Finance Recap

Here you can find a small recap on portfolios and metrics used to compare portfolios.

Portfolio

A portfolio is a combination of assets with respective shares. In order to ease our process, we will represent shares as a percentage of the portfolio. We will represent a portfolio P as follows :

- P is of size N if it has N different asset
- list of assets: $(a_i)_{i \in [[1,N]]}$
- list of weights: $(w_i)_{i \in [[1,N]]}$

To optimize a portfolio, we have to find the best combination of weights and assets. Suppose we have M assets and we want to create the best portfolio of size N. It can be represented by the following function:

$$F : (a_i)_{i \in [[1,M]]} \mapsto \begin{pmatrix} a_i \\ w_i \end{pmatrix}_{i \in [[1,N]]}$$

Then again, this function is supposed to give us the optimal list of assets ande their respective weights. We don't know yet what optimal means and how to compute it. Here comes the metrics.

Metrics

In order to compare portfolios, we need to pass by some metrics. These metrics are defined for one asset and can be generalized to a portfolio. In this section we will define the following metrics:

- Return (Rendement en français)
- Volatility (Volatilité en français)
- Sharpe

Return -- Rendement (Fr)

Return of an asset:

The Return value of an asset is an evaluation of the performance of that asset on a certain period. Let a be an asset, and V_i be the Value of a. The return value R of a on the period $T = [t_{i-1}, t_i]$ is defined as:

$$R(a,T) = \frac{V_i - V_{i-1} + Cash\ flow\ received}{V_{i-1}}$$

Return of a portfolio:

The Return value of a portfolio is an evaluation of the performance of that portfolio on a certain period. It is actually the weighted average of its assets. Let P a portfolio of size N. Let $(a_i)_{i\in[[1,N]]}$ the list of assets in P and their respective weights $(w_i)_{i\in[[1,N]]}$. The return value R of P is defined as:

$$R(P,T) = \sum_{i=1}^{N} w_i R(a_i, T)$$

Volatility -- Volatilité (Fr)

Volatility of an asset:

volatility σ is the degree of variation of a trading price series over time as measured by the standard deviation of logarithmic returns *(Wikipedia)*. In *JUMP*'s slides, it is the standard deviation of the value. We will follow this definition since we are using their API. Let a an asset, σ_a its volatility and V_a its Variance on a period of size n. Let $\overline{a}$ the mean value of a on that period and a_i the value of a at an instant i. The Volatility of a is defined as:

$$\sigma_a = \sqrt{V_a}$$

$$\sigma_a = \sqrt{\frac{\sum_{i=1}^{n}(a_i - \overline{a})^2}{n}}$$

Volatility of a portfolio:

The volatility of a portfolio is standard deviation of the value (or return) of that portfolio. Since any portfolio is a weighted sum of assets and we can compute the variance-covariance matrix of all assets, it is easy to compute the volatility of a portfolio.

Let P a porfolio of size N sush as :

- $(a_i)_{i\in[[1,N]]}$ is the list assets composing P
- $(w_i)_{i\in[[1,N]]}$ is the list of respective weights
- The values of the assets are taken over a period T of size n
- $a_{i,j}$ is the value of asset a_i at the instant j in T
- $\overline{a}_i$ is the mean value of the asset a_i over T
- $Cov(a_i, a_j)$ the covariance between assets a_i and a_j over T

The mean and covariance over T are defined as:

- $\overline{a}_i = \frac{1}{n}\sum_{j=1}^{n} a_{i,j}$
- $Cov(a_i, a_j) = \frac{1}{n}\sum_{k=1}^{n}(a_{i,k} - \overline{a}_i)(a_{j,k} - \overline{a}_j)$

Let p the value of P, we have $p = \sum_{i=1}^{N} w_i a_i$. The volatility σ_P of the portfolio is simply the standard deviation of p, there for $\sigma_P = \sqrt{Var(p)}$ with $Var(p)$ the variance of p over the period T. The volatility of a the portfolio is defined as:

$$\sigma_P = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov(a_i, a_j)}$$

Sharpe ratio

In finance, the Sharpe ratio is a way to examine the performance of an investment by adjusting for its risk. The ratio measures the excess return per unit of deviation in an investment asset, typically referred to as risk.

Let a an asset, R_a the asset return and $Var(a)$ the asset variance. Let b a benchmark asset, R_b the benchmark return and $Var(b)$ the benchmark variance. The sharpe of a is defined as:

$$S_a = \frac{E[R_a - R_b]}{\sqrt{Var(a) - Var(b)}}$$

If we take the asset benchmark as R_b as the *constant* risk-free return, noted R_f, we have $Var(R_f) = 0$ since R_f is a constant. The volatility of a is $\sigma_a = \sqrt{Var(R_a)}$
the sharpe becomes:

$$S_a = \frac{E[R_a - R_f]}{\sigma_a}$$

Conclusion

In order to optimize a portfolio P of size N (already specified assets), we need to find the weights $(w_i)_{i \in [[1,N]]}$ that maximizes $R(P)$ and minimizes σ_P