Concordia University
Department of Mechanical, Industrial and Aerospace Engineering

Aeroelasticity MECH 6481 - Fall 2024

Handout #2 - The $p - k$ method

Instructor: ©Dr. Mojtaba Kheiri

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For a pitching-plunging airfoil section, the flutter determinant based on the p-k method becomes as follows:

$$\begin{vmatrix} p^2 + f_{11} & p^2 x_\theta + f_{12} \\ p^2 x_\theta + f_{21} & r^2 p^2 + f_{22} \end{vmatrix} = 0.$$

The expansion of the above determinant yields a polynomial of degree 4 in p:

$$(r^2 - x_\theta{}^2)p^4 + (f_{22} + f_{11} r^2 - f_{21} x_\theta - f_{12} x_\theta)p^2 + f_{11} f_{22} - f_{12} f_{21} = 0,$$

where f_{ii} $(i = 1, 2)$ may be written as:

$$\begin{aligned}
f_{11} &= \frac{\sigma^2}{V^2} - \frac{k^2}{\mu} + \frac{2ikC(k)}{\mu}, \\
f_{12} &= \frac{k(i + ak) + [2 + ik(1 - 2a)]C(k)}{\mu}, \\
f_{21} &= \frac{ak^2 - ik(1 + 2a)C(k)}{\mu}, \\
f_{22} &= \frac{\dfrac{8\mu r^2}{V^2} + 4i(1 + 2a)[2i - k(1 - 2a)]C(k) - k[k - 4i + 8a(i + ak)]}{8\mu}
\end{aligned}$$

where $C(k)$ or Theodorsen's function may be approximated as

$$C(k) = \frac{0.01365 + 0.2808ik - \dfrac{k^2}{2}}{0.01365 + 0.3455ik - k^2}.$$

The solution begins with assuming an arbitrary value for k or the reduced frequency. For a given system parameters, i.e. μ, σ, r^2, a, and x_θ, and for a given value of the reduced velocity, V, we can find values of f_{ii} $(i = 1, 2)$ and thus we will be able to find the roots of p. For the system in hand, we will have four roots. We choose, for example, the first root, i.e. p_1, and using the following equations, we obtain the new value of k and the value of damping ratio γ:

$$k = Im(p_1), \quad \gamma = \frac{Re(p_1)}{k}$$

With the new value of k, we repeat the procedure outlined above until k is converged to a value. This way, γ will also get converged automatically. The obtained values of k and γ are only for one of the modes – here the first root (or mode). We need to repeat the whole procedure for the other three roots.

After finding converged values of k and γ for all four roots (or modes), we change the value of V and repeat all the procedure outlined above.

You may use 'sort' function of MATLAB to sort the roots. You may sort the roots according to the imaginary part of the roots, which is a measure of the frequency of vibration.