Assignment 4: HW – Trees – 20 pts
This week we examine tree-related data types.

Due Date
Monday 2020/02/24 at 4:00pm

Instructions
Questions below marked with an asterisk (*) are group problems, and are intended to be worked on as a group. You may speak to anyone in class about the problem, ask questions, or help your classmates. However, you must write up and submit your own solution to the problem. All other questions are intended as individual work. You may not discuss them with anyone other than the Instructor or the TA. This includes questioning what the problem means, or what is expected.

Problems

1. (2 pts.) Define the **degree** of a node as the number of of its non-empty children. Prove by induction that the number of degree 2 nodes in any binary tree is one less than the number of leaves.
2. *Show:
   a. (2 pt.) the BST that results from inserting the values 35, 40, 38, 6, 49, 17, 33, 32, 16, 50 (in that order). Show the state of the tree after the insertion of 38, 33, and 50.
   b. (2 pt.) your BST from a) above after removing the value 35.
   c. (2 pt.) the enumerations for the tree of a) above that result from doing a pre-order traversal, an in-order traversal, and a post-order traversal.
3. Show:
   a. (2 pts.) the Min-Heap that results from inserting the values 35, 40, 38, 6, 49, 17, 33, 32, 16, 50 (in that order). Show the state of the heap after the insertion of values 38, 33, and 50.
   b. (2 pt.) the Min-Heap in a) above after a single remove (getMin) operation.
4. *(3 pts.) Write a recursive function named **smallCount** that, given the pointer to the root of a BST and a key *K*, returns the number of nodes having key values less than or equal to *K*. Function **smallCount** should visit as few nodes in the BST as possible.
5. Huffman Encoding:
   a. (2 pts.) Build the Huffman encoding tree and determine the codes for the message "AARDVARKS ARE ARDENT ARTISTS"
   b. (1 pt.) How much space would the message require on disk if stored using 8-bit ASCII?
   c. (1 pt.) How much space would the message require on disk if the stored using the minimum number of bits possible (but where each character had the same number of bits)?
   d. (1 pt.) How much space is required to store the message on disk using it's Huffman Encoding from a) above (for simplicity, ignore the cost of storing the encoding itself).