“The Plan” for Rope Failure Rates:

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Definition of climbing terms used in literature review (please let me know if I need to clarify any):

Second: The person on the ground, not climbing, but instead holding the rope. When the climber falls, this person uses a belay device to help keep the rope from slipping. The rope then catches the climber and ideally keeps the climber from hitting the ground.

Running belay: The last carabiner on the rock face – often the one that takes the most weight from a climbing fall.

Literature Review:

Experimental and theoretical simulations of climbing falls by Martyn Pavier

In his paper, Pavier creates a simulation to calculate the strain on a climbing rope after a climber falls, given certain parameters about the system. He then compared the results of the simulation against experimentally measured rope tensions. After, he ran further experiments to measure the number of falls a rope needed to take under certain conditions until it failed (broke). From these results he created a failure curve, and combined with the theoretical simulations results, he went to predict the number of falls any general rope would need to take before failure.

The experimental apparatus Pavier created consisted of a trolley attached to steel bars as seen in the figure below (taken from Pavier’s paper). He could add extra weight bars to the trolley to increase its mass for different trials. It was held in a raised position by a catch between tests. Two belay mounts were equipped with strain gauges - the top belay represented the running belay, and the bottom one representing the second’s belay. The output of the strain gauges was recorded and the tensions of the two rope segments were derived from the loads measured by the belay mounts.

Since this is a replication study, I will go into detail about the theoretical simulation when presenting my conceptual model. For a review at a high level, Pavier created a base tension using a very simplified model, and then added incremental strains to this base. The incremental strains were calculated for each piece of rope between carabiners, with special cases for the last carabiner and the second’s belay (essentially the first carabiner). At the end of the paper, Pavier noted several potential extensions to his model – including requiring a more sophisticated model for the second’s belay. In my study, I will attempt to create this expansion.

Conceptual Model:

Problem Overview:

The ropes a climber uses are critical to outdoor rock climbing. In a typical climb, there are two individuals – a climber (or a first) and a belayer (or a second). A rope is attached to the belayer’s harness through a belay device, and is tied to the climber. The belay device is specially created to help the belayer hold the rope steady and keep the climber from falling far distances. The climber then climbs the rock, placing carabiners on pieces of protection in the rock, and threading the rope through these carabiners. In the even that the climber falls, the belayer holds down on the rope to keep it from slipping, and the rope catches on the last carabiner places, keeping the climber from falling to the ground. In multi-pitch climbs, when the climber reaches a safe place to stop, they set up an anchor, and top-belay while the (previous) belayer climbs up after them (hence the terms first and second).

When a climber falls, the rope they are using sustains some amount of damage proportional to the strain on the rope. If the rope takes too many hard falls, the rope can break. This is dangerous for obvious reasons, and thus it is important for a climber to know how many falls a rope can take. The simulation I will run is to calculate how many falls a rope could take before failure, thus creating some measure of the rope's initial 'life'.

Customers:

Rock climbers and climbing rope manufacturers.

Modeling Approach:

First, I will list all variables used in calculations (taken from Pavier):

Notation

d	Vertical distance above last runner
g	Acceleration due to gravity
k	Modulus of rope for linear elastic model
k_1, k_2	Moduli of rope for visco-elastic model
l_i	Length of rope segment i
n	Number of rope segments
r	Runner radius
s_i	Total slip at runner i
x_i, y_i	Co-ordinates of runner i
$\mathbf{C}$	Matrix relating incremental slips to incremental strains
$\dot{E}$	Rate of energy dissipation at a runner due to friction
F	Fall factor
$\mathbf{I}$	Unit matrix
$\mathbf{K}$	Matrix relating incremental strains to incremental tensions
L	Length of rope run-out
$\mathbf{L}$	Matrix of slip conditions
M	Mass of leader
S	Critical tension in rope segment 1 for slip at belay
T_i	Total tension in rope segment i
$\mathbf{T}$	Vector of total tensions
ε_i	Total strain in rope segment i
η_i	Maximum tension ratio at runner i
λ	Viscous coefficient of rope for visco-elastic model
μ_i	Coefficient of friction at runner i
θ_i	Angle of lap at runner i
σ_i	Slip condition at runner i
Δs_i	Incremental slip at runner i
$\Delta \mathbf{s}$	Vector of incremental slips
Δt	Duration of the increment of time
ΔT_i	Incremental tension in rope segment i
$\Delta \mathbf{T}$	Vector of incremental tensions
$\Delta \varepsilon_i$	Incremental strain in rope segment i
$\Delta \boldsymbol{\varepsilon}$	Vector of incremental strains

The first step is to find the base maximum tension using the fall factor method which demonstrates the basic mechanics of a climbing fall. Fall factor F is found through:

$$F = \frac{2d}{L}$$

for L being the total length of the live rope, and 2d being the distance the climber falls before the rope begins to catch them. A simplified calculation of the maximum tension in the rope can then be written as:

$$T = \sqrt{2kFMg}$$

. To calculate the incremental strains on the rope, first it is important to know the placement of each carabiner in the pitch. The user would input the pitch's geometry in a matrix, and from there the simulation will calculate the length of each rope segment, and the angle between the different rope segments. From this, we can find the maximum tension ratio between two segments of rope:

$$\eta_i = e^{\mu_i \theta_i}$$

. Then the strain in each rope segment is calculated from the total slip at each carabiner, and an incremental version is derived from that:

$$\Delta s_i(1+\varepsilon_i) - \Delta s_{i-1}(1+\varepsilon_i) = \Delta\varepsilon_i(l_i + s_{i-1} - s_i)$$

. This model is also valid for the slip at the first rope segment if the incremental slips there are the slips are assumed to be slips through the belay device. From this, a matrix can be created to get incremental strains in each rope segment from all incremental slips:

$$\mathbf{C}\Delta\mathbf{s} = \Delta\boldsymbol{\varepsilon}$$

where C relates the strain from one rope segment to the slips at each of its runners. An example of this is:

$$\begin{bmatrix} & & 0 & 0 & & \\ & & \vdots & \vdots & & \\ 0 & \cdots & \frac{-1-\varepsilon_4}{l_4+s_3-s_4} & \frac{1+\varepsilon_4}{l_4+s_3-s_4} & \cdots & 0 \\ & & \vdots & \vdots & & \\ & & 0 & 0 & & \end{bmatrix} \begin{bmatrix} \Delta s_0 \\ \vdots \\ \Delta s_3 \\ \Delta s_4 \\ \vdots \\ \Delta s_n \end{bmatrix} = \begin{bmatrix} \Delta\varepsilon_1 \\ \vdots \\ \Delta\varepsilon_4 \\ \vdots \\ \Delta\varepsilon_n \end{bmatrix}$$

Given this, the incremental tensions at each segment can be taken to be:

$$\Delta T_i = k_1 \Delta \varepsilon_i + \frac{\Delta t}{\lambda}[k_1 k_2 \varepsilon_i - T_i(k_1 + k_2)]$$
$$= k_i \Delta \varepsilon_i + \Delta T_i^0$$

for k1,k2, and lambda being parameters of the rope's material. This can be turned into a matrix of equations for the incremental strains by using:

$$\mathbf{K}\Delta\boldsymbol{\varepsilon} = \Delta\mathbf{T} - \Delta\mathbf{T}^0$$

where **K** = (k1)**I**. For each carabiner, slip can occur depending on the rope tensions on either side of the carbiner. The condition of slip at any given carabiner is:

$$\begin{aligned}
&\Delta T_i = \eta_i \Delta T_{i+1};\ \Delta s_i < 0;\ \sigma_i = -1 \text{ if } T_i = \eta_i T_{i+1} \\
&\Delta s_i = 0;\ \sigma_i = 0;\ \text{ if } \eta_i T_{i+1} < T_i < \frac{T_{i+1}}{\eta_i} \qquad (13) \\
&\Delta T_i = \frac{\Delta T_{i+1}}{\eta_i};\ \Delta s_i > 0;\ \sigma_i = +1 \text{ if } T_i = \frac{T_{i+1}}{\eta_i}
\end{aligned}$$

Slip ocurrs at the bleay device is the tension at the first segment is equal to some critical condition:

$$\begin{aligned}
&\Delta s_0 = 0;\ \sigma_0 = 0 \text{ if } T_1 < S \\
&\Delta T_1 = 0;\ \Delta s_0 > 0;\ \sigma_0 = +1 \text{ if } T_1 = S
\end{aligned}$$

for S as the critical tension.

If slip occurrs at the belay device, tension is recalculated, and then there is a chance that slip at the belay device can happen again. With each further belay slip, the chance of the next slip lessens – in other words, the critical tension goes up.

From the above equations, a matrix form can be derived, for **L** being a matrix that corresponds to a matrix of slip conditions:

$$\mathbf{L}\Delta\mathbf{T} = 0$$

(eq 15) An example being:

$$\begin{bmatrix} & & 0 & 0 & & \\ & & \vdots & \vdots & & \\ 0 & \dots & 1 & -1/\eta_3 & \dots & 0 \\ & & \vdots & \vdots & & \\ & & 0 & 0 & & \end{bmatrix} \begin{bmatrix} \Delta T_1 \\ \vdots \\ \Delta T_3 \\ \Delta T_4 \\ \vdots \\ \Delta T_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Pavier then notes that a modified version of the above equation is necessiary to avoid instabilities in the event that the increment is too large, and says that the two are identical assuming that (eq 15) is satisfied for all previous increments:

$$\mathbf{L}\Delta\mathbf{T} = -\mathbf{L}\mathbf{T}$$

(eq 16) for **T** being a vector of rope tensions by segment.

Now previous equations are combined to solve for the vector of incremental slips:

$$\mathbf{LKC\Delta s} = -\mathbf{L}[\mathbf{T} + \Delta\mathbf{T}^0]$$

From this, the incremental strains can be calculated. Then the current rope tensions (from the base that we calculated at the begenning) can be incremented noting that the tension on any given segment cannot be less than zero.

From all of this, the new velocity of the climber can be found from:

$$v = \dot{s}_n + (g - T_n/M)\Delta t.$$

Pavier then proposes that the amount of damage that happens to a rope is proportional to the maximum value of tension generated during the fall.

Thus, to determine the damage, the simulation should find the largest value of tension during the fall. This maximum will then be compared to an experimentally found Maximum tension vs. Fall-to-failure chart, to predict roughly how long of a life the rope initially has.

A secondary thing that this simulation could find is the carabiner with the most tension force pulling on it. This would be the area of the rope most damaged by the fall, and the area that would likely break. This has other applications as well, such as testing different clipping paths, but that will only be explored if I have extra time.

Assumptions in Model:

This implementation neglects several things. One that Pavier notes is that the model does not take into account any potential tension waves generated. Other assumptions are thus: the model does not take into account environmental issues that the rope encounters, such as sharp rock edges that might cut at and weaken the rope. It also cannot predict any past damage so a rough count of falls-to-failure is only applicable for a new rope. Further, it takes in 2d positions, though climbing is plainly a 3d sport. While there are many climbing routes that roughly follow a vertical plane, there are many that overhang. This however, would not necessarily change the prediction of a rope's general lifespan, but this model could not be used to predict a fall-to-failure rate for a rope when falling on a given route.

Implementation Plan:

Language: C++

External Libraries: Eigen (for Matrices)

Goals:

Part C:

- Have most equations coded as functions
- May or may not have extensions fully complete/integrated
- Code compiles

Part D:

- Take in user input (may have a few hard coded values for rope constants)
- Extensions to base model completely integrated

- Compile, run, give back some sort of result
- Minor bugs allowed

Part E:

- As few hard coded values as possible
- No bugs
- Started running tests and collecting data for final paper