

Reflexive ✓
Symmetric ✓
Antisymmetric ✓
Transitive ✓

aRb $a=b$

Recurrence Relations

$$T(n) = \begin{cases} T(n-1) + 3 & n \geq 1 \\ 5 & n = 1 \end{cases}$$

$$T(n) = T(n-1) + 3$$

$$T(n-1) = T(n-2) + 3$$

$$T(n) = T(n-3) + 3 + 3$$

$$T(n) = T(n-2) + 6$$

$$T(n) = T(n-3) + 9$$

$$T(n-2) = T(n-3) + 3$$

$$T(n) = T(n-x) + 3x$$

$$\text{General: } T(n) = T(n-x) + 3x$$

$$x = n-1 \quad x - (x+1)$$

$$T(n) = T(n-n+1) + 3(n-1)$$

$$T(n) = T(1) + 3n-3$$

$$= 5 + 3n - 3$$

$$\text{Closed: } T(n) = 3n + 2$$

Conjecture: The Rec. Rel.

$$T(n) = \begin{cases} T(n-1) + 3 & n > 1 \\ 5 & n = 1 \end{cases}$$

is represented by the closed form $T(n) = 3n + 2$

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Proof (induction):

Base Step:

$$T(1) = S, \text{ per Relat.}$$



$$T(1) = 3(1) + 2$$

$$= 3 + 2$$

$= S$, Perf

$S = S$, So the Base Step holds.

Inductive Hypothesis:

$$\text{If } T(n) = \begin{cases} T(n-1) + 3 & n > 1 \\ 5 & n = 1 \end{cases} \text{ is equivalent to}$$

the closed form $T(n) = 3n + 2$, then it can be

$$\text{Said that } T(k) = 3k + 2 \text{ and } T(k+1) = 3(k+1) + 2$$

$$\hookrightarrow 3k + 3 + 2$$

$$\hookrightarrow 3k + 5$$

Assume $T(k) = 3k + 2$

Recursively

Use Definition for $T(n)$ by ~~Definition~~

$$T(k+1) = 3 + T((k+1)-1)$$

$$\hookrightarrow T(k+1) = 3 + T(k)$$

from assumption $= P(k+1) =$

$\hookrightarrow 3 + 3k + 2 \leq$ assumption
about the closed
form $=$

$$\hookrightarrow T(k+1) = 3k + 5$$

\hookrightarrow equal to hypothesis $=$
Inductive Step holds! $(3k + 5 = 3k + 5) =$

Basis Step holds, IFF holds, Inductive Step holds
QED \Rightarrow make that a sentence $=$