Conjecture: given a ~~set~~ $S$ with $n$ elements, $|\wp(S)| = 2^n$

Proof (induction):

basis step:

let $S = \{a\}$; $\wp(S) = \{\{\}, \{a\}\}$

$|\wp(S)| = 2$

$|\wp(S)| = 2^1 = 2$

$2 = 2$

so basis step holds!

Inductive Step: I.H.

if Given a set $S$ of $n$ elements $|\mathcal{P}(S)| = 2^n$, then set $T$ with $n+1$ elements is $|\mathcal{P}(T)| = 2^{n+1}$

So, let $T = S \cup \{a\}$

$\underbrace{K \in \mathcal{P}(S),\ K \cup \{a\}}_{Z}$

$|\mathcal{P}(T)| = |\mathcal{P}(S)| + |Z|$

$= 2^n + 2^n$

$= 2^{n+1}$

$$T = S \cup \{a\} \qquad \mathcal{P}(T) = \{\{\}, \{a\}, \{b\}, \{ab\}\}$$

$$S = \{a\}. \qquad \mathcal{P}(S) = \{\{\}, \{a\}\}$$

$$\cup \{b\} = \underbrace{\{\{b\}, \{a,b\}\}}_{z}$$

$= 2^{n+1}$

Because $2^{n+1}$ was our expectation, the Basis Step, Inductive Step and Inductive Hypothesis all hold, our conjecture is true via Induction

Qed.

$$T(n) = \begin{cases} 3T(\frac{n}{2}) + n & n > 1 \\ 2 & n = 1 \end{cases}$$

FTP

$\frac{n}{2} \cdot \frac{1}{2} = \frac{n}{4}$

$$T(n) = 3T\left(\frac{n}{2}\right) + n$$

$$T\left(\frac{n}{2}\right) = 3T\left(\frac{n}{4}\right) + \frac{n}{2}$$

$$T(n) = 3\left(3T\left(\frac{n}{4}\right) + \frac{n}{2}\right) + n$$

$$= 9T\left(\frac{n}{4}\right) + \frac{3n}{2} + n$$

$$T\left(\frac{n}{4}\right) = 3T\left(\frac{n}{8}\right) + \frac{n}{4}$$

$$T(n) = 9\left(3T\left(\frac{n}{8}\right) + \frac{n}{4}\right) + \frac{5n}{2}$$

$$= 27T\left(\frac{n}{8}\right) + \frac{19n}{4}$$

We need Master Theorem! ;)

$$T(n) = 3T\left(\frac{n}{2}\right) + n$$

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + cn^d$$

$a \; ? \; b^d$

$3 > 2^1$

$a = 3$

$b = 2$

$c = 1$

$d = 1$

$$T(n) = \begin{cases} O(n^d) & a < b^d \\ O(n^d \log_2 n) & a = b^d \\ \underline{O(n^{\log_b a})} & \underline{a > b^d} \end{cases}$$

$n^{\log_b a}$

$$O\left(n^{\log_2 3}\right)$$