{S_n} = L$

Convergence of a Sequence

If $\lim_{n\to\infty} a_n = L$ then the sequence $\{a_n\}$ converges to L.

For a geometric sequence $\{ar^n\}$...

$$\lim_{n\to\infty} r^n = \begin{cases} 0, & \text{if } |r|<1 \\ 1, & \text{if } r=1 \\ \text{dne}, & \text{if } r\le -1 \text{ or } r>1 \end{cases}$$

you can also use the squeeze theorem

if $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ are sequences
if $a_n \le b_n \le c_n$
if $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} c_n = L$
then $\lim_{n\to\infty} b_n = L$

Convergence of a series is a lot more complicated

Improper Integrals

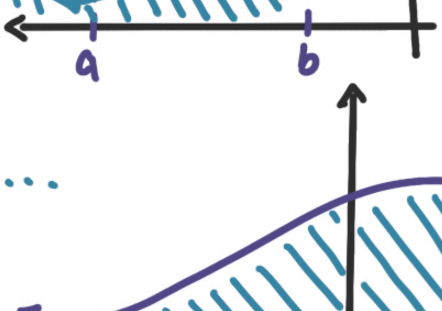
{8.9}

useful for determining convergence of series

Infinite Integrals

f is cont. on $[a, \infty)$ then...

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$$



f is cont. on $(-\infty, b]$ then...

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to\infty} \int_a^b f(x)\,dx$$

f is cont. on $(-\infty, \infty)$ then...

$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{a\to-\infty} \int_a^c f(x)\,dx + \lim_{b\to\infty} \int_c^b f(x)\,dx$$

Series Convergence Test

{10.3, 10.4, 10.5, 10.6, 10.7, 10.8}

Properties of convergent series →

- if $\sum a_k$ converges to A, $\sum ca_k$ converges to cA
- if $\sum a_k$ diverges, $\sum ca_k$ diverges
- if $\sum a_k$ converges to A, and $\sum b_k$ converges to B, then $\sum a_k \pm b_k$ converges to $A \pm B$
- if $\sum a_k$ diverges and $\sum b_k$ converges, then $\sum a_k \pm b_k$ diverges
- $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=M}^{\infty} a_k$ either both converge or both diverge

Test	Form	Convergence	Divergence	Why?
geometric series	$\sum_{k=0}^{\infty} ar^k$	$\lvert r\rvert<1$	$\lvert r\rvert \ge 1$	
p series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	$p>1$	$p<1$	
divergence test	$\sum_{k=1}^{\infty} a_k$	NA	$\lim_{k\to\infty} a_k \ne 0$	For the series to converge it can't have a "last term" that is still adding to it, we need to stop adding at some point, so the "last terms" we're adding have to be Ø.
integral test	$\sum_{k=1}^{\infty} a_k$ ☐ cont. ☐ pos. ☐ decr.	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	if we imagine adding together an infinite series graphically it looks a lot like an integral. And remember our idea of an integral comes from a reiman sum. So if they're very similar (but not the same), we can say that if an integral converges, so does a matching series.
ratio test	$\sum_{k=1}^{\infty} a_k$	$\lim_{k\to\infty} \left\lvert\frac{a_{k+1}}{a_k}\right\rvert<1$	$\lim_{k\to\infty} \left\lvert\frac{a_{k+1}}{a_k}\right\rvert>1$	very simply, it's about the magnitude decreasing as the series goes on infinitly.
root test	$\sum_{k=1}^{\infty} a_k$	$\lim_{k\to\infty} \sqrt[k]{\lvert a_k\rvert}<1$	$\lim_{k\to\infty} \sqrt[k]{\lvert a_k\rvert}>1$	
comparison test	$\sum_{k=1}^{\infty} a_k$ $a_k>0$	$a_k \le b_k$ & $\sum_{k=1}^{\infty} b_k$ converges	$b_k \le a_k$ & $\sum_{k=1}^{\infty} b_k$ diverges	if we imagine a sequence a_k + b_k graphically we can apply the fact that a bounded monotonic sequence converges. And if we know that the terms of a series stops we know the series stops as well.
limit comparison test	$\sum_{k=1}^{\infty} a_k$ ☐ $a_k>0$ ☐ $b_k>0$	$0 \le \lim_{k\to\infty} \frac{a_k}{b_k} < \infty$ & $\sum_{k=1}^{\infty} b_k$ converges	$\lim_{k\to\infty} \frac{a_k}{b_k} > 0$ & $\sum_{k=1}^{\infty} b_k$ diverges	
alternating series test	$\sum_{k=1}^{\infty} -1^k a_k$ ☐ $a_k>0$	$\lim_{k\to\infty} a_k = 0$ & $0 < a_{k+1} \le a_k$ (nonincreasing)	$\lim_{k\to\infty} a_k \ne$	
absolute convergence	$\sum_{k=1}^{\infty} a_k$	$\sum_{k=1}^{\infty} \lvert a_k\rvert$ converges	NA	

i had a really bad migraine so i stopped doing these "why" sections

Bonus →

Telescoping Series

Converges by folding in on itself

Expands to the form...

$$S_n = (a_1 - a_2) + (a_2 - a_3) + (a_3 - a_4) + \ldots + (a_{n-1} - a_n)$$

and since all this folds in on itself we can test for convergence by testing..

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} (a_1 - a_n)$$

... for convergence

Remainders →

Remainder aka Error aka Amount left over after approximating

$$R_n = \sum_{k=1}^{\infty} a_k - \sum_{k=1}^{n} a_k = a_{n+1} + a_{n+2} + a_{n+3} + \ldots$$

We can bound the remainder by thinking about the remainder and integrals graphically

$$R_n < \int_n^{\infty} f(x)\,dx$$

And we can bound the series with...

$$L_n < S < U_n$$

$$S_n + \int_{n+1}^{\infty} f(x)\,dx < \sum_{k=1}^{\infty} a_k < S_n + \int_n^{\infty} f(x)\,dx$$

↓

$$S_n + R_n = S$$

Remainder in Alternating Series

the magnitude of the remainder is less than or equal to the magnitude of the first neglected term...

$$|R_n| \le a_{n+1}$$

i feel like this should apply to all types of series but honestly idk.

this got really lazy, I have a headache + wana go to bed. NE WAYS GOOD LUCK <3<3<3