Project Proposal

Non-equilibrium dynamics and thermalisation in simple quantum systems

To understand and model the behaviour of a simplified quantum system as it thermalises. We will investigate the behaviour of a spin-1/2 system using a coumpter simulation. The Eigenstate thermalization hypothesis (ETH) suggests that for a system prepared in some initial state where the expectation value of an observable $\hat{O}$ is far from that given by the microcanonical ensemble of this system, the expectation value of $\hat{O}$ will ultimately evolve in time to its value predicted by a microcanonical ensemble, without the invocation of any random processes. We shall simulate non-equlibrium quantum systems and hope to demonstrate this process.

States

You can have a spin-1/2 system in the Sz basis, with up and down eigenstates going as

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

With eigenstates

$$up = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, down = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

IF you add anouther spin state, you gain the combined systems:

$$|\uparrow\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |\uparrow\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |\downarrow\uparrow\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |\downarrow\downarrow\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The combined S^{tot} is given by the operation of $S_{z1} + S_{z2}$ only on the correct part of the state - so S_{z1} sees the first spin, and S_{z2} the second.

$$S^{tot}|\uparrow\uparrow\rangle = (S_{z1}+S_{z2})(|\uparrow\rangle_1+|\uparrow\rangle_2) = S_{z1}|\uparrow\rangle_1+0+S_{z2}|\uparrow\rangle_2+0 = \frac{\hbar}{2}|\uparrow\rangle_1+\frac{\hbar}{2}|\uparrow\rangle_2 = \hbar|\uparrow\uparrow\rangle$$

And the matrix form of the operator S^{tot} is given by

$$S^{tot} = \begin{bmatrix} \langle\uparrow\uparrow| S^{tot} |\uparrow\uparrow\rangle & \langle\uparrow\uparrow| S^{tot} |\uparrow\downarrow\rangle & \langle\uparrow\uparrow| S^{tot} |\downarrow\uparrow\rangle & \langle\uparrow\uparrow| S^{tot} |\downarrow\downarrow\rangle \\ \langle\uparrow\uparrow| S^{tot} |\uparrow\uparrow\rangle & \langle\uparrow\downarrow| S^{tot} |\uparrow\downarrow\rangle & \langle\downarrow\uparrow| S^{tot} |\downarrow\uparrow\rangle & \langle\downarrow\downarrow| S^{tot} |\downarrow\downarrow\rangle \\ \langle\uparrow\uparrow| S^{tot} |\uparrow\uparrow\rangle & \langle\uparrow\downarrow| S^{tot} |\uparrow\downarrow\rangle & \langle\downarrow\uparrow| S^{tot} |\downarrow\uparrow\rangle & \langle\downarrow\downarrow| S^{tot} |\downarrow\downarrow\rangle \\ \langle\uparrow\uparrow| S^{tot} |\uparrow\uparrow\rangle & \langle\uparrow\downarrow| S^{tot} |\uparrow\downarrow\rangle & \langle\downarrow\uparrow| S^{tot} |\downarrow\uparrow\rangle & \langle\downarrow\downarrow| S^{tot} |\downarrow\downarrow\rangle \end{bmatrix} = \frac{\hbar}{2} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

This matrix can be found by a summation over the Kronecker products of the basis matricies and the Identity matrix.

```
ArrayFlatten[
  Outer[Times, PauliMatrix[3], IdentityMatrix[2]] +
   Outer[Times, IdentityMatrix[2], PauliMatrix[3]]
  ] // MatrixForm
```

Tha hamiliton of this system can be formed using the Kronecker product of the individual basis elements. This is implimented in Mathematica as Outer[Times, Sz, Sz], and a custom impl was used in C. This matrix is diagonal, and as such the eigenvalues are the diagonal elements and the eigenvectors are just unit vectors.

$$H^{tot} = S_z \otimes S_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Hamiltonion for a state mesured in a inconsistant basis, for example $S_{z2} \otimes S_{y1}$, is not diagonal.

A good primer is http://electron6.phys.utk.edu/qm1/modules/m10/twospin.htm.