Define the basis of system and bath

$$|\psi\rangle = \sum_{ij} C_{ij} |S_i\rangle |B_j\rangle$$

take the time evolution of the state

$$|\psi(t)\rangle = \sum_{ij} C_{ij} e^{\frac{-iE_{ij}t}{\hbar}} |S_i\rangle |B_j\rangle$$

Introduce $\rho(t) = |\psi(t)\rangle \langle\psi(t)|$

$$\rho(t) = (\sum_{ij} C_{ij} e^{\frac{-iE_{ij}t}{\hbar}} |S_i\rangle |B_j\rangle)(\sum_{i'j'} C^{\star}_{i'j'} e^{\frac{iE_{i'j'}t}{\hbar}} \langle B_{j'}| \langle S_{i'}|)$$

Simplify by introducing the partial trace, $\sum_n \langle B_n| \rho |B_n\rangle$, and using the presemed orthonormality of the basis to eliminate the sums over j, j'.

$$\sum_n \langle B_n| \rho |B_n\rangle = \langle B_n| (\sum_{ij} C_{ij} e^{\frac{-iE_{ij}t}{/}\hbar} |S_i\rangle |B_j\rangle)(\sum_{i'j'} C^{\star}_{i'j'} e^{\frac{iE_{i'j'}t}{/}\hbar} \langle B_{j'}| \langle S_{i'}|) |B_n\rangle$$

$$= \sum_n \sum_{ii'} C_{in} C^{\star}_{i'n} e^{\frac{-i(E_{in}-E_{i'n})t}{\hbar}} |S_i\rangle \langle S_{i'}|$$

Now introduce the element of the reduced density matrix, $\hat{\rho}^{\text{reduced}}_{S_1,S_2}(t) = |S_1\rangle \sum_n \langle B_n| \rho |B_n\rangle \langle S_2|$

$$\hat{\rho}^{\text{reduced}}_{S_1,S_2}(t) = |S_1\rangle \sum_n \langle B_n| \rho |B_n\rangle \langle S_2|$$

$$\hat{\rho}^{\text{reduced}}_{S_1,S_2}(t) = \langle S_1| (\sum_n \sum_{ii'} C_{in} C^{\star}_{i'n} e^{\frac{-i(E_{in}-E_{i'n})t}{\hbar}} |S_i\rangle \langle S_{i'}|) |S_2\rangle$$

use orthonormaity again to eliminate sums over i, i'

$$\hat{\rho}^{\text{reduced}}_{S_1,S_2}(t) = \sum_n C_{1n} C^{\star}_{2n} e^{\frac{-i(E_{1n}-E_{2n})t}{\hbar}}$$