

from assumption $= T(k+1) =$

$$\hookrightarrow 3 + 3k + 2 \leq \text{assumption about the closed form} =$$

$$\hookrightarrow T(k+1) = 3k + 5$$

\hookrightarrow equal to hypothesis \Rightarrow

Inductive Step holds! $(3k + 3 = 3k + 5) \Rightarrow$

Basis Step holds, IH holds, Inductive Step holds



QED

\nwarrow make that a sentence \Rightarrow

Reflexive ✓
Symmetric ✓
Antisymmetric ✓
Transitive ✓

$aRb \quad a=b$

Recurrence Relations

$$T(n) = \begin{cases} T(n-1) + 3 & n > 1 \\ 5 & n = 1 \end{cases}$$

$$T(n) = T(n-1) + 3$$

$$1 \quad T(n-1) = T(n-2) + 3$$

$$T(n) = T(n-3) + 3 + 3$$

$$T(n) = T(n-2) + 6$$

$$T(n) = T(n-3) + 9$$

$$T(n-2) = T(n-3) + 3$$

$$T(n) = T(n-x) + 3x$$

$$\text{General: } T(n) = T(n-x) + 3x$$

$$x = n-1 \quad n - (n+1)$$

$$T(n) = T(n-n+1) + 3(n-1)$$

$$T(n) = T(1) + 3n - 3$$

$$= 5 + 3n - 3$$

$$\text{Closed: } T(n) = 3n + 2$$

Conjecture: The Rec. Rel.

$$T(n) = \begin{cases} T(n-1) + 3 & n > 1 \\ 5 & n = 1 \end{cases}$$

is represented by the closed
form $T(n) = 3n + 2$

proof (induction):

Base Step: $T(1) = 5$, per R relat.

$$T(1) = 3(1) + 2$$

$$= 3 + 2$$

$$= 5, \text{ per c f}$$

$5 = 5$, so the Base Step holds.



Inductive Hypothesis:

If $T(n) = \begin{cases} T(n-1)+3 & n > 1 \\ 5 & n = 1 \end{cases}$ is equivalent to

the closed form $T(n) = 3n + 2$, then it can be

said that $T(k) = 3k + 2$ and $T(k+1) = 3(k+1) + 2$

$$\hookrightarrow 3k + 3 + 2$$

$$\hookrightarrow 3k + 5$$

Assume $T(k) = 3k + 2$ ^{rewritten relation}
Use Definition for $T(n)$ by ~~Definition~~

$$T(k+1) = 3 + T((k+1)-1)$$

$$\hookrightarrow T(k+1) = 3 + T(k)$$