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2-3-4 Tree and RB Tree Equivalency

As we know, a 2-3-4 tree of Order 2 is equivalent to a Binary Tree; expanding on this however, a 2-3-4 Tree can also be mapped to a Red-Black Tree. As we know, the nodes of a 2-3-4 Tree can only take three forms: 2-Node, 3-Node, and 4-Node. If we can map a Red-Black Tree that is equivalent to all these cases, we can conclude that a 2-3-4 Tree is equivalent to a Red-Black Tree.

In the first case, the 2-3-4 tree is a 2-Node with one element. We can represent this in a Red-Black tree as a single Black node with the appropriate red children. Thus, this case is true, and a 2-3-4 Tree can be said to be equivalent if the node is a 2-Node.

In the second case, the 2-3-4 Tree is a 3-Node with two elements. We can represent it as a Red-Black tree by having the two elements split into separate nodes. This can be accomplished by having the first element be the root black node and having its child be the right red node, or inversely by having the second element is the root and the first element is the left child. The children of the 2-3-4 tree would then follow suit, being connected to the appropriate nodes depending on which structure was chosen. Thus, this case is true, and a 2-3-4 Tree can be said to be equivalent if the node is a 3-Node.

Lastly there is the third case, where the 2-3-4 Tree is a 4-Node with three elements. We can represent this as a Red-Black Tree by having the second element be the root black node and the other two elements as the left-right children of the root. Then, the four child nodes would be put to the left and right of the red nodes depending on how many children of the original 2-3-4 Tree there are. Since this is a representation of an RB Tree, this case is true, and a 2-3-4 Tree can be said to be equivalent if the node is a 4-Node.

Therefore, since all cases have been shown to be equivalent to a Red-Black Tree we can say that all combinations of these cases can be represented as Red-Black Trees. Thus, we have shown that 2-3-4 Trees are equivalent to Red-Black Trees.