The Documentation of Graph Track in Core Challenge

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1 Graph Structure

Consider a piece consisting of 5 vertices and 2 tokens (Fig. 1). This piece can switch between 0-state and 1-state via transition-state as shown in Figure 2.
Here, if vertex 1 is blocked by an external token in the 0-state or 1-state state, this piece will not be able to switch states (Fig. 3). By preparing piece-1 and piece-2 and connecting them with edges as shown in Figure 4, it is possible to make the possibility of transition of piece-2 depend on piece-1.
Prepare n pieces piece-1, piece-2 ... piece-n. For $i(2 \leq i \leq n)$, stretch the edges so that piece-i is transitive only when piece-$i-1$ is 1-state and piece-$j(1 \leq j \leq i-2)$ is 0-state. The initial token placement state is such that each piece is 0-state. The target token placement state is set so that only piece-n is 1-state and all other pieces are 0-state.
Suppose there are m pieces, and the state of each piece is represented by an 01-string of length m. In the transition from the initial state to the target state, this 01-string changes by one character, and all 2^m possible 01-strings are transitioned as in the Gray code.

2 Reconfiguration Sequence Length

Each piece requires 3 transitions to change its state. There are $|V|/5$ pieces in the graph, therefore the reconfiguration sequence length is $3 \times (2^{|V|/5} - 1)$.

10 vertices: 2 pieces, so the length is $3 \times (2^2 - 1) = 9$.

50 vertices: 10 pieces, so the length is $3 \times (2^{10} - 1) = 3069$.

100 vertices: 20 pieces, so the length is $3 \times (2^{20} - 1) = 3145725$.
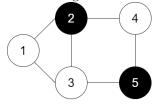
Fig. 1: A piece consisting of 5 vertices and 2 tokens

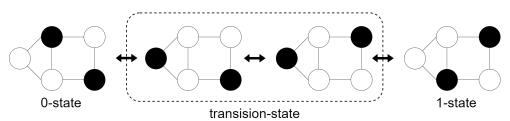Fig. 2: Transition of a piece

0-state   transision-state   1-state

Fig. 3: Blocking transitions by external tokens

0-state   transision-state   1-state

Fig. 4: Blocking transitions depending on the state of the external piece

only if piece-1 is 0-state,
piece-2 can change its state

piece-1 → piece-2
is 0, then

only if piece-1 is 1-state,
piece-2 can change its state

piece-1 → piece-2
is 1, then

Fig. 5: Connection of each piece

piece-0 | is 1 | piece-1 | is 1 | piece-2 | is 1 | piece-3 | is 1 | ... | is 1 | piece-$n-1$ | is 1 | piece-n

is 0

is 0

is 0

is 0