Phase P1: KernelMargin Inequality & Margin Existence

Version: 2025-09-22 07:15 KST
Location: lean/src/UEM/YeobaekOverlap.lean
Status: Formal proof completed in Lean 4 (tools/proofcoverage.sh status=SORRYFREE).

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0. Abstract
Phase P1 of the UEM project formalizes the interaction between a layered projection system and a
positive-semidefinite kernel. The goal is to show that the residual margin between the domain and the
projection image is non-empty and carries a quantitative lower bound determined by the kernel
thickness hypothesis. All statements summarized below are implemented and verified in Lean without
sorry placeholders.

1. Mathematical Setting

1.1 Layered geometry
Structure: YeobaekLayeredSpace (lean/src/UEM/Structure.lean:27) packages three spaces:
- Internal space Internal: complex normed additive group; represents latent states.
- External space External: real normed additive group with measurable structure; represents
observable configurations.
- Boundary space Boundary: topological space embedded into External for observable boundaries.

Key components:
- embedInternal, embedBoundary, projectionCR describe how each layer sits inside the external
world.
- observable External together with observablemeasurable encodes the measurable region where observations take place.
- measureExternal : Measure External provides the base measure used in P1; the projection hypotheses require this to agree with the measure used downstream.

1.2 Kernel and projection hypotheses
Kernel block: YeobaekOverlapHypotheses K (lean/src/UEM/YeobaekOverlap.lean:46) assumes
- symmetry K x y = K y x;
- measurability of each section K x and fun x K x y;
- PSD condition: K x y f x f y 0 for all measurable f :   0;
- finiteness of the left integrals, a uniform essential upper bound, and a global thickness lower
bound min such that min   K x y d y.

Projection block: YeobaekProjectionHypotheses layer (lean/src/UEM/YeobaekOverlap.lean:70)
requires
- is measurable with measurable image;
- the whole space has finite measure and the image measure is strictly smaller: (’’ Set.univ)
< Set.univ;
- agrees with the layered projection and fixes boundary embeddings;