Cordell Stocker

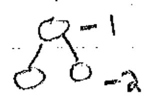
The height of a complete binary tree with n nodes is $\log_2(n+1)$

Conjecture: The height of a complete binary tree T with n nodes is $\log_2(n+1)$

Proof (induction)

Basis step:

Let a complete binary tree T with 3 nodes be given.



height = 2

Height $= \log_2(3+1) = \log_2 4$
$= 2$

$2 = 2$

Basis step holds

Inductive step

IH: Assume conjecture is true for heights $\leq k$. We want to show that a height of $k+1 = \log_2(n+1)$ for some $n \in \mathbb{Z}^+$.

h=2  3 nodes

h=3 

$1 + 2$ (height 2 trees)

$1 + 2(3) = 7$ nodes

$n = 1 + n_{left} + n_{right}$

$n = 1 + 2n_{left}$ (complete tree)

[Note: $k = \log_2(n_{left}+1)$ so $n_{left} = 2^k - 1$]

$n = 1 + 2(2^k - 1)$

$n = 2^{k+1} - 1$

$n + 1 = 2^{k+1}$

$\log_2(n+1) = k+1$

Inductive step holds

$k = \log_2(n_{left}+1)$
$2^k = n_{left}+1$

Because the basis and induction steps hold, the height of a complete binary tree with n nodes is equal to $\log_2(n+1)$ QED