Find ~~Closed~~ Closed form of

$$T(n) = \begin{cases} T(n-1)+n & n>1 \\ 5 & n=1 \end{cases}$$

$T(n) = \underbrace{T(n-1)}+n$

$T(n-1) = T(n-1-1)+(n-1)$

$T(n) = \underbrace{T(n-2)}+(n-1)+n$

$T(n-2) = T(n-2-1)+(n-2)$

$T(n) = T(n-3)+(n-2)+(n-1)+n$

$n-1 = k$

GF: $T(n) = T(1) + \sum_{k=0}^{n-2} (n-k)$

given

$\Rightarrow 1 + \frac{n^2+n-2}{2}$

5

$$\sum_{k=0}^{n-2} (n-k) = \frac{n^2+n-2}{2}$$

Conjecture: ~~T~~he RR $T(n) = \begin{cases} T(n-1)+n & n \geq \\ 5 & n = 1 \end{cases}$

is equivalent to $T(n) = 5 + \frac{n^2+n-2}{2}$

Proof (induction):

Basis Step: $T(1) = 5$ by RR

$T(1) = 5 + \frac{1^2+1-2}{2} = 5 + \frac{0}{2} = 5$ by CF

The basis step holds $5 = 5$

Inductive Step:

IH: If the RR $T(K)=\begin{cases} T(K-1)+K & K>1 \\ 5 & K=1 \end{cases}$

is equivalent to $T(K)=5+\frac{K^2+K-2}{2}$

then $T(K+1)=5+\frac{(K+1)^2+K+1-2}{2}=5+\frac{K^2+3K}{2}$

math

Assume IH:

$T(K+1)=T(K+1-1)+K+1$

$T(K+1)=T(K)+K+1$

$=5+\frac{K^2+K-2}{2}+K+1$

$=5+\frac{K^2+K-2}{2}+\frac{2K+2}{2}$

$\frac{3K+K^2}{2}+5$

Ind. step holds

Because the Basis Step, Inductive Hypothesis and Inductive Step all hold, the Conjecture is

TRUEEEEEEE by Induction.

QED

Conjecture:

$$\sum_{k=0}^{n-2}(n-k) = \frac{n^2+n-2}{2}, \quad n>1$$

proof (Induction)

Basis Step: $n=2$

$$\sum_{k=0}^{n-2}(2-k) = 2.$$

$$\frac{2^2+2-2}{2}$$

$$\frac{4+2-2}{2}$$

$$\hookrightarrow \frac{6-2}{2}$$

$$\hookrightarrow \frac{4}{2} = 2$$

$2=2$ Basis Step holds.