ch. 1.1, 1.2, 1.3, 1.4, 1.5, 1.6
2.1, 2.2, 2.3

probability

Midterm 1 study guide

Notations & Definitions

ch 1.1, 1.2

$x \in S$

"x is in set S"

element

$x \notin S$

"x is NOT in set S"

Set

a collection of objects we call elements

$S = \{\ldots\}$ → elements

$S = \{x \mid x \text{ satisfies } P\}$ → "such that"

Empty Set $\emptyset$
no elements, $\{\}$

Finite
a limited # of elements, $\{x_1, x_2, \ldots, x_n\}$

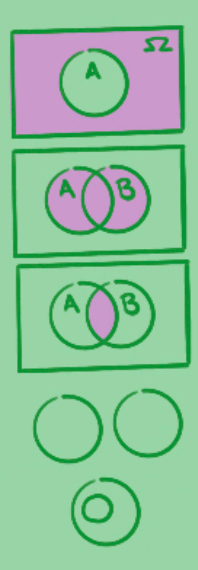
Countably Infinite
a known never-ending # of elements, $\{x_1, x_2, \ldots\}$

Uncountable
a continuous range of values that can't be written out, $\{x : 0 \le x \le 1\}$

Universal Set
a set w/ all elements, Ω

good luck

Set Operations & Algebra



Complement
all elements in the Ω not in S. S^c

Union
all elements in A or B or both. $A \cup B$ aka $\bigcup_{n=1}^{\infty} S_n$

Intersection
all elements in both A and B $A \cap B$ aka $\bigcap_{n=1}^{\infty} S_n$

Disjoint
two sets whose intersection is empty

Contained
aka subset, where every element of A is also an element of B $A \subset B$

$S \cup T = T \cup S$

$S \cap (T \cup V) = (S \cap T) \cup (S \cap V)$

$(S^c)^c = S$

$S \cup \Omega = \Omega$

$S \cap \Omega = S$

$S \cup (T \cup V) = (S \cup T) \cup V$

$S \cap S^c = \emptyset$

De Morgan's Law

$(S \cup T)^c = S^c \cap T^c$

$\left(\bigcup_n S_n\right)^c = \bigcap_n S_n^c$

Probability Definitions

What is probability? It's a way of describing uncertanty, we can do that w/ probablistic models. Some parts of a probablistic model...

- **sample space** → Ω set of all possible outcomes
- **probability law** → $P(A)$ assigns to an event a number that reps. the likelihood of that event.
- **experiment** → the process behind a probablistic model
- **outcomes** → what the experiment produces
- **event** → subset of sample space, a collection of outcomes.

Basics of Probability

basic idea of probility, aka discrete probability law...

$$P(A) = \frac{\#\text{ of times A could happen}}{\#\text{ of possible things that could happen}}$$

We use various strategies to visualize this idea, find easier ways to count occurences, perform algebra on probabilities, and find more specific probabilities.

Probability Axioms

something we assume to be true

Nonnegativity
$P(A) \ge 0$

Additivity
if A & B disjoint
$P(A \cup B) = P(A) + P(B)$

Normalization
$P(\Omega) = 1$

More propertys
- $P(\emptyset) = 0$
- if $B \subseteq A$, then $P(B) \le P(A)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B) \le P(A) + P(B)$
- $P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)$

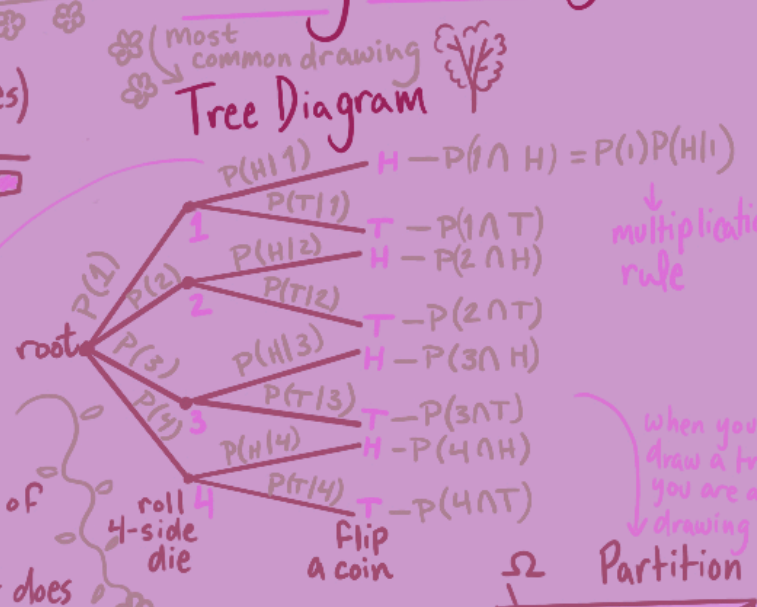
Drawing Probability Models

common for drawing dice rolls

$P(\text{doubles}) = \frac{\square}{\square + \square}$

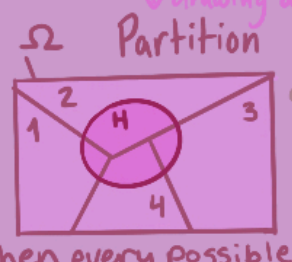
most common drawing

Tree Diagram



multiplication rule

when you draw a tree you are always drawing a Partition



when every possible outcome is included in one of your events.

Conditional Probability

what does this mean?

- $P(A|B)$ means probability of "A given B".
- We know B happened, what does that tell us about the probability A will happen?

$$P(A|B) = \frac{\#\text{ of elements in both A\&B}}{\#\text{ of elements of B}} = \frac{P(A \cap B)}{P(B)}$$

- we could also reverse the information to get

$P(A \cap B) = P(B)P(A|B)$ → multiplication rule

what happens when $P(A|B) = P(A)$

Independence

- when $P(A|B) = P(A)$, A and B are independent.
- when independent ⇝ $P(A \cap B) = P(A)P(B)$
- **Independent does not mean Disjoint!**
- Independent events will never be disjoint.
- compliments will never be independent.
- an event can be not independent but maybe **Conditionally Independent** if...

$P(A \cap B | C) = P(A|C)P(B|C)$

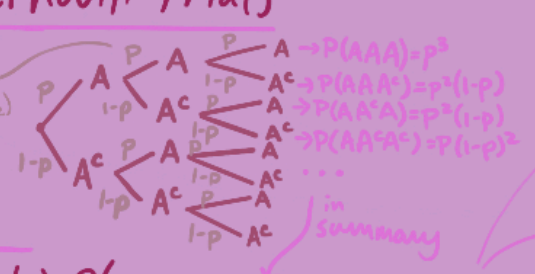
or...

$P(A \cap B | C) = P(B|C)P(A|B \cap C)$

- if an experimet has independent but identical stages we say they are **independent trials**, and if there are only 2 possible outcomes at each stage we have...

Bernoulli Trials

typically be $P(A_2|A_1)$ but indp. so $P(A_2|A_1) = P(A_2)$



in summary

$p(k) = P(k \text{ sucesses in } n \text{ trials}) = \binom{n}{k} p^k (1-p)^{n-k}$

→ binomial probabilities

but what's this mean?

Counting

- for Bernoulli Trials (and other probability equations it can be impractical to draw or write out all possible combinations, so we use counting.
  - basic principle...

n options 1st stage
n-1 options 2nd stage
...
n-(k-1) options kth stage

$\frac{n!}{(n-k)!}$ options

- this is exactly...

K-permutations
- count how many ways we can pick k out of n objects and arrange in a sequence

$$\frac{n!}{(n-k)!}$$

Combinations
- very similar
- count how many unique ways we can pick k out of n objects
- no ordering, no repeats.

$$\frac{n!}{k!(n-k)!} = \binom{n}{k}$$

Partitions
- we have a combination, but split it into subsets
- ussually done to account for repeats

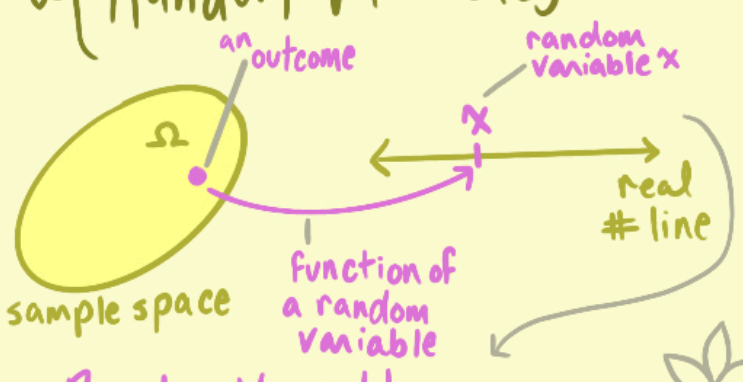
$$\frac{n!}{n_1! n_2! \ldots n_r!} = \binom{n!}{n_1, n_2, \ldots, n_r}$$

Distributions
- distributing n objects among k places, w/ random amout to each place.

$\binom{n+k-1}{k-1}$ ⇝ each handout plus a break between each place ⇝ breaks between places

ch 2.1, 2.2, 2.3

Discrete Random Variables



Random Variable

- a real-valued fuction of the outcome of the experiment
- assigning numerical values to an outcome.

discrete r.v.

$$\underbrace{P_X(x)}_{\text{probability mass function (pmf)}} = P(\underbrace{\{X = x\}}_{\text{probability of event } X = x})$$

- we can also write functions of pmfs.

$Y = g(X) \quad P_Y(y) = \sum_{\{x | g(x) = y\}} P_X(x)$

Brand-Name Random Variables

Bernoulli RV $P_X(k) = \begin{cases} p, & \text{if } k = 1 \\ p-1, & \text{if } k = 0 \end{cases}$
an experiment w/ only 2 outcomes, "sucess" and "failure", prob of each outcome.

Binomial RV $P_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$
an experiment w/ only 2 outcomes that we run n times. prob of k sucesses in those n times

Geometric RV $P_X(k) = (1-p)^{k-1}(p)$
an experiment w/ only 2 outcomes, prob of exactly k trials until first sucess.

less import. lowkey

Negative Binomial $P_X(k) = \binom{k+r-1}{k}(1-p)^k p^r$
run experiment until r sucesses. k fails until

Hypergeometric $P_X(k) = \left(\binom{w}{k}\binom{B}{n-k}\right) / \binom{w+B}{n}$
w marked balls & B norm. balls in an urn, draw n. prob. of exactly k marked balls.

Uniform $P_X(k) = 1/n$
n outcomes each w/ equal prob. prob of outcome k

Poisson RV $P_X(k) = (e^{-\lambda}\lambda)^k / k!$
used to approx. binomial w/ small k and very big n