MATH 616 HOMEWORK
DUE ~~4/6/16~~ 4/13/16

Problems 1-2 should be set up and written on paper (though you may use a calculator/computer to aid in computations). Use Mathematica to solve Problems 3-4 (or 3-5), and turn in your Mathematica notebook at https://dbinbox.com/Math615.

(1) (a) Find the least squares solution of

$$\begin{aligned} x_1 + x_2 &= 4 \\ 2x_1 + x_2 &= -2 \\ x_1 - x_2 &= 1. \end{aligned}$$

(b) Use your answer to find the point on the plane spanned by $(1, 2, 1)$ and $(1, 1, -1)$ that is closest to $(4, -2, 1)$.

(2) Find the best parabola $y = ax^2 + bx + c$ that fits the four data points $(-1, 0), (0, 1), (1, 3), (2, 5)$.

(3) Problem 7 from p. 287 in the textbook (included on following page).

(4) Find the values a_0, a_1, b_1 for which $a_0 + a_1 \cos(x) + b_1 \sin(x)$ best fits the following data points.

x	y
-3	-1.5
-2	-1
-1	-.5
0	0
1	.5
2	1
3	1.5

(5) Bonus: (Problem 4 continued) The data points in problem 4 were obtained by taking values $(x, f(x))$, where $f(x) = \frac{1}{2}x$, at sample values of x.

(a) Redo problem 4, but using the data points for all x-values in the list

$$\{-3.1, -3, -2.9, -2.8, \ldots, 2.9, 3.0, 3.1\}.$$

(b) Compare your answer from the previous part to the Fourier coefficients a_0, a_1, b_1 for $f(x) = \frac{1}{2}x$ on the interval $[-\pi, \pi]$.

(c) Can you find the function

$$a_0 + a_1 \cos(x) + b_1 \sin(x) + a_2 \cos(2x) + b_2 \sin(2x)$$

that best fits the data points from problem 5a?

Date: March 30, 2016. Material covered during April 6, 2016 lecture.

4 Find the line of best fit for the records for women's mile.

5 Do the lines of best fit for the men's and women's miles cross?

6 *(This illustrates that there are data sets for which a linear model is not right, and that the line of best fit doesn't in that case have any predictive value.)* In a highway restaurant a trucker told me that his boss often sends him by a roundabout route, using more gas but paying lower bridge tolls. He said that New York State calibrates the toll for each bridge across the Hudson, playing off the extra gas to get there from New York City against a lower crossing cost, to encourage people to go upstate. This table, from [Cost Of Tolls] and [Google Maps], lists for each toll crossing of the Hudson River, the distance to drive from Times Square in miles and the cost in US dollars for a passenger car (if a crossings has a one-way toll then it shows half that number).

Crossing	*Distance*	*Toll*
Lincoln Tunnel	2	6.00
Holland Tunnel	7	6.00
George Washington Bridge	8	6.00
Verrazano-Narrows Bridge	16	6.50
Tappan Zee Bridge	27	2.50
Bear Mountain Bridge	47	1.00
Newburgh-Beacon Bridge	67	1.00
Mid-Hudson Bridge	82	1.00
Kingston-Rhinecliff Bridge	102	1.00
Rip Van Winkle Bridge	120	1.00

Find the line of best fit and graph the data to show that the driver was practicing on my credulity.

7 When the space shuttle Challenger exploded in 1986, one of the criticisms made of NASA's decision to launch was in the way they did the analysis of number of O-ring failures versus temperature (O-ring failure caused the explosion). Four O-ring failures would be fatal. NASA had data from 24 previous flights.

temp $^\circ F$	53	75	57	58	63	70	70	66	67	67	67		
failures	3	2	1	1	1	1	1	0	0	0	0		
	68	69	70	70	72	73	75	76	76	78	79	80	81
	0	0	0	0	0	0	0	0	0	0	0	0	0

The temperature that day was forecast to be 31°F.

(a) NASA based the decision to launch partially on a chart showing only the flights that had at least one O-ring failure. Find the line that best fits these seven flights. On the basis of this data, predict the number of O-ring failures when the temperature is 31, and when the number of failures will exceed four.

(b) Find the line that best fits all 24 flights. On the basis of this extra data, predict the number of O-ring failures when the temperature is 31, and when the number of failures will exceed four.

Which do you think is the more accurate method of predicting? (An excellent discussion is in [Dalal, et. al.].)

Least Squares/Data Fitting Homework (MTH 616)

1. Question: Find the least squares solution to
x1 + x2 = 4
2x1 + x2 = -2
x1 - x2 = 1.

In[258]:=
```
(*Here are the matrices a, b*)
a = {{1, 1}, {2, 1}, {1, -1}};
b = {{4}, {-2}, {1}};
MatrixForm[a]
MatrixForm[b]
```

Out[260]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 1 & -1 \end{pmatrix}$$

Out[261]//MatrixForm=

$$\begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

In[262]:=
```
(*Solution is given by*)
(soln = Inverse[Transpose[a].a].Transpose[a].b ) // MatrixForm
```

Out[262]//MatrixForm=

$$\begin{pmatrix} \frac{1}{14} \\ \frac{2}{7} \end{pmatrix}$$

The least squares solution is x1 = 1/14, x2 = 2/7.

1b. Find the point on the plane spanned by (1,2,1) and (1,1,-1) that is closest to (4,-2,1).

In[263]:=
```
N[a.soln]
```

Out[263]= {{0.357143}, {0.428571}, {-0.214286}}

2. Question: Find the best parabola y=ax^2 + bx +c that best fits the data (-1,0), (0,1), (1,3), (2,5).

In[264]:=
```
a = {{1, -1, 1}, {0, 0, 1}, {1, 1, 1}, {4, 2, 1}};
b = {0, 1, 3, 5};
MatrixForm[a]
MatrixForm[b]
(soln = Inverse[Transpose[a].a].Transpose[a].b ) // MatrixForm
N[soln]
```

Out[266]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix}$$

Out[267]//MatrixForm=

$$\begin{pmatrix} 0 \\ 1 \\ 3 \\ 5 \end{pmatrix}$$

Out[268]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4} \\ \frac{29}{20} \\ \frac{23}{20} \end{pmatrix}$$

Out[269]= {0.25, 1.45, 1.15}

Best fit parabola is (1/4)x^2 + (29/20)x + 23/20.

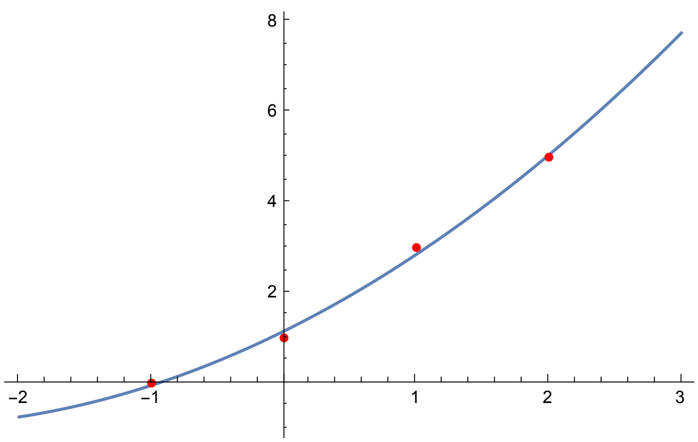
In[270]:=
```
(*Alternate Method*)
data1 = {{-1, 0}, {0, 1}, {1, 3}, {2, 5}};
quad = Fit[data1, {1, x, x^2}, x]
```

Out[271]= $1.15 + 1.45\,x + 0.25\,x^2$

Note: Here is a graph of the data points and the best fit parabola.

In[272]:=
```
Show[Plot[quad, {x, -2, 3}], ListPlot[data1, PlotStyle → Red]]
```

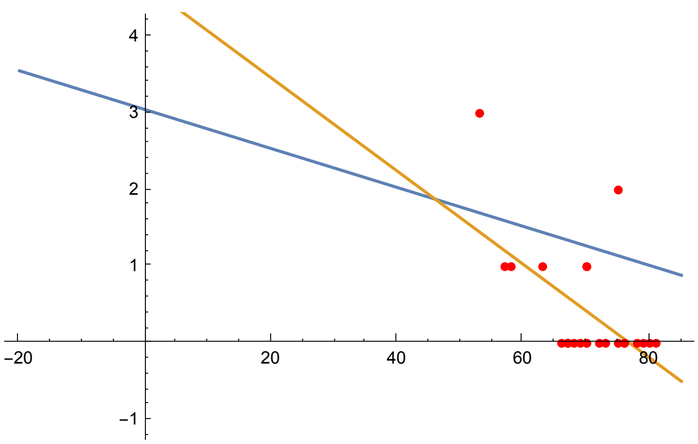
Out[272]=



3.

```
In[273]:= data1 = {{53, 3}, {75, 2}, {57, 1}, {58, 1}, {63, 1}, {70, 1}, {70, 1}};
data2 = {{53, 3}, {75, 2}, {57, 1}, {58, 1}, {63, 1}, {70, 1}, {70, 1}, {66, 0},
    {67, 0}, {67, 0}, {67, 0}, {68, 0}, {69, 0}, {70, 0}, {70, 0}, {72, 0},
    {73, 0}, {75, 0}, {76, 0}, {76, 0}, {78, 0}, {79, 0}, {80, 0}, {81, 0}};

lin1 = Fit[data1, {1, x}, x]
lin2 = Fit[data2, {1, x}, x]

lin1 /. (x → 31) (*Plug x=31 into lin1*)
lin2 /. (x → 31) (*Plug x=31 into lin2*)

Solve[lin1 == 4, x] (*Solve for when average number of failures is 4*)
Solve[lin2 == 4, x]

Show[Plot[{lin1, lin2}, {x, -20, 85}],
 ListPlot[data2, PlotStyle → Red], PlotRange → {-1, 4}]
Out[275]= 3.04649 - 0.0253934 x

Out[276]= 4.675 - 0.0608333 x

Out[277]= 2.2593

Out[278]= 2.78917

Out[279]= {{x → -37.5493}}

Out[280]= {{x → 11.0959}}
```

Out[281]=



For part (a), the linear regression using only the data points where a failure occurs gives y = 3.04 - .025x, which predicts an average of 2.26 failures at x=31 degrees. When x = -37 degrees F, the average number of failures is 4.

For part (b), the linear regression using all the data points gives an average failure of y=4.675 - .06x, predicting an average of 2.78 failures when x=31 degrees. When x=11 degrees F, there will be an average of 4 failures.

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4. Question: Find the values a0, a1, b1 for which a0 + a1 cos(x) + b1 sin(x) best fits the given data points (see data below)

In[282]:=
```
(* One Method *)
data2 = {{-3, -1.5}, {-2, -1}, {-1, -.5}, {0, 0}, {1, .5}, {2, 1}, {3, 1.5}};
MatrixForm[data2]
```

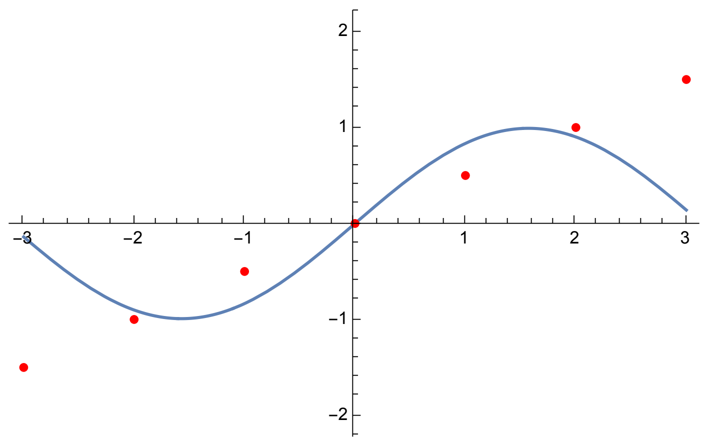
Out[283]//MatrixForm=

$$\begin{pmatrix} -3 & -1.5 \\ -2 & -1 \\ -1 & -0.5 \\ 0 & 0 \\ 1 & 0.5 \\ 2 & 1 \\ 3 & 1.5 \end{pmatrix}$$

In[284]:=
```
trig = Fit[data2, {1, Cos[x], Sin[x]}, x]
Show[Plot[trig, {x, -3, 3}],
 ListPlot[data2, PlotStyle → Red], PlotRange → {-2, 2}]
```

Out[284]= $-1.58494\times10^{-16} + 1.75954\times10^{-18}\,\mathrm{Cos}[x] + 0.991576\,\mathrm{Sin}[x]$

Out[285]=



In[286]:=
```
(* Direct Method *)
a = {{1, Cos[-3], Sin[-3]},
   {1, Cos[-2], Sin[-2]}, {1, Cos[-1], Sin[-1]}, {1, Cos[0], Sin[0]},
   {1, Cos[1], Sin[1]}, {1, Cos[2], Sin[2]}, {1, Cos[3], Sin[3]}};
b = {-1.5, -1, -.5, 0, .5, 1, 1.5};
MatrixForm[a]
MatrixForm[b]
MatrixForm[Inverse[Transpose[a].a].Transpose[a].b]
```

Out[288]//MatrixForm=

$$\begin{pmatrix} 1 & \mathrm{Cos}[3] & -\mathrm{Sin}[3] \\ 1 & \mathrm{Cos}[2] & -\mathrm{Sin}[2] \\ 1 & \mathrm{Cos}[1] & -\mathrm{Sin}[1] \\ 1 & 1 & 0 \\ 1 & \mathrm{Cos}[1] & \mathrm{Sin}[1] \\ 1 & \mathrm{Cos}[2] & \mathrm{Sin}[2] \\ 1 & \mathrm{Cos}[3] & \mathrm{Sin}[3] \end{pmatrix}$$

Out[289]//MatrixForm=

$$\begin{pmatrix} -1.5 \\ -1 \\ -0.5 \\ 0 \\ 0.5 \\ 1 \\ 1.5 \end{pmatrix}$$

Out[290]//MatrixForm=

$$\begin{pmatrix} 0. \\ 0. \\ 0.991576 \end{pmatrix}$$

The best fit trig function is, up to rounding of very small numbers, .99 sin(x).

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5. (See sheet for problem)

In[291]:=
```
a = Table[{1, Cos[x], Sin[x]}, {x, -3.1, 3.1, .1}];
b = Table[x / 2, {x, -3.1, 3.1, .1}];
MatrixForm[Inverse[Transpose[a].a].Transpose[a].b]
```

Out[293]//MatrixForm=

$$\begin{pmatrix} 1.07553 \times 10^{-16} \\ -2.35922 \times 10^{-16} \\ 1.00039 \end{pmatrix}$$

In[294]:=
```
data = Table[{x, x / 2}, {x, -3.1, 3.1, .1}];
Fit[data, {1, Cos[x], Sin[x]}, x]
```

Out[295]= $6.72756 \times 10^{-17} - 1.18468 \times 10^{-16}\,\mathrm{Cos}[x] + 1.00039\,\mathrm{Sin}[x]$

5a,b. Up to very small rounding, we get the function 1.00039 sin(x), which was very close to .991sin(x) from #4. Note this is very close to the Fourier Coefficients a0 = 0, a1 = 0, b1 = 1 for f(x) = x/2.

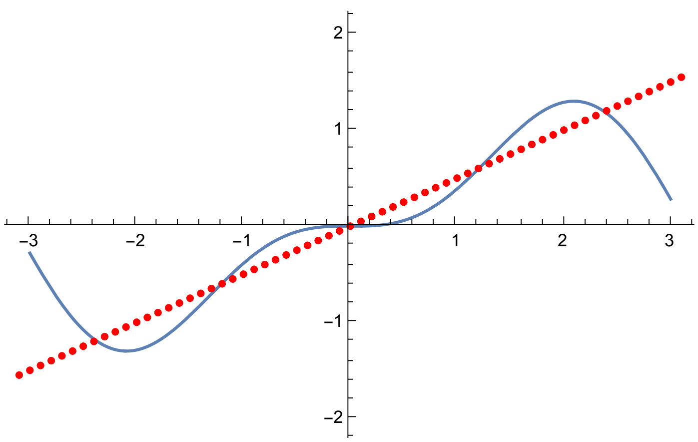
In[296]:=
```
(*Part c, done two different ways *)
a = Table[{1, Cos[x], Sin[x], Cos[2 x], Sin[2 x]}, {x, -3.1, 3.1, .1}];
b = Table[x / 2, {x, -3.1, 3.1, .1}];
MatrixForm[Inverse[Transpose[a].a].Transpose[a].b]

data = Table[{x, x / 2}, {x, -3.1, 3.1, .1}];
trig = Fit[data, {1, Cos[x], Sin[x], Cos[2 x], Sin[2 x]}, x]

Show[Plot[trig, {x, -3, 3}], ListPlot[data, PlotStyle → Red], PlotRange → {-2, 2}]
```

Out[298]//MatrixForm=

$$\begin{pmatrix} 1.11022\times10^{-16} \\ -1.31839\times10^{-16} \\ 1.00039 \\ 1.52656\times10^{-16} \\ -0.500785 \end{pmatrix}$$

Out[300]= $6.57691\times10^{-17} - 2.58046\times10^{-16}\,\mathrm{Cos}[x] - 1.69136\times10^{-17}\,\mathrm{Cos}[2\,x] + 1.00039\,\mathrm{Sin}[x] - 0.500785\,\mathrm{Sin}[2\,x]$

Out[301]=



The resulting trigonometric polynomial is, up to a few decimal places, Sin(x) - .5 Sin(2x), which is the second-order Fourier approximation.