

The following is a function SymbolicRR that makes row reduction more compatible with symbolic algebra.

Author: Corbett Redden (corbett.redden@liu.edu)

```
SymbolicRR[matrix_, symbols_ : {}] :=
  FixedPoint[Function[mtrx, Simplify[RowReduce[mtrx,
    ZeroTest → (Apply[Or, Table[!FreeQ[#, symb], {symb, symbols}]] ||
    PossibleZeroQ[#] &)]]], matrix, 10]
```

SymbolicRR::usage =

"SymbolicRR[matrix, {var₁, var₂, ...}] row reduces a matrix without
assuming expressions involving variables are non-zero.";

In ordinary RowReduce, *Mathematica* will implicitly assume all expressions involving variables are invertible. While we get the “desired answer” in this first case, ...

```
RowReduce[ $\begin{pmatrix} 1 & 1 & 3 & x \\ 2 & 0 & 2 & y \\ 1 & 1 & 5 & z \end{pmatrix}$ ] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{2}(x+y-z) \\ 0 & 1 & 0 & \frac{1}{2}(4x-y-2z) \\ 0 & 0 & 1 & \frac{1}{2}(-x+z) \end{pmatrix}$$

... in this next case RowReduce assumes that x+y-z is invertible.

```
RowReduce[ $\begin{pmatrix} 1 & 1 & 3 & x \\ 2 & 0 & 2 & y \\ 3 & 1 & 5 & z \end{pmatrix}$ ] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

To correct this, SymbolicRR gives the type of answer one would arrive at when doing row reduction by hand.

```
SymbolicRR[ $\begin{pmatrix} 1 & 1 & 3 & x \\ 2 & 0 & 2 & y \\ 3 & 1 & 5 & z \end{pmatrix}$ , {x, y, z}] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 1 & \frac{y}{2} \\ 0 & 1 & 2 & x - \frac{y}{2} \\ 0 & 0 & 0 & -4(x+y-z) \end{pmatrix}$$

Note the importance of including the correct variables.

```
SymbolicRR[ $\begin{pmatrix} 1 & 1 & 3 & x \\ 2 & 0 & 2 & y \\ 3 & 1 & 5 & z \end{pmatrix}$ , {a, b, c}] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The variables are optional, so SymbolicRR can be used in place of RowReduce.

```
SymbolicRR[ $\begin{pmatrix} 3 & 5 & 1 \\ -2 & 3 & 8 \\ 3 & 1 & 9 \end{pmatrix}$ ] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

SymbolicRR will attempt to make simplifications such as $\sin^2(x) + \cos^2(x) = 1$ and continue with row reduction.

```
SymbolicRR[{{1, 1, 2, Sin[x]^2}, {1, 1, 2, -Cos[x]^2}, {7, 8, 9, z}}, {x, y, z}] //  
MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 7 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This process of simplifying and continuing with row reduction will only occur 10 times, though that can be easily changed. This is to prevent *Mathematica* from entering an infinite loop when an incorrect matrix is inserted.

```

a = {{3, 4}, {5, 6}};
SymbolicRR[a, {x, y, z}] // MatrixForm
SymbolicRR[A, {x, y, z}] // MatrixForm

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

RowReduce[RowReduce[RowReduce[RowReduce[
  RowReduce[RowReduce[RowReduce[RowReduce[A, ZeroTest →
    (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] || PossibleZeroQ[#1] &)],
    ZeroTest → (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] ||
      PossibleZeroQ[#1] &)], ZeroTest →
    (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] || PossibleZeroQ[#1] &)],
    ZeroTest → (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] ||
      PossibleZeroQ[#1] &)], ZeroTest →
    (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] || PossibleZeroQ[#1] &)],
    ZeroTest → (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] ||
      PossibleZeroQ[#1] &)], ZeroTest →
    (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] || PossibleZeroQ[#1] &)],
    ZeroTest → (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] ||
      PossibleZeroQ[#1] &)], ZeroTest →
    (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] || PossibleZeroQ[#1] &)],
    ZeroTest → (Or @@ Table[! FreeQ[#1, symb], {symb, {x, y, z}}] ||
      PossibleZeroQ[#1] &)]

```

Documentation can be viewed using...

? SymbolicRR

SymbolicRR[matrix, {var₁, var₂, ...}] row reduces a matrix without assuming expressions involving variables are non-zero.