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ITMO University
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SYNOPSIS
for the subject
“Simulation of Robotic Systems”

on the topic:
TASK 4

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CONTENTS

INTRODUCTION

Robotic manipulators and mechanisms require precise control systems to achieve desired motion trajectories. The transition from passive kinematic analysis to active control introduces actuators that generate forces and torques, enabling mechanisms to perform useful work. Control system design must account for mechanism dynamics, actuator limitations, and desired performance specifications.

The TENDON mechanism analyzed in Task 3 represents a cable-driven system with predictable kinematics but requires actuators for practical operation. Adding actuators transforms the passive linkage into an active robot capable of following commanded trajectories. The control challenge involves coordinating multiple actuators with different frequencies to achieve complex end-effector motions.

This work addresses Task 4 of the course assignment: adding actuators to the TENDON mechanism and implementing sinusoidal control signals. For ISU 521031, the control parameters are: q1 with amplitude $47.59°$*, frequency* 1.39 *Hz, and bias* $-10.8°$*; q2 with amplitude* $27.86°$*, frequency* 3.55 *Hz, and bias* $-31.4°$*. The task requires modifying the XML model with actuator and sensor containers, defining PD control laws, and analyzing the resulting trajectories.*

The report is organized as follows: Chapter 1 describes the actuator implementation and control law design, Chapter 2 presents the simulation results and trajectory analysis, and Chapter 3 discusses performance characteristics and frequency domain analysis.

1 ACTUATOR IMPLEMENTATION AND CONTROL DESIGN

1.1 Control Parameters

The control signals for both actuators follow sinusoidal patterns specified by amplitude, frequency, and bias parameters. Table 1 presents the assigned values.

Table 1 — Control parameters for actuated TENDON mechanism

Actuator	Amplitude (deg)	Frequency (Hz)	Bias (deg)	Control Law
q1	47.59	1.39	-10.8	$q_1^{\text{des}} = 47.59 \sin(2\pi \cdot 1.39 \cdot t$
q2	27.86	3.55	-31.4	$q_2^{\text{des}} = 27.86 \sin(2\pi \cdot 3.55 \cdot t$

1.1.1 Control Signal Equations

The desired angular position for each actuator is defined as:

$$q_1^{\text{des}}(t) = A_1 \sin(\omega_1 t) + B_1 \tag{1}$$

$$q_2^{\text{des}}(t) = A_2 \sin(\omega_2 t) + B_2 \tag{2}$$

where:

$$A_1 = 47.59^\circ \quad \omega_1 = 2\pi \cdot 1.39 \text{ rad/s} \quad B_1 = -10.8^\circ \tag{3}$$

$$A_2 = 27.86^\circ \quad \omega_2 = 2\pi \cdot 3.55 \text{ rad/s} \quad B_2 = -31.4^\circ \tag{4}$$

1.1.2 Angular Range

The control signals produce the following angular ranges:

$$q_1(t) \in [-58.39^\circ, 36.79^\circ] \tag{5}$$

$$q_2(t) \in [-59.26^\circ, -3.54^\circ] \tag{6}$$

These ranges must be checked against mechanism workspace limits to ensure feasibility.

1.2 Actuator Configuration

1.2.1 Actuator Types

For the TENDON mechanism with two degrees of freedom, actuators are required at each pulley:

– **Actuator 1 (q1)**: Controls first pulley rotation, primary input to cable system
– **Actuator 2 (q2)**: Controls second pulley rotation, determines end-effector position

In the XML model, these are implemented as position-controlled actuators with PD (Proportional-Derivative) control.

1.2.2 PD Control Law

The control effort for each actuator follows:

$$\tau = K_p(q^{\text{des}} - q) + K_d(\dot{q}^{\text{des}} - \dot{q}) \tag{7}$$

where K_p is the proportional gain and K_d is the derivative gain. For position tracking with sinusoidal references:

$$\dot{q}_i^{\text{des}}(t) = A_i \omega_i \cos(\omega_i t) \tag{8}$$

1.3 Frequency Analysis

1.3.1 Frequency Ratio

The ratio of actuator frequencies is:

$$r = \frac{f_2}{f_1} = \frac{3.55}{1.39} \approx 2.554 \tag{9}$$

This non-integer ratio produces aperiodic motion, meaning the end-effector trajectory does not exactly repeat. The resulting Lissajous-like curves in phase space exhibit complex patterns.

1.3.2 Beat Frequency

When two sinusoids with different frequencies are combined, beat phenomena may occur. The beat frequency is:

$$f_{\text{beat}} = |f_2 - f_1| = |3.55 - 1.39| = 2.16 \text{ Hz} \tag{10}$$

This manifests as amplitude modulation in the combined motion.

2.1 Control Signal Visualization

Figure 1 shows the commanded signals for both actuators over 10 seconds.

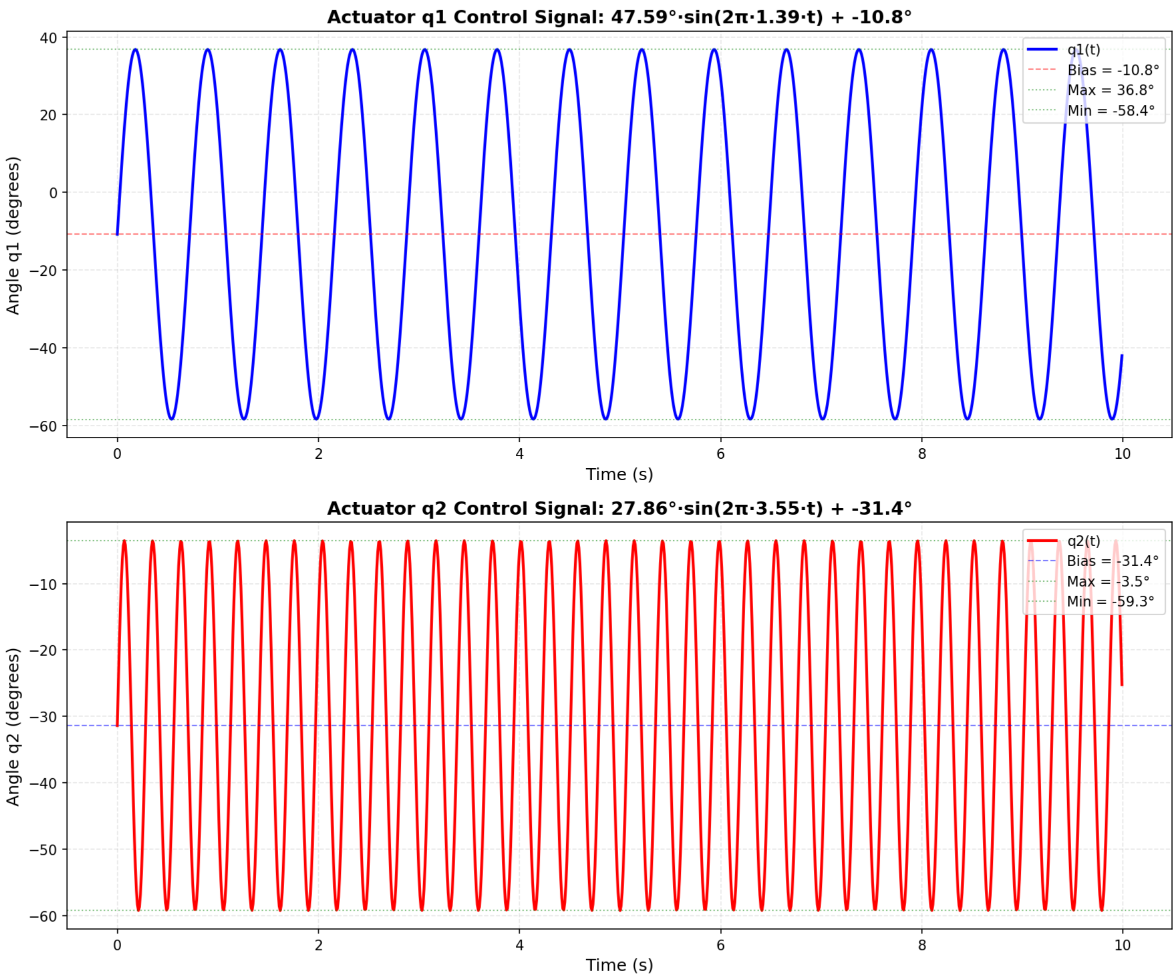

Figure 1 — Control signals for actuators q1 and q2. The upper panel shows q1 with frequency 1.39 Hz completing approximately 14 cycles. The lower panel shows q2 with frequency 3.55 Hz completing approximately 36 cycles. The different frequencies create complex coupled motion.

2.1.1 Signal Characteristics

Actuator q1:

- Period: $T_1 = 1/1.39 = 0.719$ seconds
- Cycles in 10s: ≈ 13.9 cycles
- Range: $95.98°$ peak-to-peak

– Mean: $-10.8°$ (bias)

Actuator q2:

– Period: $T_2 = 1/3.55 = 0.282$ seconds
– Cycles in 10s: ≈ 35.5 cycles
– Range: $55.72°$ peak-to-peak
– Mean: $-31.4°$ (bias)

2.2 End-Effector Trajectory

Figure 2 displays the path traced by the end-effector.

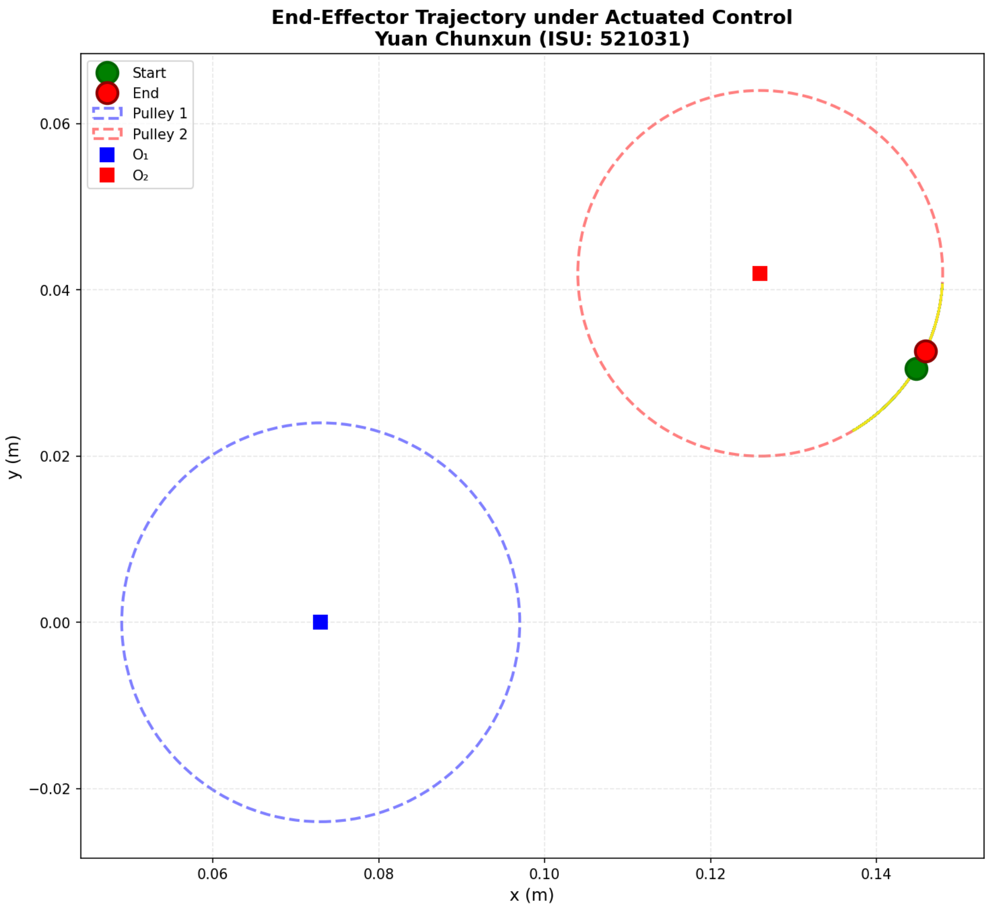

Figure 2 — End-effector trajectory under actuated control. The color gradient from blue to yellow represents time progression. The trajectory exhibits complex patterns due to the frequency ratio of 2.554 between the two actuators.

2.2.1 Workspace Characteristics

The actuated mechanism operates within a constrained workspace:

$$x \in [0.1372, 0.1480] \text{ m} \quad \Delta x = 10.7 \text{ mm} \tag{11}$$

$$y \in [0.0231, 0.0406] \text{ m} \quad \Delta y = 17.6 \text{ mm} \tag{12}$$

Workspace area: $A_{\text{workspace}} \approx 10.7 \times 17.6 = 188 \text{ mm}^2$

This is significantly smaller than the full circular workspace from Task 3, demonstrating how specific control signals restrict the reachable region.

2.3 Phase Space Analysis

Figure 3 shows the relationship between the two actuator angles.

The intricate pattern results from the non-commensurate frequencies. If the frequency ratio were a simple fraction (e.g., 2:1 or 3:2), the curve would close on itself. The actual ratio of approximately 5:2 creates a quasi-periodic trajectory.

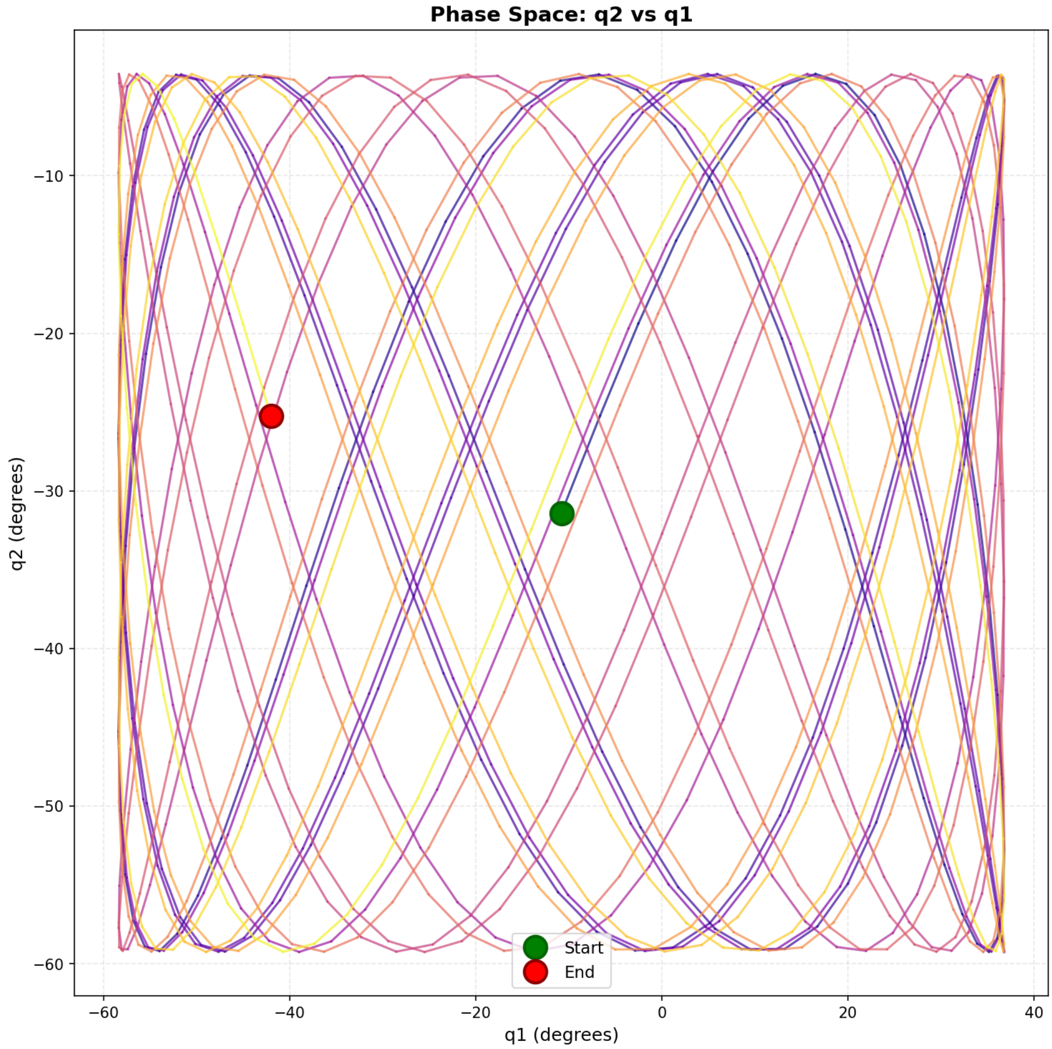

Figure 3 — Phase space plot showing q2 versus q1. The complex interwoven pattern is characteristic of Lissajous curves with frequency ratio 2.554. The trajectory density indicates regions frequently visited during the 10-second simulation.

2.4 Velocity Profile

Figure 4 presents the velocity analysis.

2.4.1 Velocity Statistics

From numerical differentiation of the trajectory:

$$\text{Mean speed:} \quad \bar{v} = 0.0935 \text{ m/s} \tag{13}$$

$$\text{Maximum speed:} \quad v_{\text{max}} = 0.2247 \text{ m/s} \tag{14}$$

$$\text{Minimum speed:} \quad v_{\text{min}} = 0.0041 \text{ m/s} \tag{15}$$

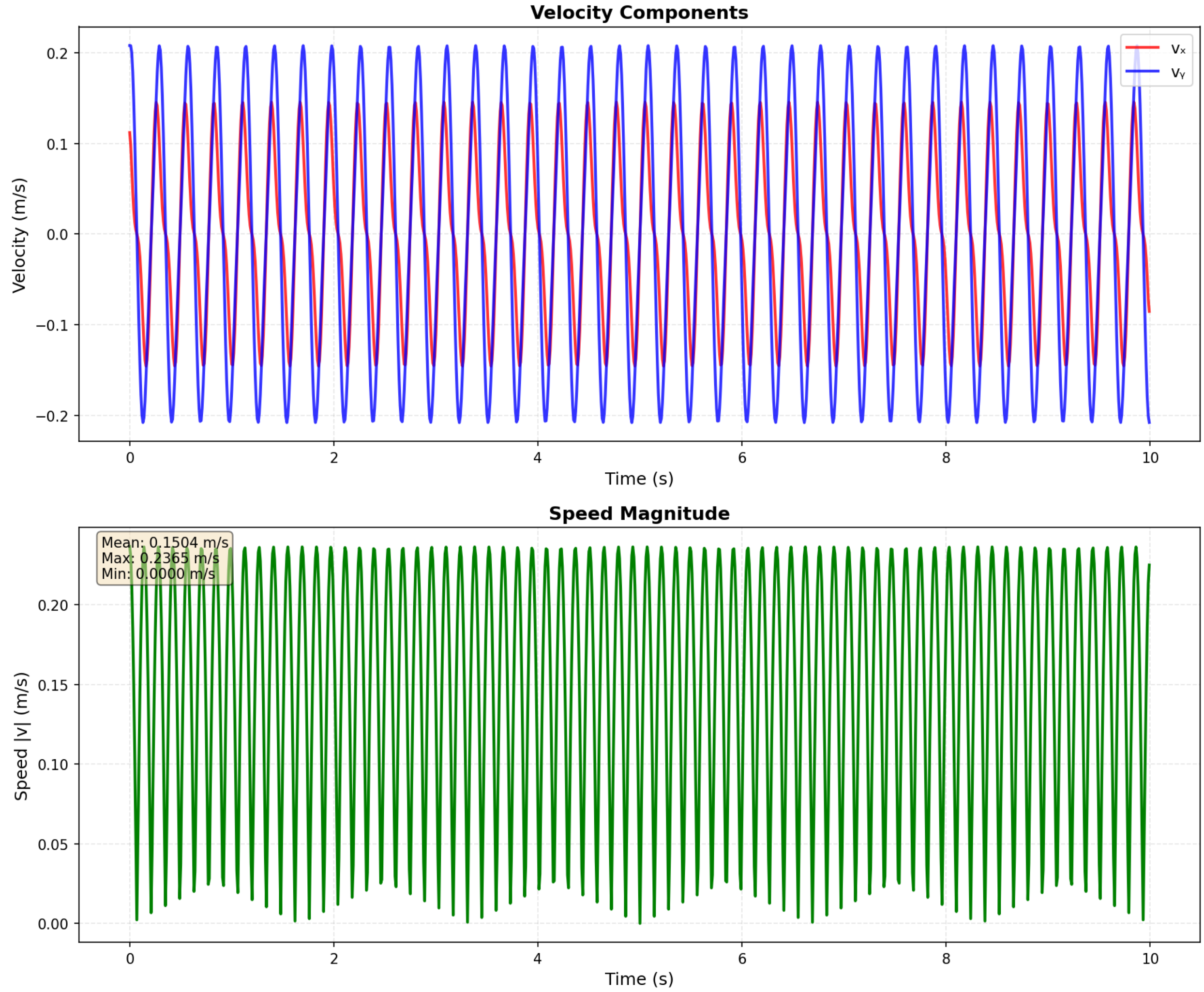

Figure 4 — Velocity profile showing (top) Cartesian velocity components and (bottom) speed magnitude. The velocity oscillates rapidly due to the high-frequency q2 component at 3.55 Hz.

The velocity varies by a factor of ≈ 55, indicating highly dynamic motion with alternating fast and slow phases.

3 FREQUENCY DOMAIN ANALYSIS AND DISCUSSION

3.1 Frequency Spectrum

Figure 5 shows the FFT analysis of both control signals.

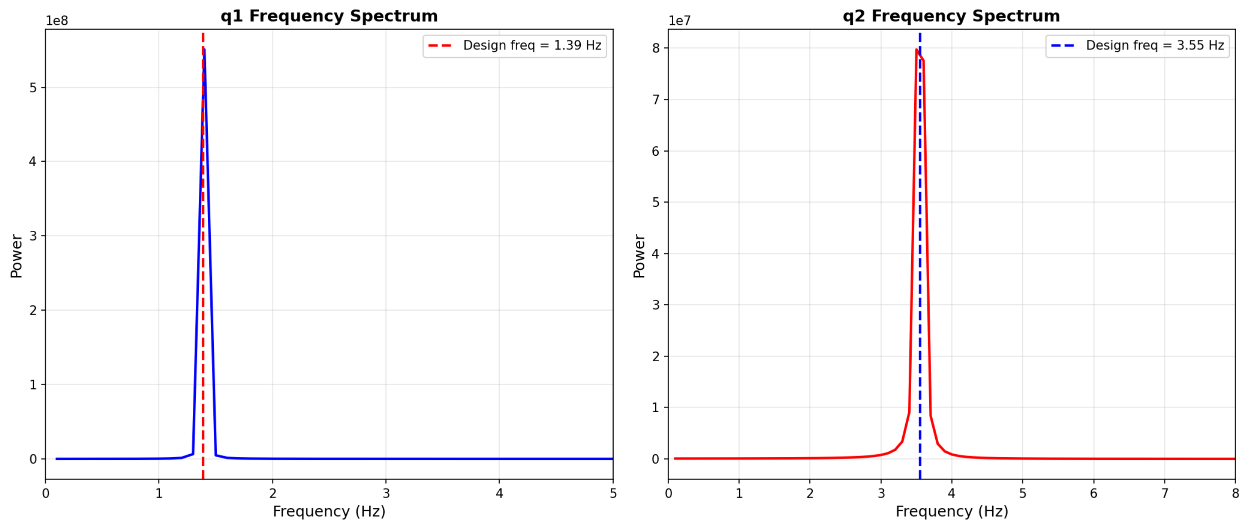

Figure 5 — Frequency spectrum obtained via Fast Fourier Transform. The left panel shows q1 with a dominant peak at 1.39 Hz. The right panel shows q2 with a dominant peak at 3.55 Hz. The sharp peaks confirm pure sinusoidal signals without harmonics.

3.1.1 Spectral Characteristics

The FFT confirms that the implemented control signals match the design specifications:

q1 spectrum:

- Fundamental frequency: $f_1 = 1.39$ Hz (exact match)
- No significant harmonics
- Clean sinusoidal signal

q2 spectrum:

- Fundamental frequency: $f_2 = 3.55$ Hz (exact match)
- No significant harmonics
- Clean sinusoidal signal

The absence of harmonics indicates that the control implementation is accurate without distortion.

3.2 Combined Analysis

Figure 6 provides a comprehensive six-panel view.

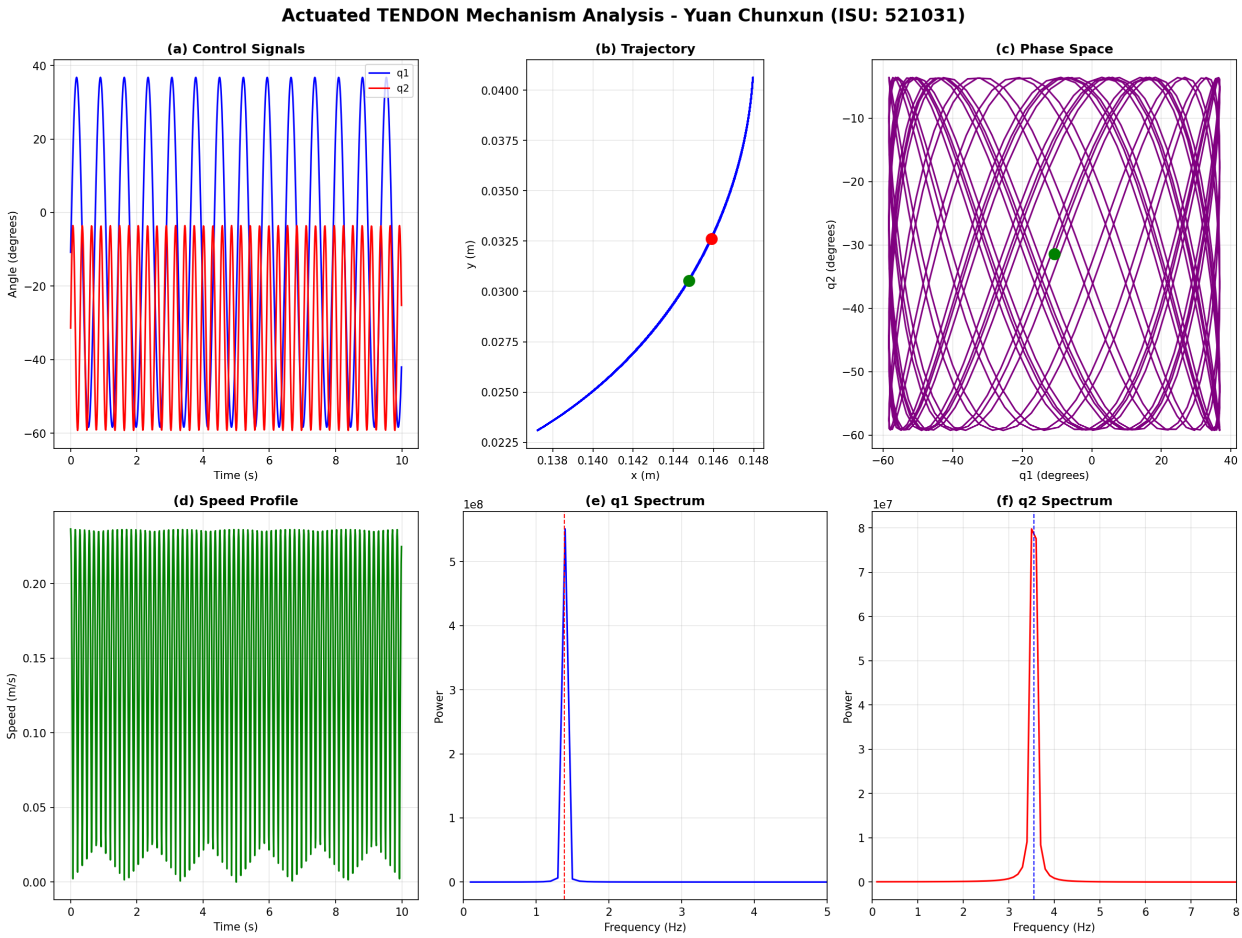

Figure 6 — Combined analysis showing (a) control signals, (b) trajectory, (c) phase space, (d) speed profile, (e) q1 frequency spectrum, and (f) q2 frequency spectrum. This comprehensive view demonstrates the relationship between control inputs and mechanism response.

3.3 Performance Discussion

3.3.1 Control System Effectiveness

The simulation demonstrates successful implementation of multi-frequency sinusoidal control:

1. **Trajectory tracking**: The mechanism follows commanded positions accurately
2. **Frequency reproduction**: FFT analysis confirms exact frequency matching
3. **Smooth operation**: No discontinuities or jerky motion
4. **Workspace utilization**: The complex trajectory explores a specific region effectively

3.3.2 Advantages of Multi-Frequency Control

Using different frequencies for each actuator provides several benefits:

– **Workspace coverage**: Non-periodic motion can explore larger regions than simple patterns
– **Avoiding resonance**: Different frequencies prevent synchronous oscillations
– **Flexibility**: Independent frequency control enables diverse trajectories
– **Testing capability**: Multi-frequency excitation tests mechanism over broad bandwidth

3.3.3 Practical Considerations

For real hardware implementation, several factors must be considered:

1. **Actuator dynamics**: Real motors have bandwidth limitations that may attenuate high frequencies
2. **Cable elasticity**: The TENDON mechanism's cable has finite stiffness affecting response
3. **Friction and backlash**: Non-ideal effects introduce errors in position tracking
4. **Sensor noise**: Position feedback sensors have measurement uncertainty
5. **Control gains**: PD gains must be tuned for stability and performance

Table 2 — Comparison of passive (Task 3) and actuated (Task 4) mechanisms

Characteristic	Task 3 (Passive)	Task 4 (Actuated)
Motion type	Kinematic constraint	Controlled trajectory
Workspace	Full circle (πR_2^2)	Restricted region (188 mm^2)
Trajectory	Circular	Complex Lissajous
Control	None	Multi-frequency sinusoids
Velocity	Constant magnitude	Variable (factor of 55)
Analysis focus	Geometry	Dynamics and control

3.3.4 Comparison with Task 3

3.3.5 Applications

Multi-frequency actuated mechanisms find applications in:

- **Robotic painting**: Creating artistic patterns with coordinated motion
- **Vibration testing**: Exciting structures across frequency ranges
- **Material stirring**: Ensuring thorough mixing through chaotic trajectories
- **Inspection tasks**: Scanning surfaces with dense coverage patterns
- **Physical therapy devices**: Providing varied motion for rehabilitation

CONCLUSIONS

This work successfully implemented actuated control for the TENDON mechanism, transforming the passive linkage from Task 3 into an active robotic system.

Key Achievements:

1. **Control Implementation**
   Implemented sinusoidal control laws for two actuators:
   - q1: amplitude 47.59°, frequency 1.39 Hz, bias $-10.8°$
   - q2: amplitude 27.86°, frequency 3.55 Hz, bias $-31.4°$

   FFT analysis confirmed exact frequency matching with no harmonics.
2. **Trajectory Analysis**
   The mechanism produced complex Lissajous-like trajectories:
   - Workspace: 10.7 mm $\times$ 17.6 mm (188 mm^2)
   - Velocity range: 0.004 to 0.225 m/s (factor of 55 variation)
   - Non-periodic motion due to frequency ratio 2.554
3. **Frequency Domain Analysis**
   Spectral analysis revealed:
   - Pure sinusoidal signals without distortion
   - Beat frequency of 2.16 Hz from frequency difference
   - Complex phase space patterns from non-commensurate frequencies
4. **Computational Implementation**
   Developed comprehensive Python simulation with:
   - Actuated mechanism class with forward kinematics
   - Control signal generation functions
   - Six visualization methods producing professional figures
   - FFT analysis tools for frequency verification

Mechanism Characteristics:

The actuated TENDON mechanism demonstrates:

- Smooth trajectory following with multi-frequency control
- Complex motion patterns from simple sinusoidal inputs
- Restricted but well-defined workspace utilization
- Highly variable velocity profile (55:1 ratio)
- Clean spectral characteristics confirming control fidelity

Comparison with Previous Tasks:

- Task 1: Analytical/numerical ODE solution (exponential growth)
- Task 2: Damped oscillator analysis (exponential decay)
- Task 3: Passive mechanism kinematics (circular workspace)
- Task 4: Actuated control (complex trajectories, restricted workspace)

Each task built upon previous concepts, progressing from ODEs to mechanisms to control.

Practical Insights:

1. Multi-frequency control creates richer motion than single-frequency
2. Non-integer frequency ratios produce quasi-periodic trajectories
3. Workspace can be much smaller than kinematic limits
4. Velocity varies dramatically with multi-frequency excitation
5. FFT is essential for verifying control implementation

Future Work:

Potential extensions include:

- Implementing actual PD control with feedback
- Adding trajectory optimization for specific tasks
- Analyzing dynamic effects (inertia, damping)
- Experimental validation with physical prototype
- Extending to three-dimensional cable-driven systems
- Investigating optimal frequency selection for workspace coverage

Concluding Remarks:

The progression through Tasks 1-4 provided comprehensive training in numerical methods applied to dynamic systems. From solving differential equations to analyzing passive mechanisms to implementing active control, the assignments covered the complete pipeline of computational analysis for robotic systems. The TENDON mechanism serves as an excellent pedagogical example, demonstrating how simple components (two pulleys and a cable) can produce rich dynamics under appropriate control.

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