Density Matrix

$$\hat{\rho}(t) = |\psi(t)\rangle \langle\psi(t)| \qquad \text{Defn of density matrix}$$

$$\hat{\rho}^{\text{reduced}}_{i,j}(t) = \sum_n |i\rangle_S |n\rangle_B \hat{\rho} \langle n|_B \langle j|_S \qquad \text{Defn of the reduced density matrix}$$

The reduced density matrix, by the ETH, goes diagonal as the bath becomes suffucently complex.

$$|\psi\rangle = \sum_n C_n \exp^{-\frac{iE_n t}{\hbar}} |A_n\rangle$$

We are looking at a 2 spin systems, where the combined system is given by:

$$S_1 \cdot S_2 = S_{1_x} \cdot S_{2_x} + S_{1_y} \cdot S_{2_y} + S_{1_z} \cdot S_{2_z} = \frac{\hbar^2}{4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The Eigensystem of this is given by

$$\begin{pmatrix} -\frac{1}{4}\left(3\hbar^2\right) & \frac{\hbar^2}{4} & \frac{\hbar^2}{4} & \frac{\hbar^2}{4} \\ \{0,-1,1,0\} & \{0,0,0,1\} & \{0,1,1,0\} & \{1,0,0,0\} \end{pmatrix}$$

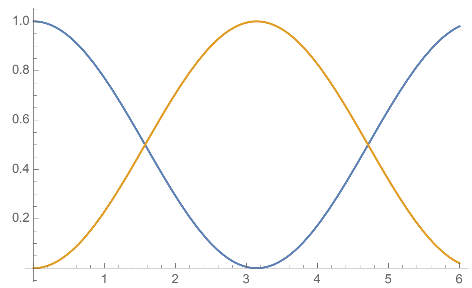
But this is only a eigensystem for S^2, not for S__z - for the simultainus mesurement of both magnitude and direction, the eigenvectors are the same (singlet and triplet) but the eigenvalues are 0 for the singlet state and $\hbar$ for the triplet states.

$$S_z^{combined} = S_{1z} + S_{2z} = \hbar \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

S__z has eigenvalues $-\hbar, 0, \hbar$ - (same as a spin-1 particle).

For a spin-up system coupled to a single spin-down bath, the probabilities of observing the system in a spin-up or spin-down state is given by the following $\rho^{reduced}$ matrix and plot.

$$\rho^{reduced}_{\uparrow\downarrow} = \begin{pmatrix} \cos^2\left(\frac{t}{2}\right) & -\frac{1}{2} i \sin(t) \\ \frac{1}{2} i \sin(t) & \sin^2\left(\frac{t}{2}\right) \end{pmatrix}$$



For the state $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle\,|\downarrow\rangle\right)_S\,|\downarrow\rangle_B$

Appendix