1.

(a) $\vec{a} = (5, -8, -6)$, $\vec{b} = (10, 0, -5)$

$\vec{a}\cdot\vec{b} = 10\times5 + (-8)\times0 + (-6)\times(-5) = 80$

$|\vec{a}| = \sqrt{5^2+(-8)^2+(-6)^2} = 5\sqrt{5}$

$|\vec{b}| = \sqrt{10^2+0^2+(-5)^2} = 5\sqrt{5}$

$\therefore \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|} = \dfrac{80}{125} = \dfrac{16}{25}$

(b) $\vec{a} = (7, 5, -5, -1)$, $\vec{b} = (-1, 7, 3, 4)$

$\vec{a}\cdot\vec{b} = 7\times(-1) + 5\times7 + (-5)\times3 + (-1)\times4 = 9$

$|\vec{a}| = \sqrt{7^2+5^2+(-5)^2+(-1)^2} = \sqrt{100} = 10$

$|\vec{b}| = \sqrt{(-1)^2+7^2+3^2+4^2} = 5\sqrt{3}$

$\therefore \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|} = \dfrac{9}{10\times5\sqrt{3}} = \dfrac{3\sqrt{3}}{50}$

(c) $\vec{a} = (-4, -2, 1, 6, 8)$, $\vec{b} = (9, 5, 2, -5, 3)$

$\vec{a}\cdot\vec{b} = (-4)\times9 + (-2)\times5 + 1\times2 + 6\times(-5) + 8\times3 = -50$

$|\vec{a}| = \sqrt{(-4)^2+(-2)^2+1^2+6^2+8^2} = 11$

$|\vec{b}| = \sqrt{9^2+5^2+2^2+(-5)^2+3^2} = 12$

$\therefore \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|} = \dfrac{-50}{11\times12} = \dfrac{-25}{66}$

2.

(a) $\begin{bmatrix} 2 & -5 \end{bmatrix} \begin{bmatrix} -2 & 3 & 4 \\ 5 & -1 & 3 \end{bmatrix} = \begin{bmatrix} -29 & 11 & -7 \end{bmatrix}$ $\begin{bmatrix} (-2)\times 2+(-5)\times 5 & 3\times 2+(-5)\times(-1) & 2\times 4+(-5)\times 3 \end{bmatrix}$

(b) $B = \begin{bmatrix} 4 & 7 \\ 0 & -5 \\ -3 & 1 \end{bmatrix}$ $B^T = \begin{bmatrix} 4 & 0 & -3 \\ 7 & -5 & 1 \end{bmatrix}$ $\begin{bmatrix} 2\times 4+(-5)\times 7 & 2\times 0+(-5)\times(-5) & (-5)\times(-3)+(-5)\times 1 \end{bmatrix}$

$\therefore AB^T = \begin{bmatrix} 2 & -5 \end{bmatrix} \begin{bmatrix} 4 & 0 & -3 \\ 7 & -5 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 25 & -11 \end{bmatrix}$

(c) $BC = \begin{bmatrix} 4 & 7 \\ 0 & -5 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} -2 & 3 & 4 \\ 5 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4\times(-2)+7\times 5 & 4\times 3+7\times(-1) & 4\times 4+7\times 3 \\ 0\times(-2)+(-5)\times 5 & 0\times 3+(-5)\times(-1) & 0\times 4+(-5)\times 3 \\ (-3)\times(-2)+1\times 5 & (-3)\times 3+1\times(-1) & (-3)\times 4+1\times 3 \end{bmatrix} = \begin{bmatrix} 27 & 5 & 37 \\ -25 & 5 & -15 \\ 11 & -10 & -9 \end{bmatrix}$

(d) $CD = \begin{bmatrix} -2 & 3 & 4 \\ 5 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 5 & 1 & 0 \\ -7 & 4 & 3 & 6 \\ 8 & -2 & 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} (-2)\times 2+3\times(-7)+4\times 8 & (-2)\times 5+3\times 4+4\times(-2) & (-2)\times 1+3\times 3+4\times 3 & -2\times 0+3\times 6+4\times(-1) \\ 5\times 2+(-1)\times(-7)+3\times 8 & 5\times 5+(-1)\times 4+3\times(-2) & 5\times 1+(-1)\times 3+3\times 3 & 5\times 0+(-1)\times 6+3\times(-1) \end{bmatrix}$

$= \begin{bmatrix} 7 & -6 & 19 & 14 \\ 41 & 15 & 11 & -9 \end{bmatrix}$

(e)

$$B^T = \begin{bmatrix} 4 & 0 & -3 \\ 7 & -5 & 1 \end{bmatrix}$$

$$B^T D = \begin{bmatrix} 4 & 0 & -3 \\ 7 & -5 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 1 & 0 \\ -7 & 4 & 3 & 6 \\ 8 & -2 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times2+0\times(-7)+(-3)\times8 & 4\times5+0\times4+(-3)\times(-2) & 4\times1+0\times3+(-3)\times3 & 4\times0+0\times6+(-3)\times(-1) \\ 7\times2+(-5)\times(-7)+1\times8 & 7\times5+(-5)\times4+1\times(-2) & 7\times1+(-5)\times3+1\times3 & 7\times0+(-5)\times6+1\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 26 & -5 & 3 \\ 57 & 13 & -5 & -31 \end{bmatrix}$$

3.

(a) $\frac{d}{dx} f(x) = \frac{d}{dx}(2x^4 - x^3 + 4x + 100) = 8x^3 - 3x^2 + 4$

(b) $\frac{d}{dt} f(t) = \frac{d}{dt}(t^7 x^3 - 7t^2 x + 30t) = 7t^6 x^3 - 14tx + 30$

(c) $\frac{d}{ds} f(s) = \frac{d}{ds}(a^2 b s^5 - b s^3 - acs^2 + bc^2 s - a^5 b^2 c^3)$

$= 5a^2 b s^4 - 3bs^2 - 2acs + bc^2$

4.

(a) $F(x) = \sqrt{x^2+1} = g(x)^{1/2} \quad (g(x) = x^2+1)$

$\frac{dF(x)}{dx} = \frac{dF(x)}{dg(x)} \times \frac{g(x)}{dx} = \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \times 2x = \frac{x}{\sqrt{x^2+1}}$

(1b) $f(x) = (2x+1)^2 (x^3-x+1)^4$

$= k(x)^2 \, h(x)^4 \quad (k(x) = 2x+1), \; (h(x) = x^3-x+1)$

$$\frac{d f(x)}{dx} = 2k'(x)k(x)h(x)^4 + 4h'(x)k(x)^2h(x)^3$$

$= 2 \times 2 \times (2x+1)(x^3-x+1)^4 + 4 \times (3x^2-1)(2x+1)^2(x^3-x+1)^3$

$= (2x+1)(x^3-x+1)^3\left(4 \times (x^3-x+1) + 4 \times (3x^2-1)(2x+1)\right)$

$= (2x+1)(x^3-x+1)^3\left(4x^3-4x+4+4(6x^3+3x^2-2x-1)\right)$

$= (2x+1)(x^3-x+1)^3(4x^3+24x^3+12x^2-4x-8x+4-4)$

$= (28x^3+12x^2-12x)(2x+1)(x^3-x+1)^3$

$= 4x(7x^2+3x-3)(2x+1)(x^3-x+1)^3$

(c) $g(x) = \cos(\sin(\tan x)) = C(B(A(x)))$

$(A(x) = \tan x \quad B(x) = \sin x \quad C(x) = \cos x)$

$$\frac{d g(x)}{dx} = \frac{d g(x)}{d(B(A(x)))} \times \frac{d(B(A(x)))}{d A(x)} \times \frac{d A(x)}{dx}$$

$= -\sin(\sin(\tan(x))) \times \cos(\tan(x)) \times \sec^2(x)$

$= -\sin(\sin(\tan(x)))\cos(\tan(x))\sec^2(x)$