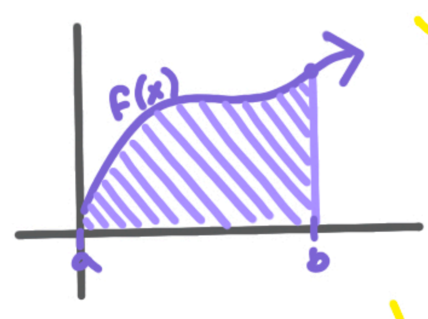
At the end of Calc One we learned that **INTEGRALS** find the area under a curve $\frac{1}{3}$ can be calculated by finding the **ANTIDERIVATIVE** of a function. **WE HAVE 2 WAYS TO GO FROM HERE**

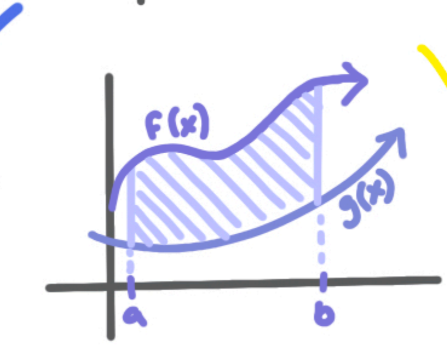
WHAT ELSE CAN WE DO WITH INTEGRALS?

Review

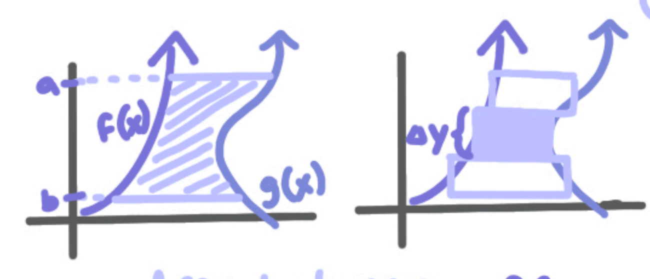


Area under curve $= \int_a^b f(x)\,dx$

6.2



Area between curves $= \int_a^b f(x) - g(x)\,dx$



origins from the reiman sum again

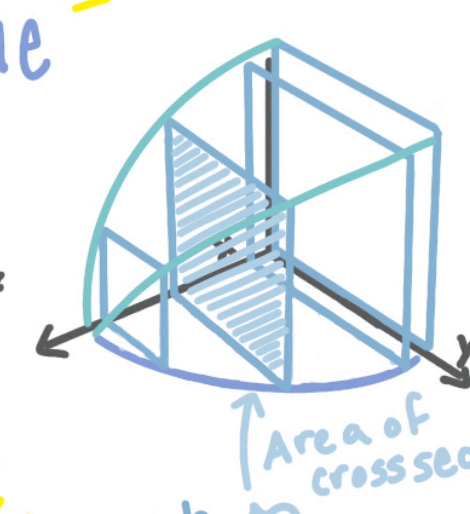
same w/ respect to y

Area between curves w/ respect to y $= \int_b^a f(y) - g(y)\,dy$

6.3

Volume

area of f(x) = base of solid

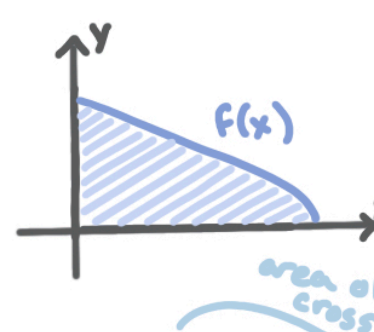


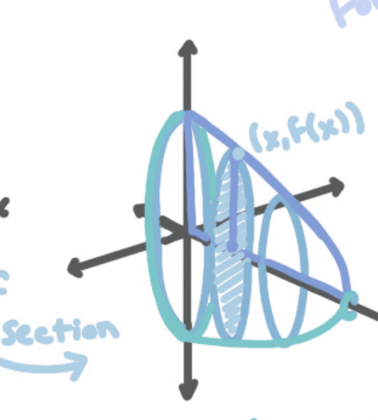
Area of cross sections

GENERAL SLICING METHOD

$$V = \int_a^b A(x)\,dx$$

basis of more formulas to follow

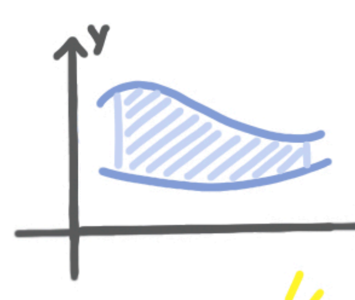


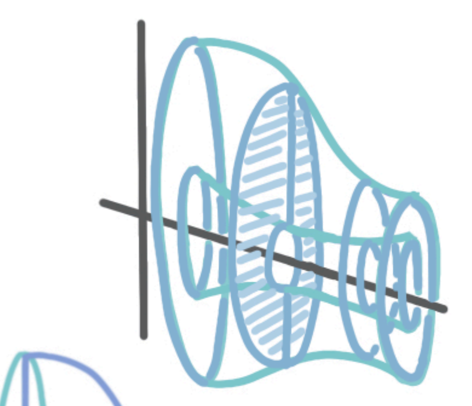


area of cross section

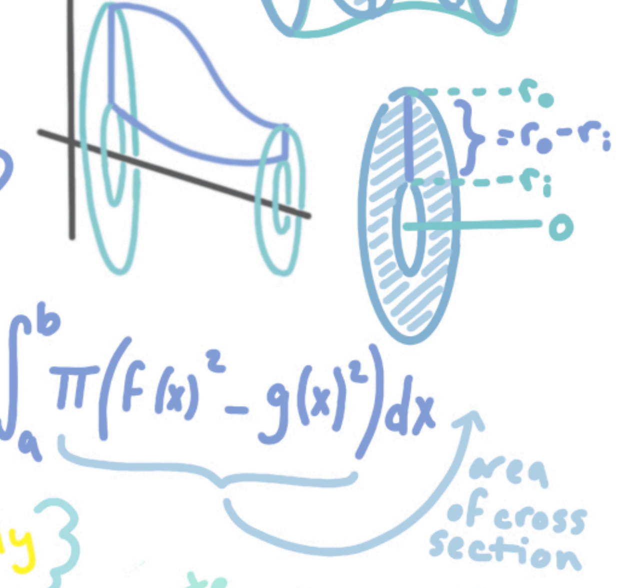
$$V = \int_a^b \pi \underbrace{f(x)}_{r=f(x)}{}^2\,dx$$

DISK METHOD





WASHER METHOD

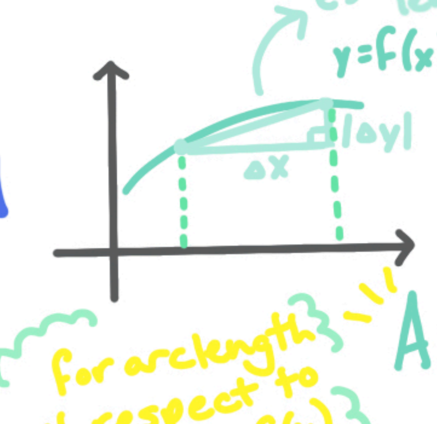


Note: Use any of these methods about the y axis by using f(y) & dy

$$V = \int_a^b \pi\left(f(x)^2 - g(x)^2\right)dx$$

area of cross section

6.5



estimate length of hypotunus

$y = f(x)$

$L \approx \sqrt{\Delta x^2 + |\Delta y|^2}$

use this to make integral for...

ARC LENGTH

$$L = \int_a^b \sqrt{1 + f'(x)^2}\,dx$$

for arclength w/ respect to y just use f(y) & dy.

BLANK SPACE
NE WAYS
GOOD LUCK ♡♡♡

WHAT OTHER STRATEGIES DO WE NEED TO SOLVE THESE INTEGRALS?

review

BASIC INTEGRATION FORMULAS

$\int k\,dx = kx + C$

$\int x^p\,dx = \frac{x^{p+1}}{p+1} + C$ $\quad \int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$ $\quad \int \sin ax\,dx = \frac{-1}{a}\cos ax + C$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$ $\quad \int \csc^2 ax\,dx = \frac{-1}{a}\cot ax + C$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax + C$ $\quad \int \frac{dx}{x} = \ln|x| + C$

$\int \csc ax \cot ax\,dx = \frac{-1}{a}\csc ax + C$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan ax + C$ $\quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a}\arcsin + C$

$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\left|\frac{x}{a}\right| + C$

8.2

INTEGRATION BY PARTS

the reverse of the product rule

$\frac{d}{dx}(u(x)v(x)) = u'(x)v(x) + u(x)v'(x)$

$\int \frac{d}{dx} uv = \int u'v + v'u$

$uv = \int u'v + \int v'u$

$\int u\,dv = uv - \int v\,du$

when picking u use...

ILATE → Inverse trig func
Logarithmic func
Algebraic func
Trig func
Exponential

in action

$\int x e^x\,dx$

$u = x \quad dv = e^x dx$
$du = dx \quad v = e^x$

$= xe^x - \int e^x\,dx$
$= xe^x - e^x$

8.3

TRIGONOMETRIC INTEGRALS

Pythagorean Identities

$\sin^2 x = 1 - \cos^2 x$
$\cos^2 x = 1 - \sin^2 x$
$\sec^2 x = \tan^2 x + 1$
$\tan^2 x = 1 - \sec^2 x$

or

Half Angle formulas

$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$
$\cos^2 x = \frac{1}{2} + \frac{\cos 2x}{2}$

Strategies for $\int \sin^m \cos^n\,dx$

m and/or n is odd → split off odd power, rewrite remaing in terms of sin or cos so all match. u sub

m and n are even → use half angle formulas till all exponents are gone

Strategies for $\int \tan^m \sec^n\,dx$

n is even → u sub, $u = \tan$
m is odd → u sub, $u = \sec$

in action

$\int \cos^5 x \sin^2 x\,dx$
$= \int \cos^4 x \sin^2 x \cos x\,dx$
$= \int (1 - \sin^2 x)^2 \sin^2 x \cos x\,dx$

$u = \sin x \quad du = \cos x\,dx$

$= \int (1 - u^2)^2 u^2\,du$
$= \int (u^2 - 2u^4 + u^6)\,du$
$= \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$
$= \frac{\sin^3 x}{3} - \frac{2\sin^5 x}{5} + \frac{\sin^7 x}{7} + C$

8.4

TRIG SUBSTITUTES

When we have equations w/ $\sqrt{a^2 + b^2}$ we really can't integrate. So we want to make $a^2 + b^2 = A^2B^2$ because... $\sqrt{A^2B^2} = |AB|$ and that's easier to integrate. we use trig to put the original formula into a product form.

basic method

$a^2 - x^2 \quad x = a\sin\theta$
$= a^2 - (a\sin\theta)^2$
$= a^2 - a^2\sin^2\theta$
$= a^2\underbrace{(1 - \sin^2\theta)}_{\text{pythagorean identity}}$
$= a^2(\cos^2\theta)$

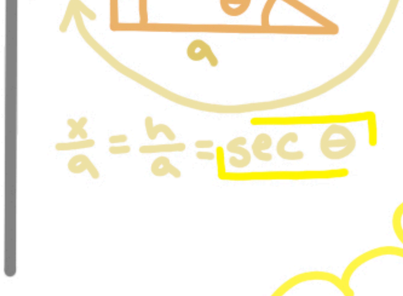
yay! a product!

what trig to use

$\frac{x}{a} = \frac{o}{h} = \boxed{\sin\theta}$

we want to draw a triangle with x and a, and then we want to make a trig formula w/o using the $\sqrt{\ }$.

$\frac{x}{a} = \frac{o}{a} = \boxed{\tan\theta}$



$\frac{x}{a} = \frac{h}{a} = \boxed{\sec\theta}$

in action

$\int \frac{x\,dx}{\sqrt{(16 - x^2)}}$ $\quad a = 4$, $x = 4\sin\theta$, $dx = 4\cos\theta\,d\theta$

$= \int \frac{4\cos\theta\, 4\sin\theta\,d\theta}{\sqrt{4^2(1 - \sin^2\theta)}}$

$= \int \frac{4\cos\theta\, 4\sin\theta}{\sqrt{4^2\cos^2\theta}}$

$= \int \frac{4\cos\theta}{4\cos\theta} \cdot 4\sin\theta\,d\theta$

$= \int 4\sin\theta\,d\theta$

$= -4\cos\theta + C$

lets draw a ref Δ to put back in terms of x

$x = 4\sin\theta$
$\frac{x}{4} = \frac{o}{h} = \sin\theta$
$o^2 + a^2 = h^2$
$a = \sqrt{h^2 - o^2}$

$\cos\theta = \frac{a}{h} = \frac{\sqrt{16 - x^2}}{4}$

$= \frac{-4\sqrt{16 - x^2}}{4} + C$
$= -\sqrt{16 - x^2} + C$

8.5

PARTIAL FRACTIONS

We can't integrate rational functions, but we can integrate sums of fractions. So lets break up some rational functions using Partial Fraction Decompisition.

NOTE: if improper simplify w/ long ÷

basic method

$\frac{3x}{x^2 + 2x - 8}$ factor denominator

$\frac{3x}{(x-2)(x+4)}$ set equal to partial fraction decomp.

$\frac{3x}{(x-2)(x+4)} = \frac{A}{x-2} + \frac{B}{x+4}$

multiply both sides by orig. denominator

$3x = A(x+4) + B(x-2)$

From here there are two ways to solve...

group like numbers

$3x + 0 = Ax + Bx + 4A - 2B$

have numbers w/ x in one equation + constants in another...

$3x + 0 = Ax + Bx + 4A - 2B$

$3x = (A+B)x$
$3 = A + B$
$3 - A = B$
$3 - 1 = B$
$\boxed{2 = B}$

$0 = 4A - 2B$
$0 = 4A - 2(3 - A)$
$0 = 4A - 6 + 2A$
$6 = 6A$
$\boxed{1 = A}$

the trick

$3x = A(x+4) + B(x-2)$

We can plug in numbers for x to find the constants. And we'll choose values of x that leave only one constant.

$3(-4) = \cancel{A(-4+4)} + B(-4-2)$
$-12 = B(-6)$
$\boxed{2 = B}$

$3(2) = A(2+4) + \cancel{B(2-2)}$
$6 = A(6)$
$\boxed{1 = A}$

then plug back into Partial Fraction Decomposition

$$\frac{3x}{x^2 + 2x - 8} = \frac{1}{x-2} + \frac{2}{x+4}$$