Optimization
with
D-WAVE
Analog
Quantum Machines

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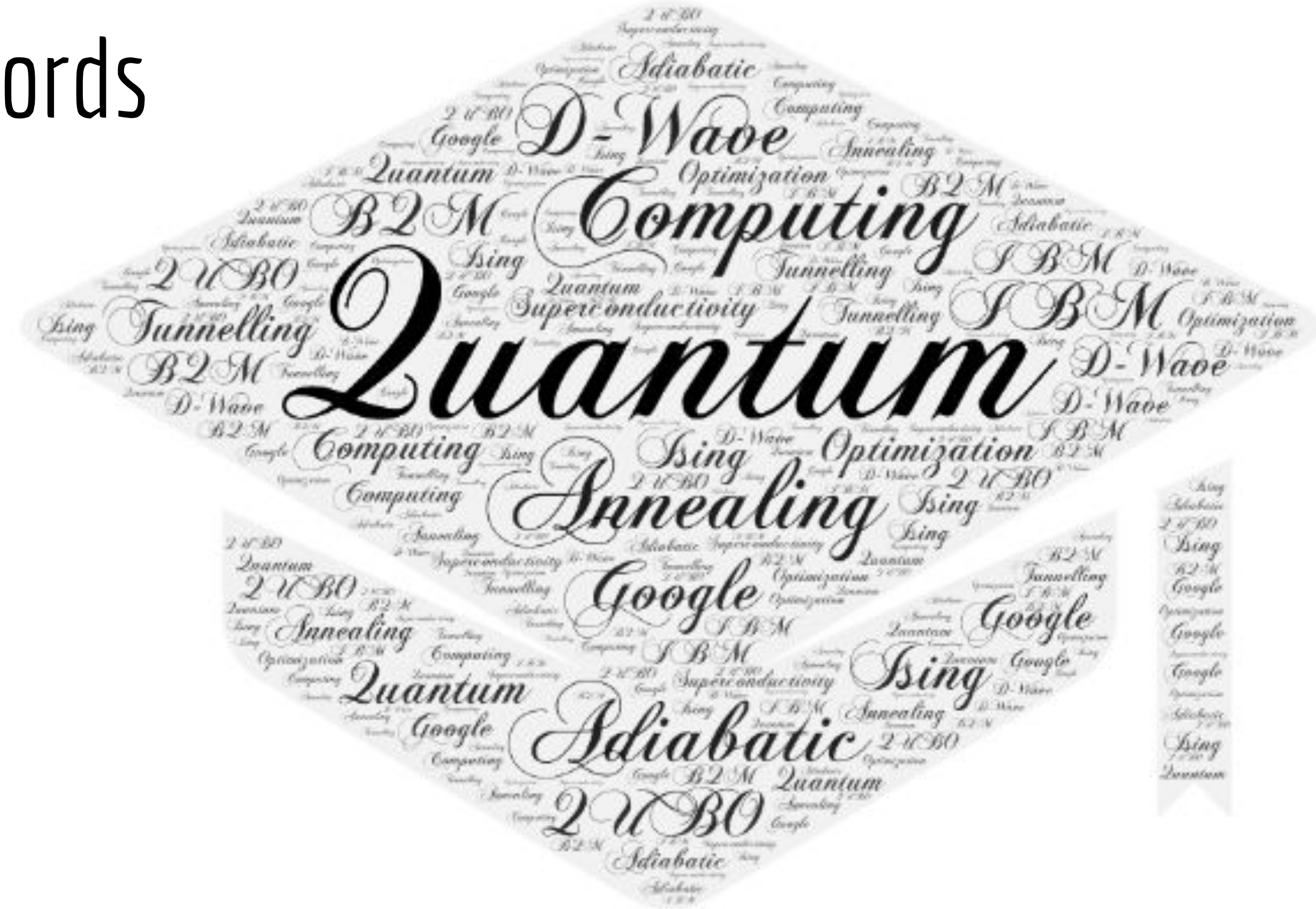
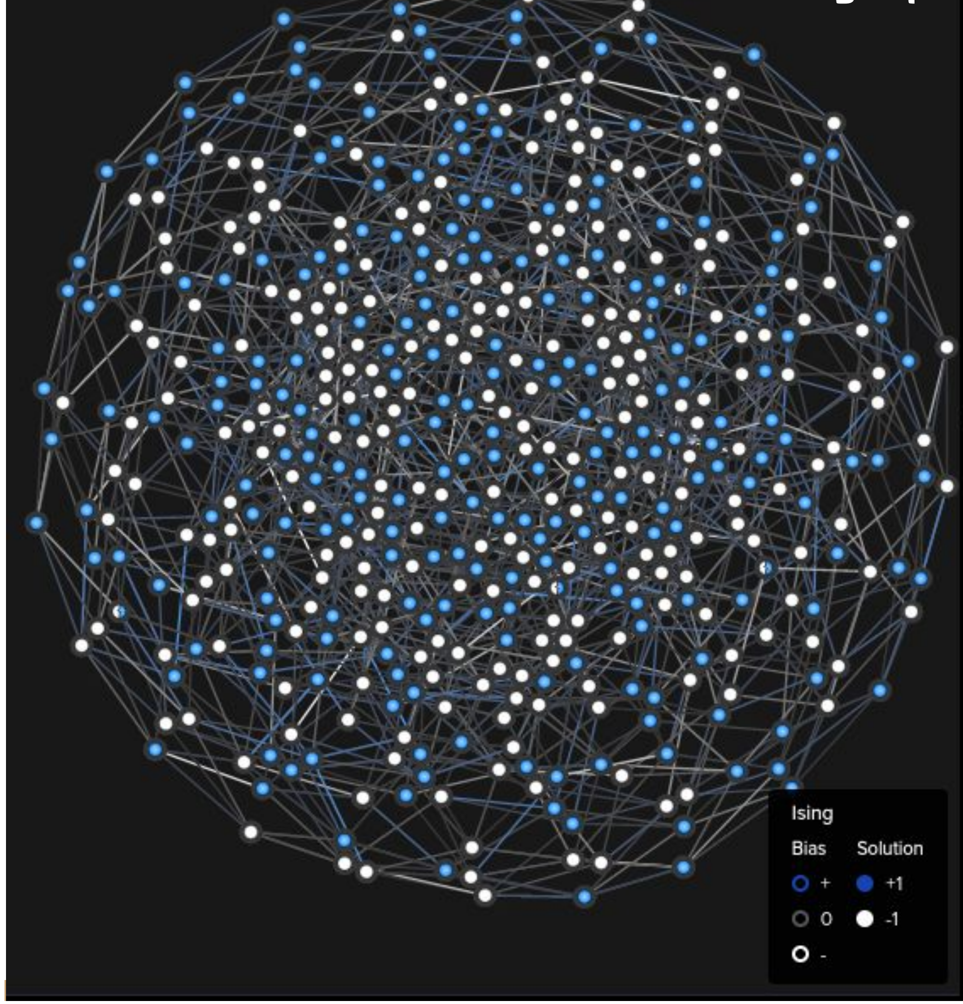
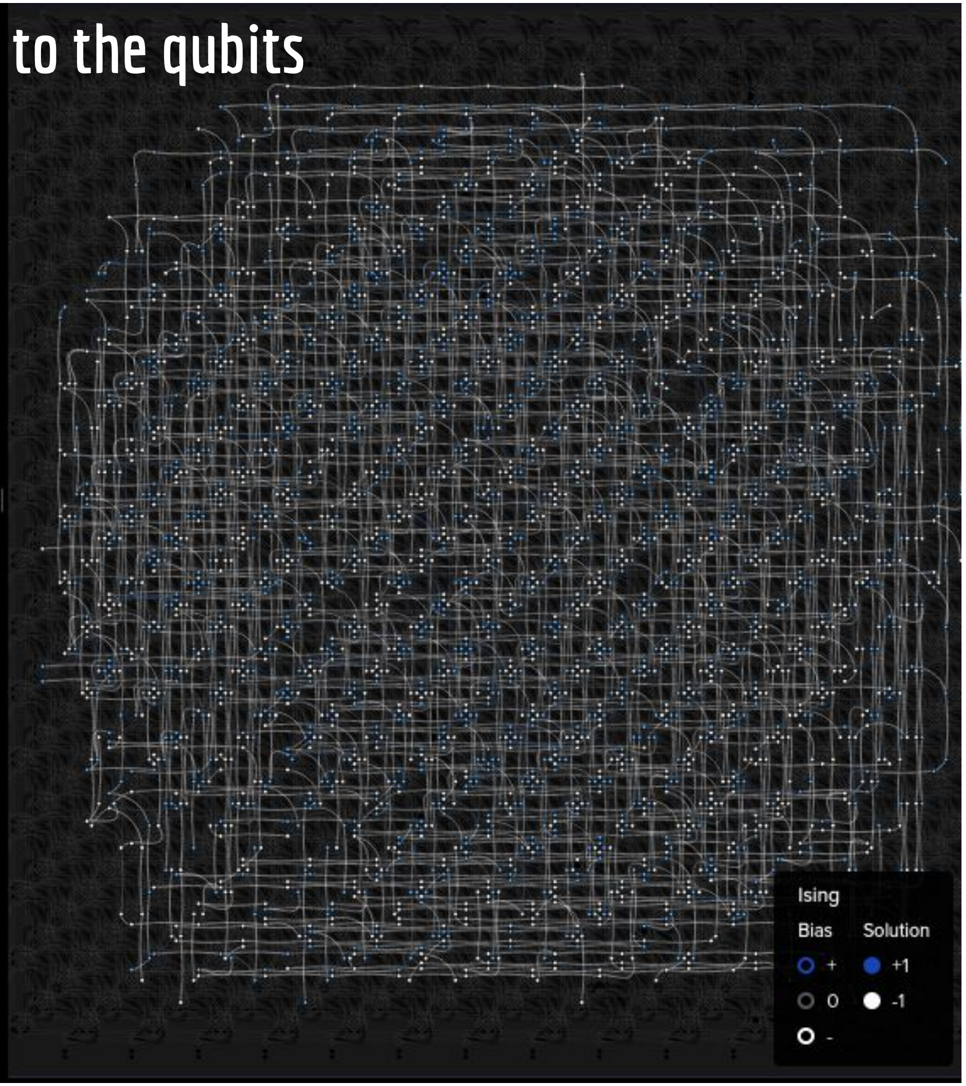
Keywords



Pic online tool: https://wordart.com/create

From a graph to the qubits





Outline

1. D-Wave technology overview
2. Optimization problems
3. Quantum Annealing
4. INPUT and General Process
5. End-to-end process: the Max-Cut Example
6. Hybrid Machines

Outline

D-Wave technology overview

- Historical
- Hardware
- Software/API

D-Wave over time

D-Wave The Quantum Computing Company™

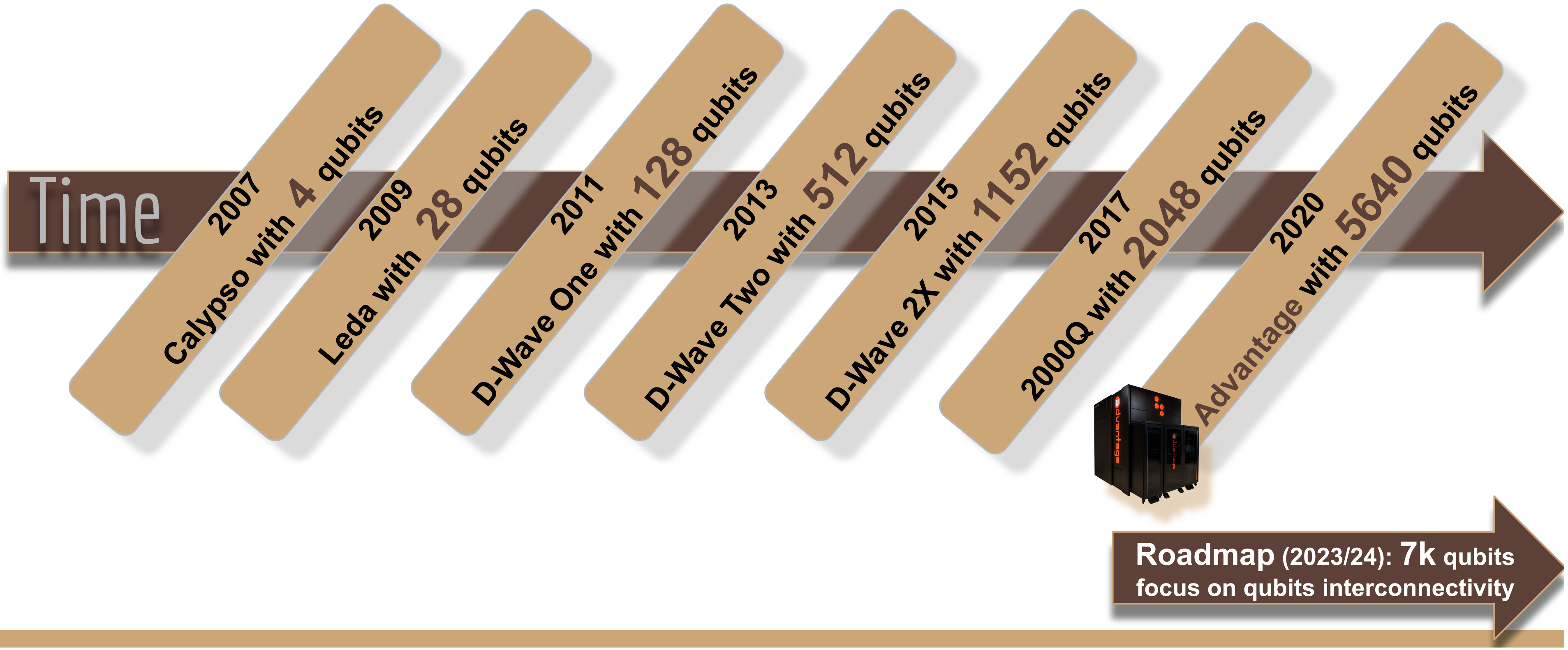
- 1999 > birth
  - 4 founding fathers: **Geordie Rose**, **Haig Farris**, **Bob Wiens** and **Alexandre Zagoski**.
- 2007 > **1st prototype** (**4 qubits**) named “Calypso”
- 2009 > New direction (**Vern Brownell** becomes CEO, only **Zagoski** stays.)
- 2011 > 1st commercialized quantum computer (128 qubits)
- 2017 > D-Wave 2000Q (cost: $ 15 million).
- end2020> The **5k** qubits **Advantage** machine is released.

Notable Funders: InQTel (CIA funds), Lockheed Martin, NSA funds (see Research & Technology Penetrating hard targets’ *Snowdew FIle*) and others.

≤ 2013

• (S//SI//REL TO USA, FVEY) Conduct basic research in quantum physics and architecture/engineering studies to determine if, and how, a cryptologically useful quantum computer can be built.

from 4 to 5640 qubits

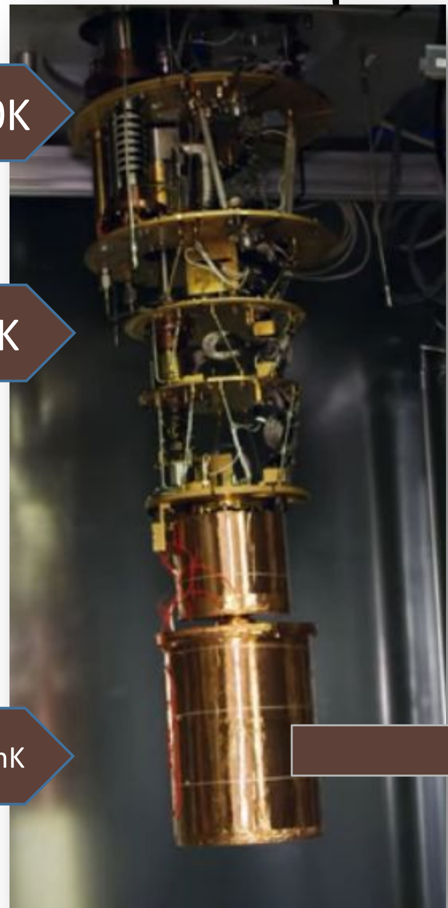


Hardware…inside a D-Wave quantum machine

≈ 70K

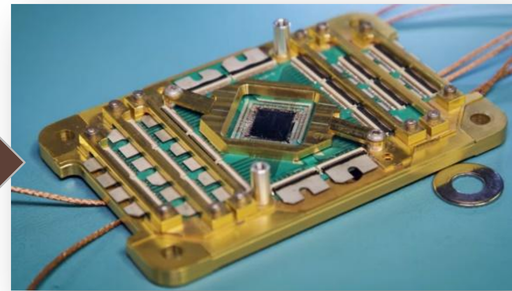
≈ 1K

≈ 12mK



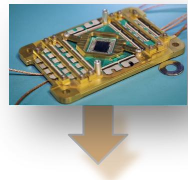
QPU
(Quantum Processing Unit)

Set of D-Wave Nodium superconductors (qubits into the QPU)



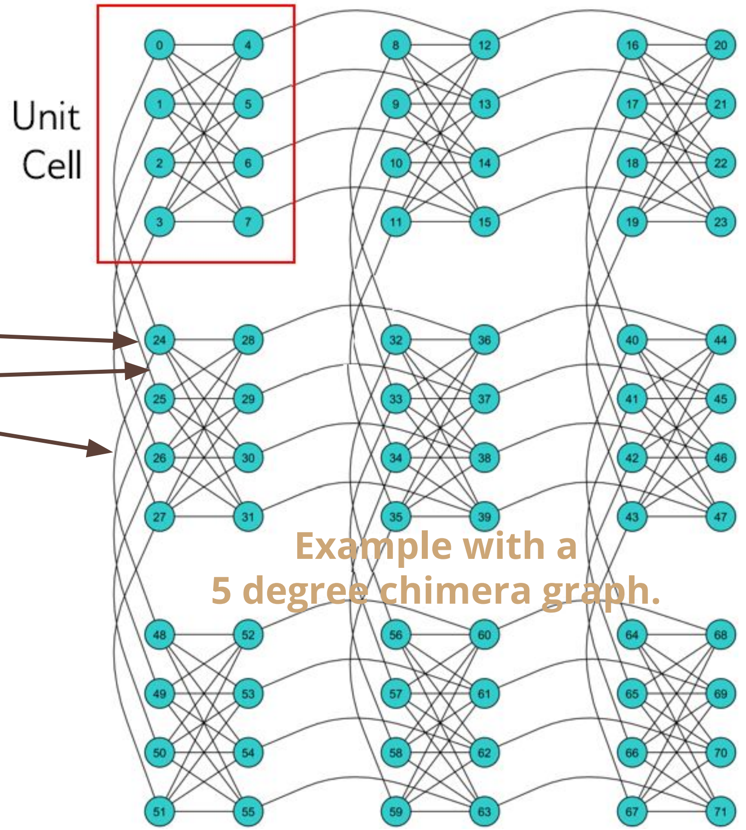
Chimera Graph
(D-Wave hardware model)

QPU

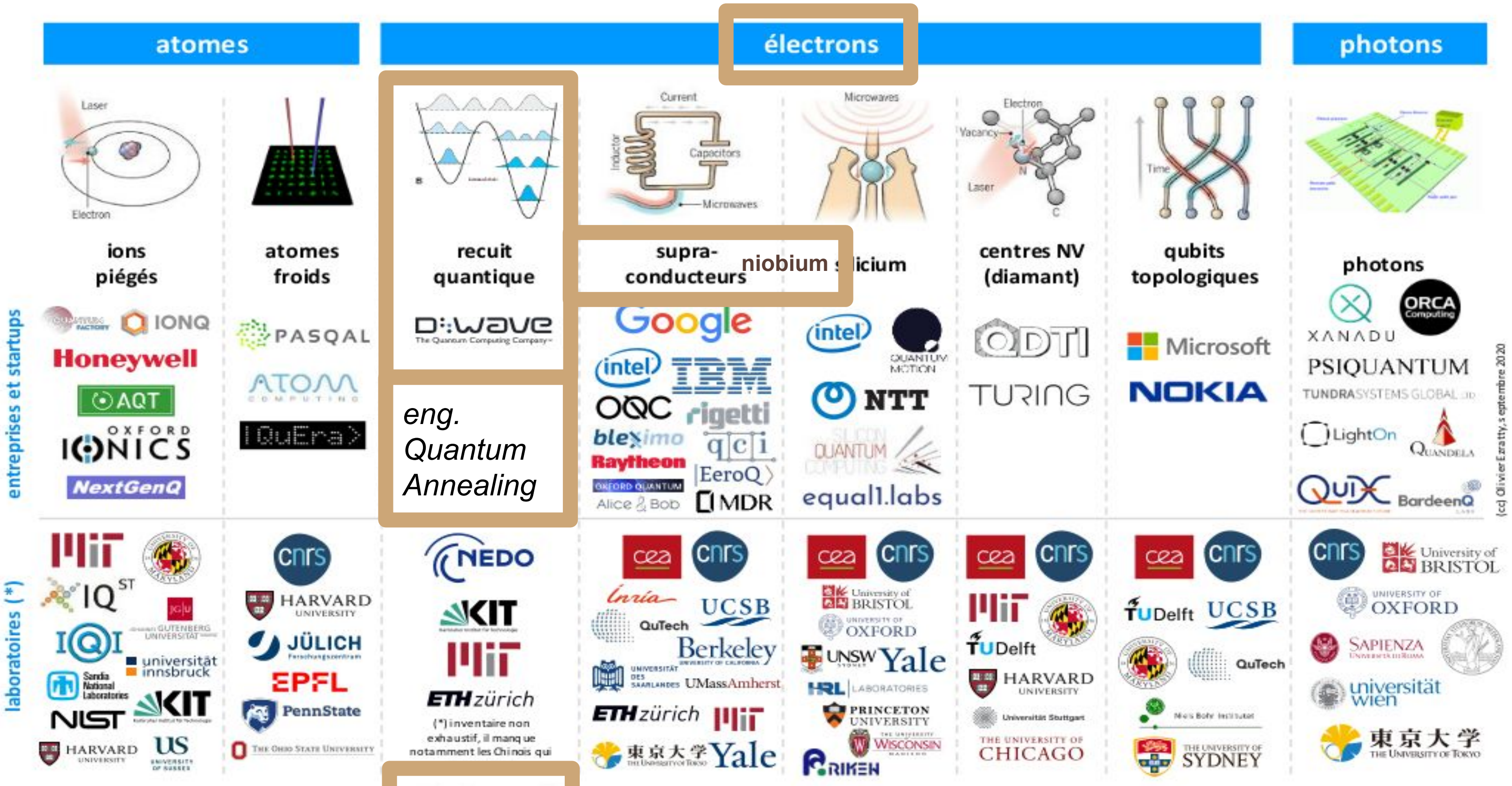


D-Wave’s quantum computer hardware can be modeled as a non-oriented graph called chimera graph. This graph has:

- **qubits** as nodes,
- quantum **couplers** as edges.
- since end of 2020 (***Pegasus*** topology), **the degree of the qubits is 15** (6 before).



D-Wave among the quantum computing world



Atos

& https://atos.net/wp-content/uploads/2020/07/PR-Atos-opens-up-a-new-path-to-quantum-annealing-simulation.pdf

& https://atos.net/fr/2020/communiques-de-presse_2020_07_07/atos-ouvre-la-voie-a-la-simulation-du-recuit-quantique

original pic: https://www.oezratty.net/wordpress/2020/comprendre-informatique-quantique-edition-2020/

Overview of the Frameworks

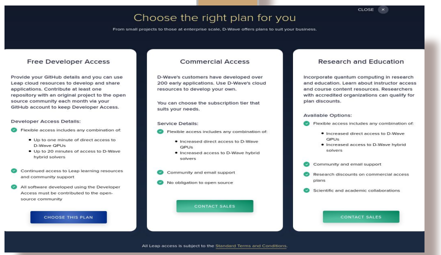
- **Cirq**
  - **Google** solution
  - https://cirq.readthedocs.io/en/stable/
- **Qiskit**
  - ***IBM*** *Quantum Experience*
  - https://quantum-computing.ibm.com/
- **Amazon Braket**
  https://aws.amazon.com/fr/braket/
- (Quirk)
  - Just a nice Graphic Editor…
  - https://algassert.com/quirk
- and few others:
  - Rigetti (Forest/PyQuil),
  - Microsoft (AzureQuantum/LiQui/Q#),
  - From labs: PyTKET, ProjectQ, QuTip etc.)

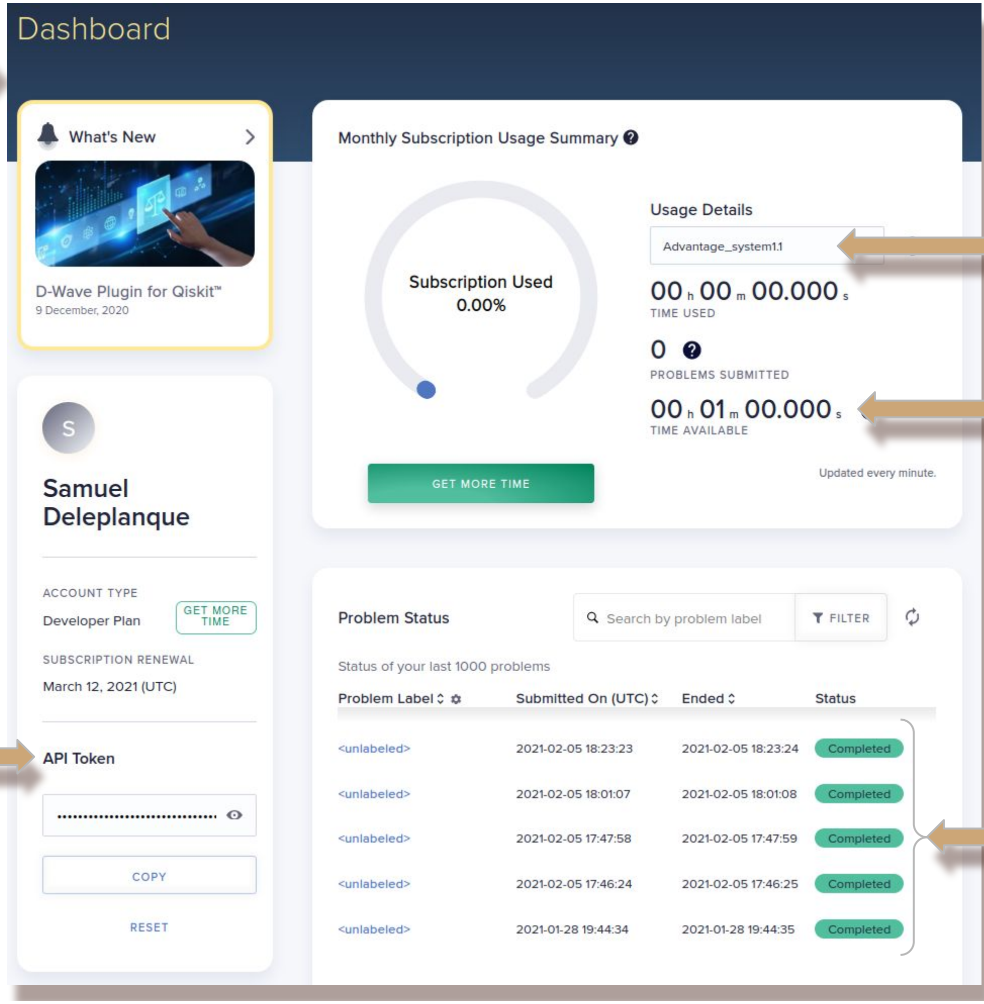
D-Wave
The Quantum Computing Company™

- **D-Wave Leap**
  - **Ocean Library (python)**
  - **Not Universal Quantum Computer**
    - **(no universal set of gate)**
  - **"High Level" programming**
  - https://www.dwavesys.com/take-leap

How can I solve a problem on a D-Wave machine? https://cloud.dwavesys.com/leap/

Subscription





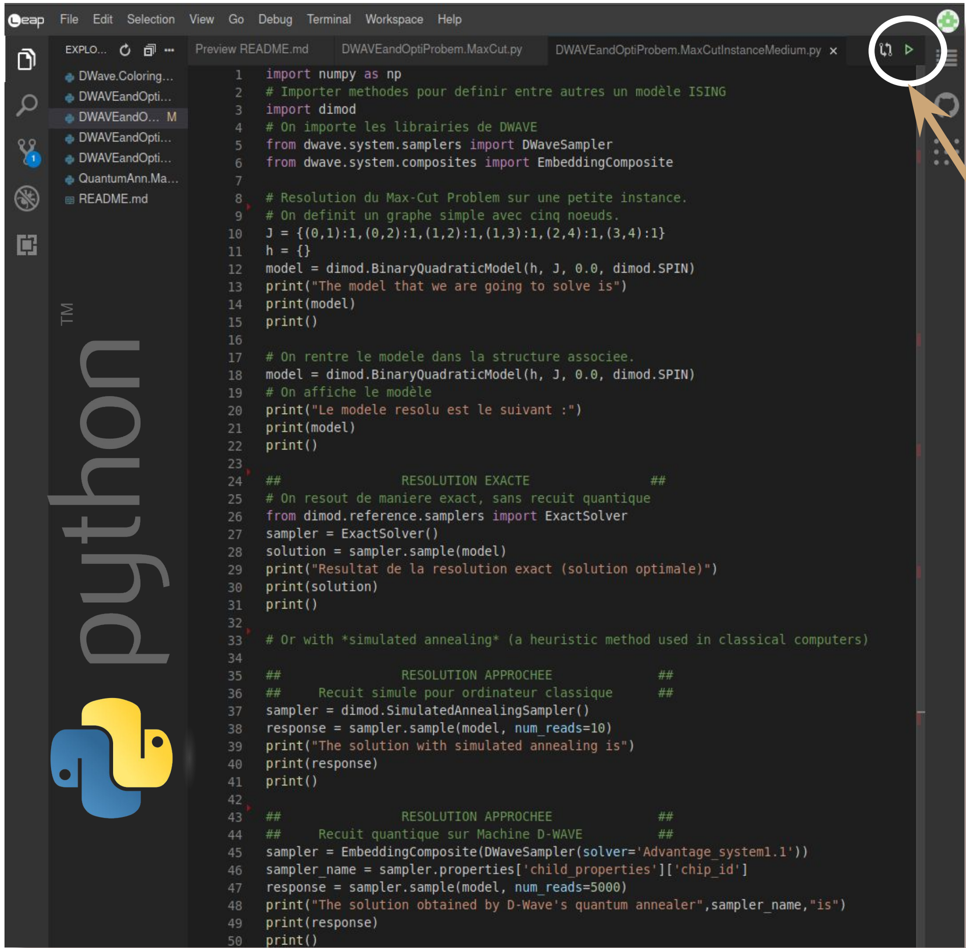
D-Wave Machine

Remaining time

Tokens to use into you local code (useless for online IDE)

Previous Problems solved

How can I solve a problem on a D-Wave machine?



1

You launch the resolution directly here, and you will have an online terminal showing the result.

2

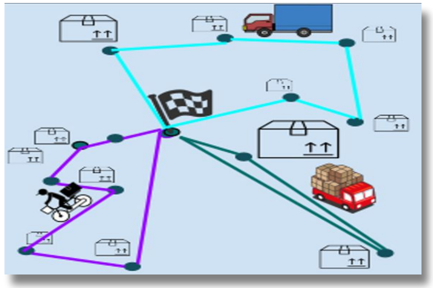
```
The solution obtained by D-Wave's quantum annealer Advantage_system1.1 is
   0  1  2  3  4 energy num_oc. chain_.
0 -1 +1 +1 -1 +1   -4.0    1398     0.0
1 +1 -1 -1 +1 -1   -4.0    1029     0.0
2 -1 +1 +1 -1 -1   -4.0    1487     0.0
3 +1 -1 -1 +1 +1   -4.0    1084     0.0
4 -1 +1 +1 +1 -1   -2.0       1     0.0
5 +1 -1 -1 -1 +1   -2.0       1     0.0
['SPIN', 6 rows, 5000 samples, 5 variables]
```

To be ready for the practical work

- Subscribe to https://cloud.dwavesys.com/leap/signup/ (free account)

- Follow this 7 minutes video in order to learn how to reach the online IDE (directly through your browser) :
    - https://www.youtube.com/watch?v=62gDQ14pjwM
    - You will have to share a folder of your github account

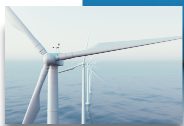
Outline

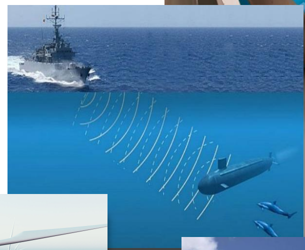


2 Optimization problems and D-Wave machines

Which systems to optimize?

Some Cases...





(very) Few words on Discrete Optimization problems

Goal:
Finding a solution of a problem that <u>minimizes the cost</u> or that <u>maximizes a performance</u>.

2 types of methods:

- an **exact method** is trying to find the best solution(s) with **the lowest cost / highest performance**.
- an **approximate method**, also named **heuristic**, will try to find a good solution with a **low cost / high performance.** A heuristic can find the best solution but it is not guaranteed and it will not be proven.

Discrete optimization problems involve <u>discrete</u> variables (**<u>integers</u> and/or <u>binaries</u>**).

Discrete Optimization problems: Example 1

Traveling Salesman Problem Example

Given a list of cities and the distances between each pair of cities, <u>what is the shortest route that visits each city exactly once and returns to the origin city</u>?

$$\min \sum_{i=1}^{n} \sum_{j\neq i, j=1}^{n} c_{ij} x_{ij}$$

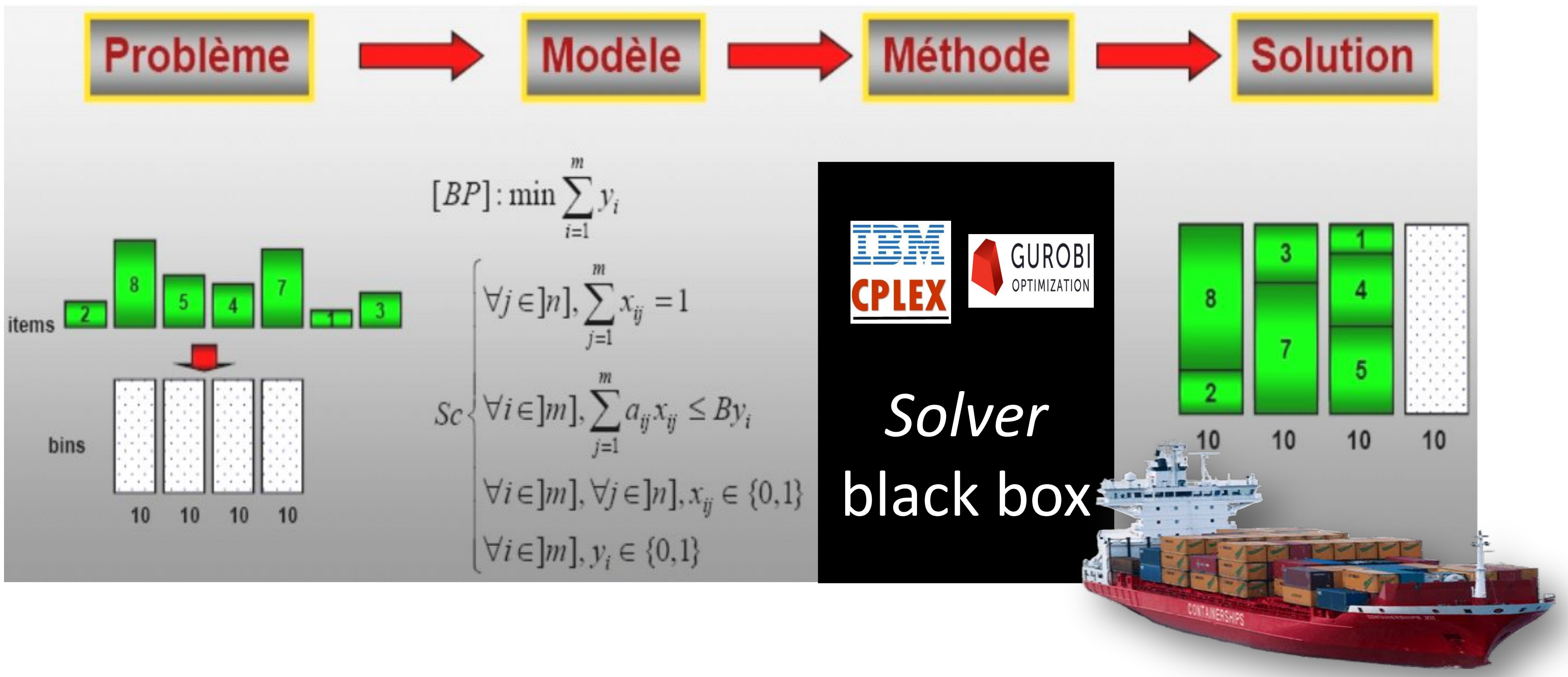
Minimize Distance (Cost) (Objective Function)

Mathematical Model

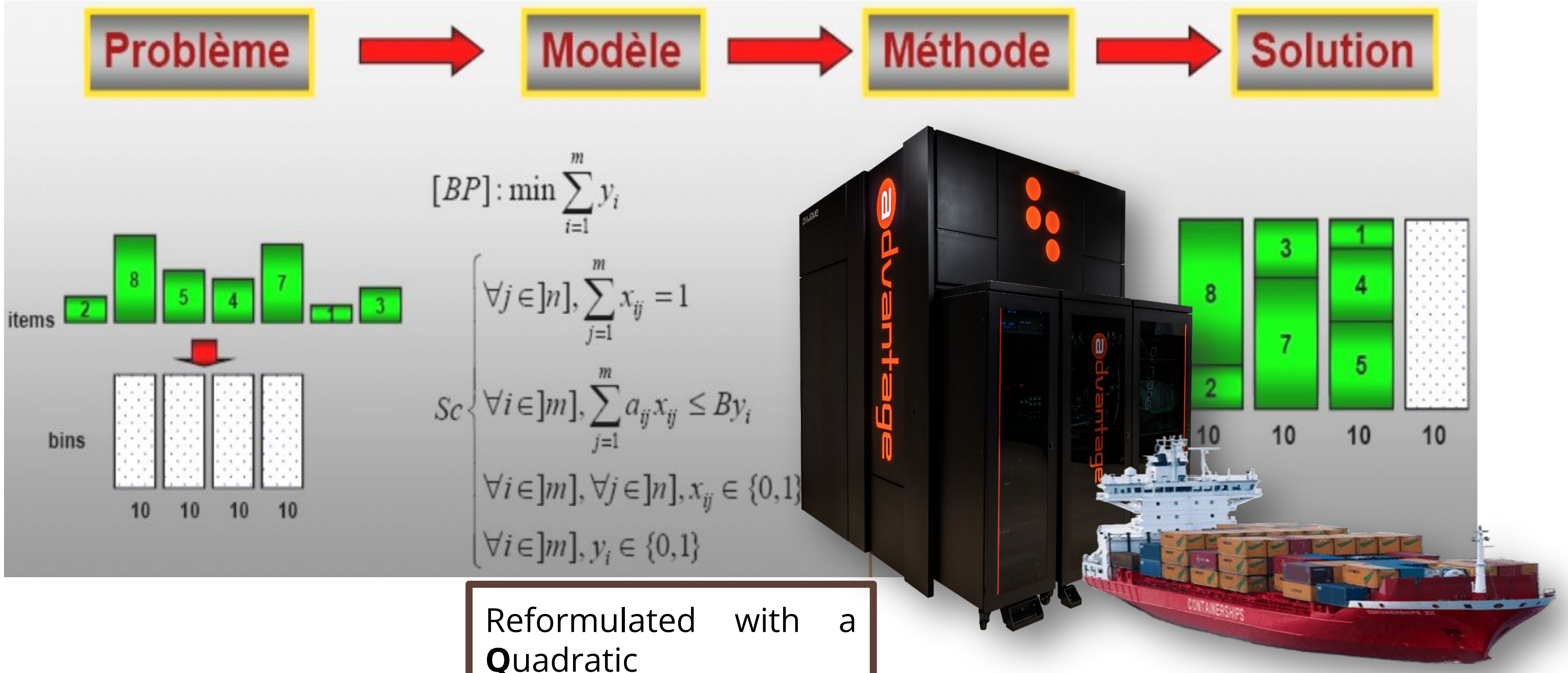
$$
\begin{array}{ll}
x_{ij} \in \{0,1\} & i,j = 1,\ldots,n; \\
u_i \in \mathbf{Z} & i = 2,\ldots,n; \\
\sum_{i=1, i\neq j}^{n} x_{ij} = 1 & j = 1,\ldots,n; \\
\sum_{j=1, j\neq i}^{n} x_{ij} = 1 & i = 1,\ldots,n; \\
u_i - u_j + n x_{ij} \leq n-1 & 2 \leq i \neq j \leq n; \\
1 \leq u_i \leq n-1 & 2 \leq i \leq n.
\end{array}
$$

Pic: https://www.math.uwaterloo.ca/tsp/poke/index.html

Discrete Optimization problems: Example 2

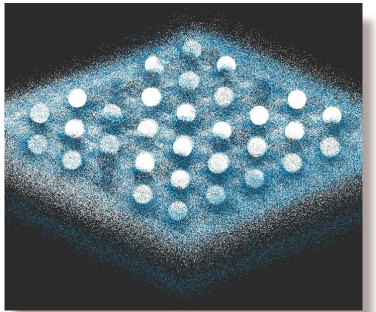


Where the quantum *magic* will appear...



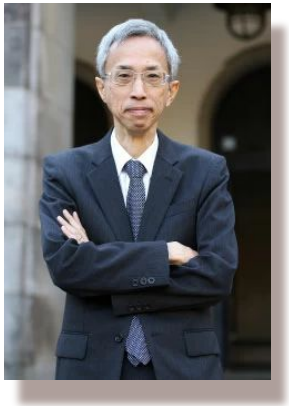
Reformulated with a **Q**uadratic **U**nconstrained **B**inary **O**ptimization model

Outline



3 Quantum Annealing on D-Wave machine

Quantum Annealing



Hidetoshi Nishimori

- Father of the **Quantum Annealing Metaheuristic**
  - Works on Statistical physics,
  - Paper: Kadowaki, Tadashi, and Hidetoshi Nishimori. "Quantum annealing in the transverse Ising model." *Physical Review E* 58.5 published in **1998**;
- Based on classical computer Metaheuristic Simulated Annealing
  - Kirkpatrick, Scott, C. Daniel Gelatt, and Mario P. Vecchi. "Optimization by simulated annealing." *science* 220.4598 (1983): 671-680;
- Prove that the Quantum Annealing gives better results that the Simulated Annealing,
- A good conference from him: https://www.youtube.com/watch?v=OQ91L96YWCk.

D:Wave The Quantum Computing Company™

- First (hardware) implementation of the Quantum Annealing,
- Starting the work in **1999**, the 1st computer based on quantum annealing released in **2007**.

Quantum annealing *(fr : recuit quantique)* & Optimization Problems

“Physics can help solve [Optimization Problems] because we can frame them as energy minimization problems. “

“The D-Wave system can be viewed as a **hardware <u>heuristic</u>** that **minimizes [..] objective functions** using a physically realized version of quantum annealing.”

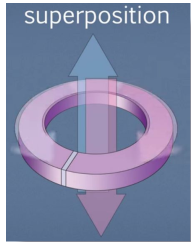
https://docs.dwavesys.com/docs/latest/c_handbook_2.html

INPUT → **Quantum Annealing** → OUTPUT

The very basic idea

First Step

The qubits start in a superposition state (“**zero and one**”).

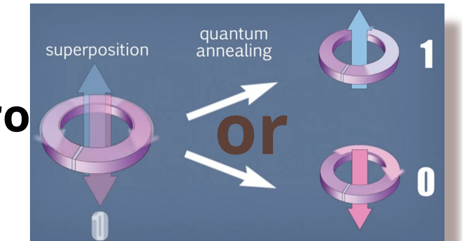


and

[...]
“Work on the 0 and 1 states (amplitudes).”
[...]

Last step

The measure of each qubit make them going to either the **zero state or** the **one state**.

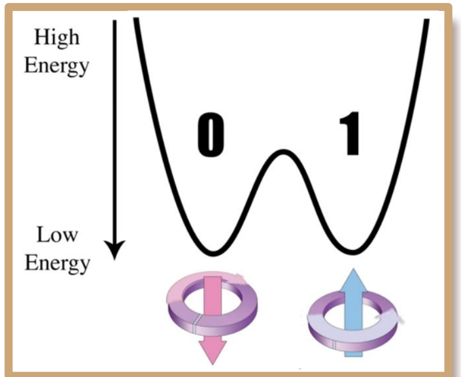


Pics: D-Wave

INPUT → **Quantum Annealing** → OUTPUT

individual qubit

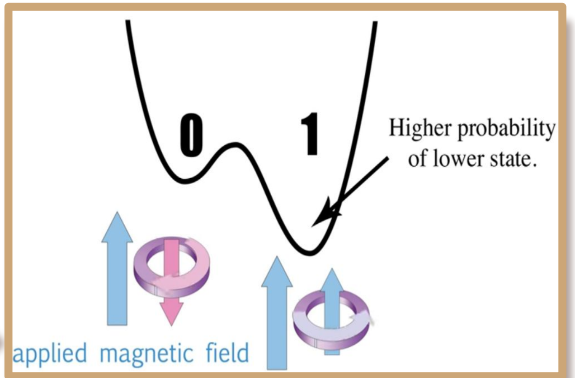
Energy Diagram (Superposition)



Starting Point.
Proba = 50%
for each state.

Control the probability of the state with a programmable external magnetic field (bias)

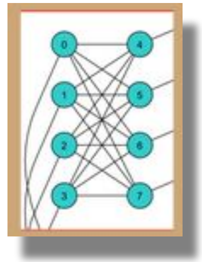
Energy Diagram (After Bias)



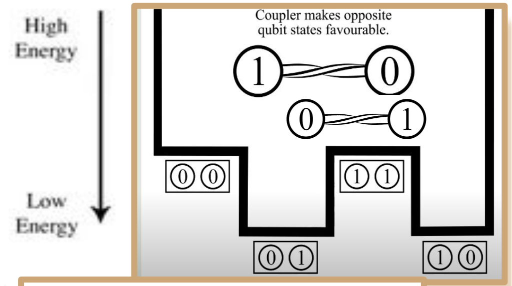
Here: higher proba to measure the state '1'.

Pics: D-Wave

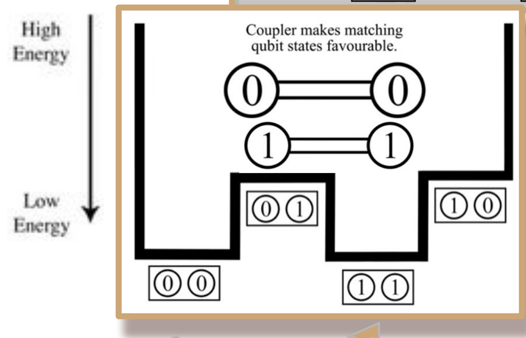
“”Entanglement”” of pair of qubits



- **The “real power” of such a system appears once we start to link the qubits together.**
  - by **couplers** (using **entanglement** behaviour).
    - **The couplers define how a pair of qubits ends up, in the same or in opposite state**, (0&0, 1&1 or 1&0, 0&1).



- **Once 2 qubits are “entangled”, they can be considered as a single object, but with 4 states.**
  - When we consider a pair of qubits to be the same (or to be opposite) **the machine will “physically” make the equality (or the inequality)** “**energetically favorable**”, i.e., to “low the energy” of those states.



This is not a strict entanglement, it is a matter of probability.

Pics: D-Wave

Pasqal & D-Wave: two analog machines which can consider an transverse field ising model

PASQAL

$$\hat{H} = \sum_{i=1}^{N} \Omega_i \hat{\sigma}_i^x - \sum_{i=1}^{N} \delta_i \hat{n}_i + \sum_{i<j=1}^{N} \frac{C_6}{r_{ij}^6} \hat{n}_i \hat{n}_j$$

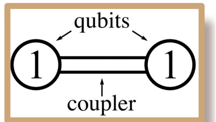
Ω = Rabi frequency

δ = Detuning

- The **Hamiltonian ($\mathcal{H}$)** includes 2 types of interaction:
  - h_i : the "programmable" external magnetic field called a **biais**,
  - J_{ij} : the interaction between each neighboring qubits *i* and *j* **(coupling)**.

D-Wave The Quantum Computing Company™

$$\mathcal{H}_s(s) = -\frac{1}{2}\sum_i \Delta(s)\sigma_i^x + \mathcal{E}(s)\left(-\sum_i \boxed{h_i}\sigma_i^z + \sum_{i<j} \boxed{J_{ij}}\sigma_i^z \sigma_j^z\right)$$



Initial Hamiltonian Hi

Final Hamiltonian Hf

pics: https://www.youtube.com/watch?v=tnikftltqE0&t=125s

Quantum Annealing: an adiabatic process

$$\mathcal{H}_S(s) = -\frac{1}{2}\sum_i \Delta(s)\sigma_i^x + \mathcal{E}(s)\left(-\sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\,\sigma_i^z\sigma_j^z\right)$$

Initial Hamiltonian H_i

Final Hamiltonian H_f

Using the *Adiabatic* Theorem (roughly) saying if we start in the ground state of H and we change H sufficiently slowly, we will stay in the ground state.

$$H(t) = (1 - (t/T))\, H_i + (t/T)\, H_f$$

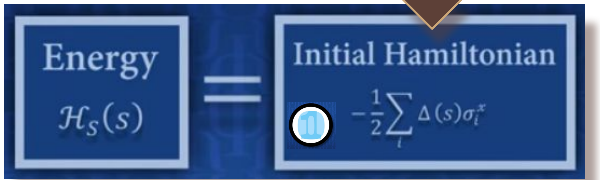
$t = 0..T$

From H_i to H_f

Using the *Adiabatic* Theorem (roughly) saying if we start in the ground state of H and we change H sufficiently slowly, we will stay in the ground state. H Time evolution:

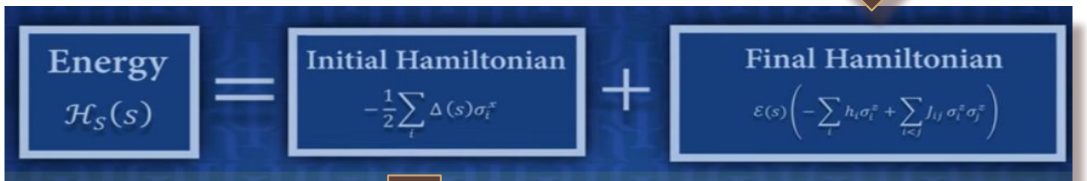
$$H(t) = (1 - (t/T))\, H_i + (t/T)\, H_f \quad \boxed{t = 0..T}$$

start



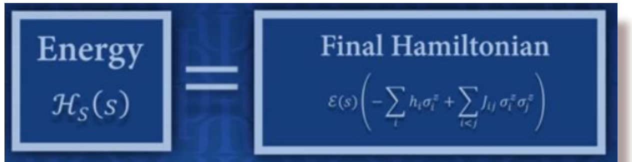
Superposition states
Ground energy state

QPU introduces our problem containing the biases and couplers.



The QA reduces the influence of the initial Hamiltonian

At the **end** (H_f) of the quantum annealing the part of the **initial Hamiltonian disappeared.** Ideally we reach the minimum energy state (i.e., optimal sol.). From a quantum object (in the initial Hamiltonian) , **each qubit ends being a classical object** (in the Final Hamiltonian).



Measure => Classical state
Lowest energy state (hopefully!)

pics: https://www.youtube.com/watch?v=tnikftltqE0&t=125s

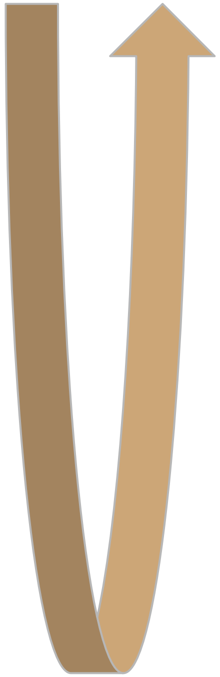
the Quantum Annealing process

(already implemented in D-Wave machines)

Initialise. Qubits are placed in an initial superposition state according to the Hamiltonian $\boldsymbol{H_i}$.

Anneal. The quantum particle process makes a transition from initial Hamiltonian $\boldsymbol{H_i}$ to the Hamiltonian $\boldsymbol{H_f}$ adiabatically (if possible).

Read. Qubits values are read, yielding a solution S.



Resampling. Since any quantum computation is probabilistic, there is a positive probability that the computation does not finish in ground state (i.e., not yielding an optimal solution), so we iterate.

MINIMIZES:

$$\boxed{\sum_{(i,j)\in E} Z_i Z_j}$$

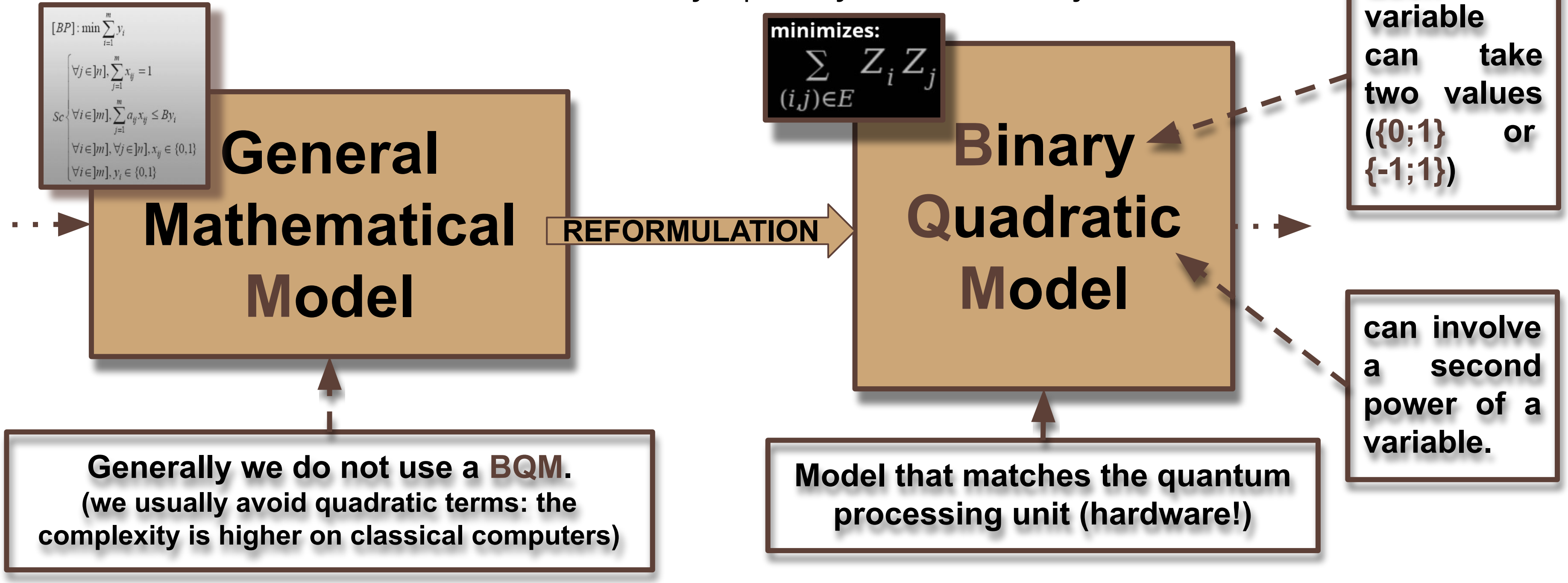
Outline

INPUT & The General Process

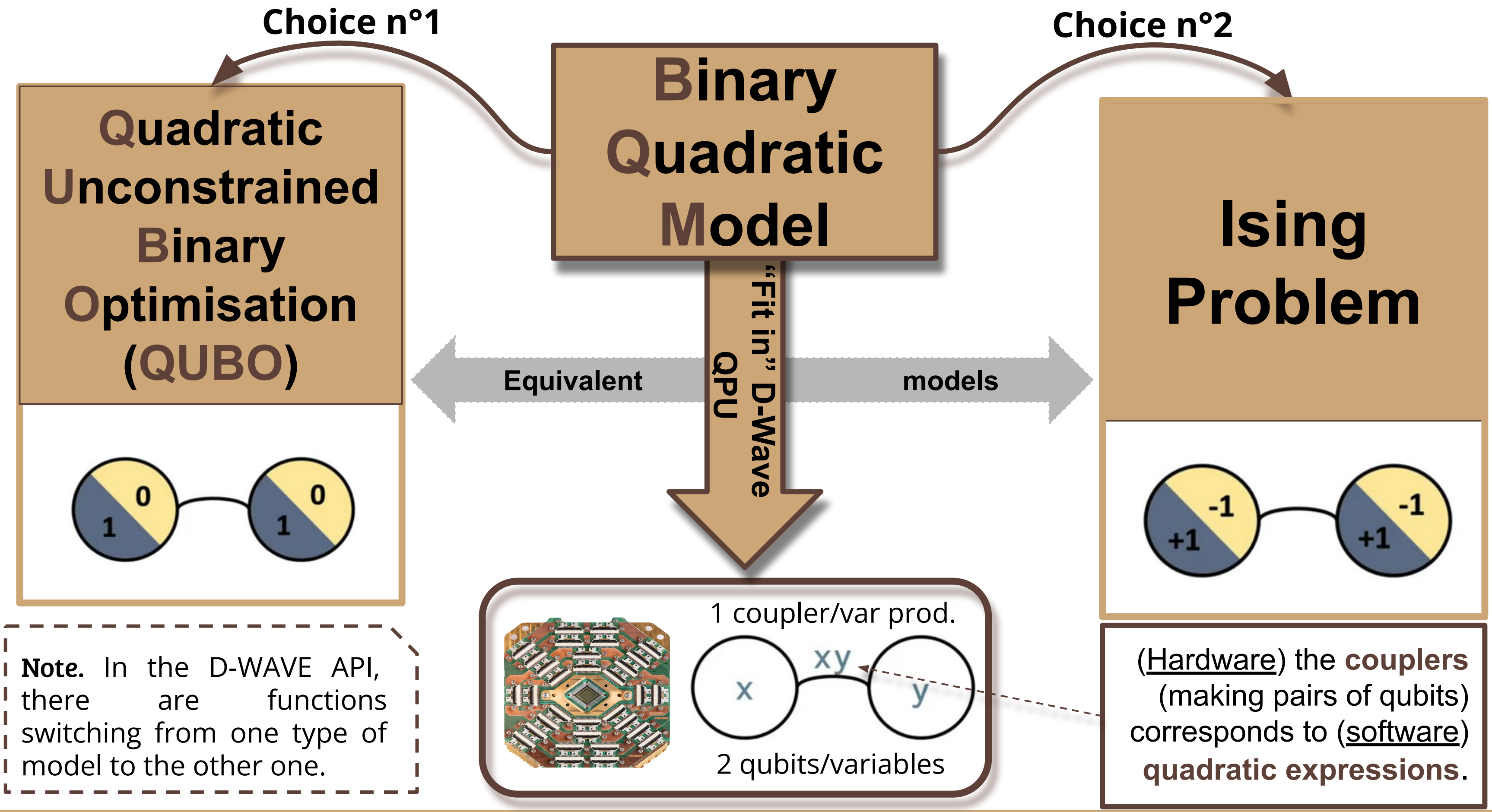
- INPUT
  - 2 equivalent Binary Quadratic Models
    - QUBO ({0,1} variables),
    - Ising Models ({-1,1} variables);

Preparing the input...

- To model an optimisation problem to a Binary Quadratic Model
  - That concerns <u>a subset</u> of optimisation problems,
  - Some constraints can be <u>relaxed</u> by a penalty in the Cost/Objective function.



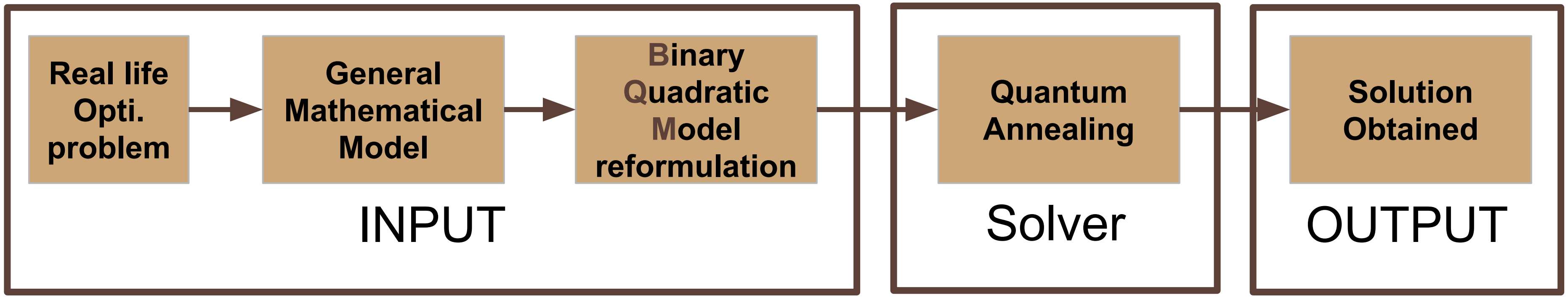
Problem formulation: 2 mathematically **equivalent** models

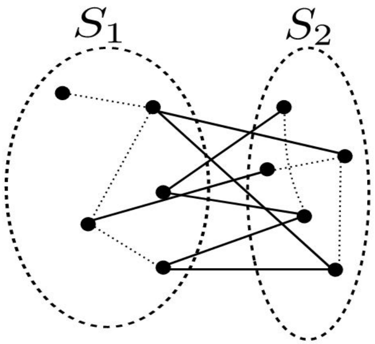


Note. In the D-WAVE API, there are functions switching from one type of model to the other one.

General Process

INPUT › QUANTUM ANNEALING › OUTPUT





Outline

5 End-to-end example: the Max-Cut Problem

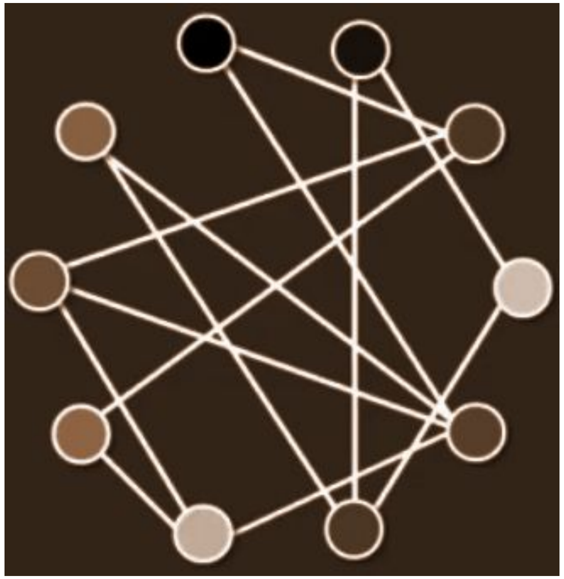
- General process
- Problem Definition
- Ising Model Formulation
- Solving one example
  - Python Code
  - Quantum Annealing Result Analysis

Context. We need to split a large group of people into two groups for going to two different bars.

The problem. Some pairs of people might fight each other after a while.

The desired solution. The best would be to separate such pairs as much as possible.

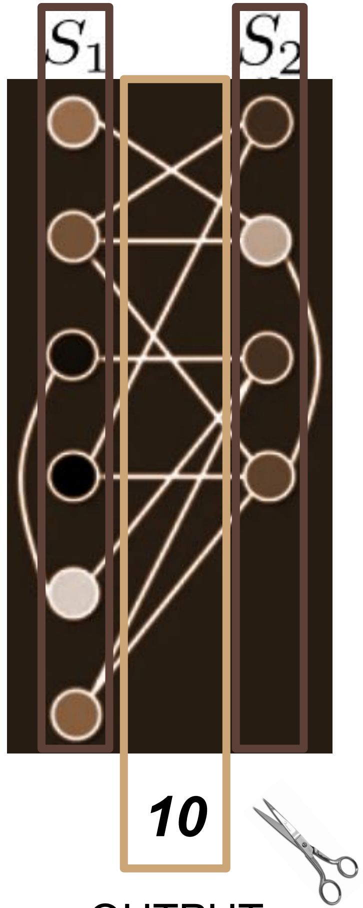
The Max-Cut Problem



INPUT

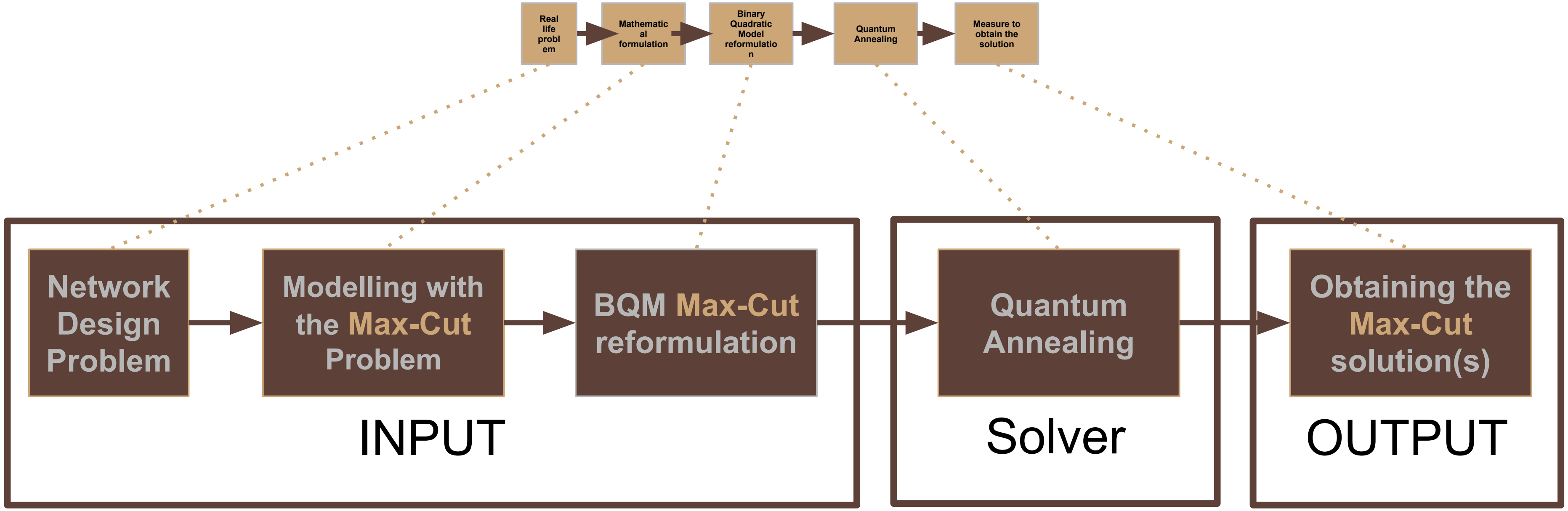
Finding 2 sets of nodes S1 and S2, such that the **number of edges** between the set S1 and the set S2 is **as large as possible**.

Complexity: NP-HARD.



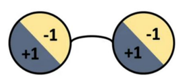
OUTPUT

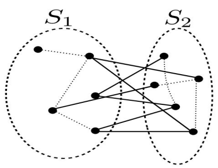
General Process to solve the Max-Cut Problem with a D-Wave machine



BQM Max-Cut reformulation

Ising Model





- Let's take a Graph **G=(X,E)**, where: **X** is the set of nodes and **E** the set of edges.
- We can identify each node **i** of the graph with a binary variable Z_i such that:
  - Z_i = **1**
    - if **i** is in one group *S*
  - Z_i = **-1**
    - if **i** is in the other group.

- **The Max-Cut Problem consists in finding the vector Z of |X| binaries which MINIMIZES:**

$$\boxed{\sum_{(i,j)\in E} Z_i Z_j}$$

- Exemple. If two nodes (**i**,**j**) are in **different** groups:
- $Z_i Z_j$ = **-1**,
  - $Z_i Z_j$ = **1** otherwise.

Let's take a Graph G=(X,E), where **X** is the set of nodes and **E** the set of edges. We can identify each node **i** of the graph with a binary variable Z_i such that:

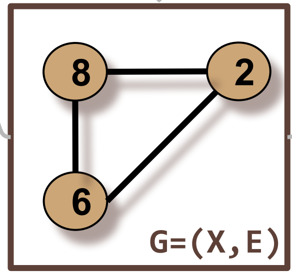
- Z_i = 1 if i is in one group,
- Z_i = -1 otherwise.

Ising Model

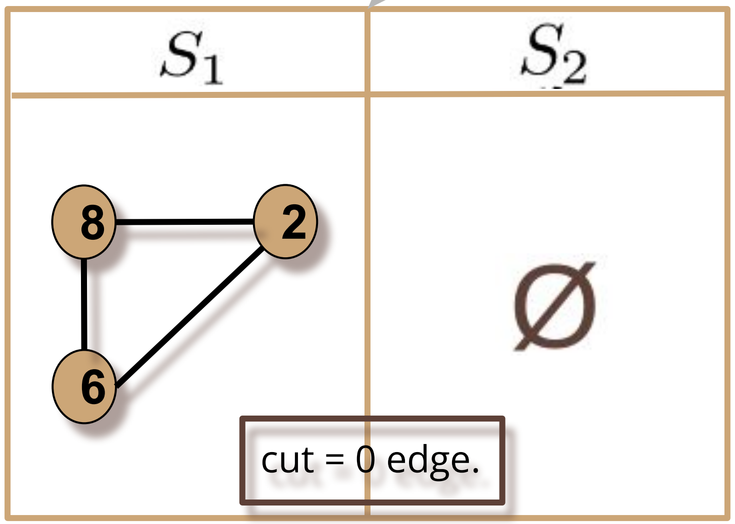
Small Example with a K3 Complete Graph

The Max-Cut Problem consists in finding the vector *Z* of |X| binaries which minimizes:

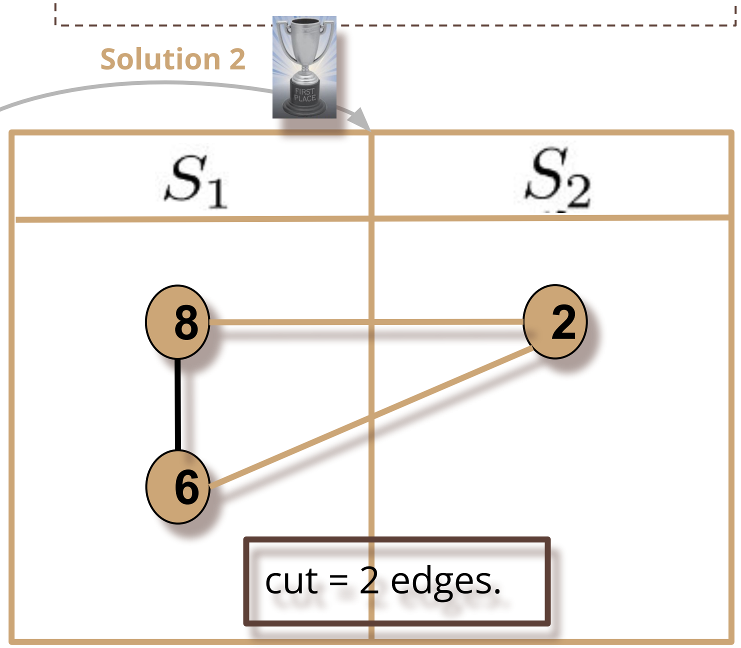
$$\sum_{(i,j)\in E} Z_i Z_j$$



Solution 1



Solution 2



f(Solution 1) = 3

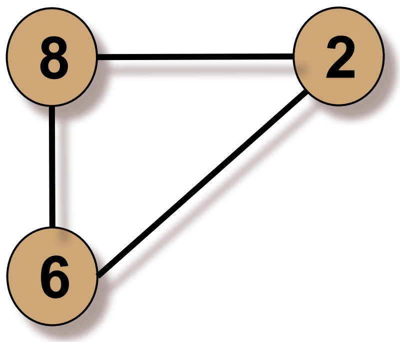
f(Solution i) = $Z_2Z_6 + Z_6Z_8 + Z_8Z_2$

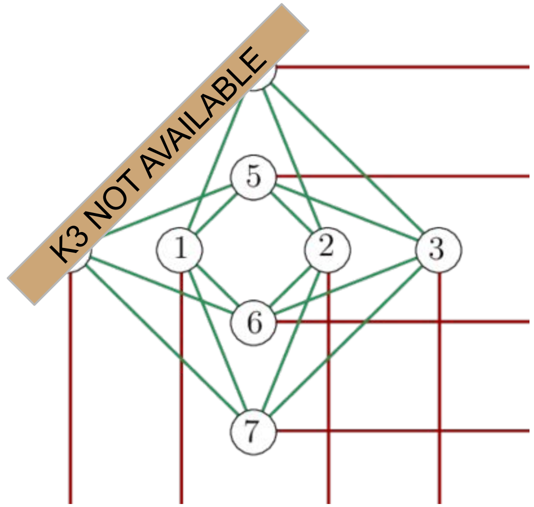
f(Solution 2) = -1

Problem of low connectivity

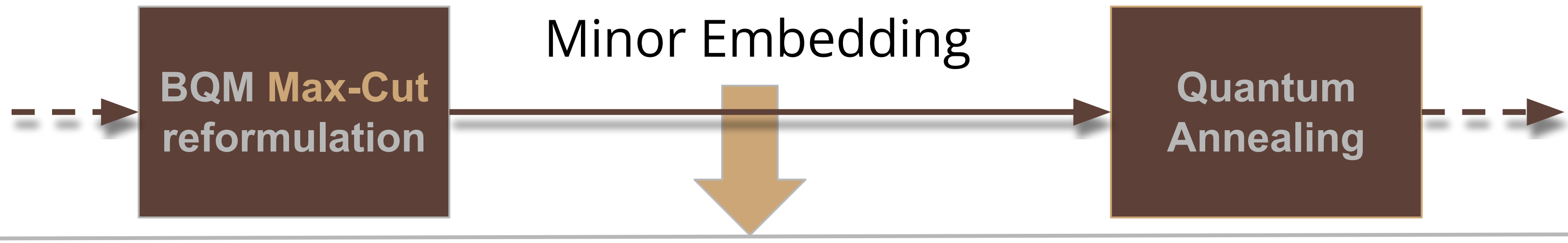
$H = Z_2 Z_6 + Z_6 Z_8 + Z_8 Z_2 +$ ...blablavar...

That requires the following K3 complete graph in the hardware:





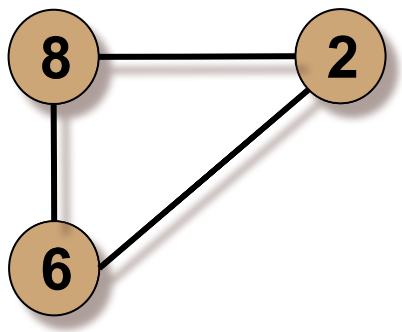
Note: From the Max-Cut Formulation as input To the (hardware) mapping into the QPU

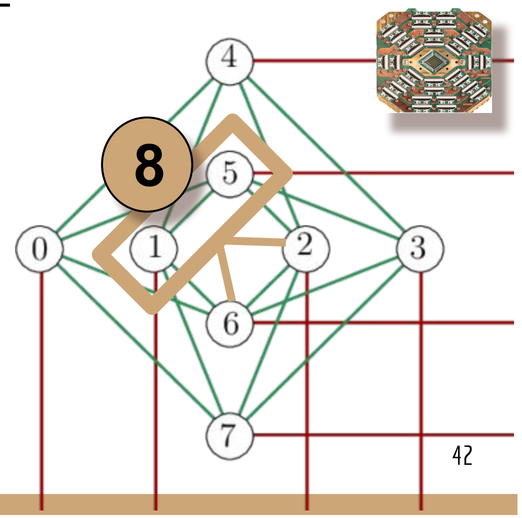


Solution: **1 logical qubit from 2 real qubits**

$H = Z_2Z_6 + Z_6Z_8 + Z_8Z_2 +$ ...blablavar...

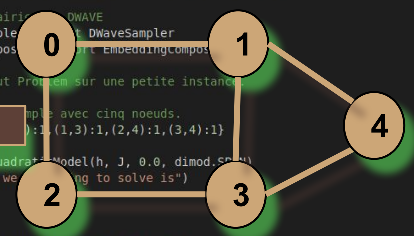
- Generation of **logical qubits**
  - using several physical qubits to “simulate” this connectivity.





Example of **code** for the Max Cut Problem

```
# We enter the 6 edges of our graph with a weight equals to 1.
J = {(0,1):1,(0,2):1,(1,3):1,(1,4):1,(2,3):1,(3,4):1}
h = {} # We do not have external magnetic field in this case
```



No external field (biais)

Couplers / edges no weight (':1')

```
model = dimod.BinaryQuadraticModel(h, J, 0.0, dimod.SPIN)
```

```
##     Quantum Annealing of D-WAVE Advantage Machine            ##
sampler = EmbeddingComposite(DWaveSampler(solver='Advantage_system1.1'))
sampler_name = sampler.properties['child_properties']['chip_id']
response = sampler.sample(model, num_reads=5000)
print("The solution obtained by D-Wave's quantum annealer",sampler_name,"is")
print(response)
```

Number of anneals

```
## Send problem to Sampler
sampleset = sampler.sample_qubo(Q)
```

Example for a QUBO.

D-Wave Quantum Annealing **results**

For sure we did not use all the qubits...

lower cost

```
The solution obtained by D-Wave's quantum annealer Advantage_system1.1 is
   0  1  2  3  4 energy num_oc. chain .
0 -1 +1 +1 -1 +1   -4.0    1398     0.0
1 +1 -1 -1 +1 -1   -4.0    1029     0.0
2 -1 +1 +1 -1 -1   -4.0    1487     0.0
3 +1 -1 -1 +1 +1   -4.0    1084     0.0
4 -1 +1 +1 +1 -1   -2.0       1     0.0
5 +1 -1 -1 -1 +1   -2.0       1     0.0
['SPIN', 6 rows, 5000 samples, 5 variables]
```

Number of occurrences during the 5000 anneals

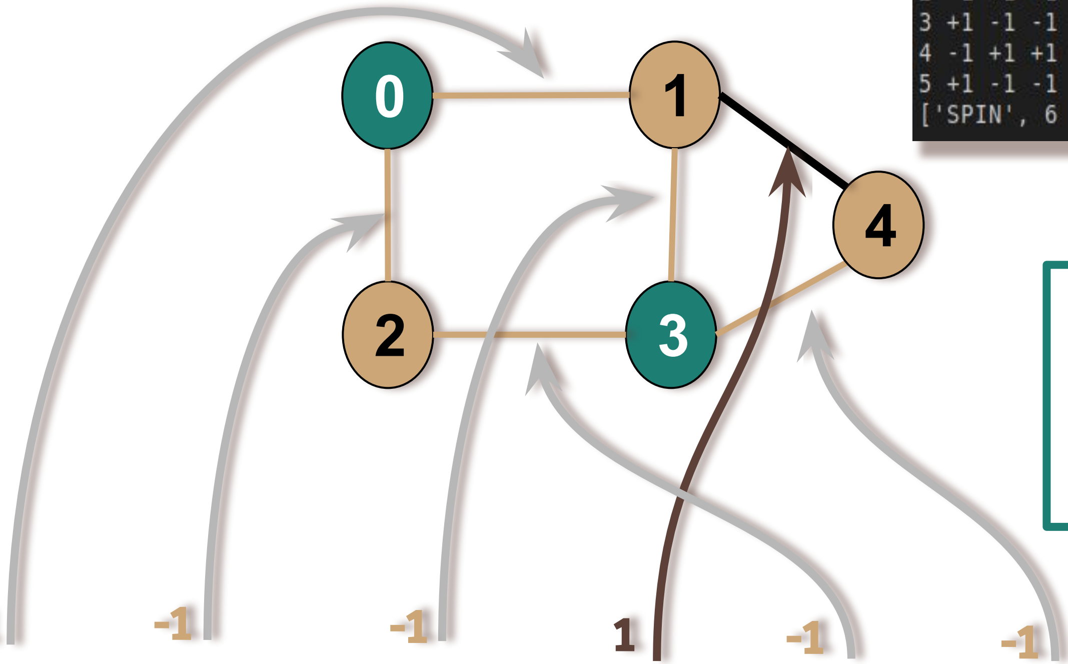
$$H = Z_0Z_1 + Z_0Z_2 + Z_1Z_3 + Z_1Z_4 + Z_2Z_3 + Z_3Z_4 = -5 + 1 = -4$$

$Z_0 = \mathbf{-1}$ $Z_1 = \mathbf{1}$ $Z_2 = \mathbf{1}$ $Z_3 = \mathbf{-1}$ $Z_4 = \mathbf{1}$

Max-cut-Problem: From the solution to the graph.

	0	1	2	3	4	energy	num_oc.	chain_.
0	-1	+1	+1	-1	+1	-4.0	1398	0.0
1	+1	-1	-1	+1	-1	-4.0	1029	0.0
2	-1	+1	+1	-1	-1	-4.0	1487	0.0
3	+1	-1	-1	+1	+1	-4.0	1084	0.0
4	-1	+1	+1	+1	-1	-2.0	1	0.0
5	+1	-1	-1	-1	+1	-2.0	1	0.0

['SPIN', 6 rows, 5000 samples, 5 variables]



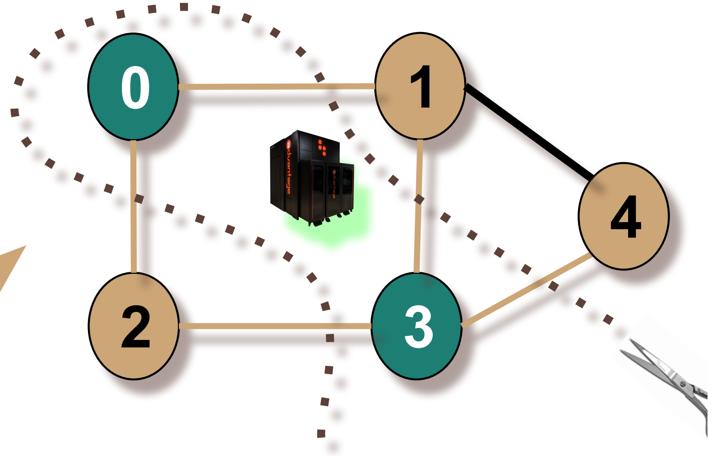
We deduce a **5 edges** cut.

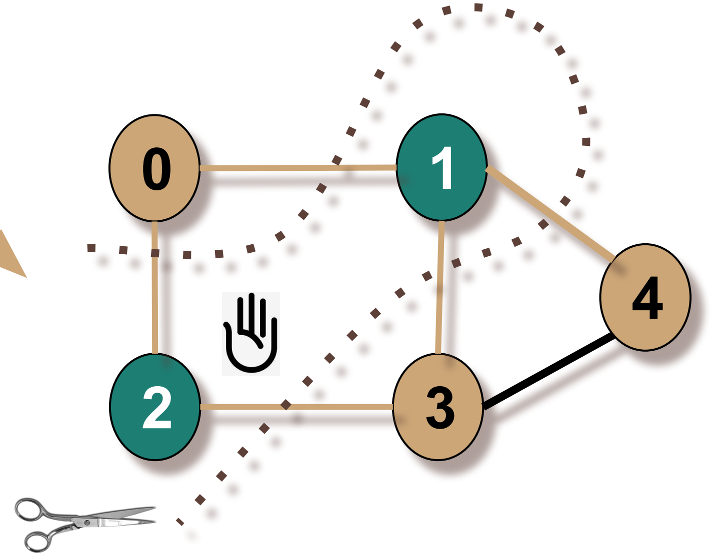
$$H = \overset{-1}{Z_0 Z_1} + \overset{-1}{Z_0 Z_2} + \overset{-1}{Z_1 Z_3} + \overset{1}{Z_1 Z_4} + \overset{-1}{Z_2 Z_3} + \overset{-1}{Z_3 Z_4} = -5+1 = -4$$

$$Z_0 = \mathbf{-1} \quad Z_1 = \mathbf{1} \quad Z_2 = \mathbf{1} \quad Z_3 = \mathbf{-1} \quad Z_4 = \mathbf{1}$$

Max-cut-Problem: several optimal solutions

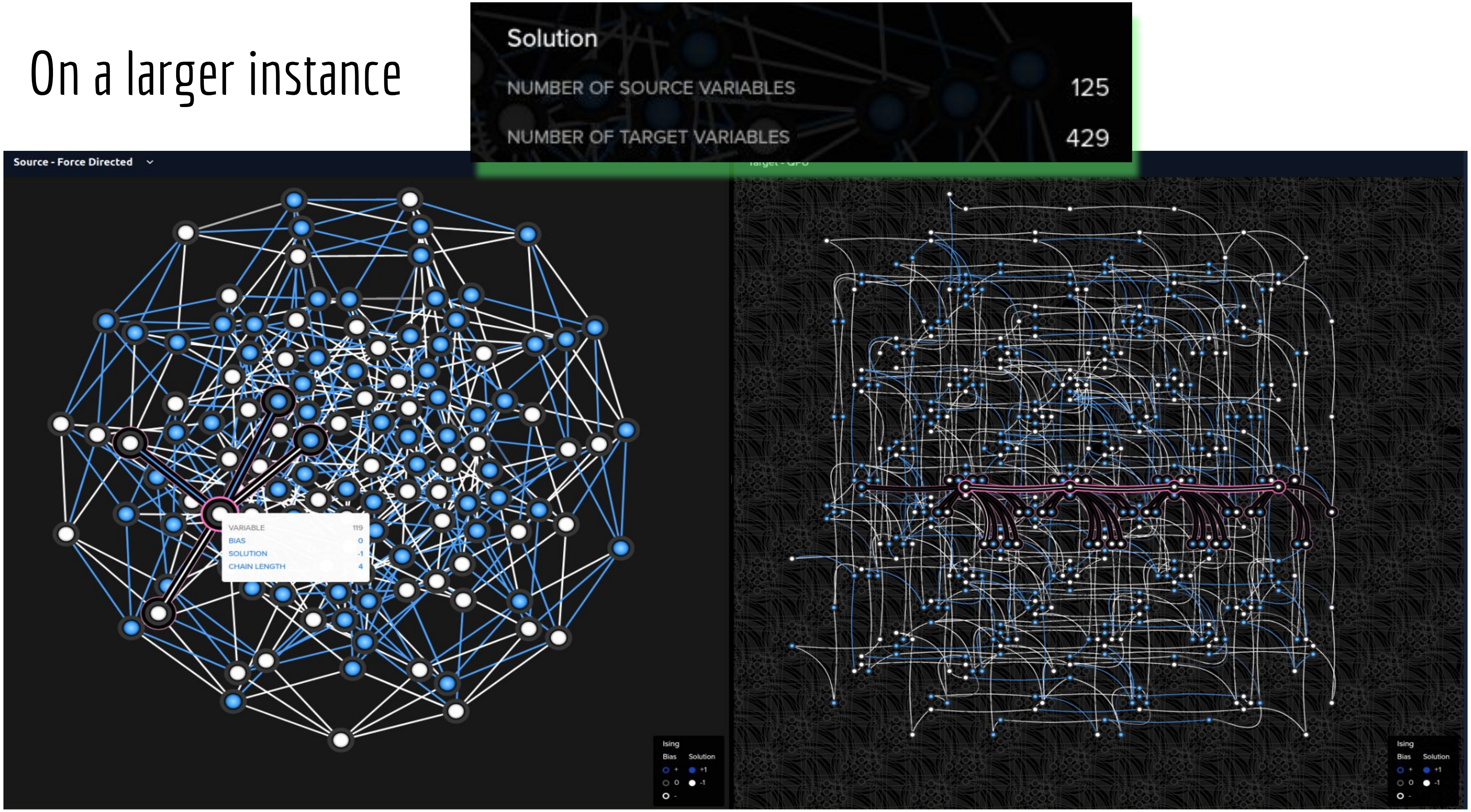
```
The solution obtained by D-Wave's quantum a
   0  1  2  3  4 energy num oc. chain .
0 -1 +1 +1 -1 +1   -4.0    1398      0.0
1 +1 -1 -1 +1 -1   -4.0    1029      0.0
2 -1 +1 +1 -1 -1   -4.0    1487      0.0
3 +1 -1 -1 +1 +1   -4.0    1084      0.0
4 -1 +1 +1 +1 -1   -2.0       1      0.0
5 +1 -1 -1 -1 +1   -2.0       1      0.0
['SPIN', 6 rows, 5000 samples, 5 variables]
```





We obtained ALL the optimal solutions.

On a larger instance



Outline

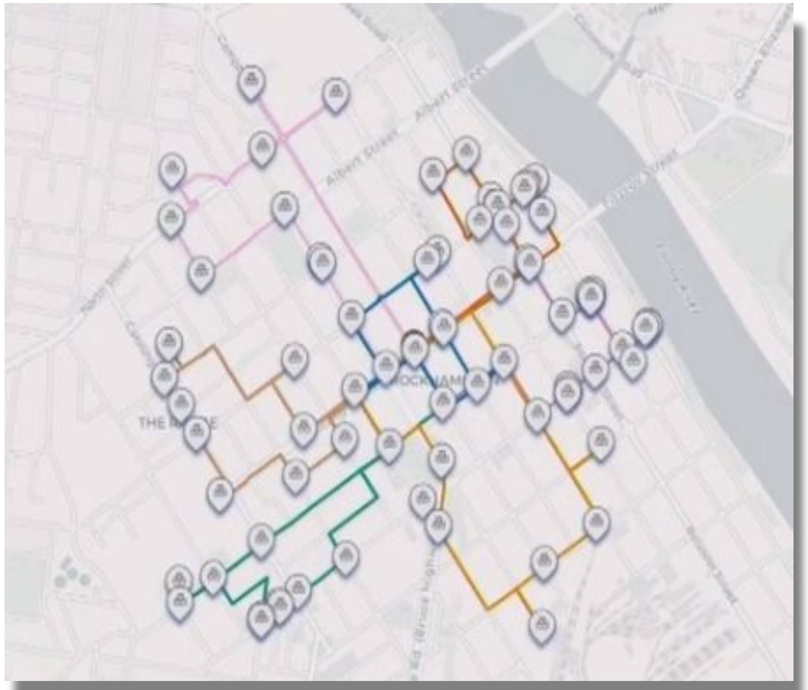
6 **Hybrid Approach**

Hybrid computers

(for both paradigms)

NEC experiment: CVRP

(David Garvin, Nec Australia at Qubits conf. 2021-10)



<u>Constraints</u>: from one depot node, supply of the other nodes with 1 of the **K vehicles** with a capacity,
<u>Objective</u>: Min total distance.

NEC experiment: CVRP

(David Garvin, Nec Australia at Qubits conf. 2021-10)

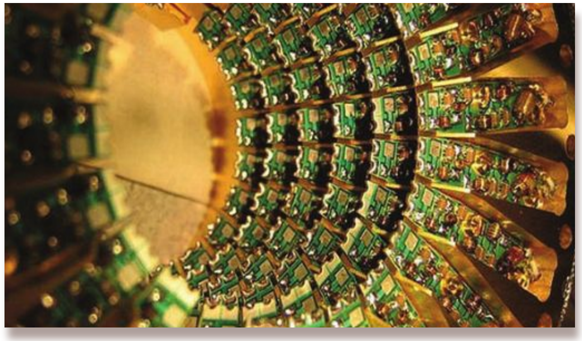
- Resolution Scheme:
  - 1. **Clustering** (for each vehicle)
    - quadratic formulation
    - quantum computer
  - 2. **Routing** (for each vehicle)
    - basic heuristic solving the TSP
    - classical computer

min problem

$$f_{clustering} = \sum_{ij} \sum_{k} c_{ij} x_{ik} x_{jk}$$

$$\sum_{i} x_{ik} u_i \leq C_k$$

$$\sum_{k} x_{ik} = 1$$



Outline

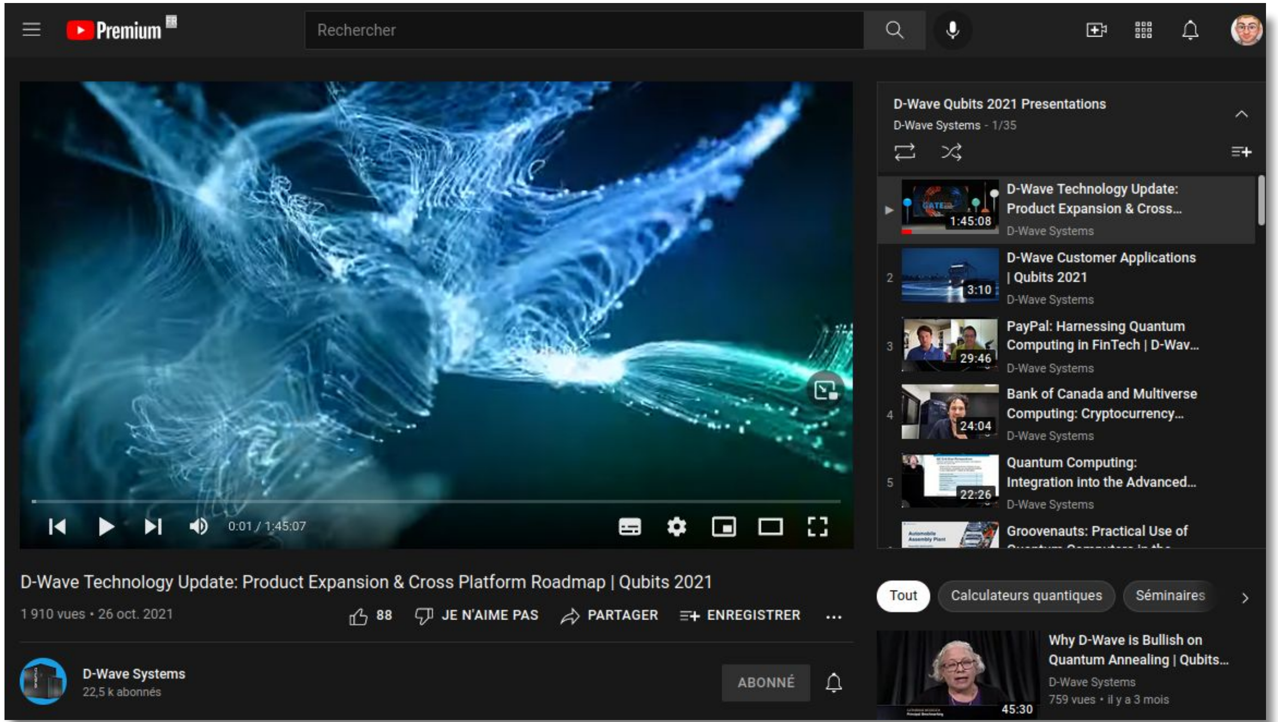
Summary,
General Results,
and Conclusions

PASQAL

Summary

- **Another paradigm**: Method NOT based on universal gates models,
  - But work on smaller number of problems as ones in machine learning and optimization fields;
- Huge number of qubits, but not controllable as with the universal quantum computers,
  - Not programmable, no universal gates to apply during the process,
  - Different connectivities between qubits,
  - Something they have in commun…a lot of errors!
- The Quantum Annealing could be called a “Hardware Heuristic” letting the physics solve the problem,
- A real potential on large scale optimization problems.

To Go Further



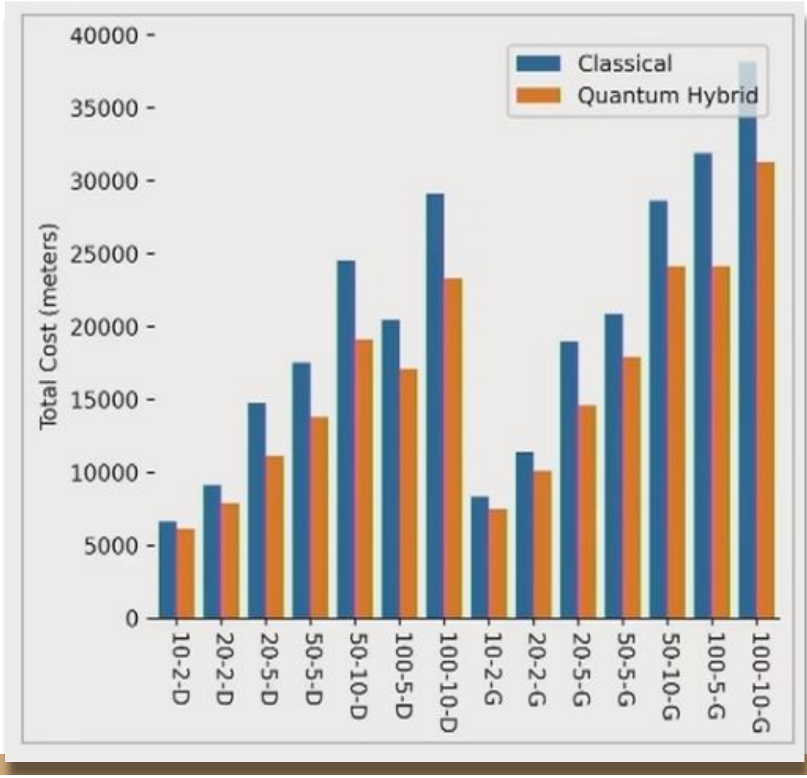
https://www.youtube.com/watch?v=lAeZ_hs9OBw

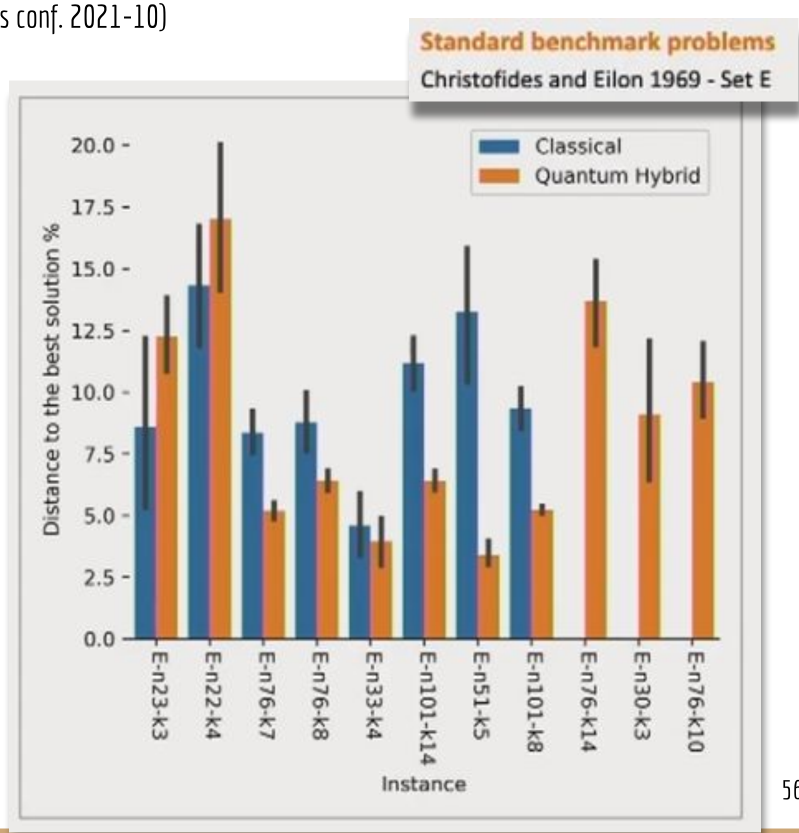
Some Results on a 1K and 2K qubits Machine

- D-Wave 2X again (1K qubits) runs up to **100M times faste**r than an optimized implementation of the quantum Monte Carlo algorithm **on a single core**.
    - Denchev, V.S., Boixo, S., Isakov, S.V., Ding, N., Babbush, R., Smelyanskiy, V., Martinis, J. and Neven, H., 2016. What is the computational value of finite-range tunneling?. Physical Review X, 6(3), p.031015.
- D-Wave newsletter of the **18/02/2021** about results on 2K machine:
    - **“**Today D-Wave marks a major milestone on the journey to quantum advantage in a new peer-reviewed paper published in *Nature Communications*. The new research uses a D-Wave lower noise system to demonstrate **3 million times speed-up over classical alternatives in a real-world problem**. [...]**”**
    - The Paper: King, A.D., Raymond, J., Lanting, T. et al. Scaling advantage over path-integral Monte Carlo in quantum **simulation of geometrically frustrated magnets**. Nature Communication 12, 1113 (2021).
    - **They compared the QA against the best known classical simulation algorithm for this problem (PIMC).**
    - To know which CPU is used in comparison (also the number of threads used), we need to go in the supplementary materials (they use a light **i7-8650U** **on one core**! To compare, if it is still possible, and if the algo is fully parallelizable, the Fujitsu A64FX will win since it has more than 7 millions of (ARM) cores)!
- Be careful with all the announcements in the media...and also from the companies!
- But still… waiting for results on **Advantage**(5k qubits and 15 connectivity) in 1 or 2 years!

Examples of the first results: a NEC experiment CVRP

(David Garvin, Nec Australia at Qubits conf. 2021-10)





Maybe not for all optimization problems…

$$
\begin{array}{llr}
\min & \sum_{r\in R} z_{n_r} & (1)\\
\text{s.t.} & & \\
& x_e^{end} = x_{e+1}^{begin}, \quad e \in RE_r, r\in R : e = 1..n_r - 1, & (2)\\
& x_e^{end} \geq x_e^{begin} + d_e, \quad e\in E, & (3)\\
& x_e^{begin} \geq \tau_e^{begin}, \quad e \in E : i_e = 1, & (4)\\
& x_{\hat{e}}^{end} - \tau_e^{end} \leq z_e, \quad e\in E, & (5)\\
& \sum_{t\in T_s} q_{et} = 1, \quad e\in SE_s, s\in S, & (6)\\
& q_{et} + q_{\hat{e}t} - 1 \leq \lambda_{e\hat{e}} + \gamma_{e\hat{e}}, \quad e,\hat{e}\in SE_s, t\in T_s, s\in S : e < \hat{e}, & (7)\\
& x_{\hat{e}}^{begin} - x_e^{end} \geq \Delta_s^M \gamma_{e\hat{e}} - M(1-\gamma_{e\hat{e}}), e < \hat{e} \text{ in } SE_s, s\in S, o_e \neq o_{\hat{e}}, & (8)\\
& x_{\hat{e}}^{begin} - x_e^{end} \geq \Delta_s^F \gamma_{e\hat{e}} - M(1-\gamma_{e\hat{e}}), e < \hat{e} \text{ in } SE_s, s\in S, o_e = o_{\hat{e}}, & (9)\\
& x_e^{begin} - x_{\hat{e}}^{end} \geq \Delta_s^M \lambda_{e\hat{e}} - M(1-\lambda_{e\hat{e}}), e < \hat{e} \text{ in } SE_s, s\in S, o_e \neq o_{\hat{e}}, & (10)\\
& x_e^{begin} - x_{\hat{e}}^{end} \geq \Delta_s^F \lambda_{e\hat{e}} - M(1-\lambda_{e\hat{e}}), e < \hat{e} \text{ in } SE_s, s\in S, o_e = o_{\hat{e}}, & (11)\\
& \lambda_{e\hat{e}} + \gamma_{e\hat{e}} \leq 1, \quad e,\hat{e}\in SE_s, s\in S : e<\hat{e}, & (12)\\
& x_e^{begin} \geq w_t^{end} q_{et} - M\alpha_e^{w_t}, \quad e\in SE_s, t\in T_s, s\in S, & (13)\\
& x_e^{end} \leq w_t^{begin} q_{et} + M(1-\alpha_e^{w_t}), \quad e\in SE_s, t\in T_s, s\in S, & (14)\\
& x_e^{begin}, x_e^{end}, z_e \geq 0, \quad e\in E, & (15)\\
& \gamma_{e\hat{e}}, \lambda_{e\hat{e}}, \in \{0,1\}, \quad e,\hat{e}\in SE_s, s\in S : e<\hat{e}, & (16)\\
& q_{et}, \alpha_e^{w_t} \in \{0,1\}, \quad e\in SE_s, t\in T_s, s\in S, & (17)
\end{array}
$$

Not a relaxing relaxation...

Future of the quantum annealing tech

- Several Projects in Europe:
  - **AVaQus**
    - the European project developing the first superconducting quantum annealer
    - https://www.quantaneo.com/AVaQus-the-European-project-to-develop-the-first-superconducting-coherent-quantum-annealer_a479.html
  - **ATOS**
    - Quantum Annealing **Simulator**
    - Scope: Machine Learning
    - https://atos.net/en/2020/press-release_2020_07_07/atos-opens-up-a-new-path-to-quantum-annealing-simulation
  - **Qilimanjaro**

Outline

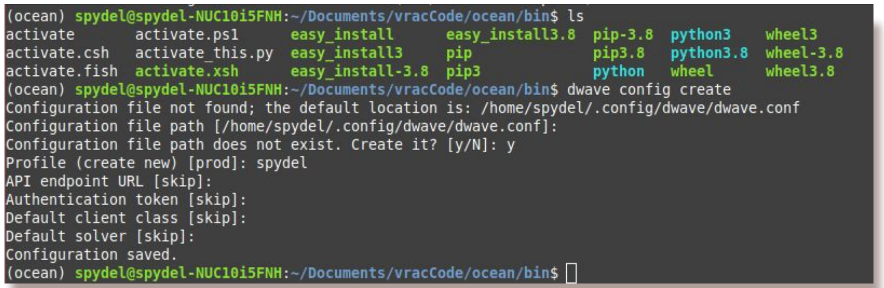
References & Co

- Some Help to install D-Wave Ocean API
- Several types of references:
  - Books, Vidéos, Conf', Papers, Courses,
  - From vulgarisation to theoretical.

Some help to install the D-Wave Ocean API on Debian based machine

- Linux commands :
  - `sudo apt-get install python<version>`
  - `sudo pip install virtualenv`
  - `virtualenv ocean`
  - `pip install dwave-ocean-sdk`
  - `git clone https://github.com/dwavesystems/dwave-ocean-sdk.git`
  - `cd dwave-ocean-sdk` ou `cd ocean`
  - `python setup.py install` ou `./python ./easy_install install`
  - Add to your cod the token available in your dashboard

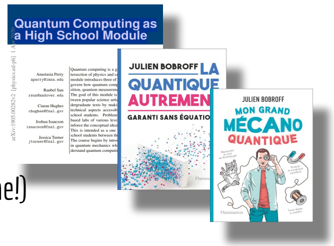
- More detail: https://docs.ocean.dwavesys.com/en/stable/overview/install.html



General popularization works

(from the easiest to the hardest)

- Article (100p): Quantum Computing as a High School Module https://arxiv.org/abs/1905.00282
- Book (fr): La quantique autrement. Julien Bobroff.
- Book (fr): Mon grand mécano quantique. Julien Bobroff.



- Video: "Les Ordinateurs Quantiques Expliqués - Limites de la technologie humaine" (so well done!) https://www.youtube.com/watch?v=JhHMJCUmq28&ab_channel=Kurzgesagt%E2%80%93InaNutshell
- Website article: "You don't need to be a mathematician to master quantum computing" https://towardsdatascience.com/you-dont-need-to-be-a-mathematician-to-master-quantum-computing-161026af8878
- Slides: US Department of energy: How about quantum computing. Bert de Jong https://cs.lbl.gov/assets/CSSSP-Slides/20190624-deJong.pdf

- Videos: Understanding Quantum Mechanics. Sabine Hossenfelder. https://www.youtube.com/watch?v=XJSfgE9LUJw&list=PLwgQsqtH9H5djIfFhXE6We207beTgUnyL
- Book: Quantum computation and quantum information. Nielsen & Chuang

Research Papers

- Simulated Annealing: Kirkpatrick, Scott, C. Daniel Gelatt, and Mario P. Vecchi. "Optimization by simulated annealing." science 220.4598 (1983): 671-680.
- Smelyanskiy, Vadim N., et al. "A near-term quantum computing approach for hard computational problems in space exploration." arXiv preprint arXiv:1204.2821 (2012).
- Hamerly, R., Inagaki, T., McMahon, P.L., Venturelli, D., Marandi, A., Onodera, T., Ng, E., Langrock, C., Inaba, K., Honjo, T. and Enbutsu, K., 2019. Experimental investigation of performance differences between coherent Ising machines and a quantum annealer. *Science advances*, *5*(5), p.eaau0823.
    - Here the MaxCut problem is solved on DW2Q has 2048 qubits (lets try with a 5K!)
- Barahona, Francisco; Grötschel, Martin; Jünger, Michael; Reinelt, Gerhard (1988). "An Application of Combinatorial Optimization to Statistical Physics and Circuit Layout Design". *Operations Research*. **36** (3): 493–513.
- Barahona. "On the Computational Complexity of Ising Spin Glass Models." J. Phys. A 15 (1982), pp. 3241–3253.
    - About the NP-Hard complexity of Ising and QUBO Problems
- Mengoni, Riccardo, Daniele Ottaviani, and Paolino Iorio. "Breaking RSA Security With A Low Noise D-Wave 2000Q Quantum Annealer: Computational Times, Limitations And Prospects." *arXiv preprint arXiv:2005.02268* (2020).
- King, James, et al. "Benchmarking a quantum annealing processor with the time-to-target metric." arXiv preprint arXiv:1508.05087 (2015).
- Analytical and numerical evidence suggests that quantum annealing outperforms simulated annealing under certain conditions. Heim, B., Rønnow, T. F., Isakov, S. V., & Troyer, M. (2015). Quantum versus classical annealing of Ising spin glasses. Science, 348(6231), 215-217.
- Venegas-Andraca, Salvador E., et al. "A cross-disciplinary introduction to quantum annealing-based algorithms." *Contemporary Physics* 59.2 (2018): 174-197.
- Albash, Tameem, Victor Martin-Mayor, and Itay Hen. "Temperature scaling law for quantum annealing optimizers." Physical review letters 119.11 (2017): 110502.

Courses

- “A practical introduction to quantum computing: from qubits to quantum machine learning and beyond (5/7)”
    - 7 courses from the CERN Quantum Technology Initiative.
    - (5/7): “Quantum algorithms for combinatorial optimization. Quantum adiabatic computing and quantum annealing. Introduction to D-Wave Leap. Quantum Approximate Optimization Algorithm.”
    - https://cds.cern.ch/record/2746545
- John Preskill lectures:
    - http://theory.caltech.edu/~preskill/ph229/
- Serge Haroche (collège de France lectures)
    - https://www.college-de-france.fr/site/serge-haroche/_course.htm

D-Wave ressource

- Leap:
  - https://cloud.dwavesys.com/leap/
- Qubo & Ising Models
  - https://docs.dwavesys.com/docs/latest/c_gs_3.html
- D-Wave examples on Github
  - https://github.com/dwave-examples
- Tutos (Videos and articles) for developers
  - https://www.dwavesys.com/practical-quantum-computing-developers
- Tuto for installing the Ocean API:
  - https://docs.ocean.dwavesys.com/en/stable/overview/install.html
- A tons of videos from D-Wave:
  - https://www.youtube.com/user/dwavesystems/playlists

This is the end

...

Thanks for your attention!

(samuel.deleplanque@junia.com)

pic: https://www.youtube.com/watch?v=JhHMJCUmq28&ab_channel=Kurzgesagt%E2%80%93InaNutshell