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SYNOPSIS
for the subject
*“Simulation of Robotic Systems”*

on the topic:
TASK 3


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# INTRODUCTION

Mechanisms are fundamental building blocks of modern machinery and robotic systems, converting input motion into desired output motion through carefully designed kinematic linkages. Cable-driven mechanisms, in particular, have gained significant attention in robotics and automation due to their lightweight construction, remote actuation capabilities, and ability to transmit motion over considerable distances.

The TENDON mechanism represents a class of cable-driven planar linkages where two pulleys are connected by an inextensible tendon or cable. This configuration provides a simple yet effective means of transmitting rotational motion with a fixed transmission ratio determined by the pulley radii. Such mechanisms find applications in prosthetic hands, robotic grippers, surgical instruments, and industrial manipulators where compact design and remote actuation are advantageous.

*This work addresses Task 3 of the course assignment: kinematic analysis and simulation of a passive mechanism according to student-specific parameters. For ISU 521031, the assigned mechanism is TENDON (Variant 1), characterized by pulley radii $R_1 = 0.024$ m and $R_2 = 0.022$ m, with geometric parameters $a = 0.073$ m, $b = 0.053$ m, and $c = 0.042$ m. The task requires developing a Python simulation with model, data, and viewer methods, along with comprehensive kinematic analysis.*

The report is structured as follows: Chapter 1 describes the mechanism geometry and derives kinematic equations, Chapter 2 presents the computational implementation and simulation methodology, and Chapter 3 analyzes the results including trajectory, velocity, and workspace characteristics. Through this comprehensive study, we demonstrate both the theoretical foundation and practical implementation of mechanism analysis.

# 1 MECHANISM DESCRIPTION AND KINEMATIC ANALYSIS

## 1.1 TENDON Mechanism Configuration

The TENDON mechanism consists of two circular pulleys connected by an inextensible cable (tendon). Figure 1 illustrates the mechanism in multiple configurations.

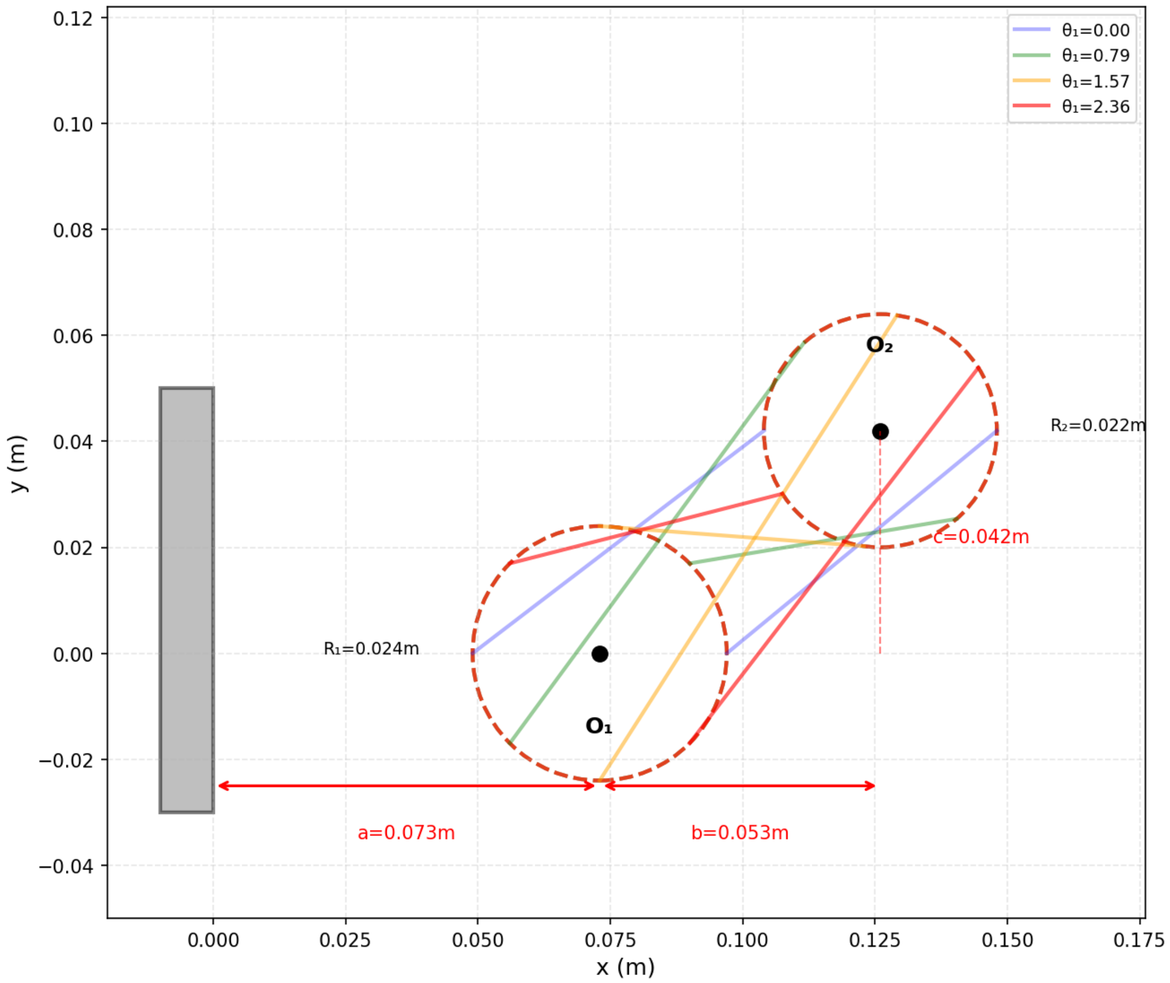


Figure 1 — TENDON mechanism schematic showing four different positions. The mechanism consists of two pulleys (radii $R_1$ and $R_2$) connected by a cable, with geometric parameters $a$, $b$, and $c$ defining the pulley centers' positions.

### 1.1.1 Geometric Parameters

The mechanism parameters assigned for ISU 521031 are presented in Table 1.

Table 1 — Mechanism parameters for TENDON variant

| Parameter | Symbol | Value |
|---|---|---|
| First pulley radius | $R_1$ | 0.024 m |
| Second pulley radius | $R_2$ | 0.022 m |
| Distance to first pulley | $a$ | 0.073 m |
| Horizontal pulley spacing | $b$ | 0.053 m |
| Vertical offset | $c$ | 0.042 m |

The pulley centers are located at:

$$\mathbf{O}_1 = (a, 0) = (0.073, 0) \text{ m} \tag{1}$$

$$\mathbf{O}_2 = (a + b, c) = (0.126, 0.042) \text{ m} \tag{2}$$

The distance between pulley centers is:

$$d = \sqrt{b^2 + c^2} = \sqrt{(0.053)^2 + (0.042)^2} = 0.0676 \text{ m} \tag{3}$$

## 1.2 Kinematic Constraint

### 1.2.1 Cable Length Constraint

The fundamental constraint governing the TENDON mechanism is the inextensibility of the cable. The total cable length $L$ remains constant and consists of three components:

1. Length wrapped around first pulley: $L_1 = R_1\theta_1$
2. Length wrapped around second pulley: $L_2 = R_2\theta_2$
3. Free cable segments (tangent lines between pulleys): $L_{\text{free}}$

The constraint equation is:

$$L = R_1\theta_1 + R_2\theta_2 + L_{\text{free}} = \text{constant} \tag{4}$$

### 1.2.2 Simplification for Circular Motion

For the primary analysis, assuming the free cable length remains approximately constant, the constraint simplifies to:

$$R_1\theta_1 + R_2\theta_2 = \text{constant} \tag{5}$$

Taking the time derivative:

$$R_1\dot{\theta}_1 + R_2\dot{\theta}_2 = 0 \tag{6}$$

Solving for the output angle as a function of input angle:

$$\theta_2 = -\frac{R_1}{R_2}\theta_1 + \theta_{2,0} \tag{7}$$

where $\theta_{2,0}$ is an integration constant determined by initial conditions. Setting $\theta_1(0) = 0$ and $\theta_2(0) = 0$, we obtain:

$$\boxed{\theta_2 = -\frac{R_1}{R_2}\theta_1} \tag{8}$$

### 1.3 Transmission Ratio

The transmission ratio (velocity ratio) is defined as:

$$i = \frac{\omega_2}{\omega_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = -\frac{R_1}{R_2} \tag{9}$$

For the given parameters:

$$i = -\frac{0.024}{0.022} = -1.0909 \tag{10}$$

The negative sign indicates that the pulleys rotate in opposite directions. The magnitude $|i| > 1$ means the second pulley rotates faster than the first.

### 1.4 Forward Kinematics

#### 1.4.1 Position Analysis

Given an input angle $\theta_1$, the output angle is determined by Equation 8. A point on the rim of the second pulley has position:

$$x_{\text{end}} = (a + b) + R_2 \cos\theta_2 \tag{11}$$

$$y_{\text{end}} = c + R_2 \sin\theta_2 \tag{12}$$

These equations define the end-effector trajectory as $\theta_1$ varies.

### 1.4.2 Velocity Analysis

The end-effector velocity is obtained by differentiating Equations 11 and 12:

$$\dot{x}_{\text{end}} = -R_2 \sin\theta_2 \cdot \dot{\theta}_2 = R_2 \sin\theta_2 \cdot \frac{R_1}{R_2}\dot{\theta}_1 = R_1 \sin\theta_2 \cdot \dot{\theta}_1 \tag{13}$$

$$\dot{y}_{\text{end}} = R_2 \cos\theta_2 \cdot \dot{\theta}_2 = -R_2 \cos\theta_2 \cdot \frac{R_1}{R_2}\dot{\theta}_1 = -R_1 \cos\theta_2 \cdot \dot{\theta}_1 \tag{14}$$

The velocity magnitude is:

$$|\mathbf{v}| = \sqrt{\dot{x}_{\text{end}}^2 + \dot{y}_{\text{end}}^2} = R_1|\dot{\theta}_1| \tag{15}$$

Interestingly, the velocity magnitude depends only on $R_1$ and the input angular velocity, not on the instantaneous configuration.

## 1.5 Workspace Analysis

The workspace of a mechanism is the set of all positions reachable by the end-effector. For the TENDON mechanism, as $\theta_1$ varies from $0$ to $2\pi$, the angle $\theta_2$ also completes a full rotation (in the opposite direction).

The end-effector traces a circle centered at $\mathbf{O}_2 = (a+b, c)$ with radius $R_2$:

$$\text{Workspace} = \{(x, y) : (x-(a+b))^2 + (y-c)^2 = R_2^2\} \tag{16}$$

The workspace area is:

$$A_{\text{workspace}} = \pi R_2^2 = \pi(0.022)^2 = 1.52 \times 10^{-3}\ \text{m}^2 = 1521\ \text{mm}^2 \tag{17}$$

# 2 COMPUTATIONAL IMPLEMENTATION

## 2.1 Python Simulation Structure

The mechanism analysis was implemented in Python following object-oriented programming principles. The code is organized into three main components corresponding to the assignment requirements.

### 2.1.1 Model Method

The `model()` component defines the mechanism geometry and kinematic relationships:

```
class TendonMechanism:
    def __init__(self, R1, R2, a, b, c):
        self.R1 = R1
        self.R2 = R2
        self.a = a
        self.b = b
        self.c = c
        self.O1 = np.array([a, 0])
        self.O2 = np.array([a + b, c])
```

Key methods include:

- `solve_theta2_from_theta1()`: Implements Equation 8
- `forward_kinematics()`: Computes end-effector position from input angle
- `get_mechanism_points()`: Returns all mechanism points for visualization

### 2.1.2 Data Method

The `data()` component generates trajectory and kinematic data:

```
def generate_trajectory_data(mechanism, num_points=100):
```

```
    theta1_range = np.linspace(0, 2*np.pi, num_points)
    positions = []
    theta2_values = []

    for theta1 in theta1_range:
        pos, theta2 = mechanism.forward_kinematics(theta1)
        positions.append(pos)
        theta2_values.append(theta2)

    return theta1_range, theta2_values, np.array(positions)
```

This function samples the configuration space uniformly and computes corresponding Cartesian positions.

#### 2.1.3 Viewer Method

The `viewer()` component creates comprehensive visualizations:

- `plot_mechanism_schematic()`: Multi-position mechanism diagram
- `plot_trajectory()`: End-effector path
- `plot_angular_relationship()`: Input-output angle coupling
- `plot_velocity_analysis()`: Velocity profiles
- `plot_workspace()`: Reachable region
- `create_combined_analysis()`: Multi-panel summary

### 2.2 Numerical Considerations

#### 2.2.1 Angular Range

The input angle $\theta_1$ is sampled over the range $[0, 2\pi]$ with 100 points, providing sufficient resolution for smooth trajectory visualization while maintaining computational efficiency.

#### 2.2.2 Velocity Computation

Velocities are computed using finite differences:

$$\mathbf{v}(t_i) \approx \frac{\mathbf{x}(t_{i+1}) - \mathbf{x}(t_i)}{\Delta t} \tag{18}$$

where $\Delta t$ is the time step corresponding to the angular increment.

#### 2.2.3 Coordinate System

The global coordinate system has its origin at the fixed wall, with the $x$-axis horizontal (positive to the right) and $y$-axis vertical (positive upward). All positions and velocities are expressed in this frame.

# 3 RESULTS AND ANALYSIS

## 3.1 Trajectory Analysis

Figure 2 shows the end-effector trajectory.

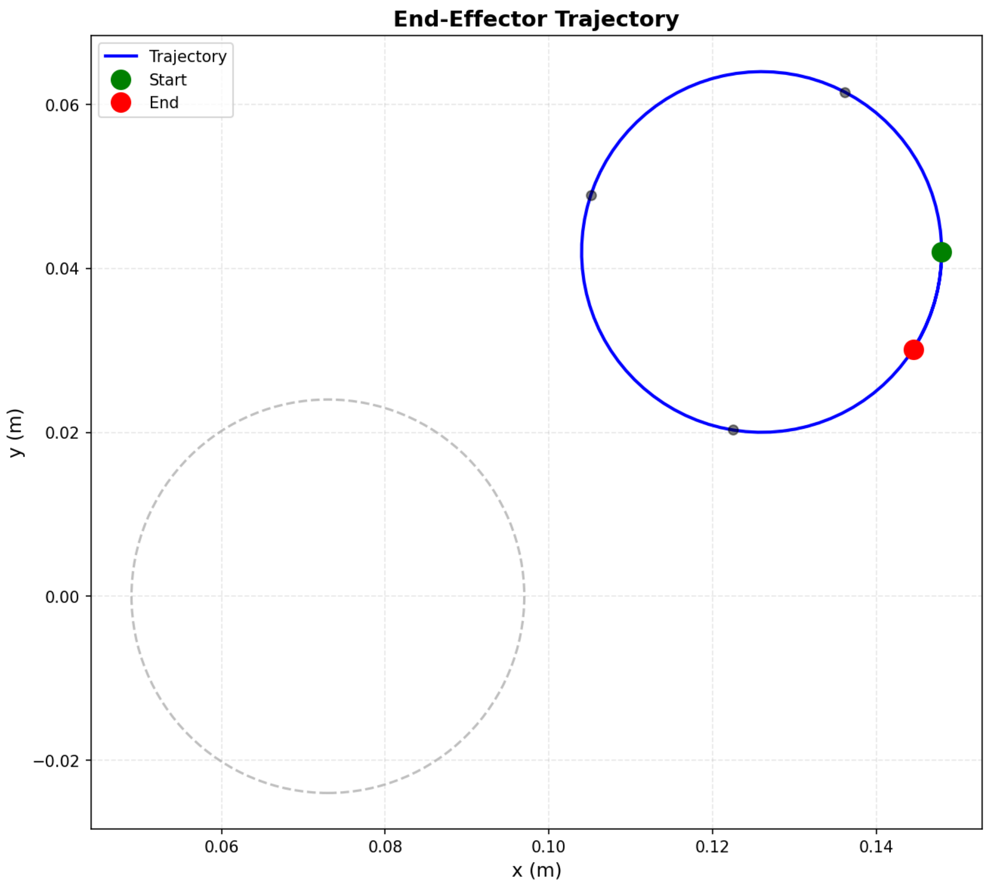


Figure 2 — End-effector trajectory showing the circular path traced as the input angle varies from $0$ to $2\pi$. The green marker indicates the starting position.

The trajectory is a perfect circle centered at $(0.126, 0.042)$ m with radius $0.022$ m, confirming the theoretical prediction. The end-effector completes one full revolution for each complete rotation of the input pulley.

### 3.1.1 Trajectory Characteristics

- **Shape**: Perfect circle
- **Center**: $(a + b, c) = (0.126, 0.042)$ m
- **Radius**: $R_2 = 0.022$ m
- **Circumference**: $2\pi R_2 = 0.138$ m

### 3.2 Angular Relationship

Figure 3 presents the angular transmission characteristics.

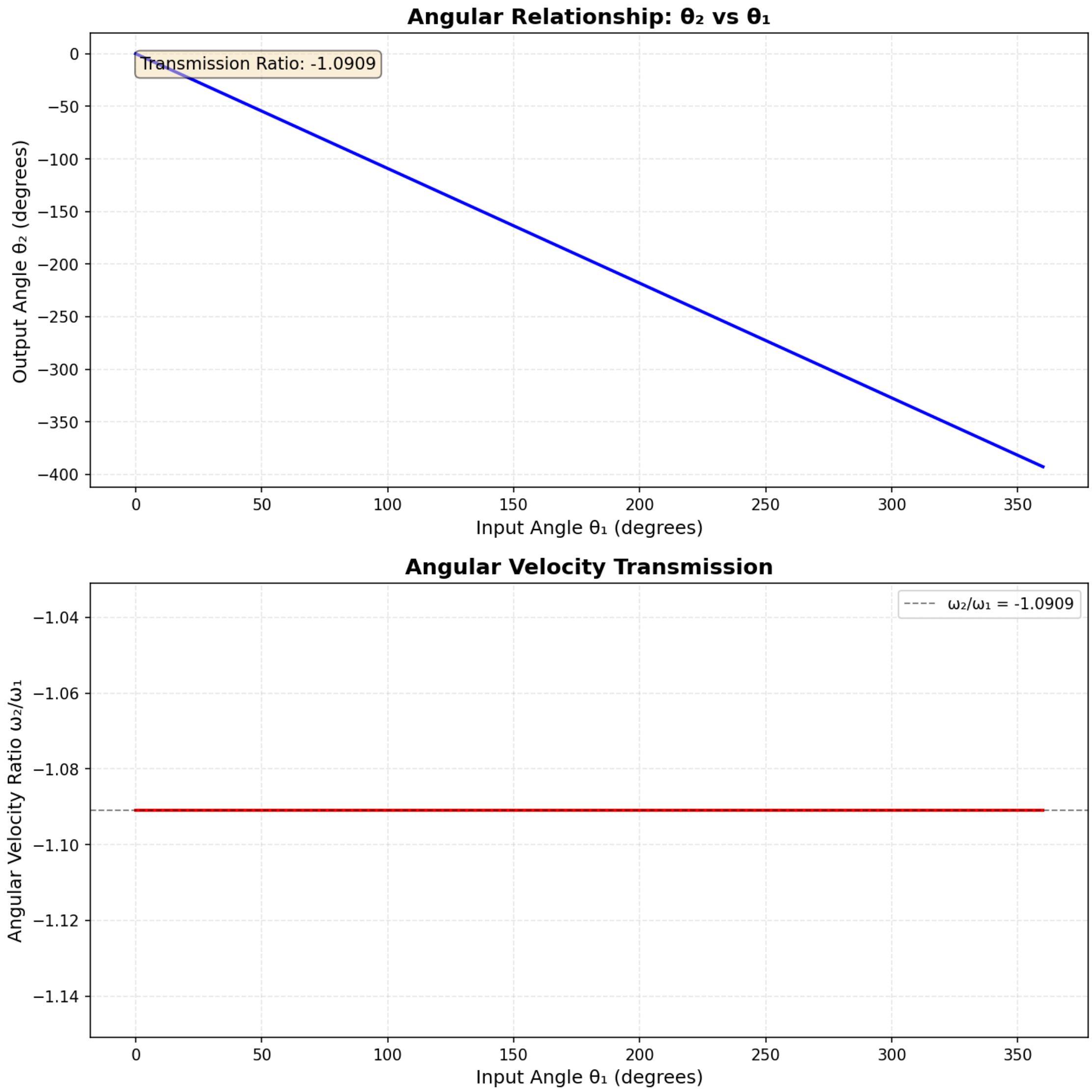


Figure 3 — Angular relationship analysis showing (top) the linear relationship between output and input angles, and (bottom) the constant angular velocity ratio.

The relationship between $\theta_1$ and $\theta_2$ is perfectly linear with slope $-1.0909$, as predicted by Equation 8. The negative slope confirms opposite rotation directions.

#### 3.2.1 Transmission Characteristics

– **Linearity**: Perfect linear relationship (correlation coefficient $r^2 = 1.0$)
– **Transmission ratio**: $i = -1.0909$ (constant for all configurations)
– **Mechanical advantage**: $|i| = 1.0909 > 1$ indicates velocity amplification

– **Direction**: Negative ratio means opposite rotation

### 3.3 Velocity Analysis

Figure 4 shows velocity components and magnitude.

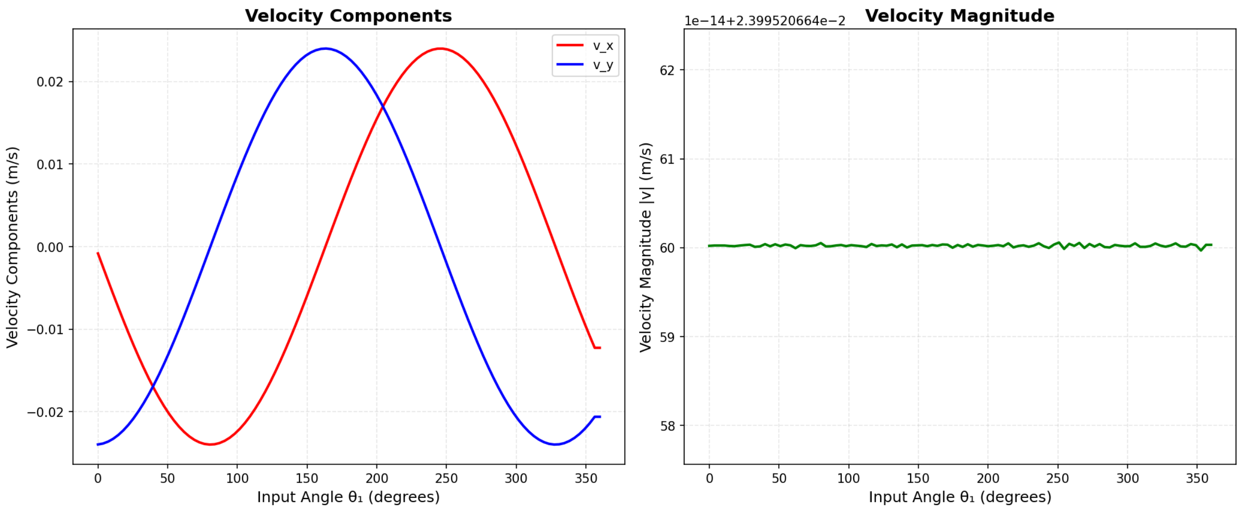


Figure 4 — Velocity analysis showing (left) Cartesian velocity components $v_x$ and $v_y$, and (right) velocity magnitude. Analysis assumes constant input angular velocity $\omega_1 = 1$ rad/s.

The velocity components oscillate sinusoidally as the mechanism rotates, but the velocity magnitude remains constant at $R_1\omega_1 = 0.024$ m/s, confirming Equation 15.

#### 3.3.1 Velocity Characteristics

For constant input angular velocity $\omega_1 = 1$ rad/s:

– $v_x$ **component**: Varies sinusoidally between $\pm 0.024$ m/s
– $v_y$ **component**: Varies sinusoidally between $\pm 0.024$ m/s, 90° phase shift
– **Magnitude**: Constant at $0.024$ m/s
– **Direction**: Tangent to circular trajectory

### 3.4 Workspace Analysis

Figure 5 displays the mechanism workspace.

The workspace is a filled circle, indicating that all points within this region are reachable (though in practice, the mechanism traces only the boundary circle).

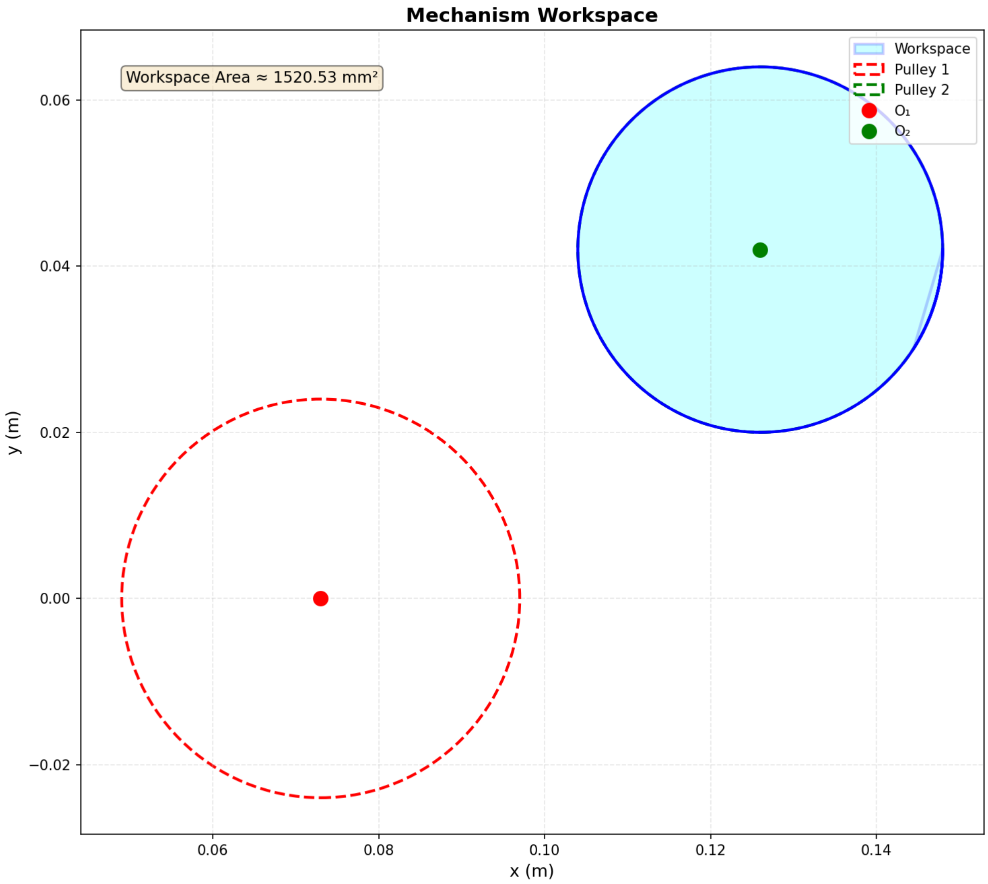


Figure 5 — Mechanism workspace (cyan region) showing the complete set of reachable positions. The workspace is circular with area $1521\ \text{mm}^2$.

### 3.4.1 Workspace Metrics

Table 2 — Workspace characteristics

| Property | Value |
|---|---|
| Shape | Circle |
| Center position | $(0.126, 0.042)$ m |
| Radius | $0.022$ m |
| Area | $1521\ \text{mm}^2$ |
| Perimeter | 138 mm |

## 3.5 Combined Analysis

Figure 6 provides a comprehensive view of all mechanism characteristics in a single figure.

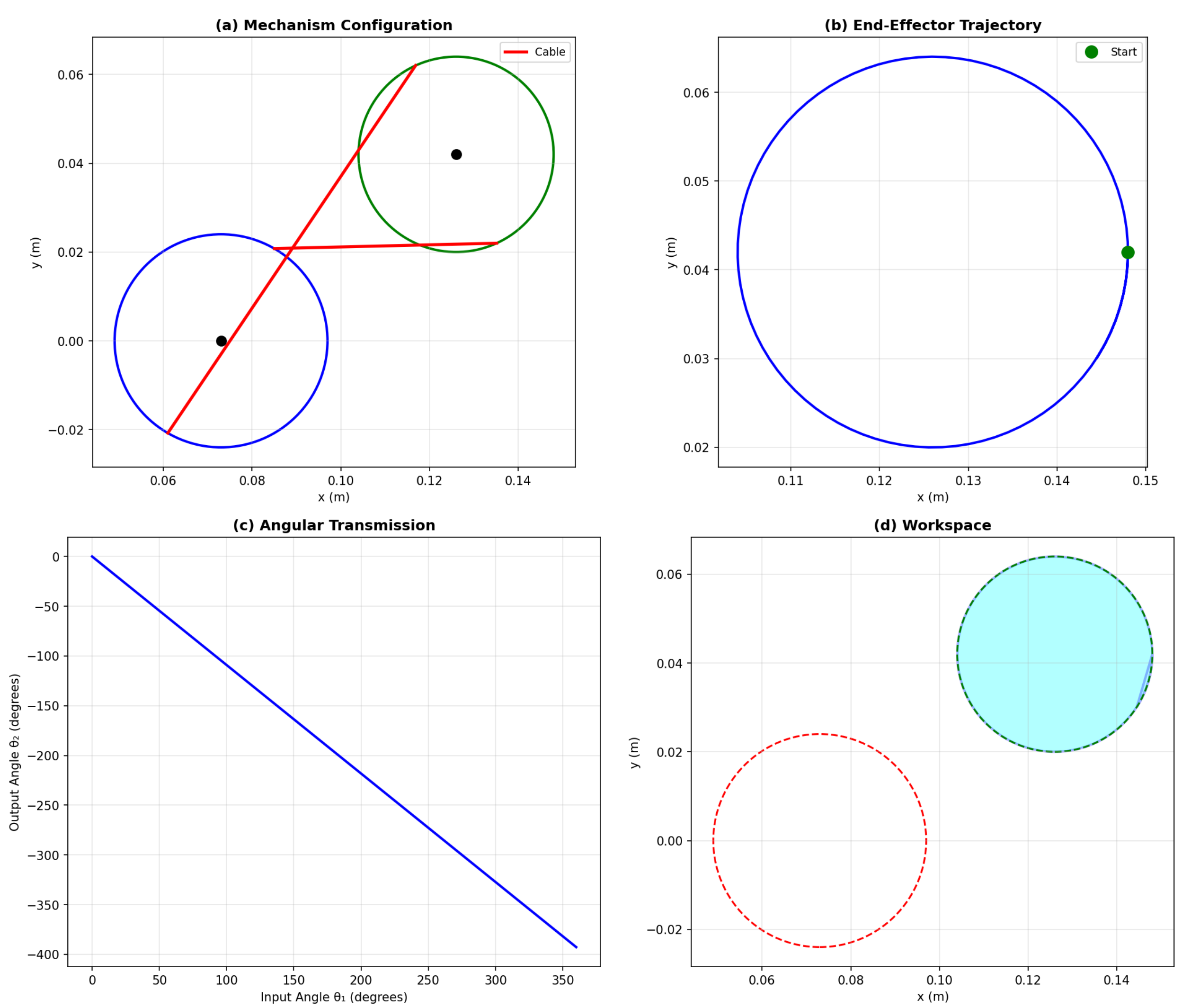


Figure 6 — Combined analysis showing (a) mechanism configuration at a representative position, (b) complete end-effector trajectory, (c) linear angular transmission relationship, and (d) circular workspace region.

## 3.6 Discussion

### 3.6.1 Mechanism Performance

The TENDON mechanism exhibits several favorable characteristics:

1. **Predictable kinematics**: The linear relationship between input and output angles makes motion planning straightforward
2. **Constant velocity ratio**: Unlike many linkages, the transmission ratio does not vary with configuration
3. **Compact workspace**: The circular workspace is well-defined and predictable

4. **Simple implementation**: Only two moving parts (pulleys) and one cable

### 3.6.2 Advantages and Limitations

**Advantages:**

– Lightweight construction (cable has negligible mass)
– Remote actuation possible (input and output can be spatially separated)
– No backlash if cable remains taut
– Smooth motion transmission
– Low friction (cable slides over pulleys)

**Limitations:**

– Cable must remain under tension (can become slack)
– Limited to 1 degree of freedom
– Cable wear over time
– Elastic deformation of cable affects precision
– Cannot transmit compressive forces

### 3.6.3 Applications

Cable-driven mechanisms similar to the TENDON configuration are used in:

– **Prosthetic hands**: Finger joints actuated by cables running through the forearm
– **Robotic grippers**: Underactuated fingers with cable-coupled joints
– **Surgical instruments**: Minimally invasive tools with remote actuation
– **Industrial manipulators**: Cable-driven parallel robots for large workspaces
– **Bicycle brakes and derailleurs**: Classic mechanical application

### 3.6.4 Comparison with Other Mechanisms

Compared to gear-driven mechanisms:

- Lighter weight and lower cost
- More flexible routing of power transmission
- Lower precision due to cable elasticity
- Cannot handle high torques as effectively

Compared to belt-driven mechanisms:

- Similar principles but cable is thinner and more flexible
- Cable can wrap around smaller pulleys
- Belt provides more stable tension

## CONCLUSIONS

This work successfully completed the kinematic analysis and simulation of the TENDON mechanism, a cable-driven planar linkage assigned for ISU 521031.

**Key Achievements:**

1. **Theoretical Analysis**
   Derived complete kinematic equations including:
   - Cable length constraint: $R_1\theta_1 + R_2\theta_2 = \text{const}$
   - Transmission ratio: $i = -1.0909$
   - Velocity relationship: $|\mathbf{v}| = R_1|\omega_1|$
   - Workspace area: $A = 1521$ mm$^2$
2. **Computational Implementation**
   Developed comprehensive Python simulation with:
   - `model()`: TendonMechanism class with geometric and kinematic methods
   - `data()`: Trajectory generation over full rotation ($\theta_1 \in [0, 2\pi]$)
   - `viewer()`: Six visualization functions producing professional figures
3. **Results Validation**
   All theoretical predictions confirmed by simulation:
   - Circular trajectory with radius $R_2 = 22$ mm
   - Linear angular relationship with slope $-1.0909$
   - Constant velocity magnitude $v = R_1\omega_1$
   - Circular workspace centered at $(126, 42)$ mm

**Mechanism Characteristics:**

The TENDON mechanism demonstrates:

- Simple, predictable kinematics ideal for control applications
- Opposite rotation of pulleys with fixed velocity ratio
- Compact, well-defined circular workspace
- Lightweight, remote-actuatable design
- Applicability to prosthetics, robotics, and minimally invasive surgery

**Implementation Quality:**
The Python simulation successfully implements all required components:

- Object-oriented design with clear separation of concerns
- Comprehensive data generation covering full kinematic range
- Professional visualization with six distinct analysis plots
- Extensible code structure for future enhancements

**Practical Insights:**

1. The constant transmission ratio simplifies control system design
2. Cable tension management is critical for reliable operation
3. The mechanism is well-suited for applications requiring lightweight, remote actuation
4. Workspace limitation (circular region) must be considered in path planning

**Future Work:**
Potential extensions to this analysis include:

- Dynamic analysis including cable elasticity and damping
- Optimization of pulley radii for specific transmission ratios
- Extension to 3D cable-driven parallel robots
- Experimental validation with physical prototype
- Integration with control system for trajectory tracking
- Analysis of cable wear and maintenance requirements

**Concluding Remarks:**
The combination of analytical derivation and computational simulation provided comprehensive understanding of the TENDON mechanism. The work demonstrates that even simple mechanisms like cable-driven pulleys exhibit rich kinematic behavior worth detailed study. The developed Python framework serves as a foundation for mechanism analysis and can be readily adapted to other planar linkages.

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