Introduction to C++ Programming
Part 2: Data representation

Course: CPSC 1050

Instructor: Dr Bijan Farrahi

Lecture 2

Learning Outcomes

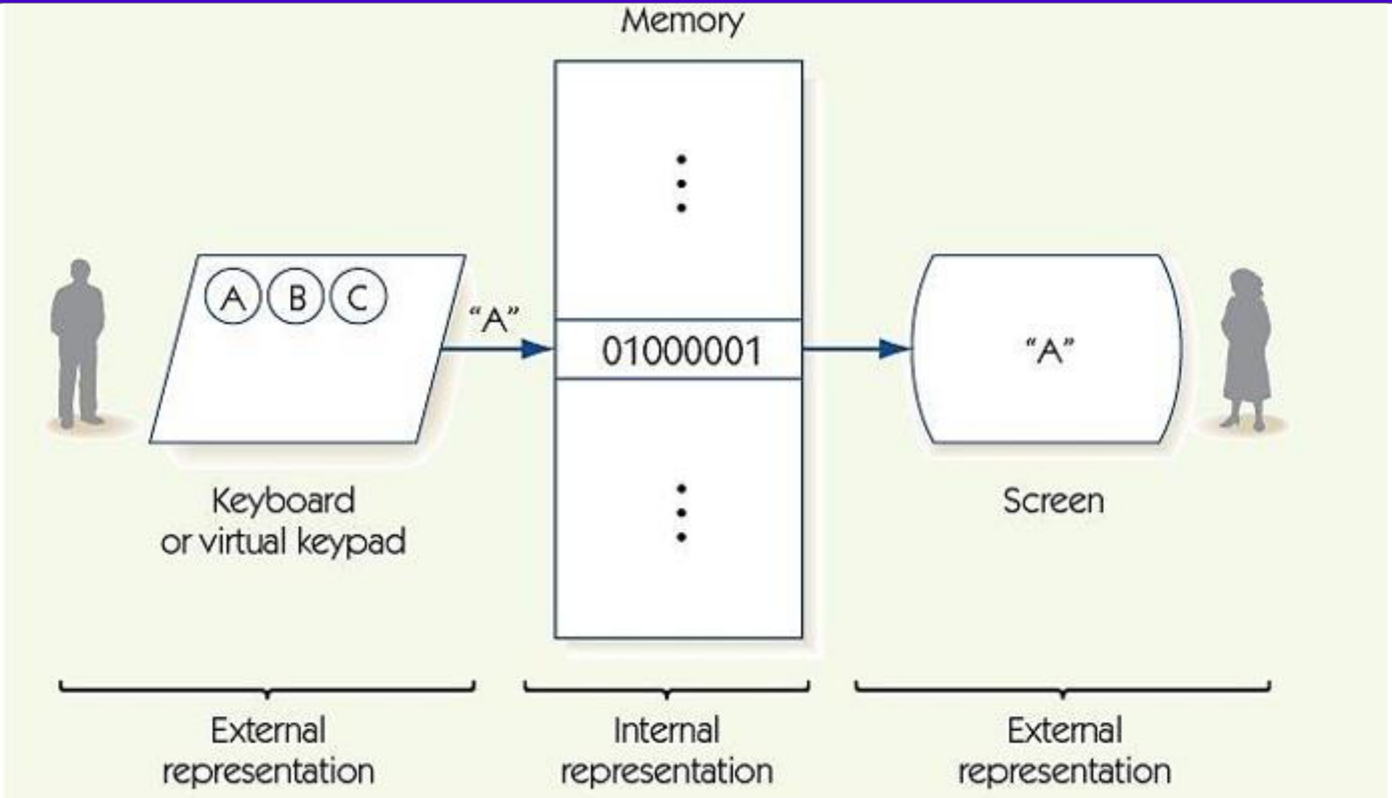
– convert numbers in different bases

– Identify how encoding works in computer

Data and Computers

- In the past, computers deal almost exclusively with **numeric** and **textual** data (a value)
- Today computers are really **multimedia** devices, dealing with a vast array of information categories such as:
  - Numbers
  - Text
  - Audio
  - Video
  - Images and graphics

Decimal Versus Binary



Distinction between external and internal representation of information

External representation is human oriented

Internal representation is computer oriented

Numbers

– Integers

Positive and negative numbers:

249, 0, - 45645, - 32

– Rational

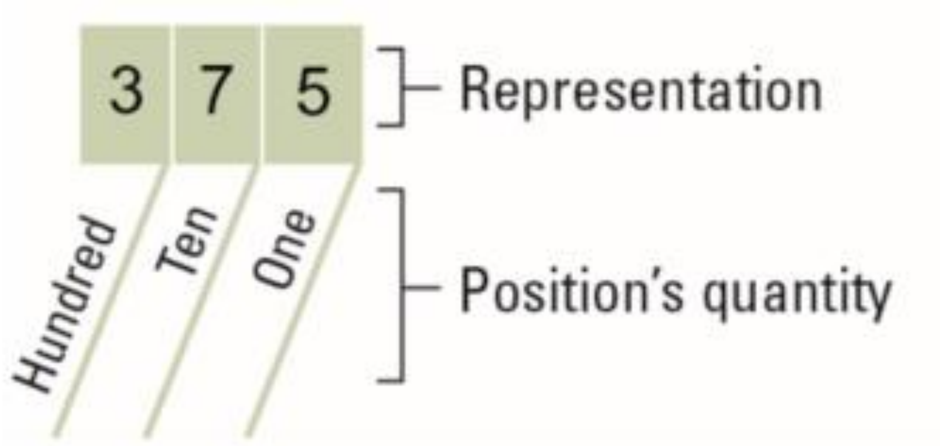
A number that can be made by dividing two integers:

- ½ or (0.5) is a rational number (1 divided by 2, or the ratio of 1 to 2)
- 0.75 is a rational number (3 / 4)
- −6.6 is a negative rational number (−66 / 10)

Positional Notation

375 is 300 + 70 + 5 in Base 10

(each digit has its own dedicated value!)



The **Base** of a number determines both:

– The number of different digit symbols

– The dedicated values of digit positions

Positional Notation

Positional notation of (642) in Base 10:

$6 \times 10^2 = 6 \times 100 = 600$

$+ 4 \times 10^1 = 4 \times 10 = 40$

$+ 2 \times 10^0 = 2 \times 1 = 2 \quad = 642$ in base 10

This number is in base 10

The power indicates the position of the number

Positional Notation

- *What if 642 has the base of 13?*

$$\begin{aligned} & 6 \times 13^2 \quad (6 \times 169 = 1014) \\ + & \; 4 \times 13^1 \quad (4 \times 13 = 52) \\ + & \; 2 \times 13^0 \quad (2 \times 1 = \;\; 2) \\ = & \;\; 1068 \text{ in base } 10 \end{aligned}$$

- 642 in base 13 is equivalent to 1068 in base 10

- *What if 642 has the base of 8 (Octal)?*

Binary System

- Decimal is base 10 and has 10 digit symbols:

  0,1,2,3,4,5,6,7,8,9

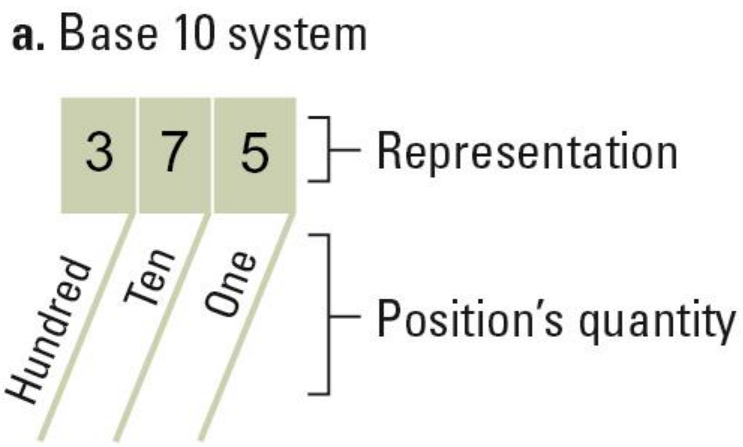
- Binary is base 2 and has 2 digit symbols:

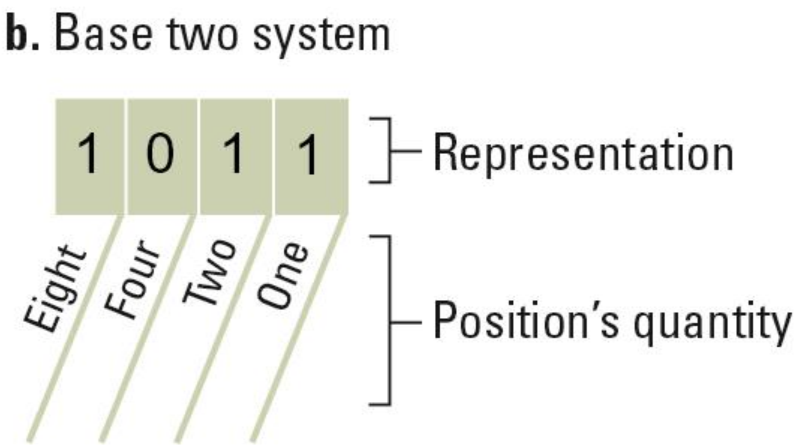
  0,1

- Why binary system is so important?

- Using **binary** instead of decimal, simplifies the design of computers (**far less expensive** and **far more reliable)**
- As only one of two values (patterns) is shown

The Base 10 and Binary System





Bases Higher than 10

- How are digits in bases higher than 10 represented?

  - Base 16 (Hexadecimal) has 16 digits:

    0,1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F

Number Systems

3 main conversion categories are:

1. Convert numbers in other bases to base 10: positional notation
2. Describe the relationship between bases 2, 8, and 16
3. Convert base 10 numbers to numbers in other bases (2, 8, 16): successive divisions

Converting Octal to Decimal

– *What is the **decimal** equivalent of the **octal** number 642?*

6×8^2 $(6 \times 64 = 384)$
$+ 4 \times 8^1$ $(4 \times 8 = 32)$
$+ 2 \times 8^0$ $(2 \times 1 = 2)$
$= 418$ in base 10

Converting Hexadecimal to Decimal

– *What is the* ***decimal*** *equivalent of the* ***hexadecimal*** *number DEF?*

$$
\begin{aligned}
& D \times 16^2 \quad (13 \times 256 = 3328) \\
+ & E \times 16^1 \quad (14 \times 16 = 224) \\
+ & F \times 16^0 \quad (15 \times 1 = 15) \\
= & \; 3567 \text{ in base 10}
\end{aligned}
$$

– Remember, the digit symbols in base 16 are:

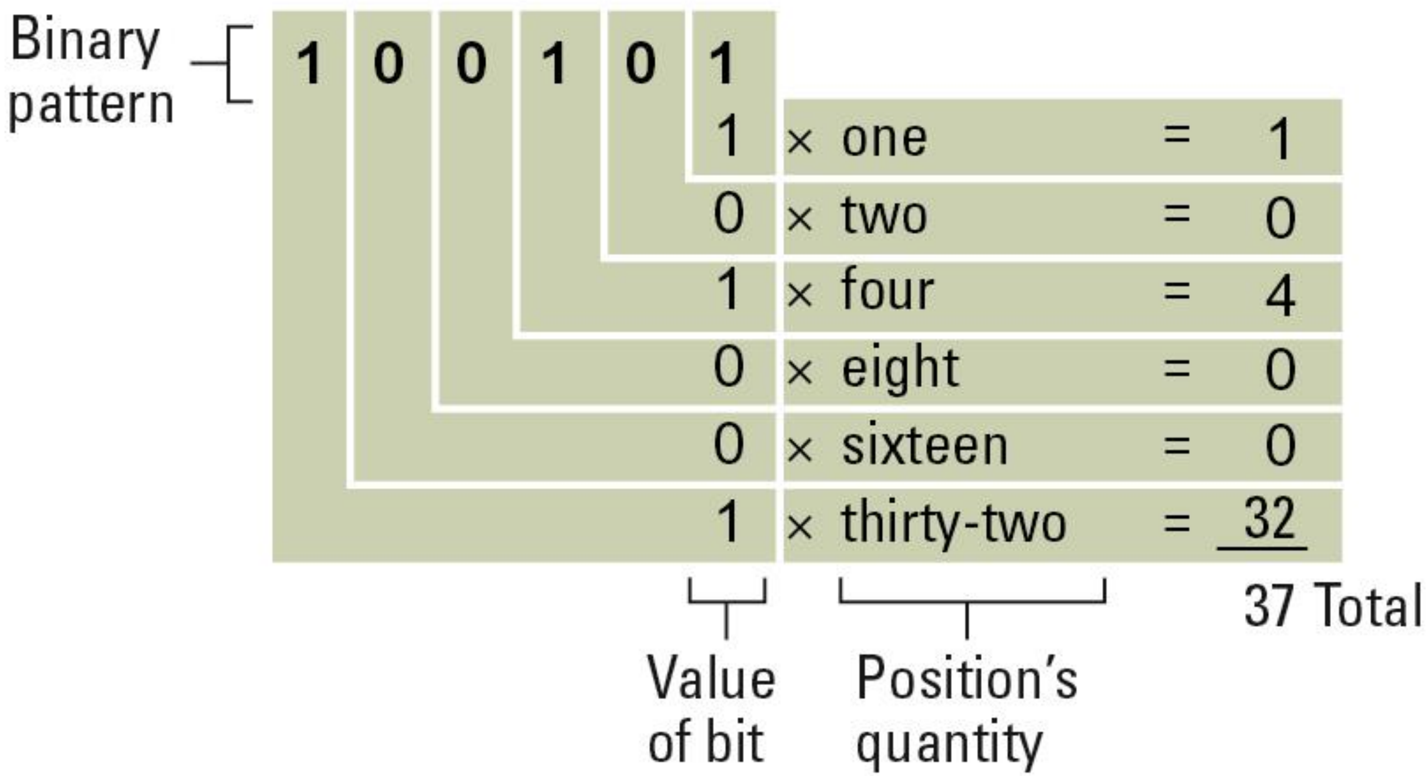
0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

| ..... | | | | | |

0, ... 10,11,12, 13, 14,15

Converting Binary to Decimal

- *What is the decimal equivalent of the binary number 100101?*



Checkpoint

- Is $136_{(7)}$ a valid number?
- Is $136_{(6)}$ a valid number?
- What is the maximum digit in octal system (base 8)?
- What is the value of $1011001_{(2)}$ in decimal system?
- What is the value of $136_{(8)}$ in decimal system?
- What is the value of $136_{(13)}$ in decimal system?

Converting Decimal to Other Bases

- An **algorithm** is a **sequence of steps** that solve a problem

- Algorithm for converting a number in base 10 to other bases:

  While (the quotient is not zero)

  1) Divide the decimal number by the new base
  2) Make the remainder the next digit to the left in the answer
  3) Replace the original decimal number with the quotient

Converting Decimal 13 to Binary

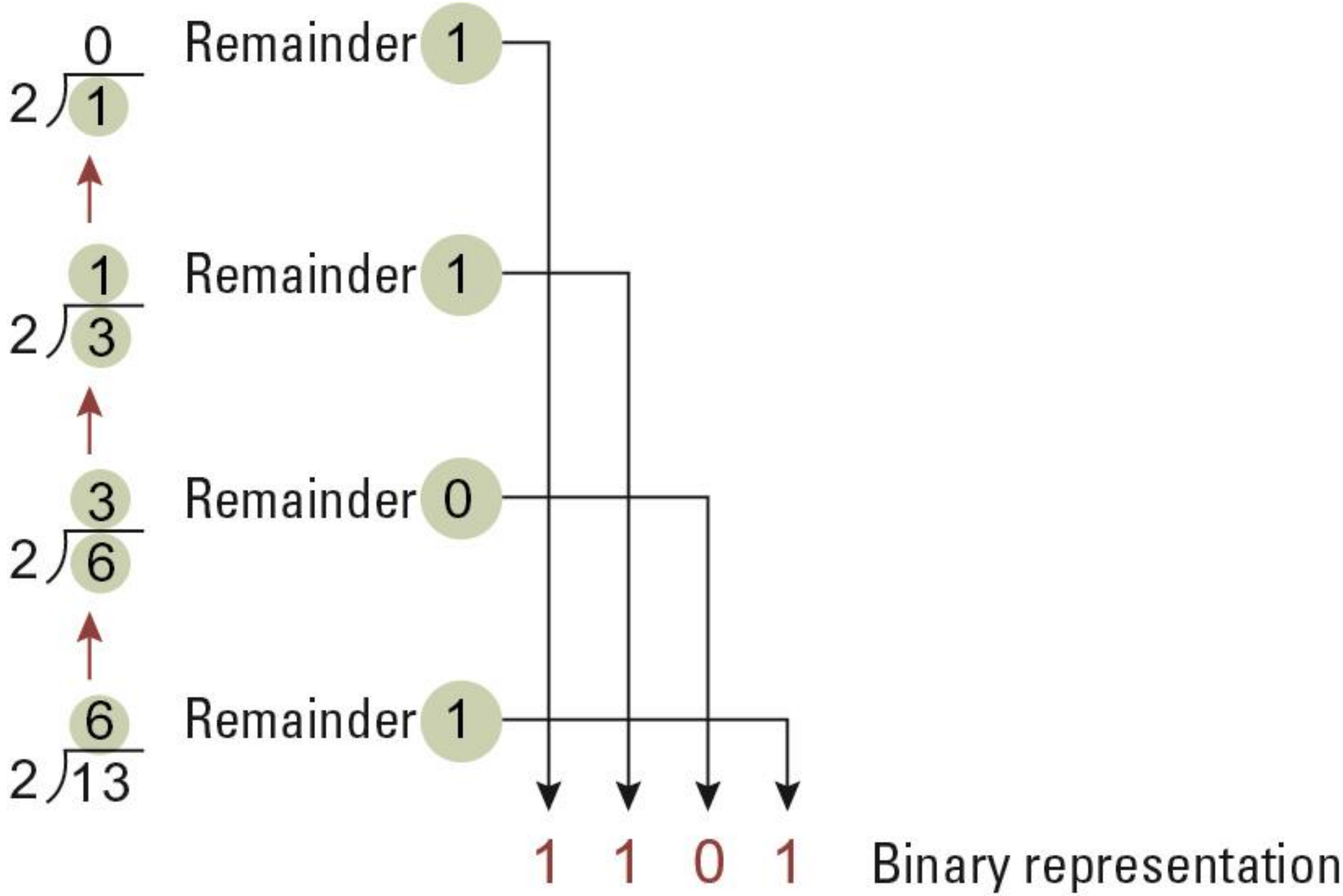
13 ÷ 2: quotient = 6 remainder 1
6 ÷ 2 : quotient = 3 remainder 0
3 ÷ 2 : quotient = 1 remainder 1
1 ÷ 2 : quotient = 0 remainder 1

order for reading the remainder digits

Stop, because the quotient is now zero

Answer: 1 1 0 1

Converting Decimal 13 to Binary



Converting Decimal to Binary - intuitive approach

– We need to decompose and represent 29 in terms of possible powers of 2

Power of 2	16	8	4	2	1
29	1	1	1	0	1
Remaider	13	5	1		0

$$29 = 16 + 8 + 4 + 0 + 1$$

$(29)_{10}$ is $(11101)_2$

Converting Decimal 93 to Octal

93 ÷ 8 quotient 11 remainder 5
11 ÷ 8 quotient 1 remainder 3
1 ÷ 8 quotient 0 remainder 1

order for reading the remainder digits

Answer: 1 3 5

Converting Decimal 93 to Hexadecimal

$93 \div 16$ quotient 5, remainder 13
$5 \div 16$ quotient 0, remainder 5

↑ order for reading the remainder digits

Answer: $5D_{(16)}$

Hint: 13 in base 16 is replaced by D

checkpoint

- What is the octal value of 984?
- What is the binary value of 984?
- What is the hexadecimal (hex) value of 984?

Converting Binary to Octal

- When it comes to conversion from Binary to Octal two steps are necessary:

Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

1) Mark groups of *three* (from right)
2) Convert each group and replace it with a corresponding octal number

10101011 → 010 101 011

2 5 3

- Add leading zeros until you have a multiple of three bits

Converting Binary to Hexadecimal

Hex	Binary			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
A	1	0	1	0
B	1	0	1	1
C	1	1	0	0
D	1	1	0	1
E	1	1	1	0
F	1	1	1	1

1) Mark groups of *four* (from right)
2) Convert each group and replace it with a corresponding hexadecimal number

10101011 → <u>1010</u> <u>1011</u>

A B

10101011 in base 2 is AB in base 16 (no leading zeros)

Conversion Between Two Bases

Exercise

Convert the following binary numbers to octal and hexadecimal:

- 1000
- 110
- 10110011
- 1101110

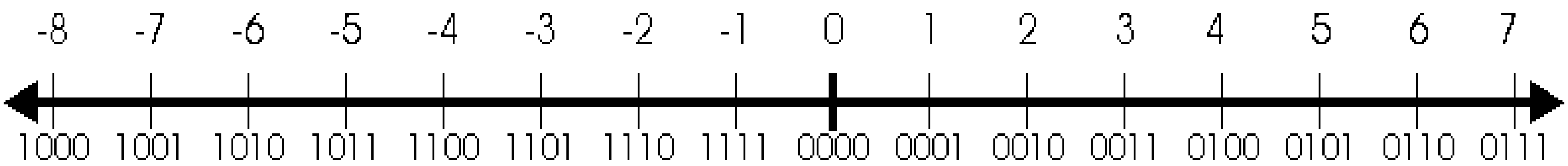
Exercise

Convert the following numbers to binary:

- $[137]_8$
- $[4E]_{16}$
- $[52]_8$
- $[A00]_{16}$

Negative Numbers : 2's Complement

– Uses all bit patterns efficiently



- Given n = 4, the range of numbers: -8 to 7
- With this notation, we can represent the negative numbers from -8 to -1, whereas with signed magnitude we could only represent -7 to -1 (do not lose any pattern here)

– Quick identification of negative numbers

- All negative numbers have the leftmost bit set to '1'

How to Calculate 2's Complement

- Take the positive representation, flip the bits and add 1.

Negative	Positive	Flip	Add 1
-7	0111	1000	1001
-3	0011	1100	1101

- What about **subtraction**? Does it work properly?

4 -7 would be: 4 + (-7) = -3

0100 (+4)
1001 (-7)

1101 (-3)

How do I know this is -3?

Again, flip and add 1

Number Overflow

- With **4 bits** we could represent **- 8 to +7** (slide 15)
- Using **8 bits** (represents numbers in range **-128** to **+127**)
- Lets sum **127 and 3**

```
 1111111      carry
 01111111     +127
 00000011     +3
 --------
 10000010
```

Sign bit

It is -126

number overflow

What number is it?

Storing Characters

- Data stored in computer must be stored as binary number
- Characters are converted to numeric code, numeric code stored in memory
  - Most important coding scheme is ASCII
    - ASCII is limited: defines codes for only 128 characters
  - Unicode coding scheme becoming standard
    - Uses 4 bytes to represent each character
    - Compatible with ASCII
    - Can represent characters for other languages

ASCII Character Set

<table>
<tr><th>Left Digit(s)</th><th>Right Digit</th><th colspan="10">ASCII</th></tr>
<tr><th></th><th></th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th></tr>
<tr><td>0</td><td></td><td>NUL</td><td>SOH</td><td>STX</td><td>ETX</td><td>EOT</td><td>ENQ</td><td>ACK</td><td>BEL</td><td>BS</td><td>HT</td></tr>
<tr><td>1</td><td></td><td>LF</td><td>VT</td><td>FF</td><td>CR</td><td>SO</td><td>SI</td><td>DLE</td><td>DC1</td><td>DC2</td><td>DC3</td></tr>
<tr><td>2</td><td></td><td>DC4</td><td>NAK</td><td>SYN</td><td>ETB</td><td>CAN</td><td>EM</td><td>SUB</td><td>ESC</td><td>FS</td><td>GS</td></tr>
<tr><td>3</td><td></td><td>RS</td><td>US</td><td>□</td><td>!</td><td>“</td><td>#</td><td>$</td><td>%</td><td>&</td><td>’</td></tr>
<tr><td>4</td><td></td><td>(</td><td>)</td><td>*</td><td>+</td><td>,</td><td>-</td><td>.</td><td>/</td><td>0</td><td>1</td></tr>
<tr><td>5</td><td></td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>:</td><td>;</td></tr>
<tr><td>6</td><td></td><td><</td><td>=</td><td>></td><td>?</td><td>@</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>7</td><td></td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td><td>K</td><td>L</td><td>M</td><td>N</td><td>O</td></tr>
<tr><td>8</td><td></td><td>P</td><td>Q</td><td>R</td><td>S</td><td>T</td><td>U</td><td>V</td><td>W</td><td>X</td><td>Y</td></tr>
<tr><td>9</td><td></td><td>Z</td><td>[</td><td>\</td><td>]</td><td>^</td><td>_</td><td>`</td><td>a</td><td>b</td><td>c</td></tr>
<tr><td>10</td><td></td><td>d</td><td>e</td><td>f</td><td>g</td><td>h</td><td>i</td><td>j</td><td>k</td><td>l</td><td>m</td></tr>
<tr><td>11</td><td></td><td>n</td><td>o</td><td>p</td><td>q</td><td>r</td><td>s</td><td>t</td><td>u</td><td>v</td><td>w</td></tr>
<tr><td>12</td><td></td><td>x</td><td>y</td><td>z</td><td>{</td><td>|</td><td>}</td><td>~</td><td>DEL</td><td></td><td></td></tr>
</table>

CR (Carriage Return) is a **control *character* or mechanism** used to reset a device's position to the beginning of a line of text

Additional Resources

- To Read!
  - Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond – Varsity Tutors
  - Computer - Number Systems – tutorialspoint
  - Number Systems: An Introduction to Binary, Hexadecimal, and More – tutsplus

This is a good one!

- To Watch!
  - Introduction to number systems and binary – Khan Academy