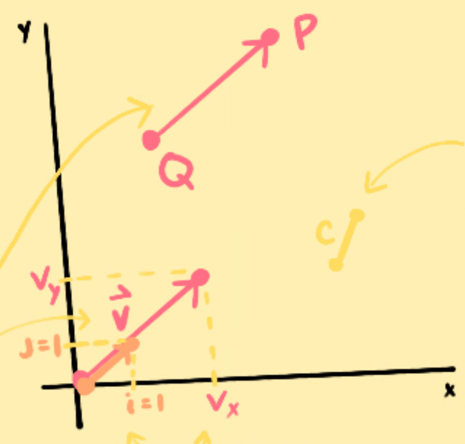
Calc 3 Exam 1 Study Guide

13.1, 13.2, 13.3, 13.4, 13.5, 13.6

13.1 → Vectors Intro



Scalars → quantities w/ magnitude and no direction

c

Scalar Multiples: vector whose length is $c\vec{V}$ and w/ direction of $\vec{V}$

$$\overrightarrow{QP} = \vec{V} = \langle V_x, V_y \rangle = V_x i + V_y j$$

tail of vector; head of vector; vector components; position vector; coordinate unit vectors

Turn $\vec{V}$ into unit vector in same direction $= \dfrac{\vec{V}}{|\vec{V}|}$

Vectors → any quantity w/ length/magnitude and direction

$$|\overrightarrow{QP}| = |\vec{V}| = \sqrt{V_x^2 + V_y^2}$$

Vector Operations

$\vec{M}+\vec{V} = \langle M_1+V_1, M_2+V_2 \rangle$

$\vec{M}-\vec{V} = \langle M_1-V_1, M_2-V_2 \rangle$

$cM = \langle cM_1, cM_2 \rangle$

pretty standard

imagine graphically

Equal Vectors → vectors w/ an equal magnitude and direction

Zero Vector → weird vector, length of Ø and no direction

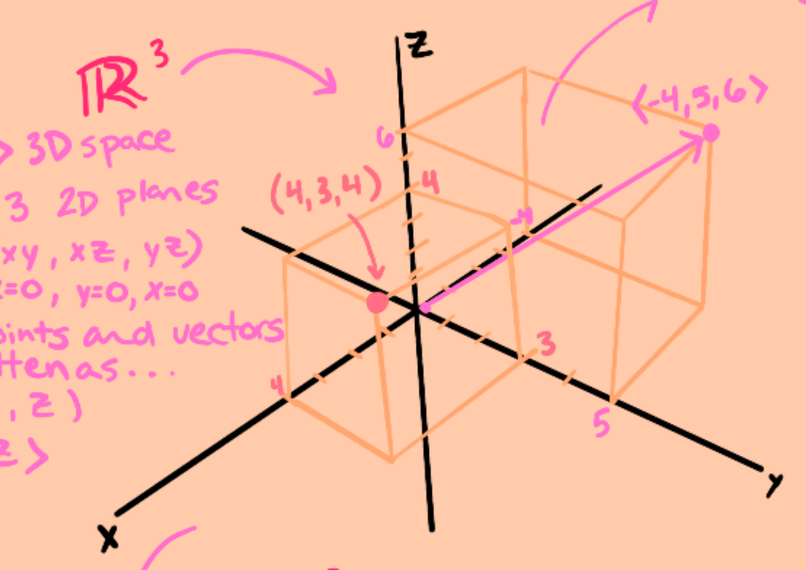
Parallel Vectors → two vectors are parallel if they are scalar multiples of each other

So... what are vectors?

13.2 → Vectors in 3D

sounds like a movie title

$\mathbb{R}^3$
> 3D space
> 3 2D planes (xy, xz, yz) z=0, y=0, x=0
> Points and vectors written as... (x, y, z) ⟨x, y, z⟩



Working w/ Vectors in $\mathbb{R}^3$
> the vector equations we've learned so far extend to $\mathbb{R}^3$, we just add a z element

$\vec{M}+\vec{V} = \langle M_1+V_1, M_2+V_2, M_3+V_3 \rangle$

$|\vec{M}| = \sqrt{M_1^2+M_2^2+M_3^2}$

... etc.

> We can also find the midpoint with...

midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{3}\right)$

this more useful later!

Balls and Spheres in $\mathbb{R}^3$

Balls: filled in; Spheres: hollow

centered @ (a,b,c), radius r

sphere: $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

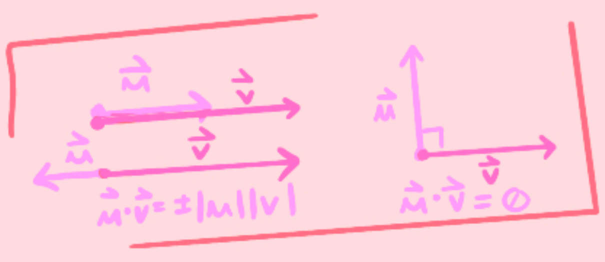
ball: $(x-a)^2+(y-b)^2+(z-c)^2 \le r^2$

13.3 → Dot Products

> Two ways to "multiply" two vectors. One gets a scalar (dot product), and one gets a vector (cross product).

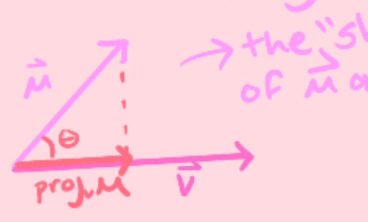
Dot Product

$$\vec{M} \cdot \vec{V} = |\vec{M}||\vec{V}|\cos\theta = M_1V_1 + M_2V_2 + M_3V_3$$



Dot Product Uses

> projections → the "shadow" of M on V.



the scalar component of M in the direction of V: $scal_v M = |M|\cos\theta = \frac{M\cdot V}{|V|}$

$proj_v M = |M|\cos\theta \frac{V}{|V|} = scal_v M \left(\frac{V}{|V|}\right) = \boxed{\left(\frac{M\cdot V}{V\cdot V}\right)V}$

> Finding work done by Force

$\boxed{W = |F||d|\cos\theta = F\cdot d}$ (F: force, d: displacement)

Common Problem

Parallel Force = projection of weight onto direction sliding down

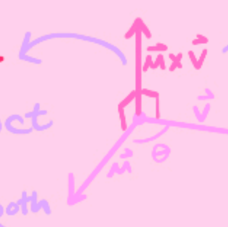
F = parallel force + normal force

How can we combine vectors for various applications?

13.4 → Cross Products

> "multiplying" two vectors but getting a vector

Cross Product



> the cross product is the line orthogonal to both vectors M and V
> if $\vec{M}\times\vec{V}=0$, the vectors are parallel.

Cross Product Equations

$$\vec{M}\times\vec{V} = \begin{vmatrix} i & j & k \\ M_1 & M_2 & M_3 \\ V_1 & V_2 & V_3 \end{vmatrix} = \begin{vmatrix} M_2 & M_3 \\ V_2 & V_3 \end{vmatrix} i - \begin{vmatrix} M_1 & M_3 \\ V_1 & V_3 \end{vmatrix} j + \begin{vmatrix} M_1 & M_2 \\ V_1 & V_2 \end{vmatrix} k$$

$$= M_2V_3 i - M_3V_2 i - M_1V_3 j + M_3V_1 j + M_1V_2 k - M_2V_1 k$$

> $|\vec{M}\times\vec{V}| = |M||V|\sin\theta$
> if M and V are two sides of a parallelogram then the area of a parallelogram is... area $= |\vec{M}\times\vec{V}| = |M||V|\sin\theta$
> if M and V are sides of a triangle area is... area $= \frac{1}{2}|\vec{M}\times\vec{V}| = \frac{1}{2}(|M||V|\sin\theta)$

Cross Product of Coord. Unit Vectors

$i\times j = k$, $j\times i = -k$; $i\times k = -j$, $k\times i = j$; $k\times j = -i$, $j\times k = i$

$i\times i = j\times j = k\times k = Ø$

Application of Cross Product

> Torque

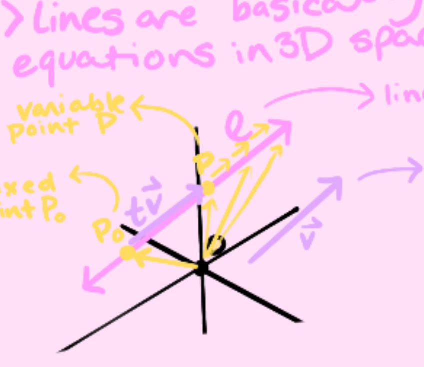
$|T| = |r\times F| = |r||F|\sin\theta$ → twisting effect produced by force F acting on intermediate r.

> max torque when F is orthogonal to r.

13.5 → Lines and Planes in Space

Lines

> Lines are basically linear equations in 3D space.



$\overrightarrow{P_0P} = t\vec{v}$

$\overrightarrow{P_0P} + \overrightarrow{OP_0} = \overrightarrow{OP}$

$t\vec{v} + \overrightarrow{OP_0} = \overrightarrow{OP} = \ell$

t and $\overrightarrow{OP}$ are variable

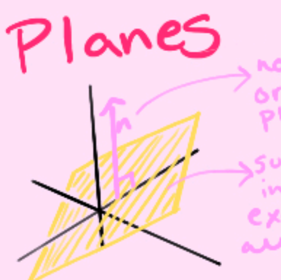
as variable P moves infinitley across ℓ, we are able to use that to track dif points as t grows

Component Form
$\langle x,y,z\rangle = \langle x_0,y_0,z_0\rangle + t\langle a,b,c\rangle$

Vector Form
$r = r_0 + tv$

Parametric Form
$x = x_0+at,\ y = y_0+bt,\ z = z_0+ct$ for $-\infty < t < \infty$

Planes



> equation: fixed point $P_0(x_0, y_0, z_0)$, normal vector $n = \langle a,b,c\rangle$

$\boxed{a(x-x_0)+b(y-y_0)+c(z-z_0)=0}$

or...

$\boxed{ax+by+cz=d=ax_0+by_0+cz_0}$

> if planes are parallel, their normal vectors are parallel
> if planes are orthogonal, their normal vectors are orthogonal.

Distance between point and line

> because of right triangles...

$d = |\overrightarrow{PQ}|\sin\theta$

$|\vec{v}\times\overrightarrow{PQ}| = |\vec{v}|\underbrace{|\overrightarrow{PQ}|\sin\theta}_{d}$ ← def of cross product

$|\vec{v}\times\overrightarrow{PQ}| = |\vec{v}|d$

$$\boxed{\frac{|\vec{v}\times\overrightarrow{PQ}|}{|\vec{v}|} = d}$$

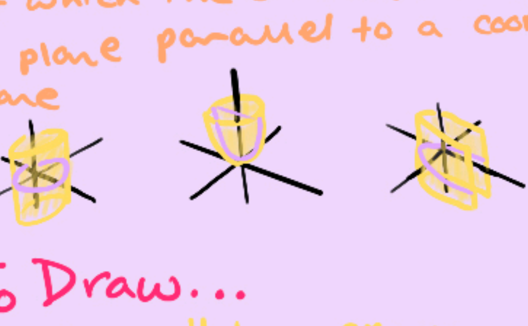
What else can we do in $\mathbb{R}^3$ (w/ vectors help)?

13.6 → Quadric Surfaces

Vocab...

cylander → a surface parallel to a line, not necessarilly

"trace" → the set of points at which the surface intersects a plane parallel to a coordinate plane

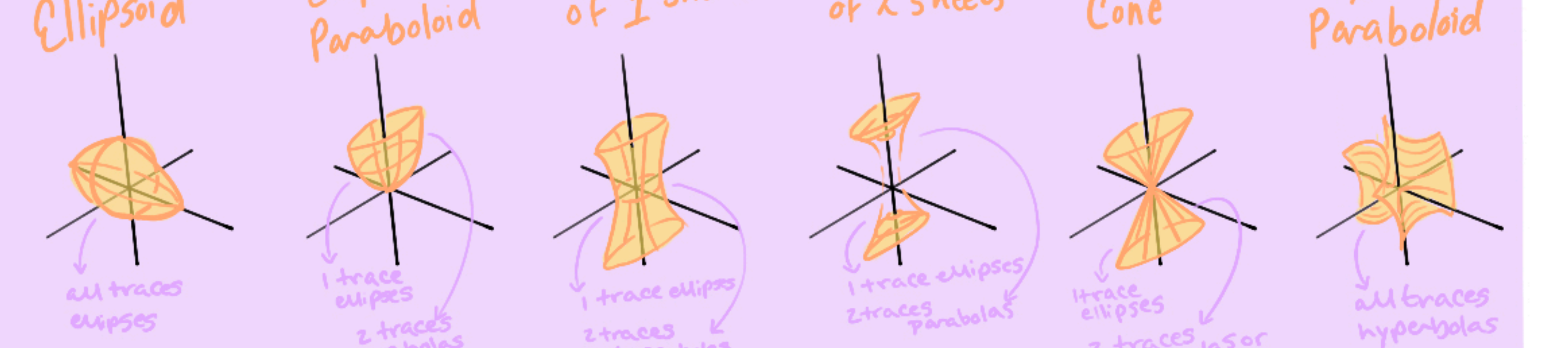


To Draw...

↳ prob. won't be on exam

1. intercepts
2. traces
3. fill in

Summary of Quadric Surfaces...



good luck