

In order to raise a matrix to some power, use the `MatrixPower` command. Here is the syntax, along with a couple of examples.

? MatrixPower

`MatrixPower[m, n]` gives the n^{th} matrix power of the matrix m .

`MatrixPower[m, n, v]` gives the n^{th} matrix power of the matrix m applied to the vector v . >>

$$\mathbf{a} = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \\ 0 & 4 & 2 \end{pmatrix};$$

`MatrixForm[a]`

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \\ 0 & 4 & 2 \end{pmatrix}$$

`MatrixForm[a.a]`

`MatrixForm[MatrixPower[a, 2]]`

`MatrixForm[a.a.a]`

`MatrixForm[MatrixPower[a, 3]]`

`MatrixForm[MatrixPower[a, 20]]`

$$\begin{pmatrix} -1 & 10 & 3 \\ -3 & 2 & -1 \\ -4 & 16 & 4 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 10 & 3 \\ -3 & 2 & -1 \\ -4 & 16 & 4 \end{pmatrix}$$

$$\begin{pmatrix} -11 & 30 & 5 \\ -5 & -6 & -5 \\ -20 & 40 & 4 \end{pmatrix}$$

$$\begin{pmatrix} -11 & 30 & 5 \\ -5 & -6 & -5 \\ -20 & 40 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 424\,004\,599 & -124\,592\,230 & 185\,511\,743 \\ -185\,511\,743 & 609\,516\,342 & 123\,903\,929 \\ 495\,615\,716 & 246\,431\,256 & 361\,708\,484 \end{pmatrix}$$

Here is the transformation matrix “trans” that we used when discussing the Markov chain for the coin flip game (discussed on p. 301 of the textbook).

$$\text{trans} = \begin{pmatrix} 1 & .5 & 0 & 0 & 0 & 0 \\ 0 & 0 & .5 & 0 & 0 & 0 \\ 0 & .5 & 0 & .5 & 0 & 0 \\ 0 & 0 & .5 & 0 & .5 & 0 \\ 0 & 0 & 0 & .5 & 0 & 0 \\ 0 & 0 & 0 & 0 & .5 & 1 \end{pmatrix};$$

MatrixForm[trans]

$$\begin{pmatrix} 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 \end{pmatrix}$$

MatrixForm[

MatrixPower[trans, 5]]

$$\begin{pmatrix} 1. & 0.6875 & 0.375 & 0.21875 & 0.0625 & 0. \\ 0. & 0. & 0.15625 & 0. & 0.09375 & 0. \\ 0. & 0.15625 & 0. & 0.25 & 0. & 0. \\ 0. & 0. & 0.25 & 0. & 0.15625 & 0. \\ 0. & 0.09375 & 0. & 0.15625 & 0. & 0. \\ 0. & 0.0625 & 0.21875 & 0.375 & 0.6875 & 1. \end{pmatrix}$$

We can use the Manipulate command to easily see how things change as some parameter changes. Below is the documentation, plus the example of the transformation matrix raised to some power.

? Manipulate

Manipulate[expr, {u, u_{min}, u_{max}}] generates a version of

expr with controls added to allow interactive manipulation of the value of u.

Manipulate[expr, {u, u_{min}, u_{max}, du}] allows the value of u to vary between u_{min} and u_{max} in steps du.

Manipulate[expr, {{u, u_{init}}, u_{min}, u_{max}, ...}] takes the initial value of u to be u_{init}.

Manipulate[expr, {{u, u_{init}, u_{lbl}}, ...}] labels the controls for u with u_{lbl}.

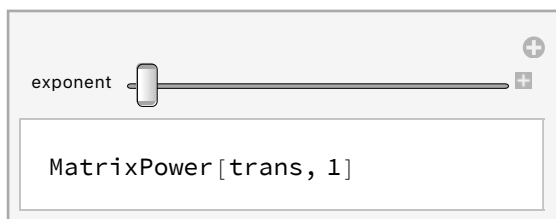
Manipulate[expr, {u, {u₁, u₂, ...}}] allows u to take on discrete values u₁, u₂,

Manipulate[expr, {u, ...}, {v, ...}, ...] provides controls to manipulate each of the u, v,

Manipulate[expr, c_u → {u, ...}, c_v → {v, ...}, ...]

links the controls to the specified controllers on an external device. >>

```
Manipulate[MatrixForm[
  MatrixPower[trans, exponent] ], {exponent, 1, 50, 1}]
```



It seems the best way to display numbers to a fixed number of decimal places is to use the `NumberForm` command, specifically the second example listed in the documentation below. (Note that it behaves a bit unexpectedly when it comes to very small numbers and scientific notation, as you will see in examples.)

? NumberForm

`NumberForm[expr, n]` prints with approximate real numbers in *expr* given to *n*-digit precision.

`NumberForm[expr, {n, f}]` prints with approximate real

numbers having *n* digits, with *f* digits to the right of the decimal point. >>

```
NumberForm[MatrixForm[
  MatrixPower[trans, 5] ], {10, 3}]
```

$$\begin{pmatrix} 1.000 & 0.688 & 0.375 & 0.219 & 0.063 & 0.000 \\ 0.000 & 0.000 & 0.156 & 0.000 & 0.094 & 0.000 \\ 0.000 & 0.156 & 0.000 & 0.250 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.250 & 0.000 & 0.156 & 0.000 \\ 0.000 & 0.094 & 0.000 & 0.156 & 0.000 & 0.000 \\ 0.000 & 0.063 & 0.219 & 0.375 & 0.688 & 1.000 \end{pmatrix}$$

This shows our transformation matrix raised to a power, and we have modified the number format and made it so that the exponent also appears to the right of the slider bar.

```
Manipulate[NumberForm[MatrixForm[
  MatrixPower[trans, exponent] ], {4, 3}],
{exponent, 1, 60, 1, Appearance -> "Labeled"}]
```

