

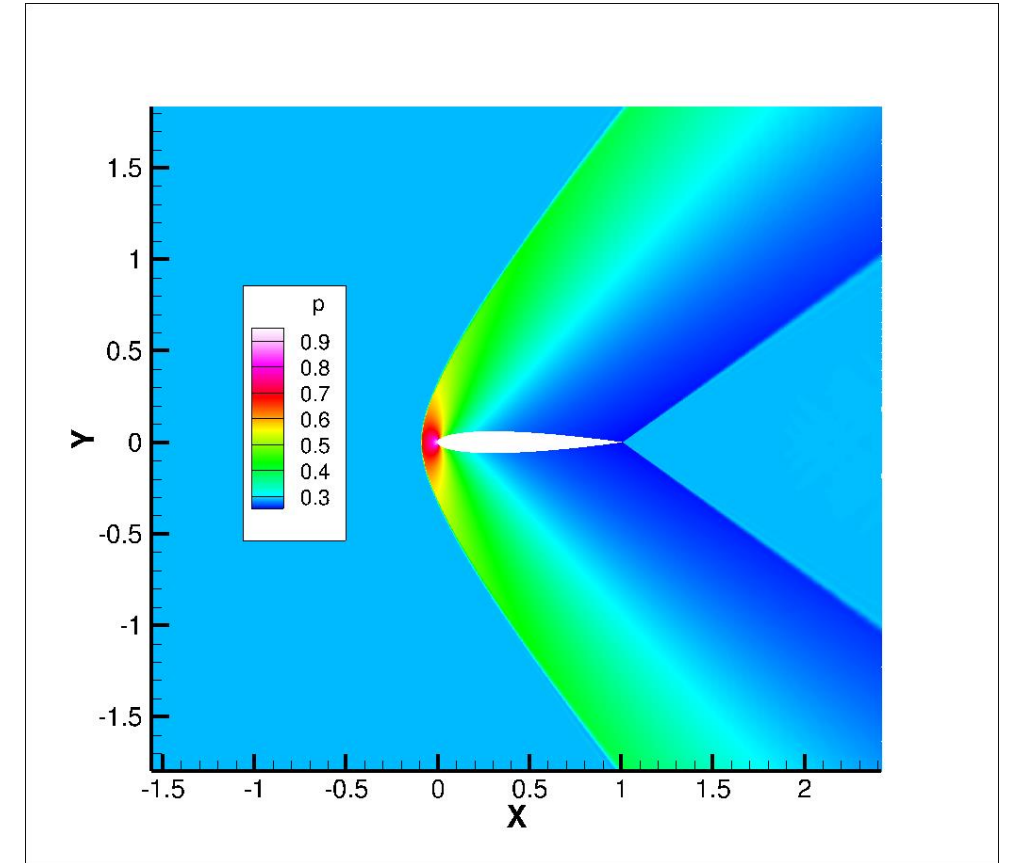


**POLYTECHNIQUE
MONTRÉAL**

Homework 3

Development of a 2D Euler CFD solver

MEC6602E – Transonic Aerodynamics



2D Euler equations

- **Local form**

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \cancel{\frac{\partial(\rho w)}{\partial z}} = 0$$

Continuity equation

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u \vec{V}) + \nabla p = 0$$

Momentum equations

$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v \vec{V}) + \nabla p = 0$$

$$\frac{\partial(\rho E)}{\partial t} + \nabla \cdot (\rho E \vec{V}) + \nabla \cdot (p \vec{V}) = 0$$

Energy equations

2D Euler equations

- **Integral form** : used in the numerical solver

$$\int_{\Omega} \frac{\partial \vec{W}}{\partial t} + \oint_S \vec{F}_c dS = 0$$

$$\vec{W} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \end{pmatrix} \quad \text{3D}$$

$$\vec{F}_c = \begin{pmatrix} \rho V_c \\ \rho u V_c + n_x p \\ \rho v V_c + n_y p \\ \rho w V_c + n_z p \\ \rho E V_c + p V_c \end{pmatrix} \quad \text{3D}$$

Contravariant velocity: $V_c = \vec{V} \cdot \vec{n}$

Code overview

- **Language** : Python
 - **Cons** :
 - Lack of low-level optimizations : No direct memory management or hardware-level optimizations.
 - Higher memory usage : Inefficient for handling large-scale simulations compared to low-level languages.
 - Interpreted language : Slower execution compared to compiled languages like C or Fortran.
 - **Pros** :
 - High level language : User-friendly.
 - Object-oriented programming language : not the case of C language
 - Possibility of relying on pre-compiled libraries (e.g. Numpy) to accelerate numerical computations
- **Structure**: Oriented object
 - **Classes** : *Mesh, ConservedVariables, FluxDiscretizationScheme, TimeIntegrationMethod, Monitor*

Code overview

- **Type of CFD solver** : structured, cell-centered
- **Numerical methods**
 - Flux discretization : **Central scheme with Artificial Dissipation** (order 1,2)
 - Time Integration : **Explicit Euler** (order 1), **RK2** (order 2)
 - Global & Local Time stepping
- **Inputs** : Mesh (plot3D format), AOA, Mach number, Numerical Parameters
- **Outputs** : Flow solution (.dat file), **CL**, **CD**, **CM_{c/4}** coefficients, residuals evolution, **Cp** curve, computational time

Validation case : NACA0012 airfoil in freestream flow

- **Symmetrical airfoil** : $CL = 0$, $CM_{c/4} = 0$ @ 0 deg. AOA
- Comparison with results from Vassbert et Jameson [1] for different flow conditions (see Tab.1)
- Family of grids used : **O-grids, from (8x8 to 2049x2049)**
- BCs : wall, farfield, connect

(M, α)	Nonlifting	Lifting
Subcritical	(0.5M, 0 deg)	(0.5M, 1.25 deg)
Transonic	(0.8M, 0 deg)	(0.8M, 1.25 deg)

Tab.1 : Comparison cases with [1]

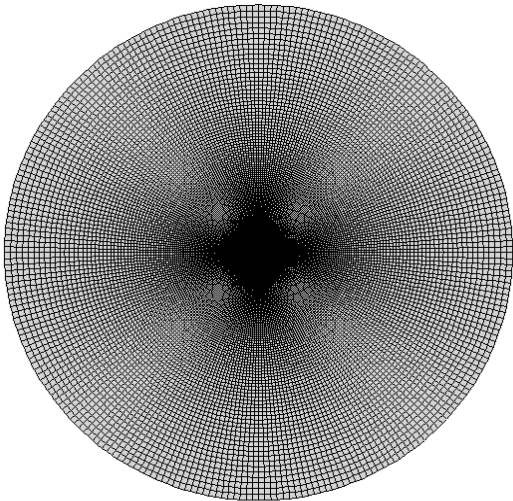


Fig.1.a : Overview of the whole mesh
(256x256)

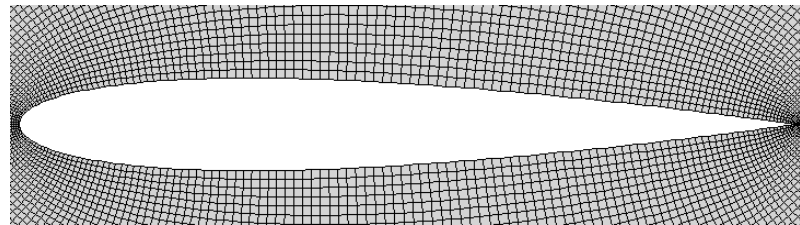
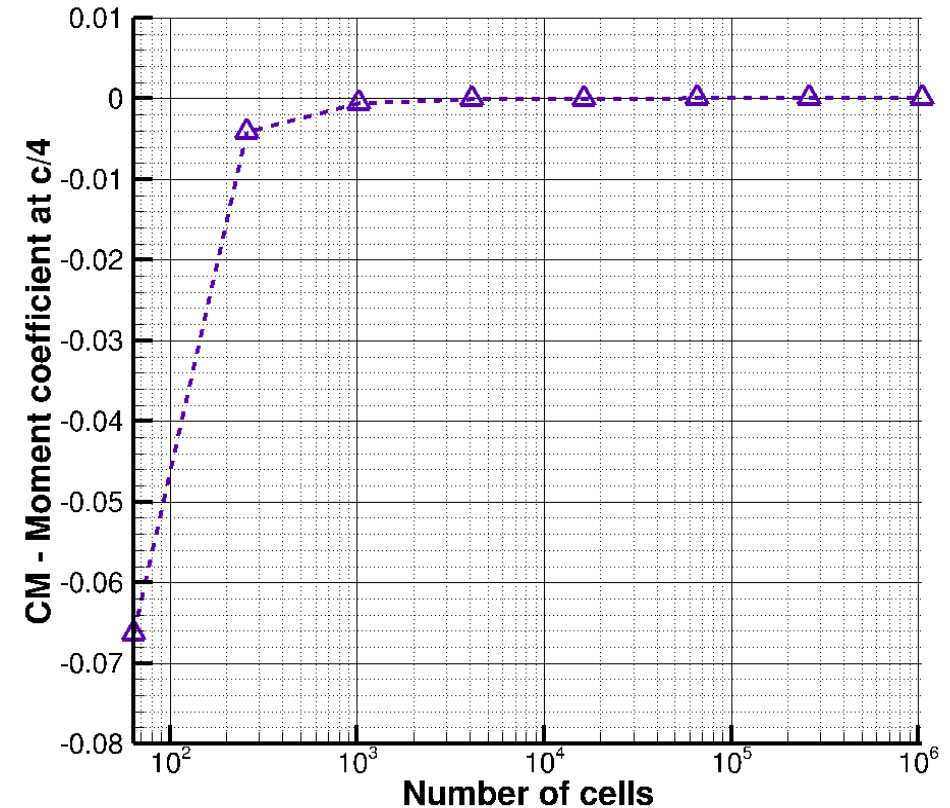
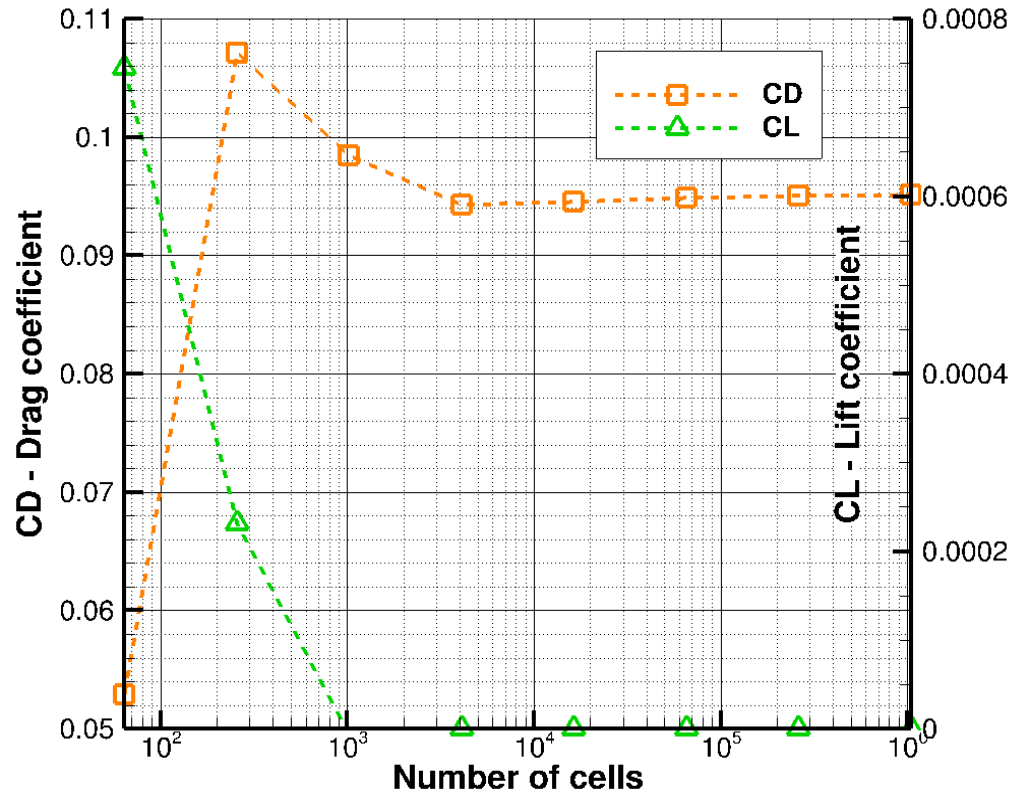


Fig.1.b : Closeup view of the mesh
near the airfoil (256x256)

Grid Convergence

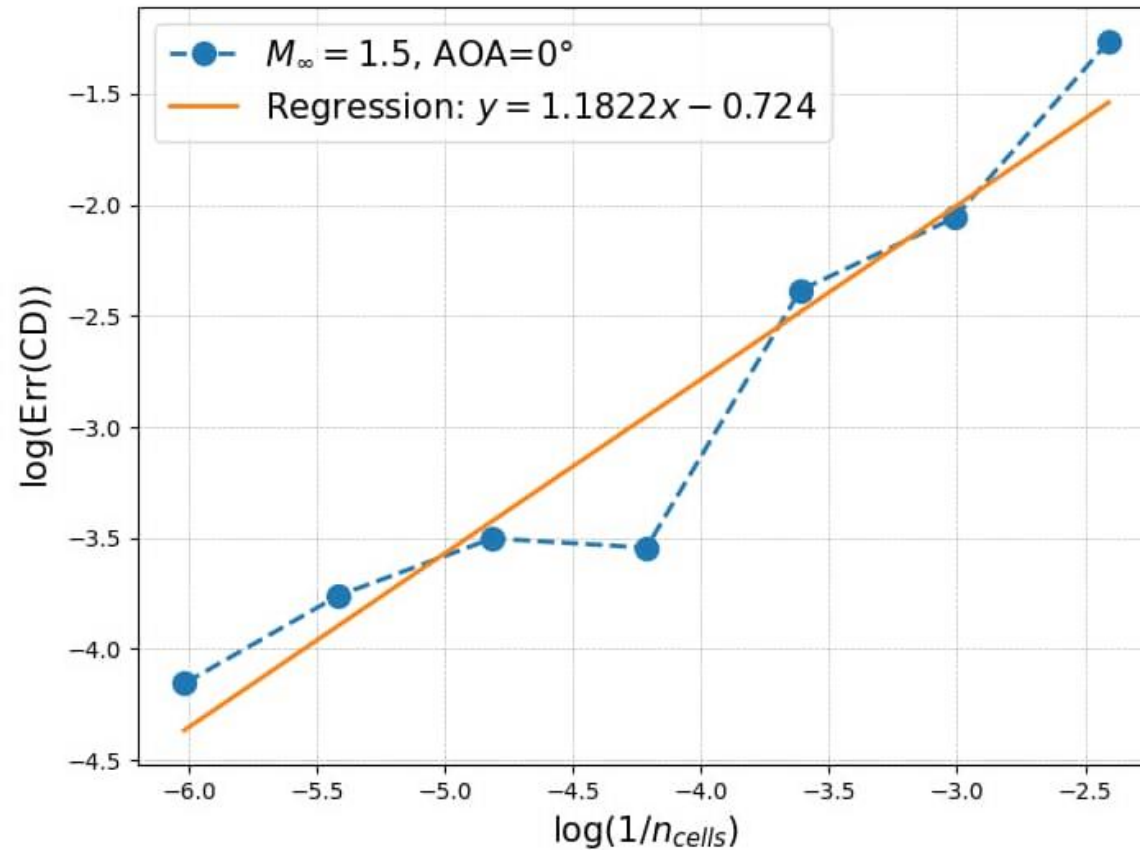
- Performed for supersonic flow : **M=1.5, AOA=0°**
- Numerical parameters (CFL, dissipation constants) are kept constant across the cases to compare convergence and computational time
- Time Integration : RK2 (CFL = 1)
- Flux discretization : Central scheme with AD ($k_2 = \frac{1}{2}$, $k_4 = 1/64$)
- Residual decrease (density) : 10^{-10} to consider the case as converged

Grid Convergence



- Convergence in aerodynamic coefficients reached for the 128x128 mesh
- Converged values : $CL = 0$, $CM_{c/4} = 0$, $CD = 0.095134$ (due to shock)

Order of Convergence



Grid Convergence

- **Exponential rise in computational time** observed at grid sizes of 1024 and 2048 cells.

Mesh Size	Convergence	Runtime [hh:mm:ss]
1024	10^{-10}	8:58:23
2048	10^{-6}	51:52:42

- Optimized **parallel processing** or **advanced algorithms** are necessary for scalability in industrial applications

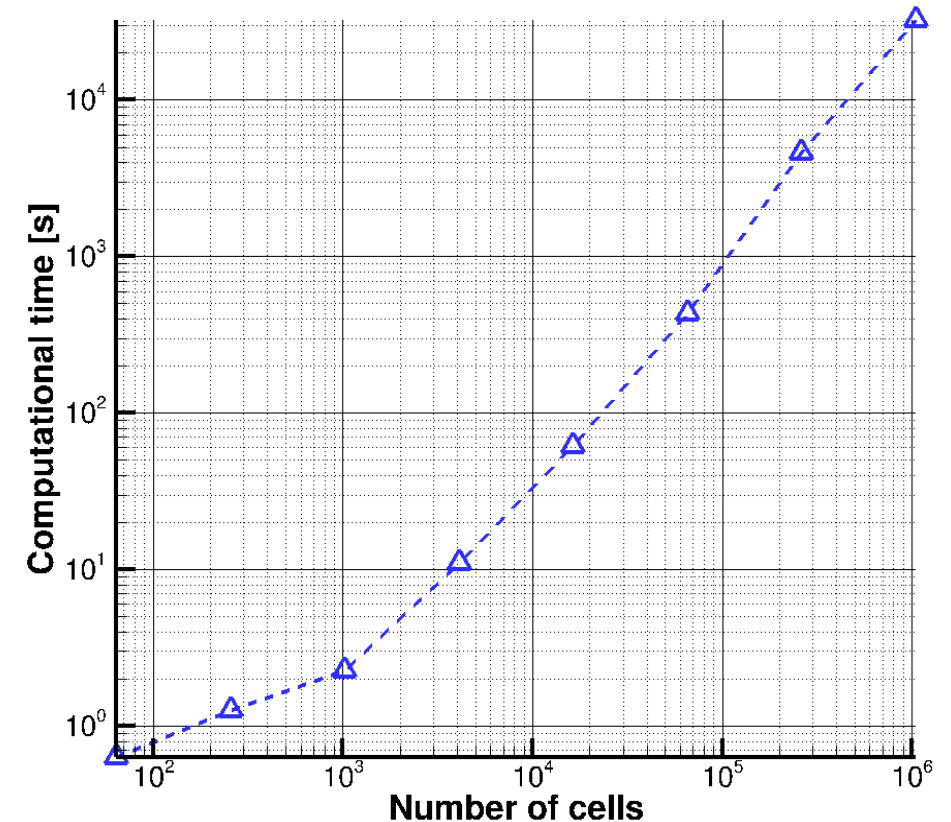
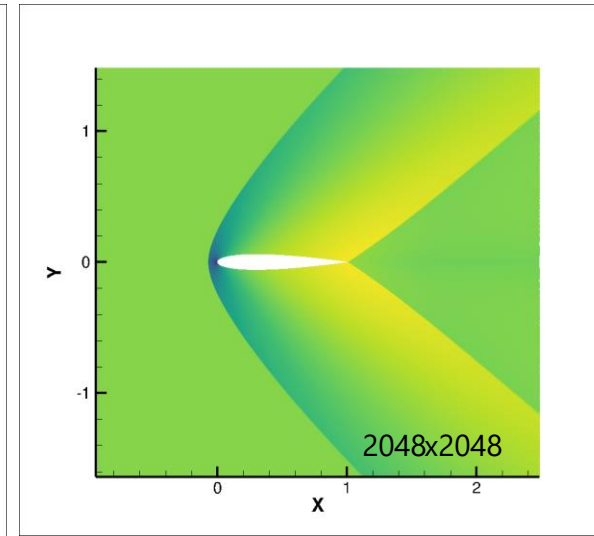
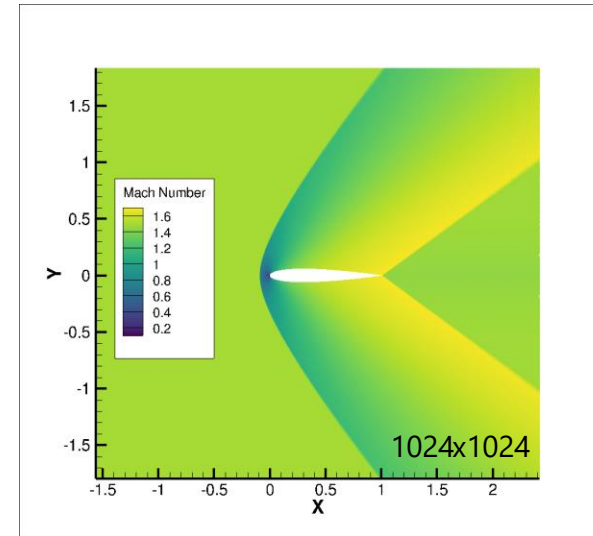
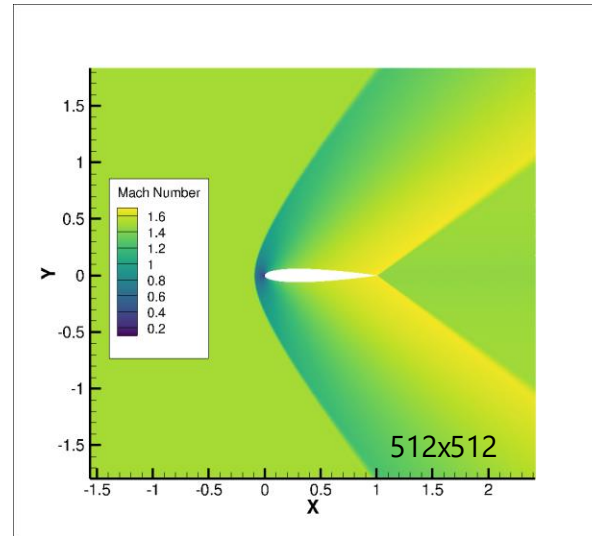
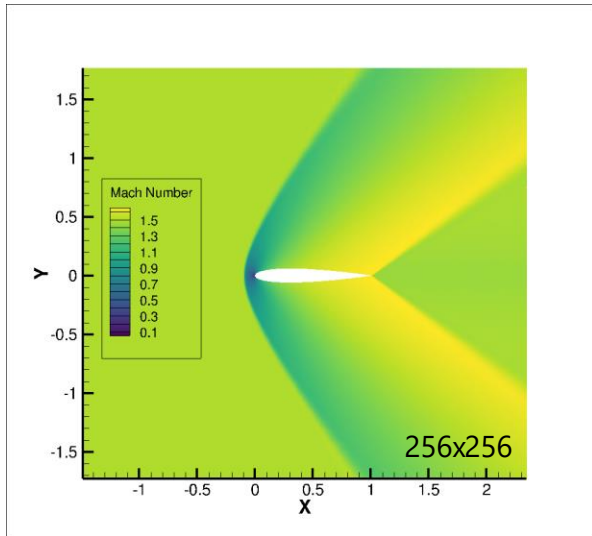
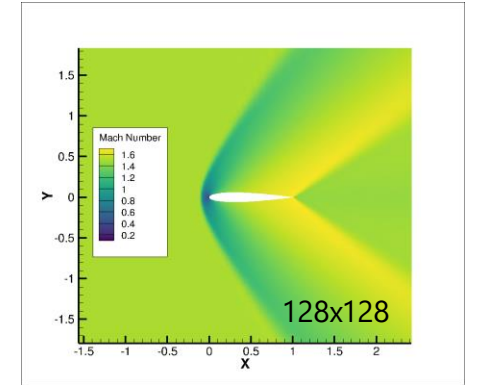
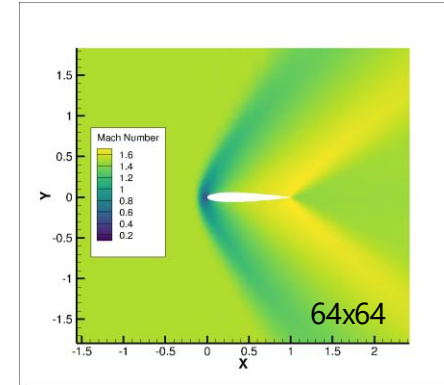
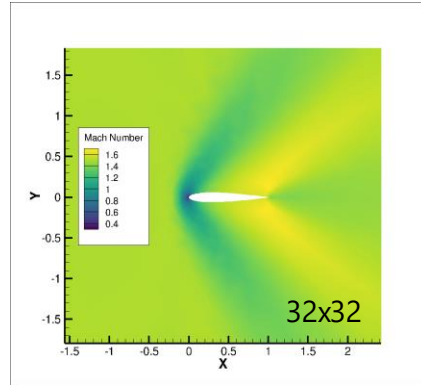
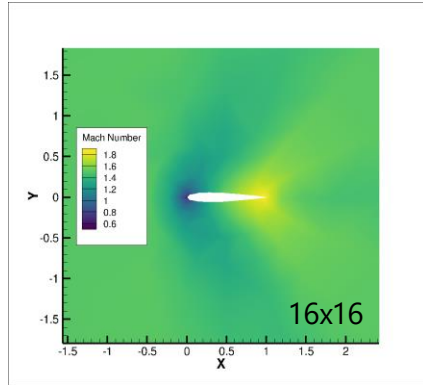
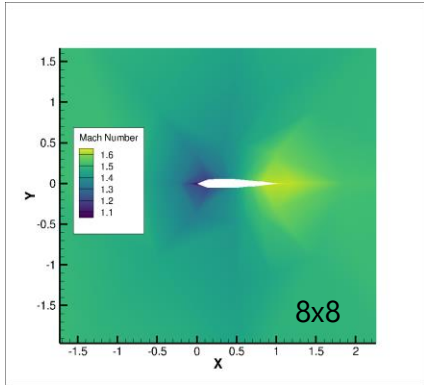


Figure: Computational Time vs. Grid Size: Impact of Increasing Resolution ¹.

¹ Results for grid convergence have not been presented for the largest mesh. That mesh was only computed till a residual of 10^{-6} .

Grid Convergence



Analysis of Results for the 1024x1024 Grid

- Bow shock at LE and shock at TE
- Symmetrical solution since the problem is symmetric

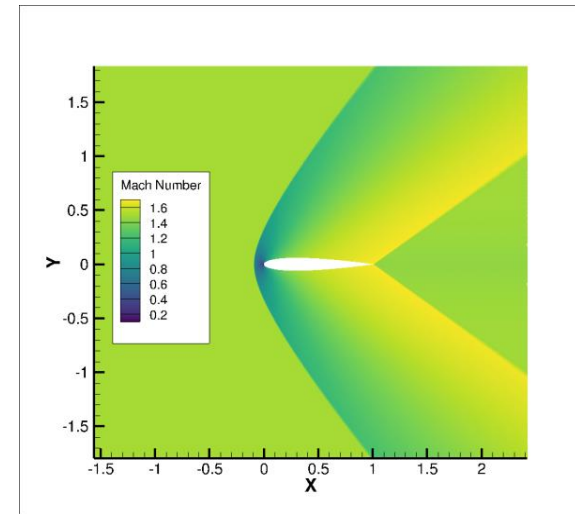
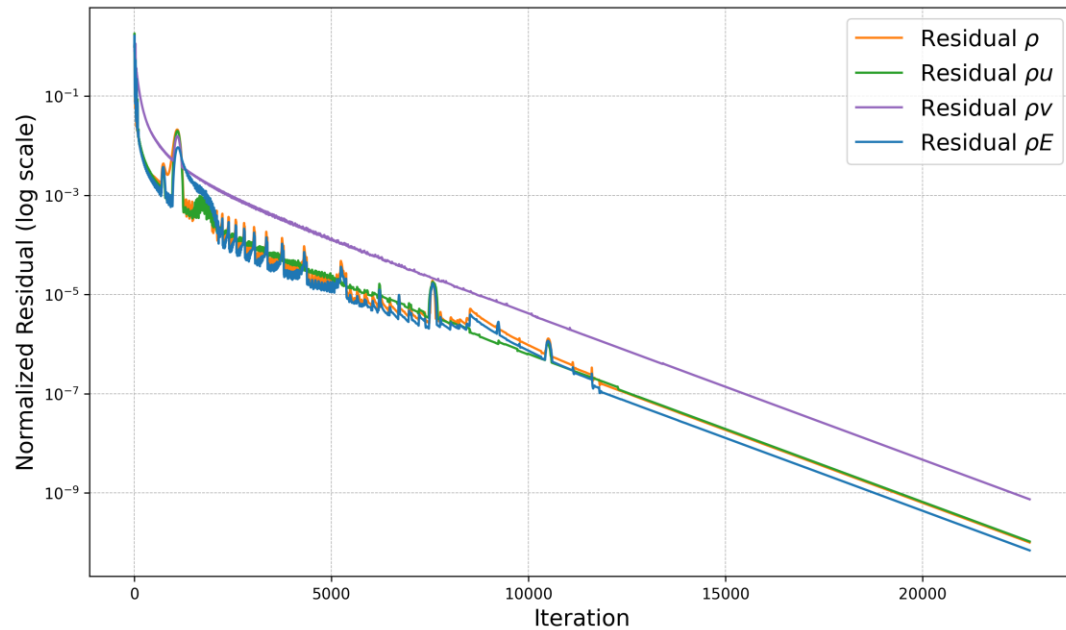


Figure: Mach Number.

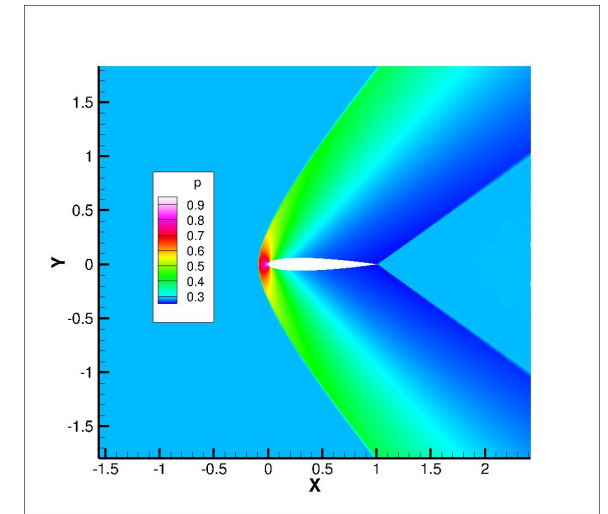


Figure: Density.

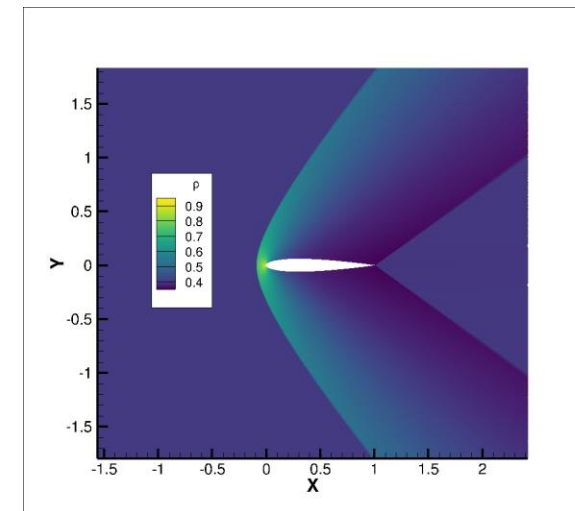


Figure: Pressure.

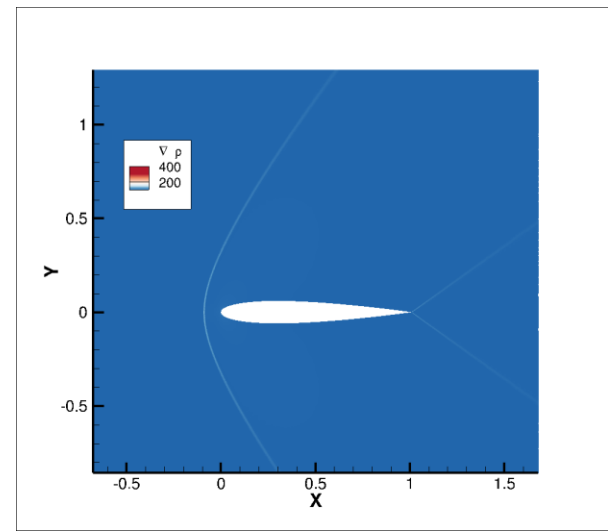
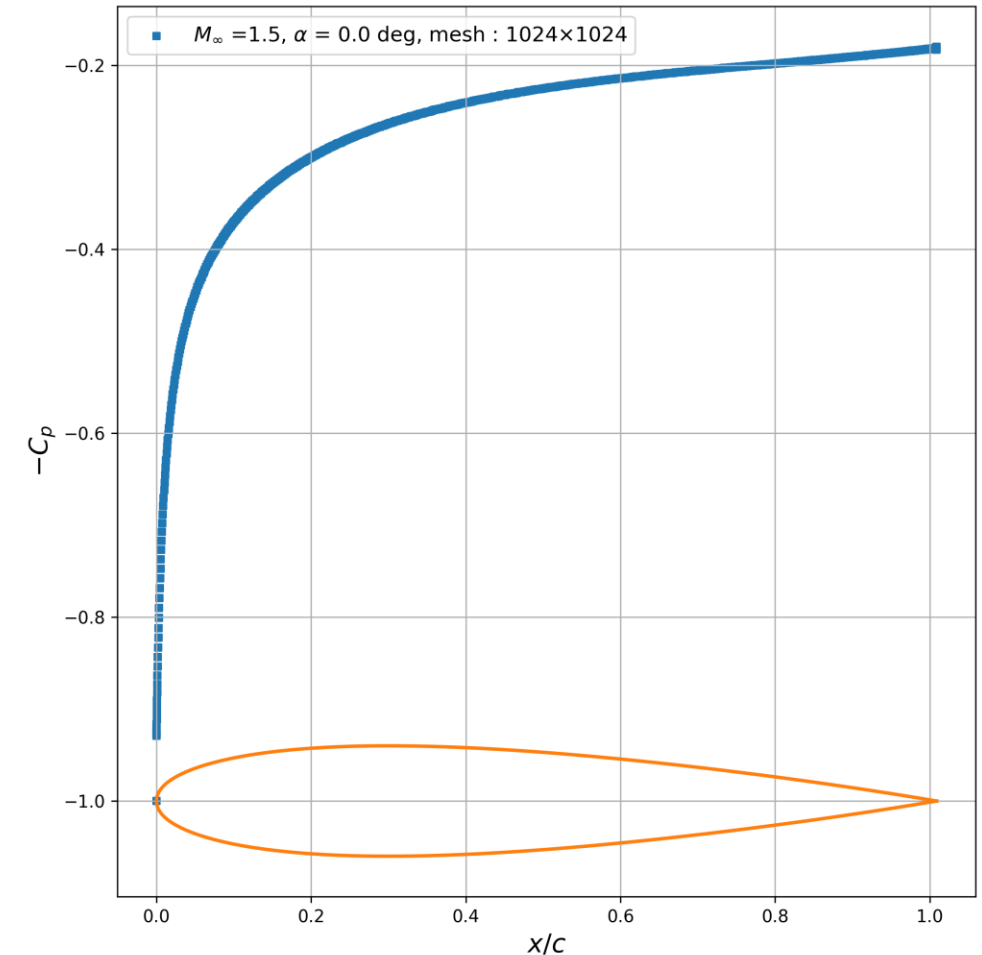


Figure: Density gradient.

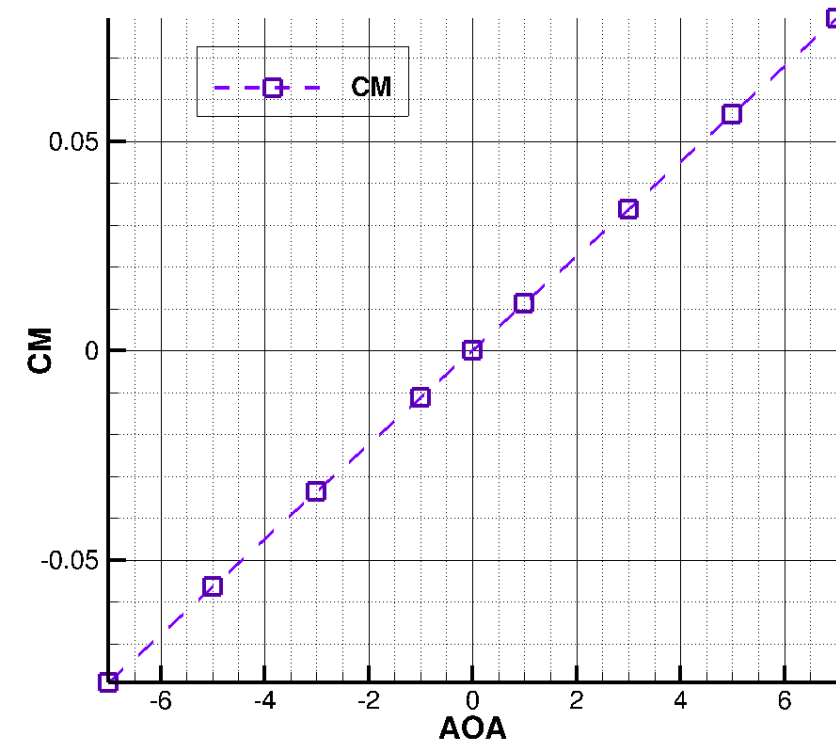
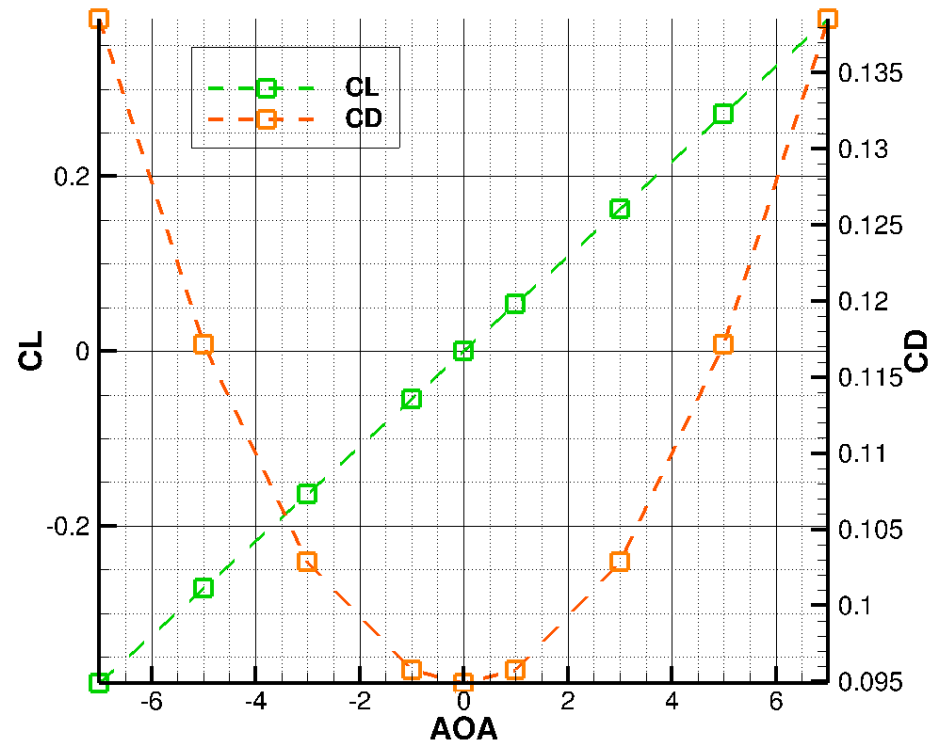
Analysis of Results for the 1024x1024 Grid

- Pressure distribution on the airfoil physically coherent with the case studied
- Pressure distribution is identical for lower and upper surfaces : $CL = 0$



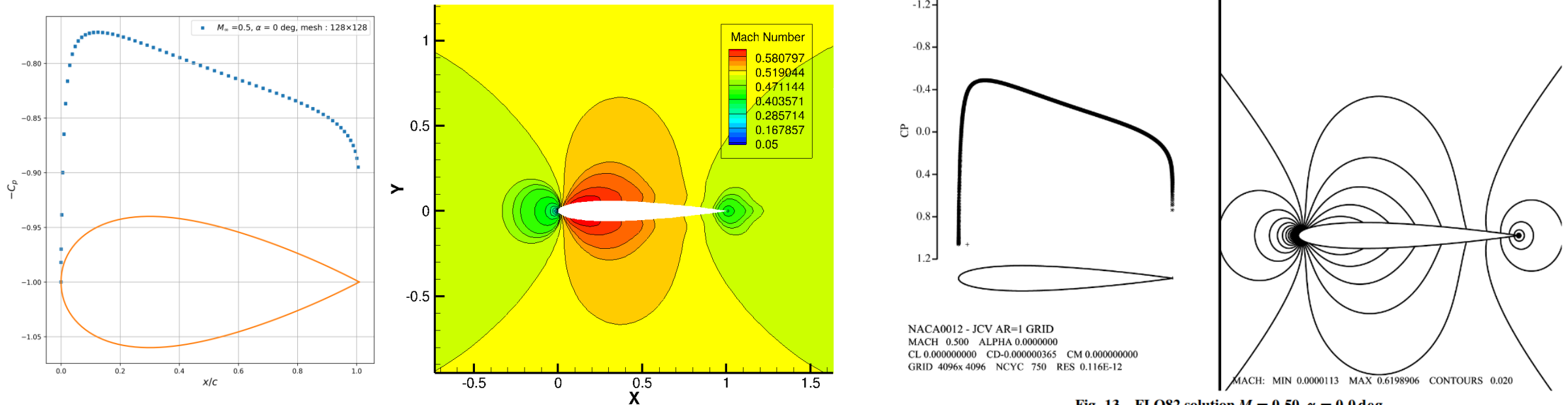
CL/CD/CM vs AOA curves

- Three flow regimes studied : subsonic ($M=0.5$), transonic ($M=0.8$), supersonic ($M=1.5$)
- Results for supersonic case ($M=1.5$) using 256×256 grid



Comparison with Jameson & Vassberg results [1]

- Results for subsonic case ($M=0.5$) using 128×128 grid – $\text{AOA} = 0^\circ$



- Results agree well with the reference publication

Comparison with Jameson & Vassberg results [1]

- Results for subsonic case ($M=0.5$) using 128×128 grid – $\text{AOA} = 1.25^\circ$

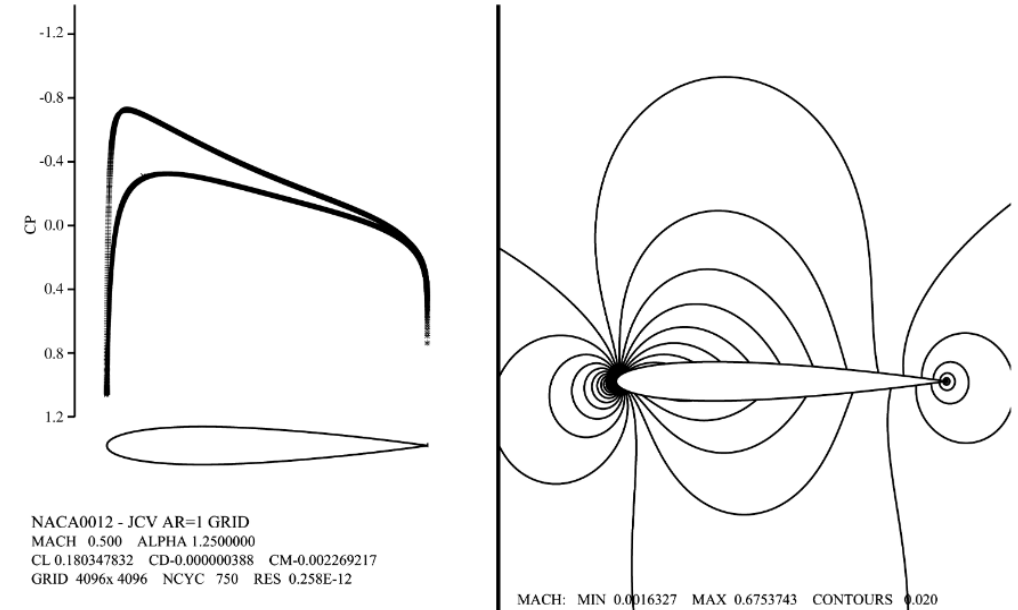
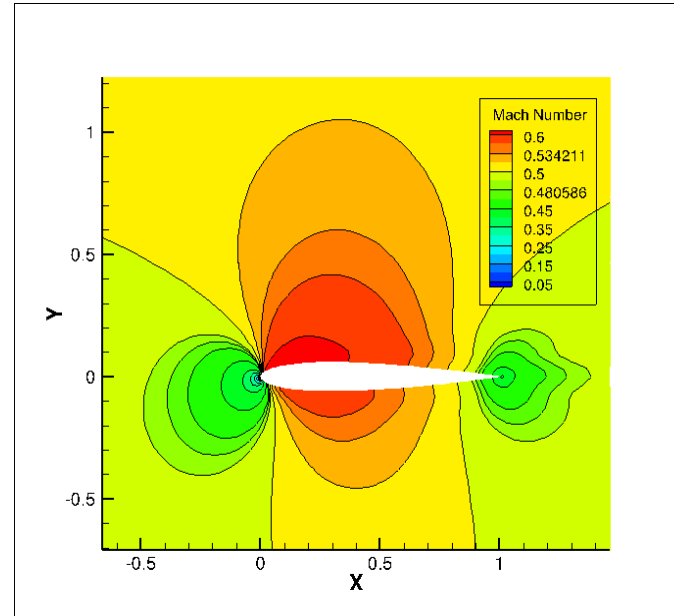
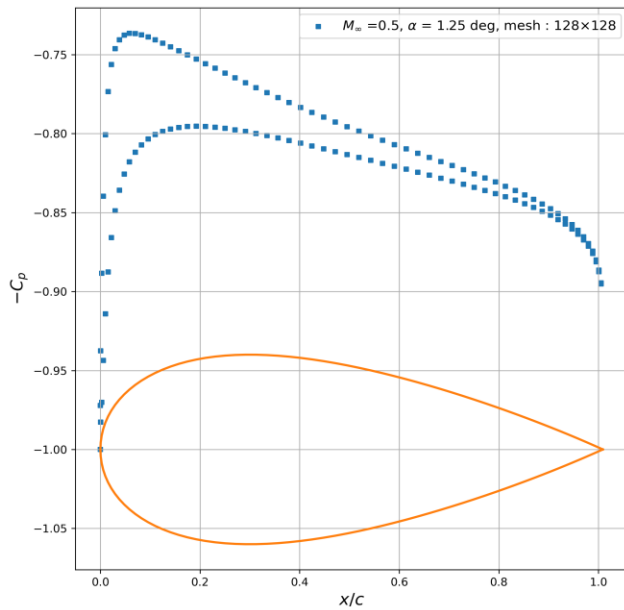
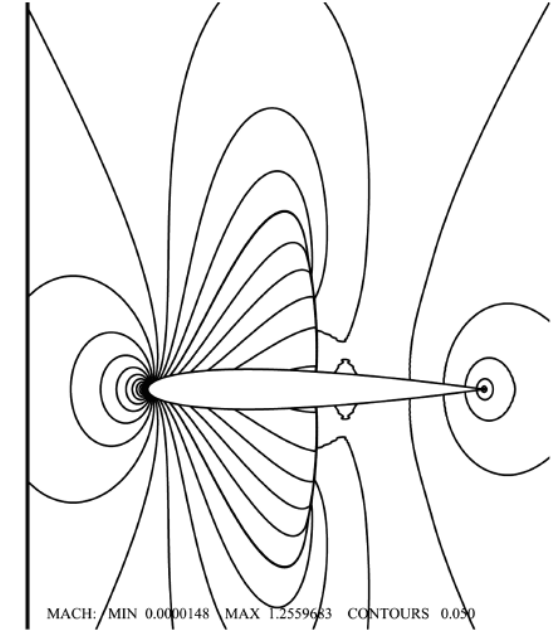
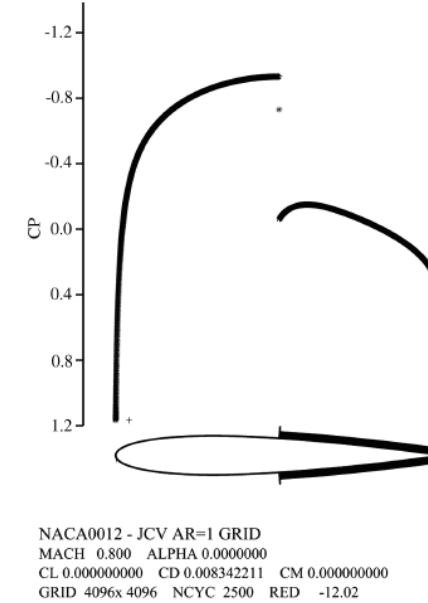
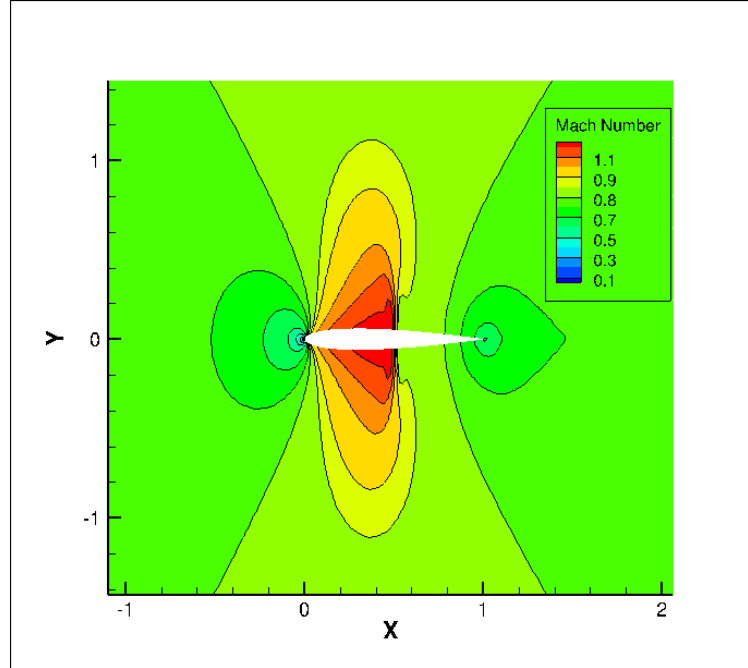
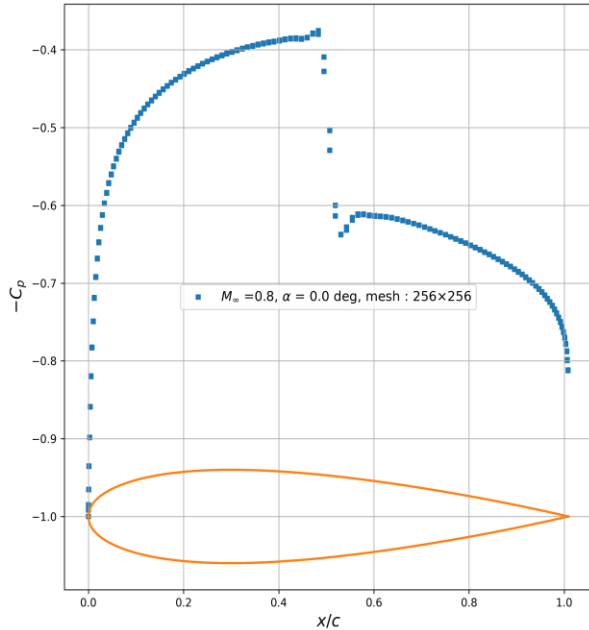


Fig. 14 FLO82 solution $M = 0.50$, $\alpha = 1.25^\circ$.

- $\mathbf{CL} = 0.177$, $\mathbf{CD} = 0.000358$, $\mathbf{CM}_{c/4} = 0.00211$
- $\mathbf{CL} = 0.180$, $\mathbf{CD} = 0.000298$, $\mathbf{CM}_{c/4} = 0.00229$
- Results agree well with the reference publication

Comparison with Jameson & Vassberg results [1]

- Results for transonic case ($M=0.8$) using 256×256 grid – $\text{AOA} = 0^\circ$



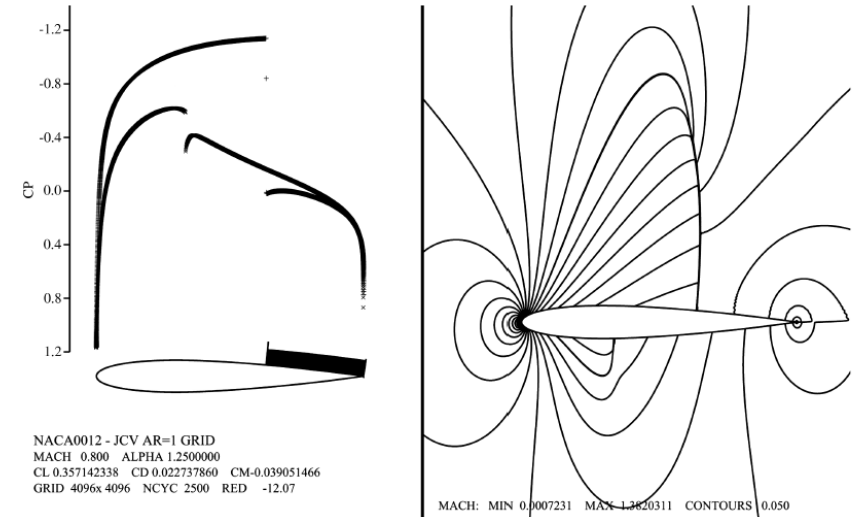
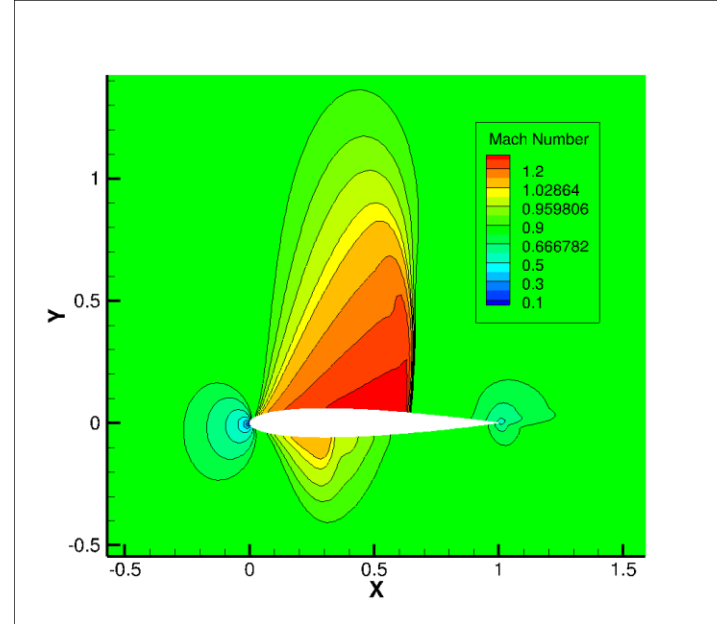
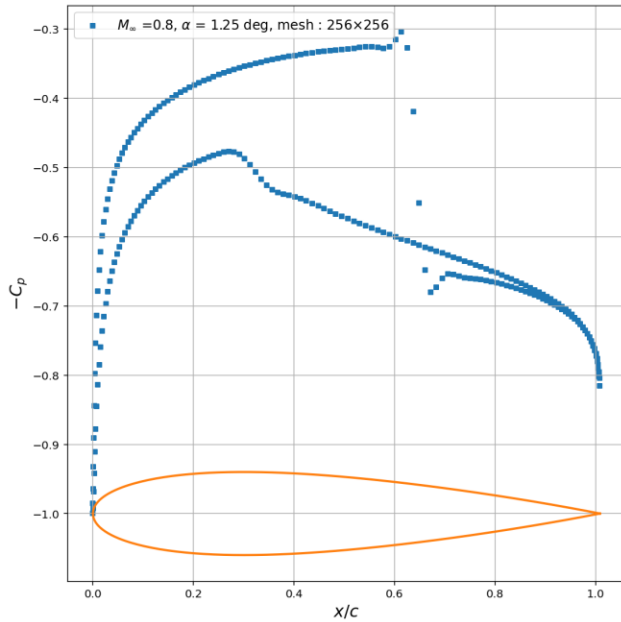
- $CD = 0.008131$

- $CD = 0.008312$

- Results agree well with the reference publication

Comparison with Jameson & Vassberg results [1]

- Results for transonic case ($M=0.8$) using 256×256 grid – $\text{AOA} = 1.25^\circ$



- $\mathbf{CL} = 0.364, \mathbf{CD} = 0.0226, \mathbf{CM}_{c/4} = 0.0415$

- $\mathbf{CL} = 0.357, \mathbf{CD} = 0.0227, \mathbf{CM}_{c/4} = 0.0391$

- Results agree well with the reference publication

Improvements

- **Development of implicit time integration schemes to enable faster convergence through the use of higher CFL numbers.**
- **Development of acceleration techniques, such as multigrid methods and preconditioning, to enhance computational efficiency.**
- **Optimization of the code, including rewriting it in a low-level language, implementing parallelization strategies, and exploring GPU acceleration to improve performance.**
- **Verify the implementation of subsonic far-field boundary conditions.**
- **Incorporate additional validation cases and design the code with greater abstraction to ensure it is not specialized solely for the NACA0012 problem.**

References

- [1] Vassberg, J. & Jameson, Antony. (2010). In Pursuit of Grid Convergence, Part I: Two-Dimensional Euler Solutions. *Journal of Aircraft - J AIRCRAFT*. 47. 1152-1166. 10.2514/1.46737.
- [2] Blazek, J. (2005). *Computational Fluid Dynamics: Principles and Applications*. 10.1016/B978-0-08-044506-9.X5000-0.