$$T(n) = \begin{cases} T(n-1) + n & n > 1 \\ 7 & n = 1 \end{cases}$$

find closed form and prove

$$T(n) = T(n-1) + n$$

$$T(n-1) = T(n-2) + (n-1)$$

$$T(n) = T(n-2) + (n-1) + n$$

$$T(n-2) = T(n-3) + (n-2)$$

$$T(n) = T(n-3) + (n-2) + (n-1) + n$$

$$\sum_{i=1}^{n} = \frac{n(n+1)}{2}$$

GF: $7 + \frac{n(n+1)}{2}$

$T(n) = \begin{cases} 7T(\frac{n}{4}) + 9n \\ 7 \end{cases}$

Prove Big-O using Master Theorem

$a = 7$

$b = 4$

$c = 9$

$d = 1$

$a \quad b^d$

$7 > 4$

$O(n^d) \quad a < b^d$

$O(n^d \log_2 n) \quad a = b^d$

$O(n^{\log_b a}) \quad a > b^d$

$O(n^{\log_4 7})$

$4^x = 7$

$O(n^{\log_4 7})$

$$T(n) = \begin{cases} T(n-1) + 4 & n > 1 \\ 3 & n = 1 \end{cases}$$

$T(n) = T(n-1) + 4$

$T(n-1) = T(n-2) + 4$

$T(n) = T(n-2) + 8$

$T(n-2) = T(n-3) + 4$

$T(n) = T(n-3) + 12$

GF: $T(n) = T(n-x) + 4x$

$n - x = 1$
$-n \qquad -n$

$-x = 1 - n$

$x = -1 + n$

or

$x = n - 1$

CF: $T(n) = T(n-(n-1)) + 4(n-$

$= T(1) + 4n - 4 = 4n - 1$

3

Conjecture: The Recurrance Relation

$$\begin{cases} T(n-1)+4 & n>1 \\ 3 & n=1 \end{cases}$$

is equivalent to $T(n)=4n-1$.

proof (Induction)

Basis Step $n=1$

by our RR, $T(1)=3$ by the closed form

$T(1)=4(1)-1=4-1=3$

$3=3$ ✓

Basis Step Holds.

Inductive Step: $k \in \mathbb{Z}^+$

IH: if our RR $T(k) = \begin{cases} T(k-1)+4 & k > 1 \\ 3 & k = 1 \end{cases}$

is equal to closed form $T(k) = 4k-1$. Then

$T(k+1) = 4 \cdot (k+1) - 1 = 4k+4-1 = 4k+3$

Assume $\underline{T(k) = 4k-1}$.

$T(k+1) = T(k+1-1)+4 = T(k)+4$

$= 4k-1+4$

$= 4k+3$

Which is equal to IH, Inductive Step holds

Since the base step holds, inductive step holds then the conjecture is true. QED

aRb if $a <= b$

Transitive? ✓

Symmetric? X

Antisymmetric? ✓

Reflexive? ✓

Equivalence Relation, buddy.

$$\log_b a + \log_b c = \log_b (a \cdot c)$$

$$\log_b a - \log_b c = \log_b \left(\frac{a}{c}\right)$$

$$\log_b a^c = c \cdot \log_b a$$

$$\log_b 1 = 0 \qquad \log_b b = 1$$

2020–03–17 03

Prove if x is even than x^2 is even

proof(direct):

Assume x is even, then

$x = 2k$, $k \in \mathbb{Z}^+$

Then,

$$x^2 = (2k)^2$$
$$= 4k^2$$
$$= 2(2k^2)$$

Because x^2 is in the form $2 \cdot$ int, and $2k^2$ is an integer, x^2 is even, Thus proving the conjecture.

QED

Pseudocode binary search
recursive ^ :)

```
search(arr, K) : int
  return search(arr, K, 0, arr.length-1)

search(arr, K, L, R)
if L is less than or equal to R
  mid <- (L+R)/2
  if arr[mid] is equal to K
    return mid
  if arr[mid] is less than K
    return search(arr, K, mid+1, R)
```

```
return search(arr, K, L, mid-1)

return -1
```