Supporting Information to: "Modelling approaches for meta-analyses with dependent effect sizes in ecology and evolution: A simulation study"

1 Distribution of meta-analyses in ecology and evolution

The data-generating mechanism of our simulation studies was based on a survey of 86 published meta-analyses in ecology and evolution Senior et al., 2016, including phylogenetic meta-analysis. Figure S1.A shows that effect sizes per study are right-skewed, ranging from 3 to 759 effect sizes, with six studies reporting outlying values of over 500 effect sizes (i.e. large meta-analysis). Of the 86 meta-analyses surveyed, 51 specified the number of species included, which ranged from 1 to 438 species. Similarly, Figure S1.B displays a right-skewed distribution in the number of species per study.

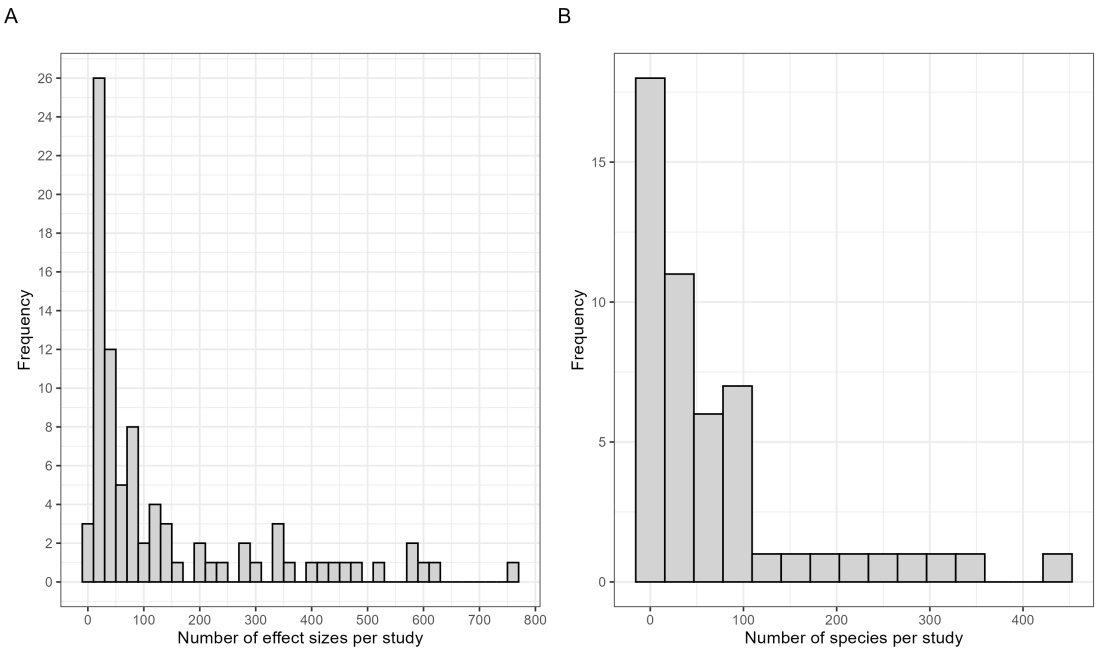


Figure S1: Overview of characteristics of 83 published meta-analytical data in ecology and evolution (Senior et al., 2016). **A.** Distribution of the number of effect sizes per study k. **B.** Distribution of the number of species per study $k_{species}$.

2 Results simulation study 1

The following are the supporting figures of the simulation study 1.

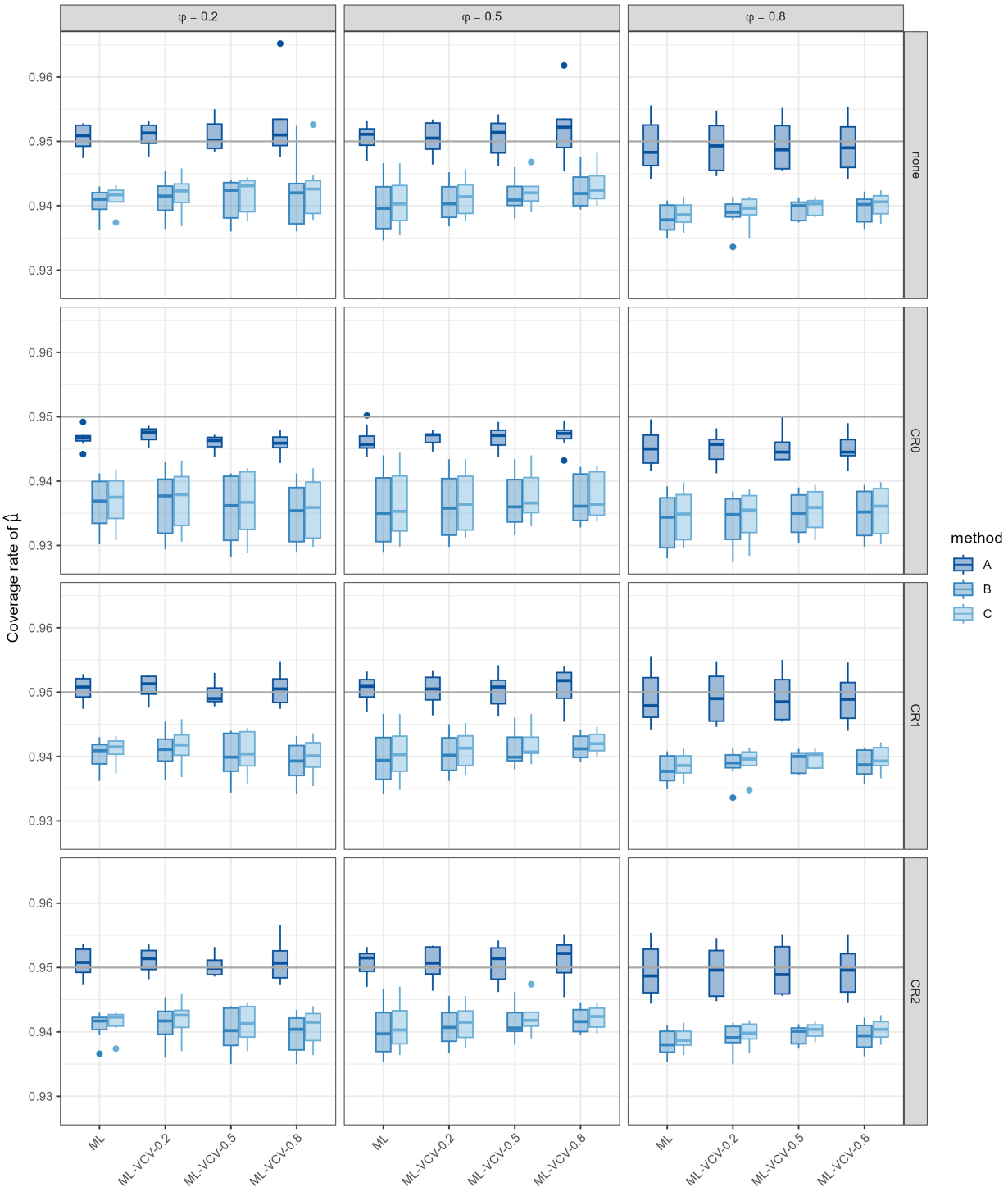


Figure S2: 95% coverage rates of multilevel models given different inference methods across different true within-study correlation, $\phi \in (0.2, 0.5, 0.8)$, and CRVE methods. **Inference method A:** T-test and adjusted degrees of freedom ($k_{studies} - 1$). **Inference method B:** Z-test statistic. **Inference method C:** T-test statistic with no adjusted to degrees of freedom ($k - 1$).

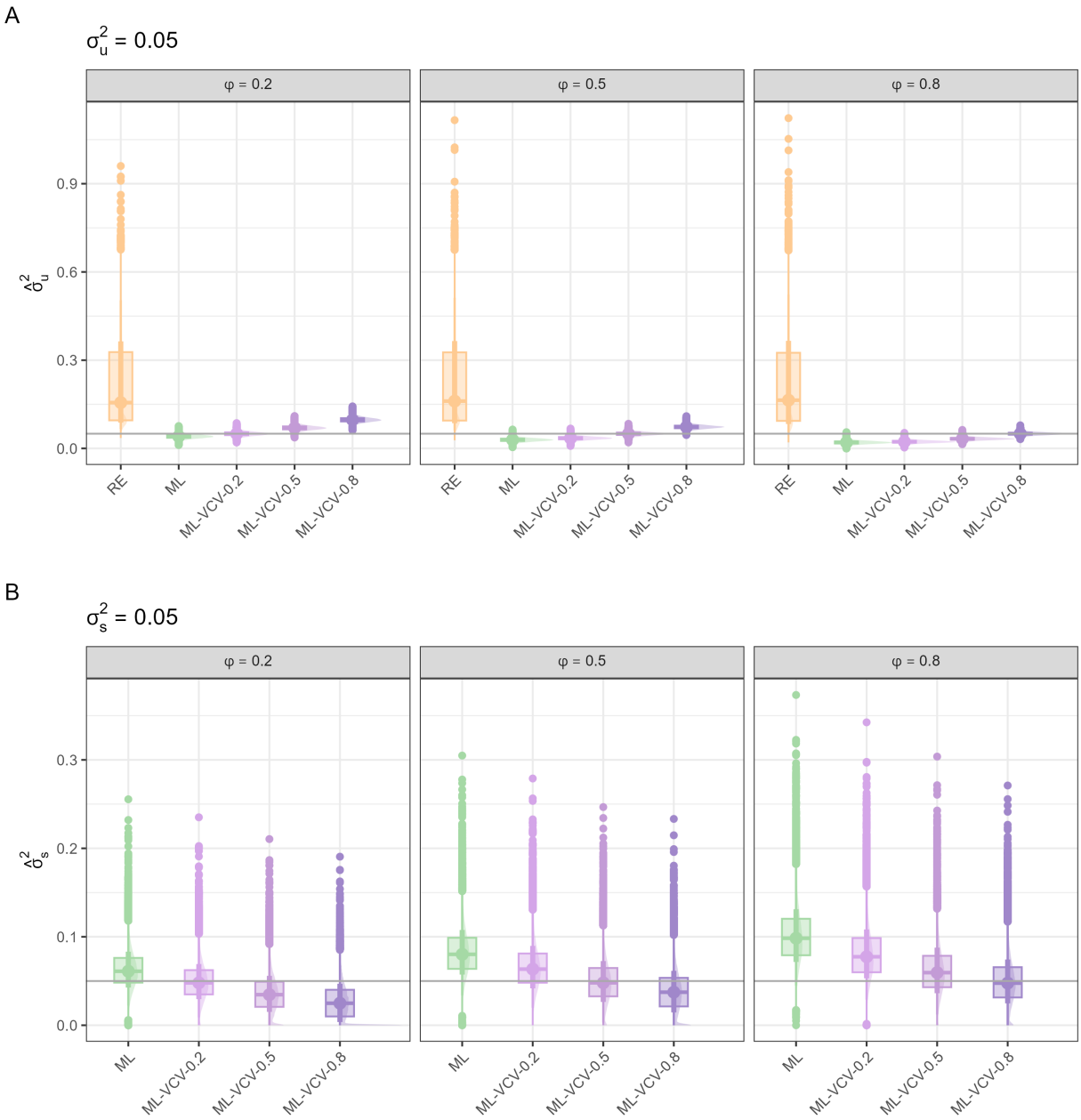


Figure S3: Boxplot of within-study conditional variance estimates ($\hat{\sigma}_u^2$) and among study ($\hat{\sigma}_s^2$) under true values of $\sigma_u^2 = 0.05$ and $\sigma_s^2 = 0.05$ and true within correlation levels of $\phi \in (0.2, 0.5, 0.8)$. The true variance is shown in the grey bolded line and the boxplots represent the variability of estimates across 5,000 simulations.

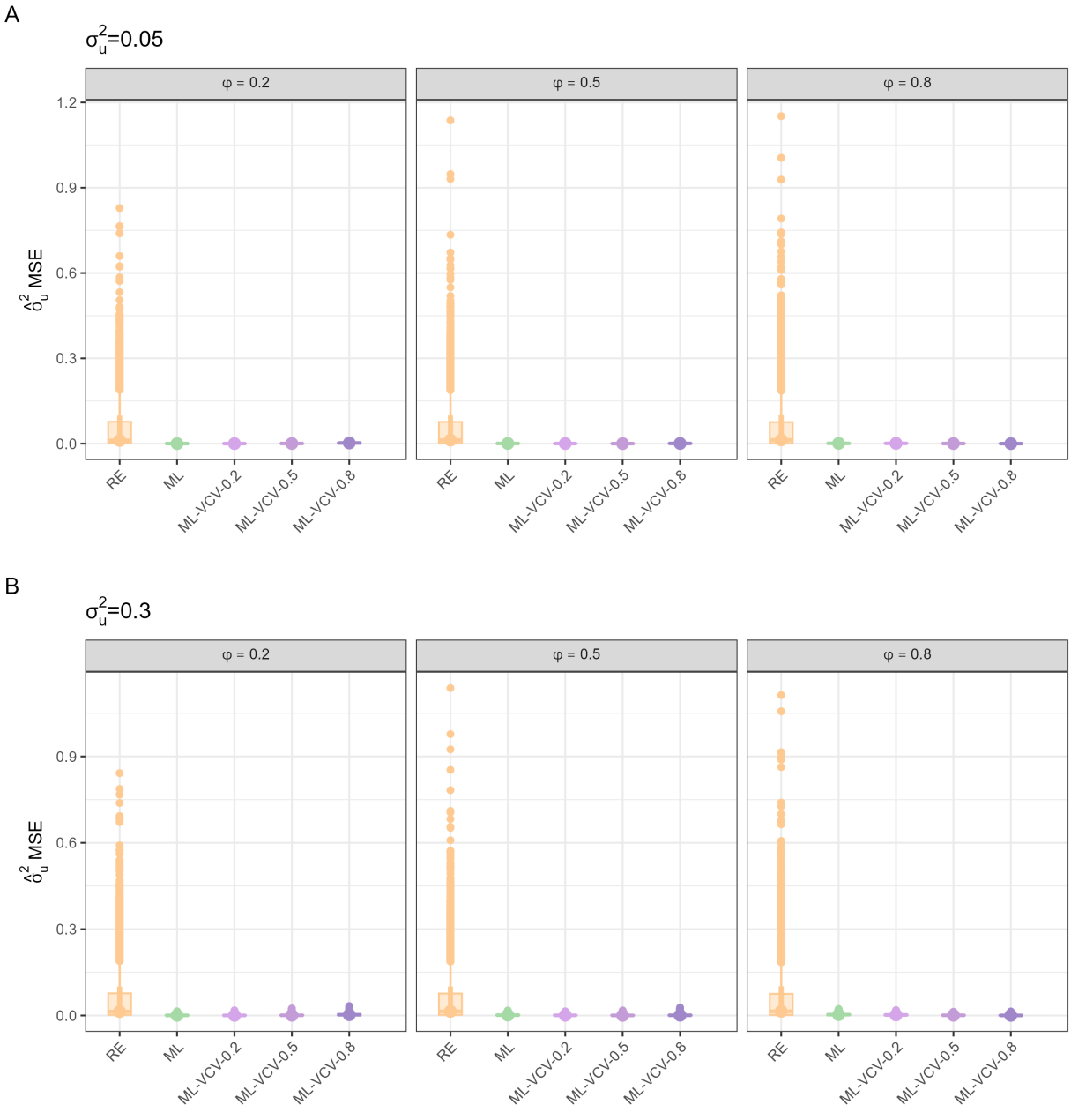


Figure S4: Distribution of the mean squared error (MSE) of the within-study conditional variance estimates ($\hat{\sigma}_u^2$) under true values of $\sigma_u^2 \in (0.05, 0.3)$ and true within study correlation levels of $\phi \in (0.2, 0.5, 0.8)$.

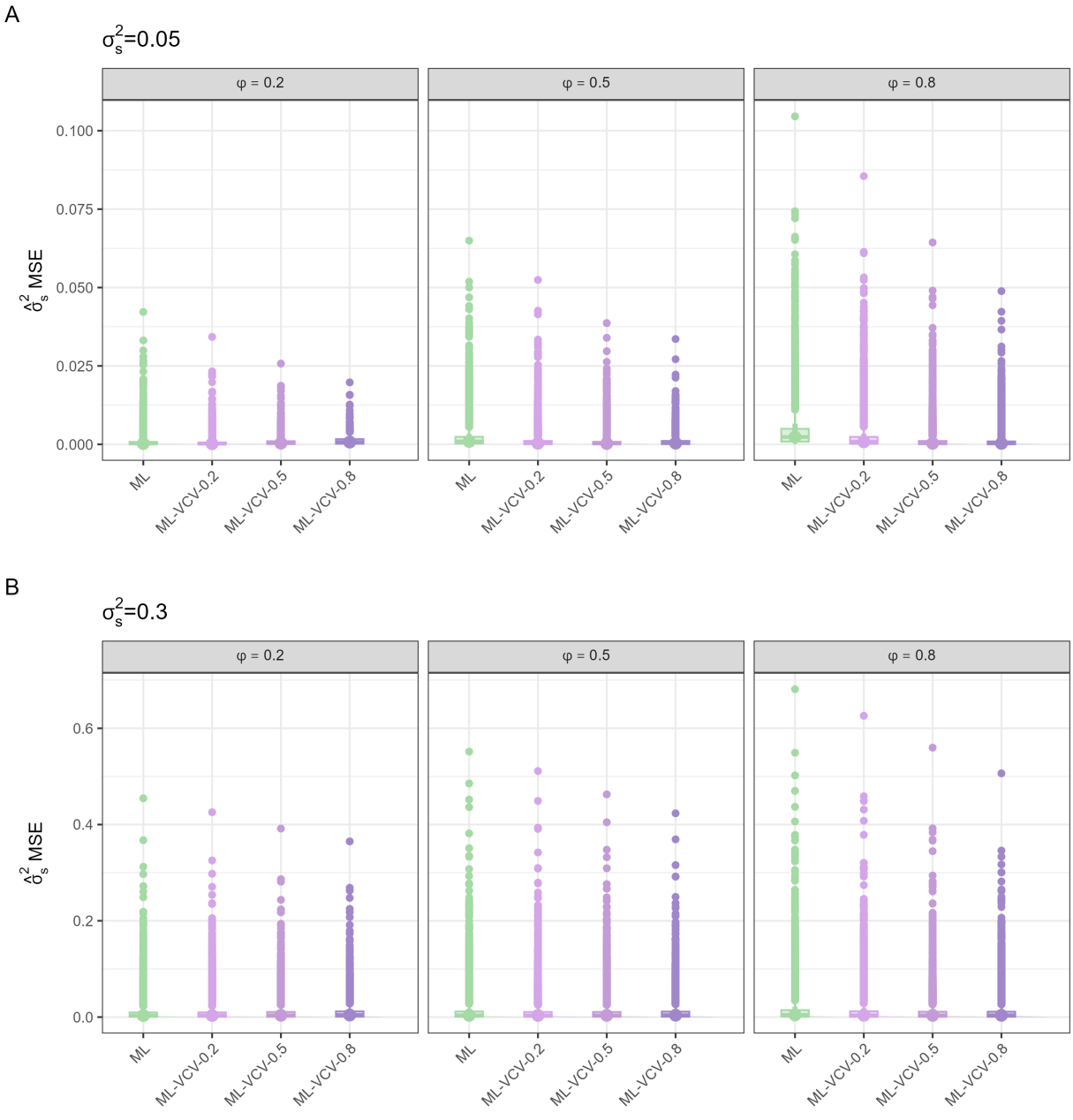


Figure S5: Distribution of the mean squared error (MSE) of the among study conditional variance estimates ($\hat{\sigma}_s^2$) under true values of $\sigma_s^2 \in (0.05, 0.3)$ and true within study correlation levels of $\phi \in (0.2, 0.5, 0.8)$.

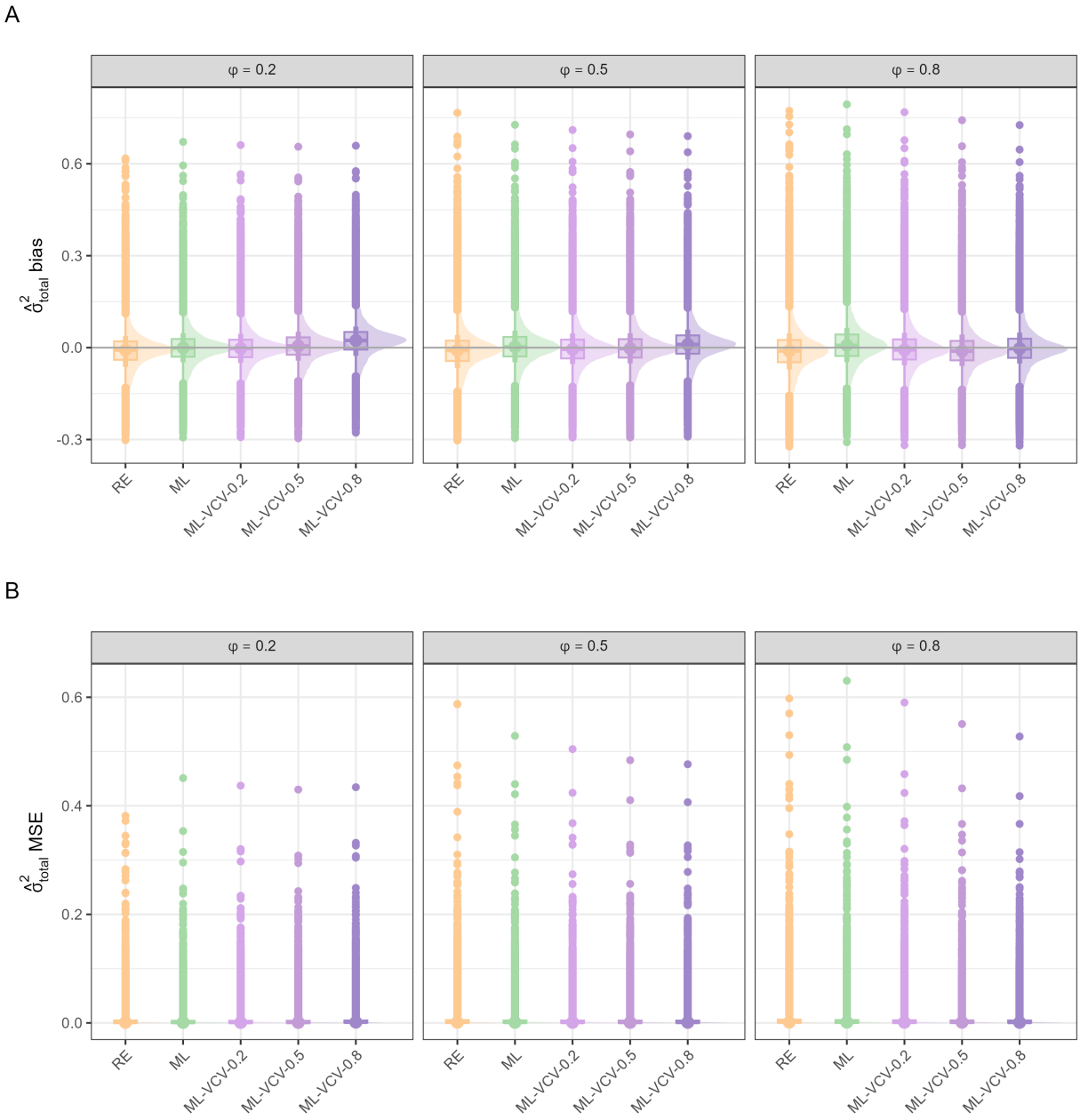


Figure S6: Distribution of bias in boxplots and the mean squared error (MSE) of the total conditional variance estimates of the models ($\hat{\sigma}^2_{total} = \hat{\sigma}^2_u + \hat{\sigma}^2_s$) under all conditions of true values of $\sigma^2_s \in (0.05, 0.3)$ and true within study correlation levels of $\phi \in (0.2, 0.5, 0.8)$. Models that did not estimate among study variation had $\hat{\sigma}^2_s = 0$.

Table S1: Monte Carlo standard errors of overall mean estimate ($\hat{\mu}$) bias across all models (with and without CRVE methods) per true within-study correlation (ϕ).

model	ϕ	$\hat{\mu}$	MCSE
FE	0.20	0.20	0.1075
RE	0.20	0.20	0.0960
ML	0.20	0.20	0.0854
ML-VCV-0.2	0.20	0.20	0.0851
ML-VCV-0.5	0.20	0.20	0.0853
ML-VCV-0.8	0.20	0.20	0.0859
FE	0.50	0.20	0.1098
RE	0.50	0.20	0.1007
ML	0.50	0.20	0.0889
ML-VCV-0.2	0.50	0.20	0.0878
ML-VCV-0.5	0.50	0.20	0.0874
ML-VCV-0.8	0.50	0.20	0.0876
FE	0.80	0.20	0.1124
RE	0.80	0.20	0.1056
ML	0.80	0.20	0.0926
ML-VCV-0.2	0.80	0.20	0.0907
ML-VCV-0.5	0.80	0.20	0.0897
ML-VCV-0.8	0.80	0.20	0.0893

Table S2: Monte Carlo standard errors of overall mean estimate ($\hat{\mu}$) of 95% coverage rate across all models (without CRVE methods) per true within-study correlation (ϕ).

model	ϕ	$Cov(\hat{\mu})$	MCSE
FE	0.20	0.204	0.1426
RE	0.20	0.546	0.1760
ML	0.20	0.951	0.0766
ML-VCV-0.2	0.20	0.951	0.0763
ML-VCV-0.5	0.20	0.951	0.0764
ML-VCV-0.8	0.20	0.953	0.0752
FE	0.50	0.195	0.1401
RE	0.50	0.511	0.1767
ML	0.50	0.951	0.0766
ML-VCV-0.2	0.50	0.950	0.0768
ML-VCV-0.5	0.50	0.951	0.0766
ML-VCV-0.8	0.50	0.952	0.0756
FE	0.80	0.188	0.1380
RE	0.80	0.486	0.1767
ML	0.80	0.949	0.0776
ML-VCV-0.2	0.80	0.949	0.0776
ML-VCV-0.5	0.80	0.949	0.0775
ML-VCV-0.8	0.80	0.949	0.0775

Table S3: Monte Carlo standard errors of the bias of the within-study variance component ($\hat{\sigma}_u^2$) and the among study variance component ($\hat{\sigma}_s^2$) estimates across all models (with and without CRVE methods) per true within-study correlation (ϕ).

model	ϕ	$\hat{\sigma}_u^2$ MCSE	$\hat{\sigma}_s^2$ MCSE
RE	0.20	0.188	
ML	0.20	0.124	0.142
ML-VCV-0.2	0.20	0.127	0.142
ML-VCV-0.5	0.20	0.129	0.141
ML-VCV-0.8	0.20	0.128	0.140
RE	0.50	0.189	
ML	0.50	0.120	0.144
ML-VCV-0.2	0.50	0.123	0.144
ML-VCV-0.5	0.50	0.126	0.143
ML-VCV-0.8	0.50	0.128	0.142
RE	0.80	0.191	
ML	0.80	0.115	0.147
ML-VCV-0.2	0.80	0.119	0.146
ML-VCV-0.5	0.80	0.123	0.145
ML-VCV-0.8	0.80	0.126	0.144

Table S4: Computational time and convergence rates (%) for models in simulation study 1.

model	k.studies	time (sec.)	%
FE	20	0.06	100.00
RE	20	0.15	100.00
ML	20	0.27	100.00
ML-VCV-0.2	20	0.29	100.00
ML-VCV-0.5	20	0.30	100.00
ML-VCV-0.8	20	0.34	100.00
FE	50	0.15	100.00
RE	50	1.39	100.00
ML	50	2.46	100.00
ML-VCV-0.2	50	2.31	100.00
ML-VCV-0.5	50	2.40	100.00
ML-VCV-0.8	50	2.54	100.00

3 Results simulation study 2

The following are the supplementary figures and tables for the simulation study 2.

3.1 Phylogenetic meta-analysis

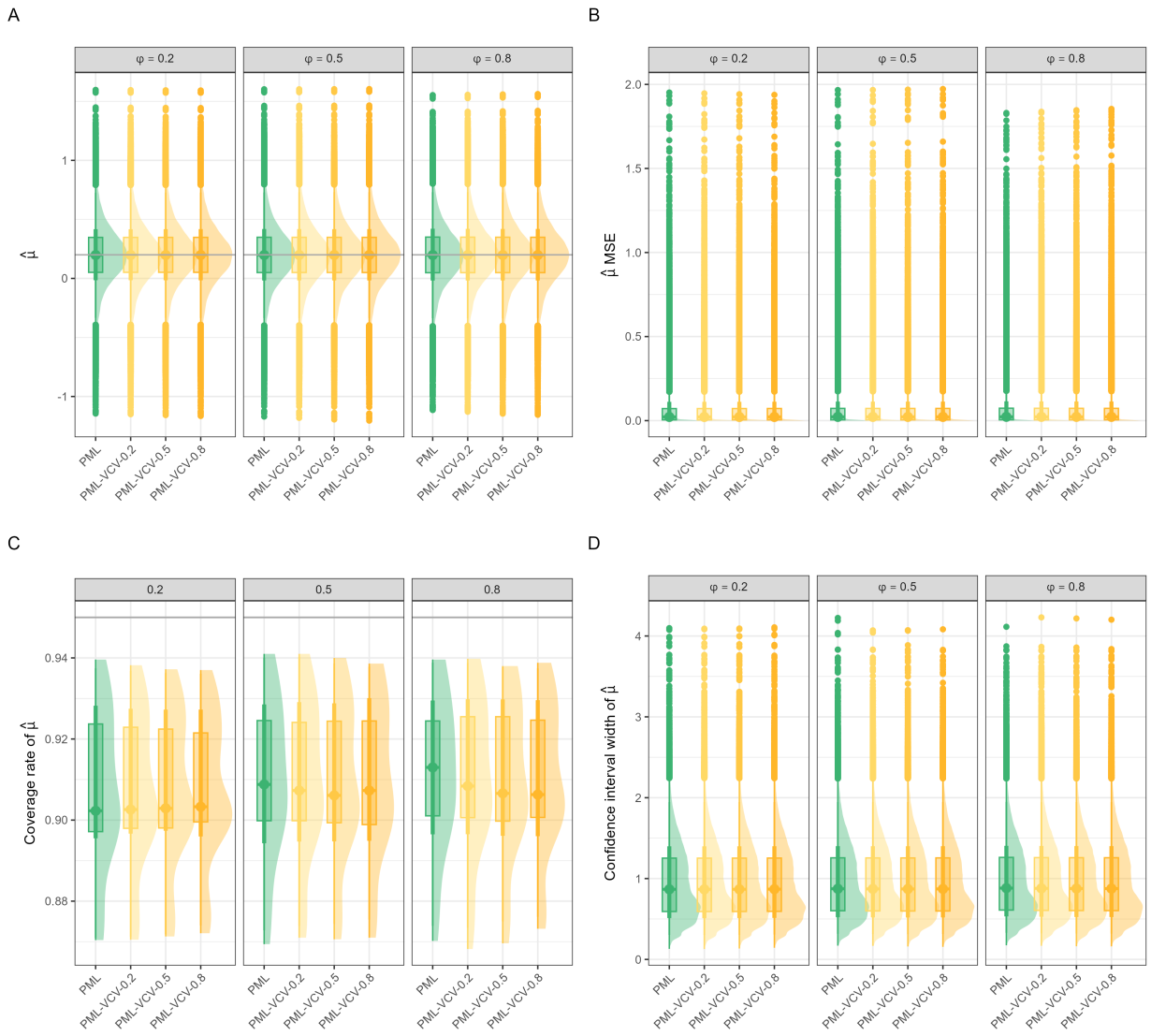


Figure S7: Overall mean estimate $\hat{\mu}$ performance across all working models and conditions assuming a true within study correlation between effect sizes of $\phi \in (0.2, 0.5, 0.8)$, evaluated over 5,000 simulation iterations. **A.** The bias of the overall mean estimate $\hat{\mu}$, reflecting the deviation from the true mean. **B.** The mean squared error (MSE) of $\hat{\mu}$, combining both bias and variance to measure accuracy. **C.** The coverage rates of the 95% confidence intervals, indicating the proportion of intervals that include the true mean μ and assessing the reliability and consistency of the interval estimates. **D.** The widths of the 95% confidence intervals, representing the precision of the estimates across different conditions.

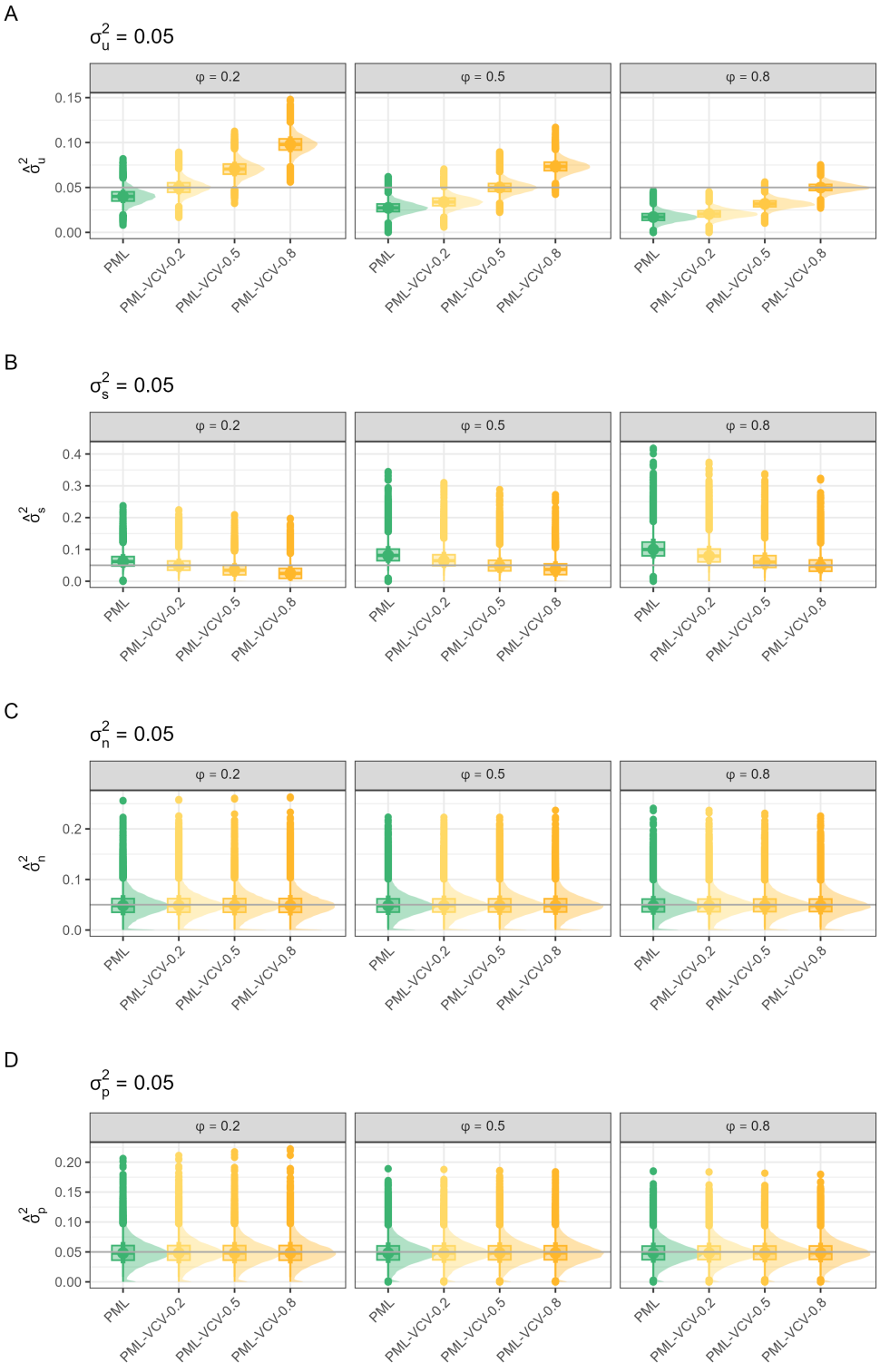
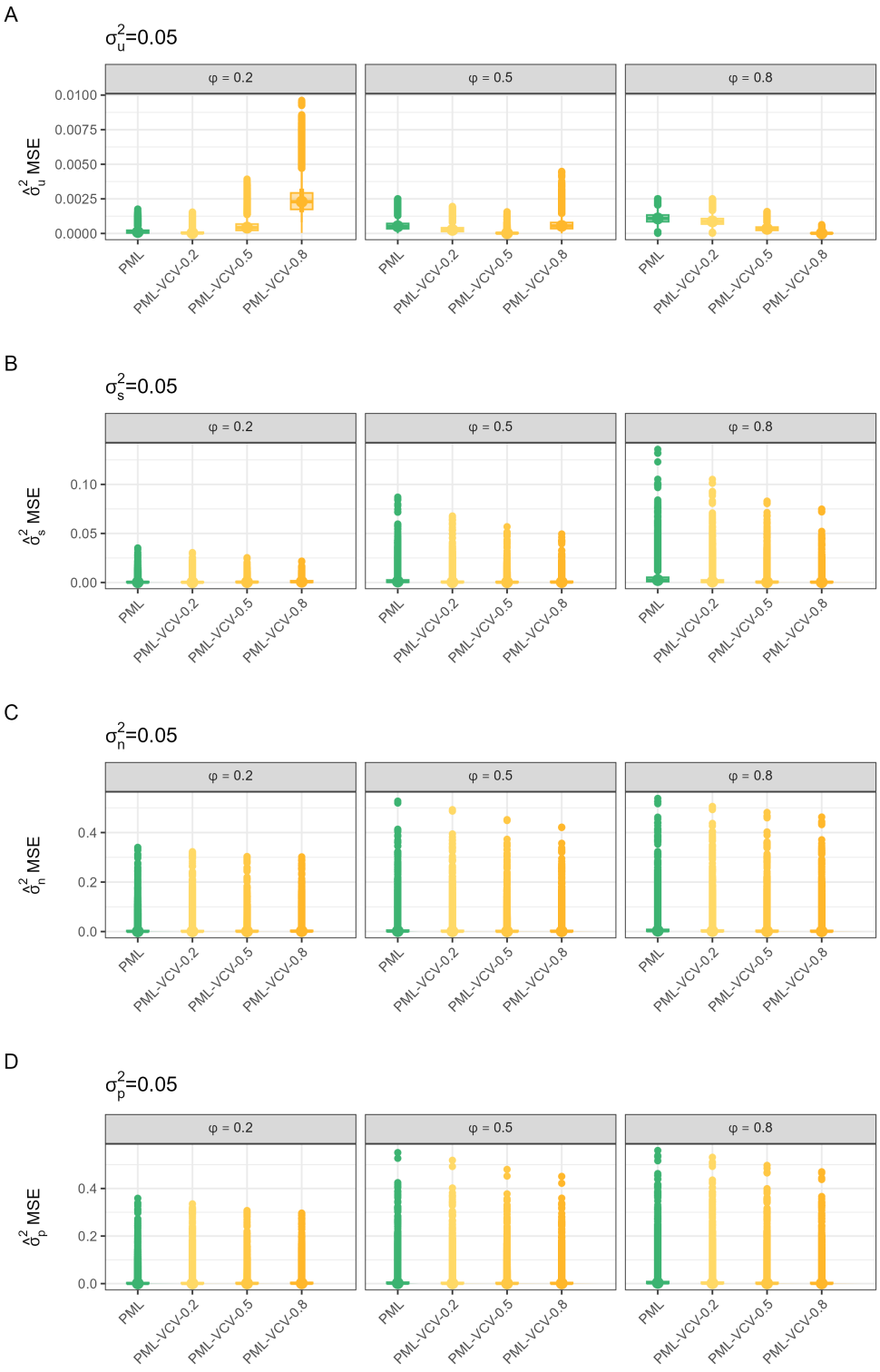
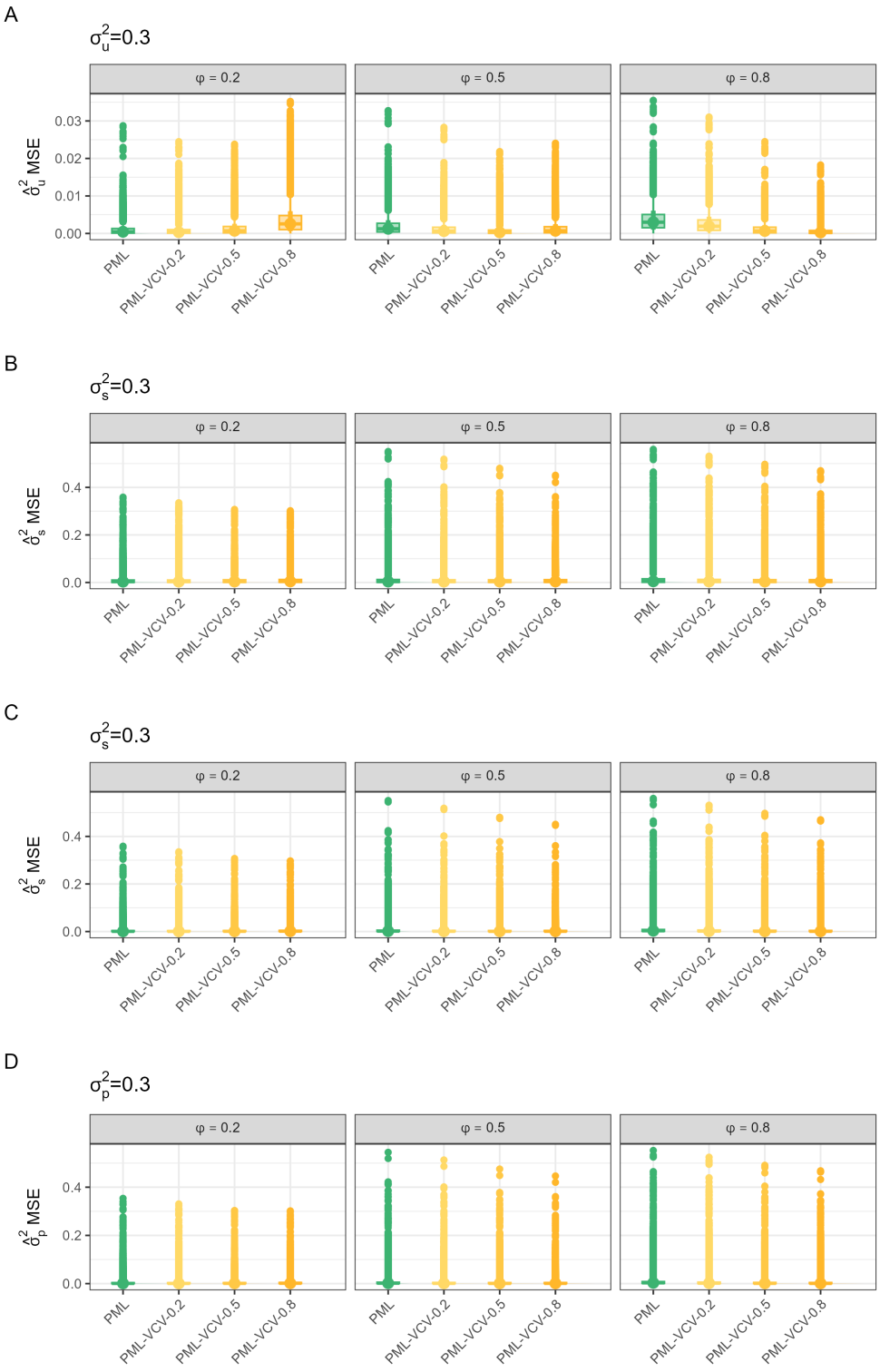


Figure S8: Boxplot of within-study conditional variance estimates ($\hat{\sigma}_u^2$) and among study ($\hat{\sigma}_s^2$) under true values of $\sigma_u^2 = 0.05$ and $\sigma_s^2 = 0.05$ and true within correlation levels of $\phi \in (0.2, 0.5, 0.8)$. The true variance is shown in the grey bolded line and the boxplots represent the variability of estimates across 5,000 simulations.



.

Figure S9: Distribution of the mean squared error (MSE) of conditional variance estimates ($\hat{\sigma}_u^2$, $\hat{\sigma}_s^2$, $\hat{\sigma}_n^2$, $\hat{\sigma}_p^2$) under true values of within study correlation levels of $\phi \in (0.2, 0.5, 0.8)$



.

Figure S10: Distribution of the mean squared error (MSE) of conditional variance estimates ($\hat{\sigma}_u^2$, $\hat{\sigma}_s^2$, $\hat{\sigma}_n^2$, $\hat{\sigma}_p^2$) under true values of within study correlation levels of $\phi \in (0.2, 0.5, 0.8)$

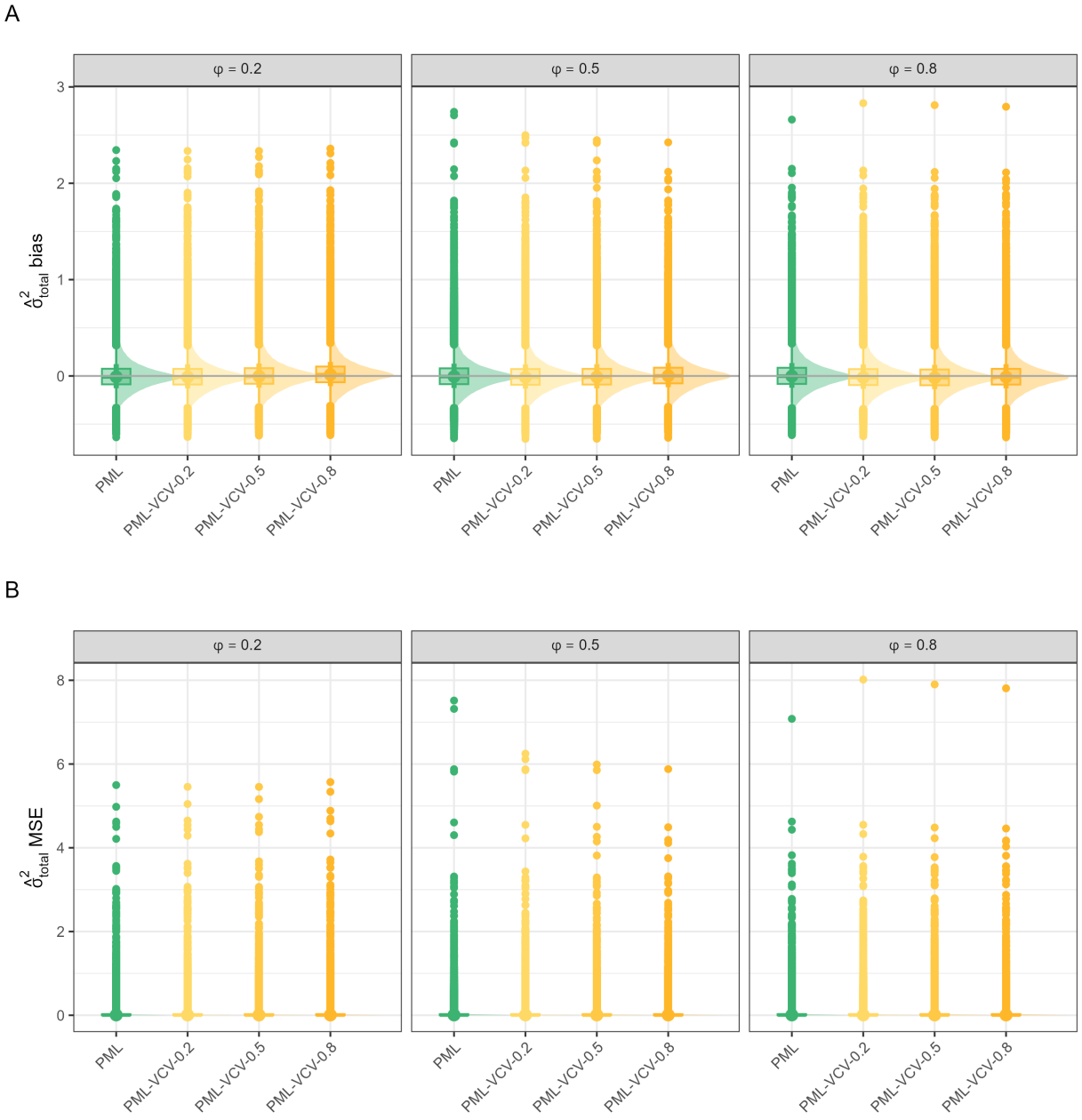


Figure S11: Distribution of bias in boxplots and the mean squared error (MSE) of the total conditional variance estimates of the models ($\hat{\sigma}^2_{total} = \hat{\sigma}^2_u + \hat{\sigma}^2_s + \hat{\sigma}^2_n + \hat{\sigma}^2_p$) under all conditions and true within study correlation levels of $\phi \in (0.2, 0.5, 0.8)$.

Table S5: Monte Carlo standard errors of overall mean estimate ($\hat{\mu}$) bias across all models (with and without CRVE methods) per true within-study correlation (ϕ).

model	ϕ	$\hat{\mu}$	MCSE
PML	0.20	0.199	0.255
PML-VCV-0.2	0.20	0.199	0.255
PML-VCV-0.5	0.20	0.199	0.255
PML-VCV-0.8	0.20	0.199	0.255
PML	0.50	0.199	0.256
PML-VCV-0.2	0.50	0.199	0.256
PML-VCV-0.5	0.50	0.199	0.256
PML-VCV-0.8	0.50	0.199	0.256
PML	0.80	0.199	0.258
PML-VCV-0.2	0.80	0.199	0.257
PML-VCV-0.5	0.80	0.198	0.257
PML-VCV-0.8	0.80	0.198	0.256

Table S6: Monte Carlo standard errors of overall mean estimate ($\hat{\mu}$) of 95% coverage rate across all models (without CRVE methods) per true within-study correlation (ϕ).

model	ϕ	$Cov(\hat{\mu})$	MCSE
PML	0.20	0.907	0.1026
PML-VCV-0.2	0.20	0.907	0.1025
PML-VCV-0.5	0.20	0.907	0.1025
PML-VCV-0.8	0.20	0.908	0.1024
PML	0.50	0.910	0.1012
PML-VCV-0.2	0.50	0.910	0.1012
PML-VCV-0.5	0.50	0.910	0.1014
PML-VCV-0.8	0.50	0.910	0.1013
PML	0.80	0.911	0.1006
PML-VCV-0.2	0.80	0.911	0.1009
PML-VCV-0.5	0.80	0.910	0.1012
PML-VCV-0.8	0.80	0.910	0.1011

Table S7: Monte Carlo standard errors of the bias of the estimates of the within-study variance component ($\hat{\sigma}^2_u$), the among study variance component ($\hat{\sigma}^2_s$), the species level non-phylogenetic component ($\hat{\sigma}^2_n$), and the species level phylogenetic variance component ($\hat{\sigma}^2_p$), across all models (with and without CRVE methods) per true within-study correlation (ϕ).

		$\hat{\sigma}^2_u$	$\hat{\sigma}^2_s$	$\hat{\sigma}^2_n$	$\hat{\sigma}^2_p$
model	ϕ	MCSE	MCSE	MCSE	MCSE
PML	0.20	0.125	0.143	0.136	0.200
PML-VCV-0.2	0.20	0.127	0.143	0.136	0.200
PML-VCV-0.5	0.20	0.128	0.142	0.136	0.200
PML-VCV-0.8	0.20	0.128	0.141	0.136	0.201
PML	0.50	0.121	0.146	0.135	0.199
PML-VCV-0.2	0.50	0.124	0.145	0.135	0.199
PML-VCV-0.5	0.50	0.127	0.145	0.136	0.199
PML-VCV-0.8	0.50	0.127	0.144	0.136	0.199
PML	0.80	0.116	0.148	0.135	0.199
PML-VCV-0.2	0.80	0.119	0.147	0.135	0.199
PML-VCV-0.5	0.80	0.124	0.146	0.135	0.198
PML-VCV-0.8	0.80	0.126	0.146	0.135	0.198

Table S8: Computational time and convergence rates (%) for phylogenetic multilevel models in simulation study 2.

model	k.studies	time (secs)	%
PML	20	0.79	99.976
PML-VCV-0.2	20	0.81	99.976
PML-VCV-0.5	20	0.82	99.976
PML-VCV-0.8	20	0.83	99.976
PML	50	5.97	99.998
PML-VCV-0.2	50	5.54	99.998
PML-VCV-0.5	50	5.36	99.998
PML-VCV-0.8	50	5.36	99.998

3.2 Phylogenetic meta-regression

The following are the supplementary figures and tables for the simulation study 2 meta-regression.

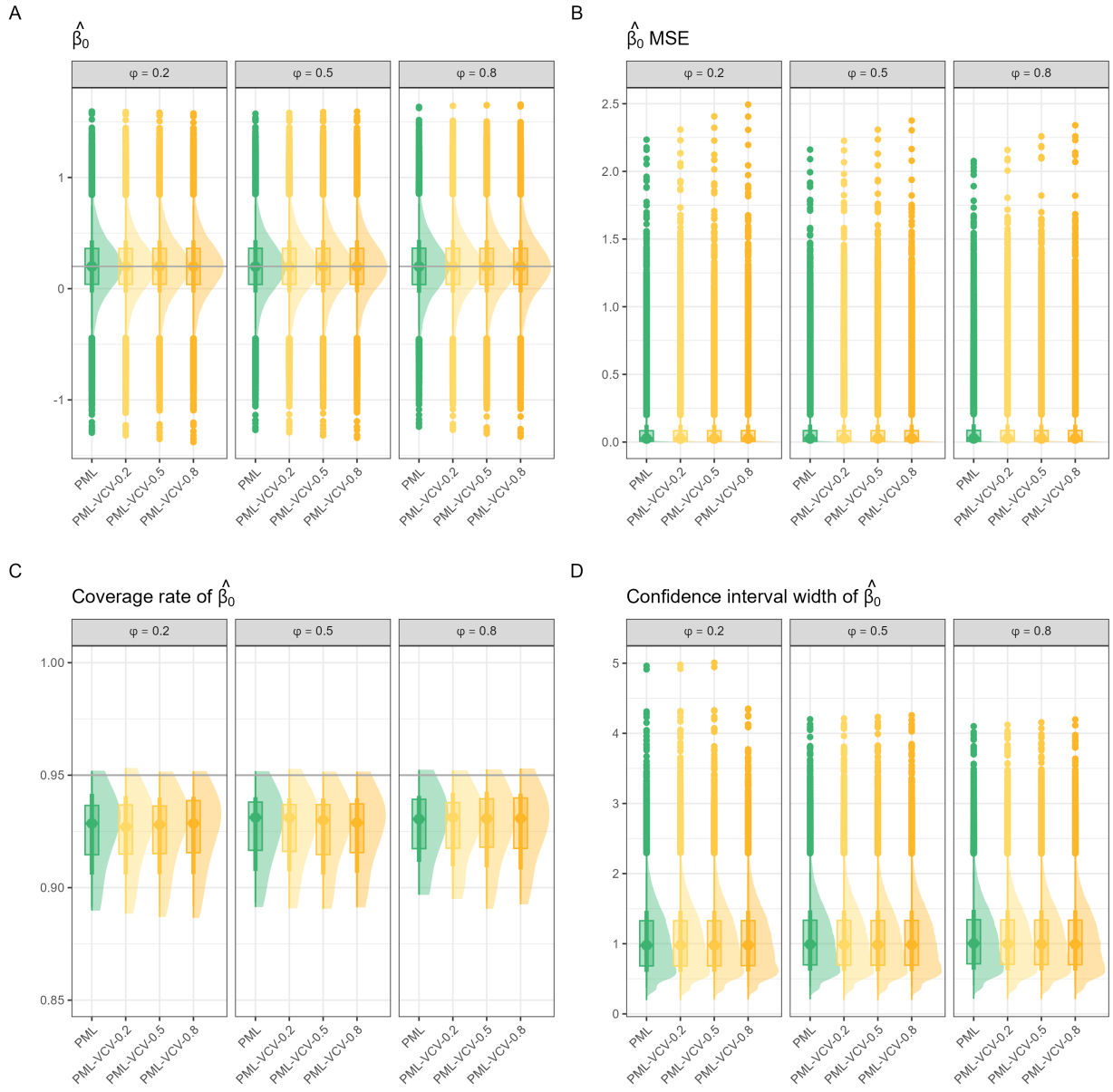


Figure S12: Regression coefficient estimate $\hat{\beta}_0$ performance across all working models and conditions assuming a true within study correlation between effect sizes of $\phi \in (0.2, 0.5, 0.8)$, without any CRVE method applied. **A.** The bias of the overall mean estimate $\hat{\beta}_0$, reflecting the deviation from the true mean. **B.** The mean squared error (MSE) of $\hat{\beta}_0$, combining both bias and variance to measure accuracy. **C.** The coverage rates of the 95% confidence intervals, indicating the proportion of intervals that include the true mean β_0. **D.** The widths of the 95% confidence intervals of $\hat{\beta}_0$.

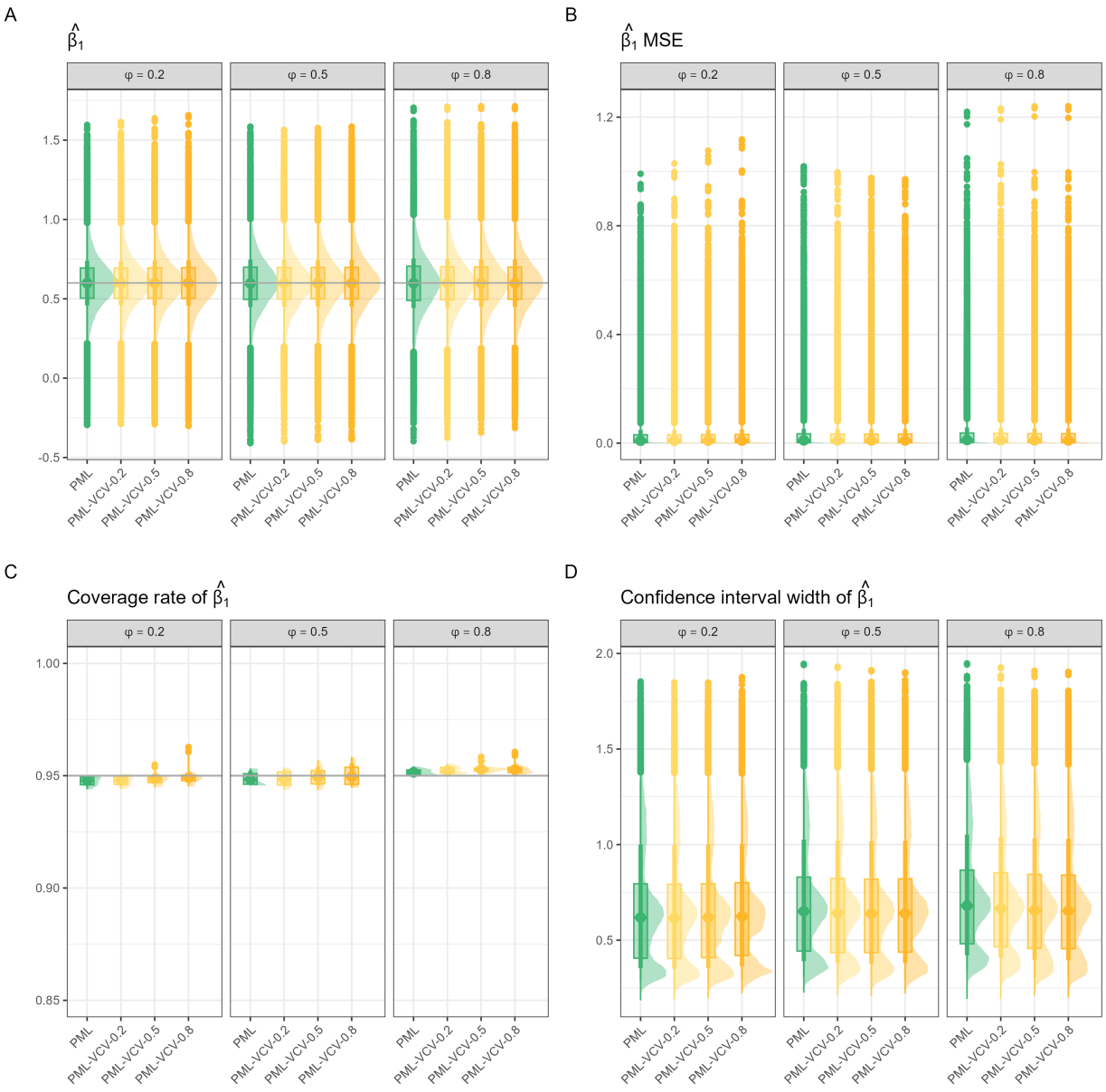


Figure S13: Regression coefficient estimate $\hat{\beta}_1$ performance across all working models and conditions assuming a true within study correlation between effect sizes of $\phi \in (0.2, 0.5, 0.8)$, without any CRVE method applied. **A.** The bias of the overall mean estimate $\hat{\beta}_1$, reflecting the deviation from the true mean. **B.** The mean squared error (MSE) of $\hat{\beta}_1$, combining both bias and variance to measure accuracy. **C.** The coverage rates of the 95% confidence intervals, indicating the proportion of intervals that include the true mean β_1. **D.** The widths of the 95% confidence intervals of $\hat{\beta}_1$.

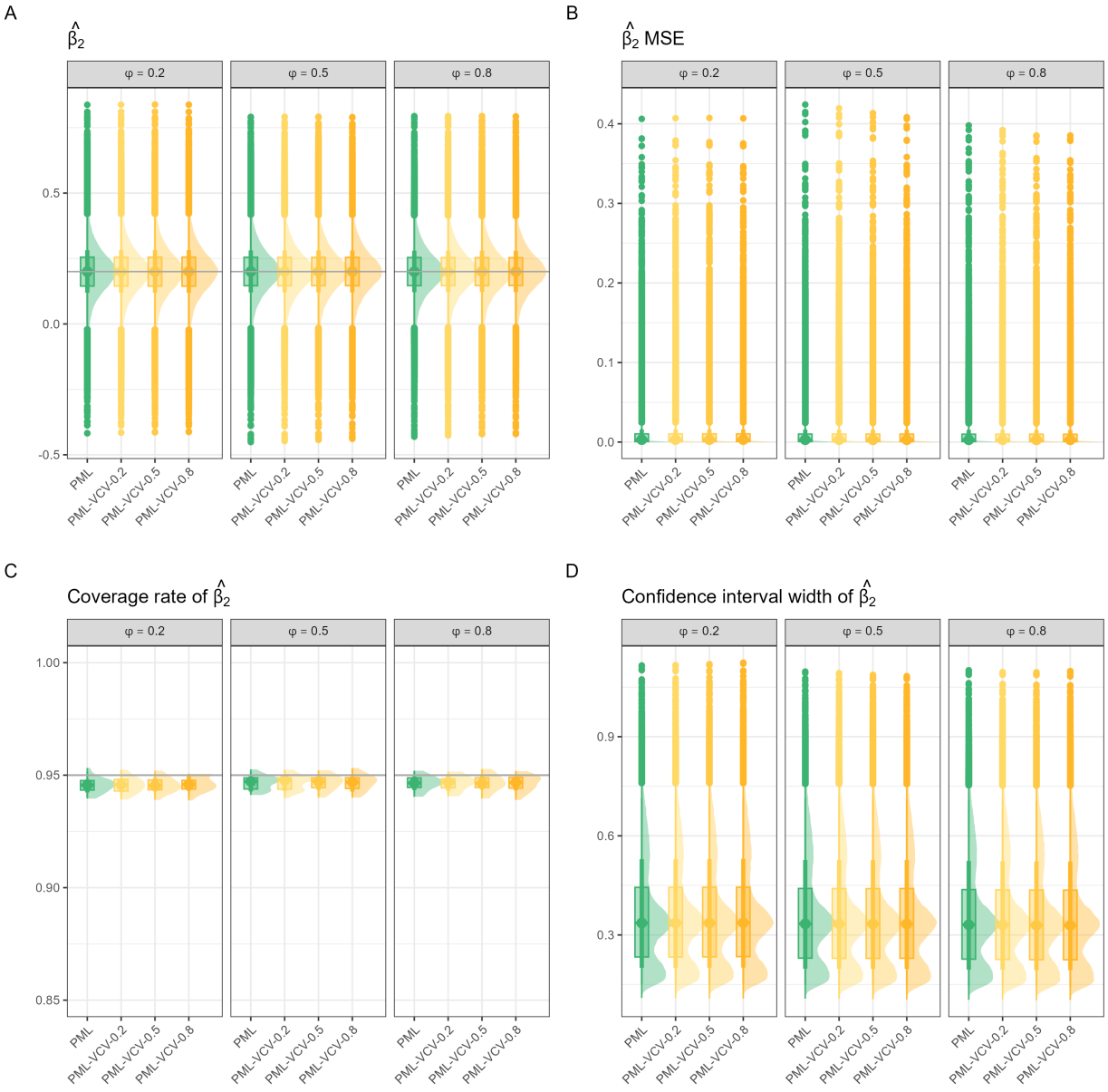


Figure S14: Regression coefficient estimate $\hat{\beta}_2$ performance across all working models and conditions assuming a true within study correlation between effect sizes of $\phi \in (0.2, 0.5, 0.8)$, without any CRVE method applied. **A.** The bias of the overall mean estimate $\hat{\beta}_2$, reflecting the deviation from the true mean. **B.** The mean squared error (MSE) of $\hat{\beta}_2$, combining both bias and variance to measure accuracy. **C.** The coverage rates of the 95% confidence intervals, indicating the proportion of intervals that include the true mean β_2. **D.** The widths of the 95% confidence intervals of $\hat{\beta}_2$.

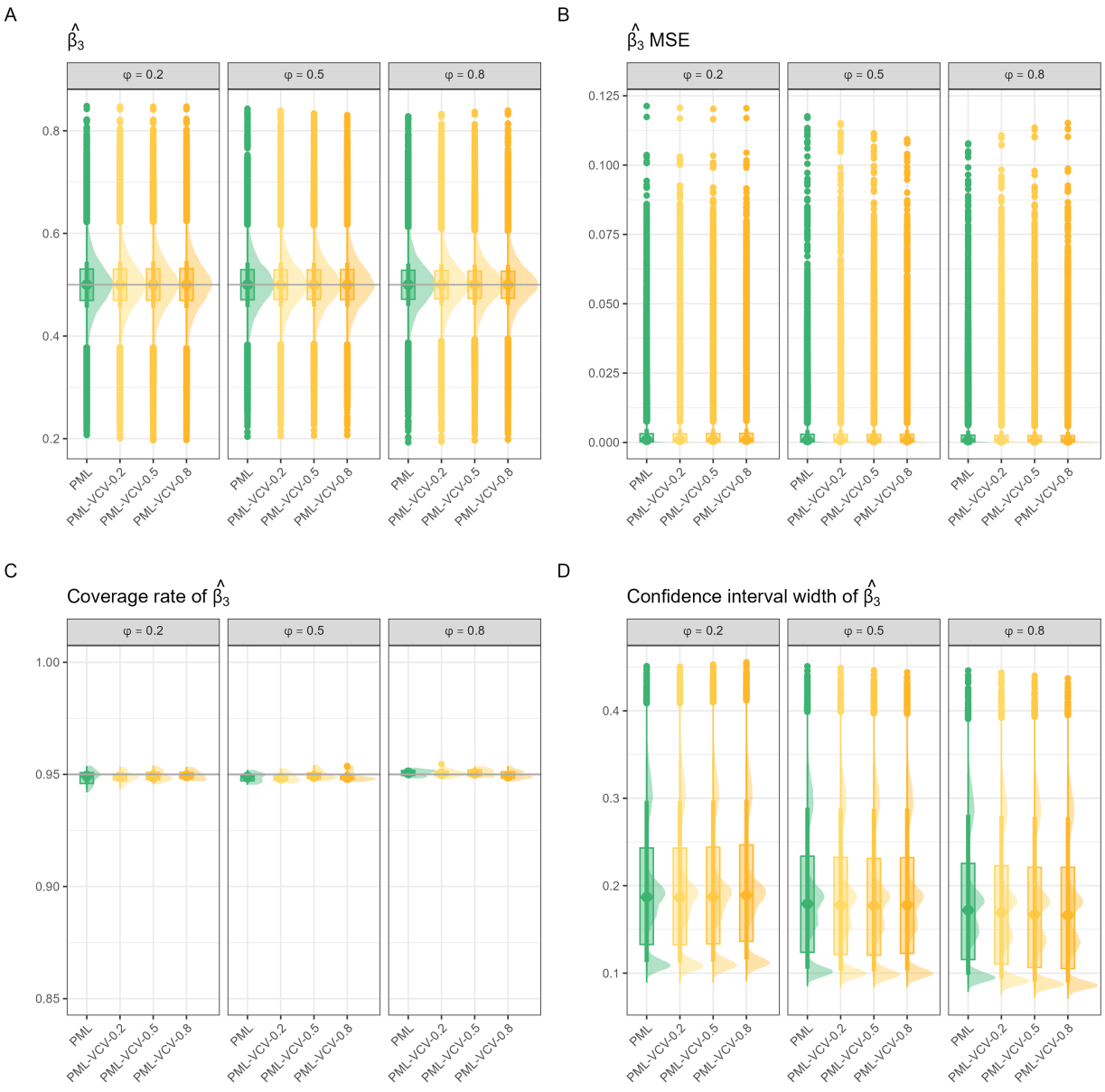


Figure S15: Regression coefficient estimate $\hat{\beta}_3$ performance across all working models and conditions assuming a true within study correlation between effect sizes of $\phi \in (0.2, 0.5, 0.8)$, without any CRVE method applied. **A.** The bias of the overall mean estimate $\hat{\beta}_3$, reflecting the deviation from the true mean. **B.** The mean squared error (MSE) of $\hat{\beta}_3$, combining both bias and variance to measure accuracy. **C.** The coverage rates of the 95% confidence intervals, indicating the proportion of intervals that include the true mean β_3. **D.** The widths of the 95% confidence intervals of $\hat{\beta}_3$.

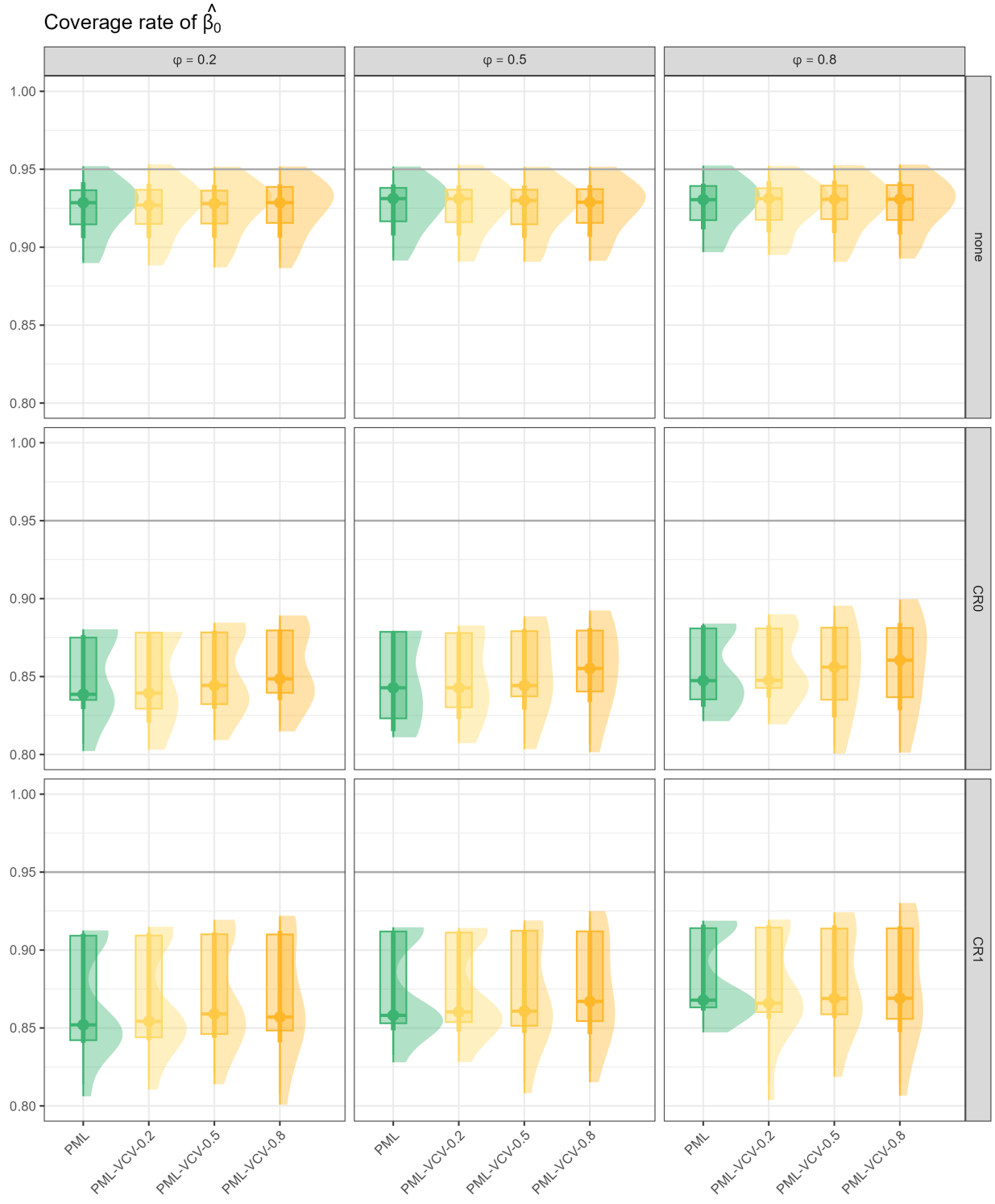


Figure S16: Regression coefficient estimate $\hat{\beta}_0$ coverage rate across CRVE methods, all working models, conditions assuming a true within study correlation between effect sizes of $\phi \in (0.2, 0.5, 0.8)$.

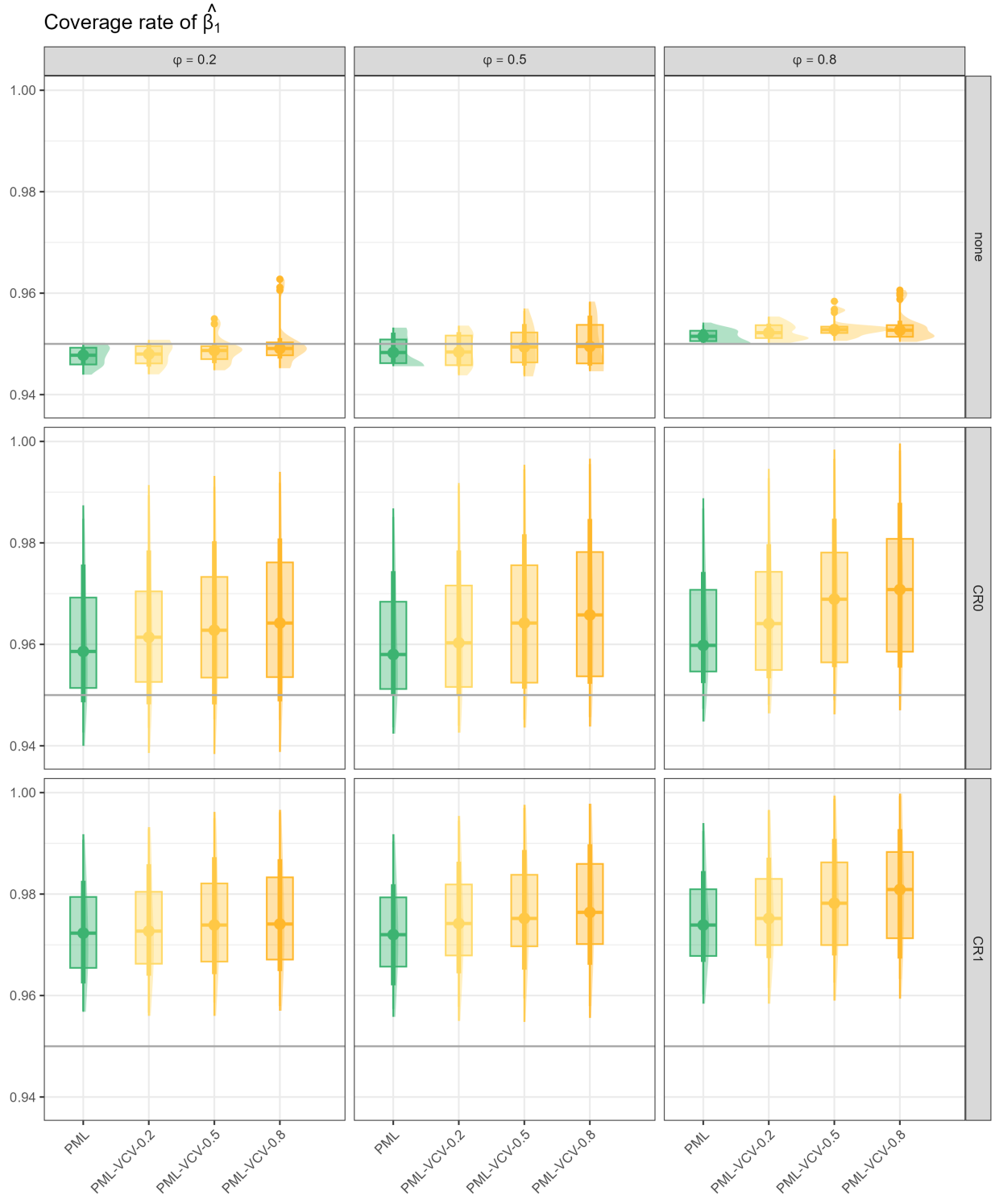


Figure S17: Regression coefficient estimate $\hat{\beta}_1$ coverage rate across CRVE methods, all working models, conditions assuming a true within study correlation between effect sizes of $\phi \in (0.2, 0.5, 0.8)$.

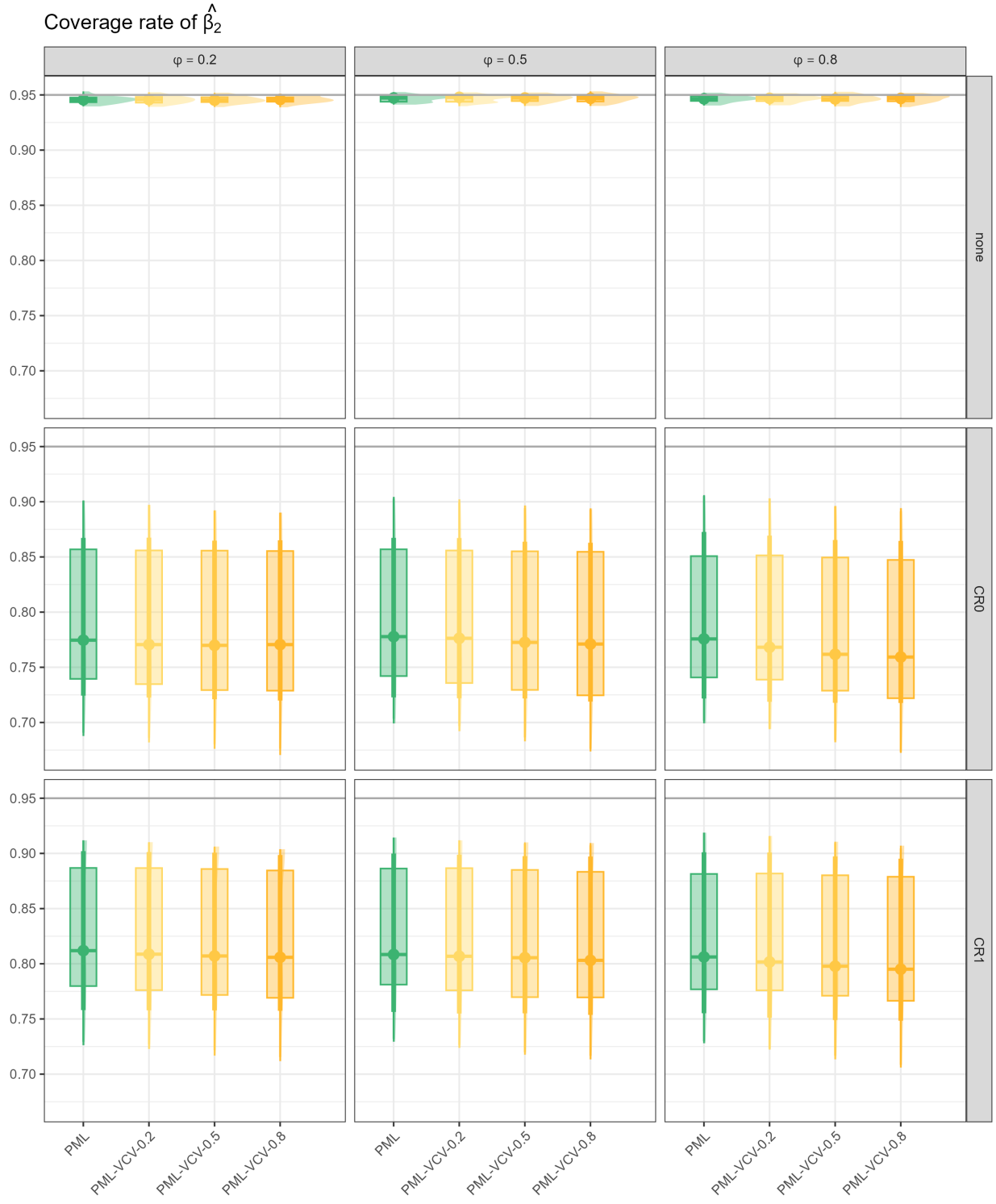


Figure S18: Regression coefficient estimate $\hat{\beta}_2$ coverage rate across CRVE methods, all working models, conditions assuming a true within study correlation between effect sizes of $\phi \in (0.2, 0.5, 0.8)$.

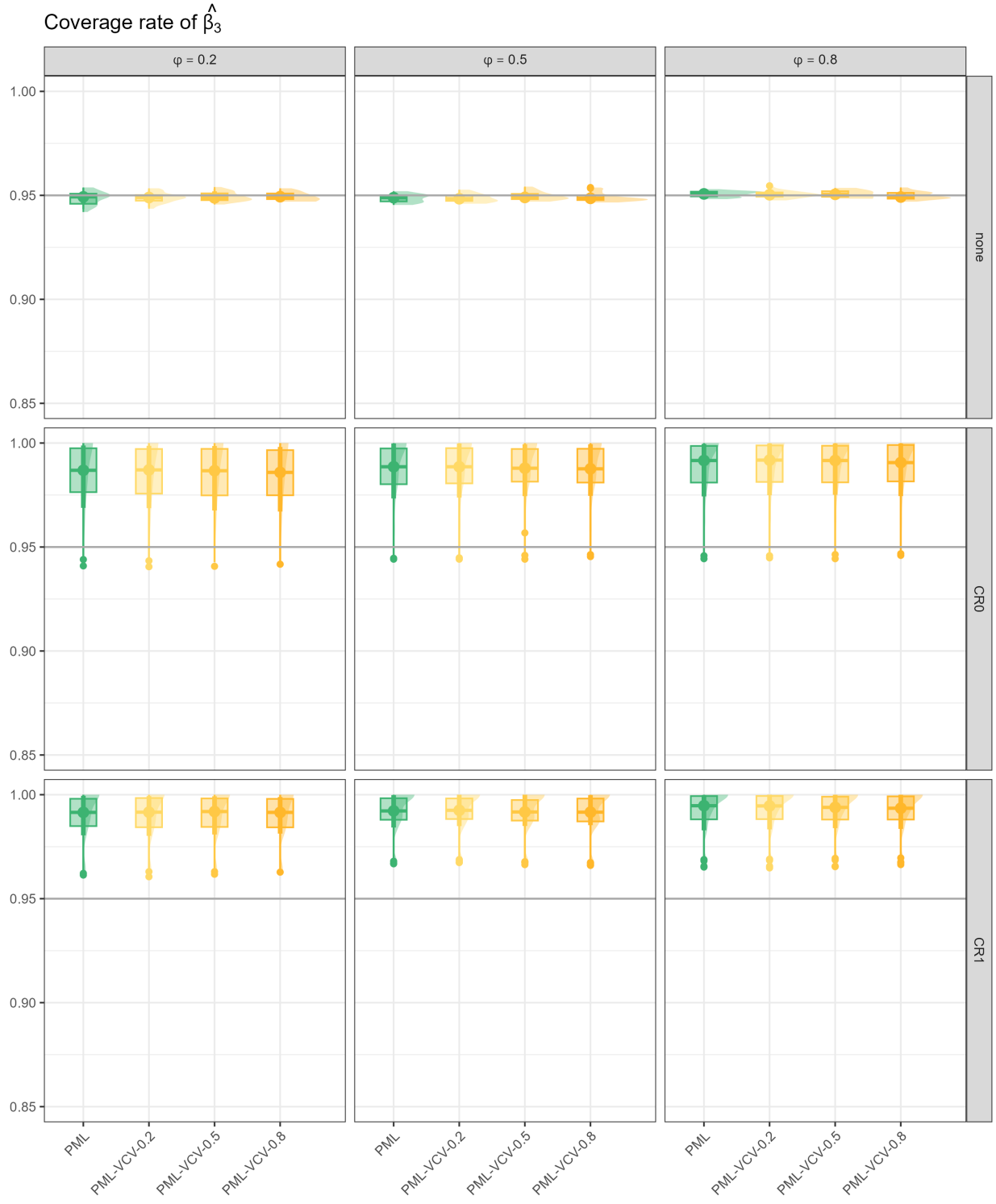


Figure S19: Regression coefficient estimate $\hat{\beta}_3$ coverage rate across CRVE methods, all working models, conditions assuming a true within study correlation between effect sizes of $\phi \in (0.2, 0.5, 0.8)$.

Table S9: Monte Carlo standard errors of bias of regression coefficient estimates ($\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$) across all models (with and without CRVE methods) per true within-study correlation (ϕ), given true values of $\beta_0 = 0.2$, $\beta_1 = 0.6$, $\beta_2 = 0.2$, $\beta_3 = 0.5$

model	ϕ	$\hat{\beta}_0$	MCSE	$\hat{\beta}_1$	MCSE	$\hat{\beta}_2$	MCSE	$\hat{\beta}_3$	MCSE
PML	0.20	0.200	0.270	0.599	0.175	0.200	0.099	0.500	0.054
PML-VCV-0.2	0.20	0.200	0.270	0.600	0.174	0.200	0.099	0.500	0.054
PML-VCV-0.5	0.20	0.200	0.270	0.600	0.175	0.200	0.099	0.500	0.054
PML-VCV-0.8	0.20	0.200	0.270	0.600	0.176	0.200	0.099	0.500	0.055
PML	0.50	0.200	0.272	0.598	0.180	0.200	0.098	0.500	0.052
PML-VCV-0.2	0.50	0.200	0.271	0.598	0.178	0.200	0.098	0.500	0.052
PML-VCV-0.5	0.50	0.200	0.270	0.599	0.178	0.200	0.098	0.500	0.052
PML-VCV-0.8	0.50	0.200	0.271	0.599	0.178	0.200	0.098	0.500	0.052
PML	0.80	0.200	0.273	0.598	0.186	0.200	0.097	0.500	0.050
PML-VCV-0.2	0.80	0.200	0.272	0.599	0.183	0.200	0.097	0.500	0.050
PML-VCV-0.5	0.80	0.200	0.271	0.599	0.181	0.200	0.097	0.500	0.049
PML-VCV-0.8	0.80	0.200	0.271	0.599	0.180	0.200	0.097	0.500	0.049

Table S10: Computational time and convergence rates (%) for phylogenetic multilevel meta-regression models in simulation study 2.sub.

model	k.studies	time (secs)	%
PML	20	0.77	99.98
PML-VCV-0.2	20	0.79	99.98
PML-VCV-0.5	20	0.80	99.98
PML-VCV-0.8	20	0.81	99.98
PML	50	6.04	100.00
PML-VCV-0.2	50	5.60	100.00
PML-VCV-0.5	50	5.42	100.00
PML-VCV-0.8	50	5.37	100.00

References

Senior, A. M., Grueber, C. E., Kamiya, T., Lagisz, M., O'Dwyer, K., Santos, E. S. A., & Nakagawa, S. (2016). Heterogeneity in ecological and evolutionary meta-analyses: Its magnitude and implications. *Ecology*, *97*(12), 3293–3299. https://doi.org/10.1002/ecy.1591