Calul

- Calcul de $\hat{u}_m$:

- $$\hat{u}_m = \sum_{n=0}^{N-1} u_n \langle U_m | D_n \rangle$$

$$= u_0 \langle U_m | T_0 \rangle + u_1 \langle U_m | T_1 \rangle + \sum_{n=2}^{N-1} u_n \langle U_m | T_n \rangle - \sum_{n=2}^{N-1} u_n \langle U_m | T_{n-2} \rangle$$

Or $\langle U_m | T_n \rangle = \dfrac{\delta_{m,n} - \delta_{m,n-2}}{2 - \delta_{n,0}}$

$$= u_0 \delta_{m,0} + u_1 \frac{\delta_{m,1}}{2} + \frac{1}{2} \sum_{n=2}^{N-1} u_n [\delta_{m,n} - \delta_{m,n-2}] - \frac{1}{2} \sum_{n=2}^{N-1} u_n [\delta_{m,n-2} - \delta_{m,n-4}]$$

$$\frac{1}{2} \sum_{n=2}^{N-1} u_n [\delta_{m,n} - 2\delta_{m,n-2} + \delta_{m,n-4}]$$

$$= \delta_{m,0}[u_0 - u_2 + \tfrac{1}{2} u_4]$$
$$+ \delta_{m,1}[\tfrac{u_1}{2} - u_3 + \tfrac{1}{2} u_5]$$
$$+ \delta_{m,2}[\tfrac{u_2}{2} - u_4 + \tfrac{1}{2} u_6]$$
$$\vdots$$
$$+ \cdots$$
$$+ \delta_{m,N-5}[\tfrac{u_{N-5}}{2} - u_{N-3} + \tfrac{1}{2} u_{N-1}]$$
$$+ \delta_{m,N-4}[\tfrac{u_{N-4}}{2} - u_{N-2}]$$
$$+ \delta_{m,N-3}[\tfrac{u_{N-3}}{2}]$$
$$+ \delta_{m,N-2}[\tfrac{u_{N-2}}{2}]$$
$$+ \delta_{m,N-1}[\tfrac{u_{N-1}}{2}]$$

$$\Rightarrow \hat{u}_m = \begin{cases} u_0 - u_2 + \frac{1}{2} u_4 & \text{si } m=0 \\ \frac{u_m}{2} - u_{m+2} + \frac{1}{2} u_{m+4} & \text{par } N-3 > m > 0 \\ \frac{u_m}{2} & \text{pour } m \geq N-3 \end{cases}$$

en comptant nulle les termes hors dom $\boxed{\hat{u}_m = \dfrac{u_m}{2 - \delta_{m,0}} - u_{m+2} + \tfrac{1}{2} u_{m+4}}$

- Calcul de $\widehat{\partial_x u_m}$:

- $\sum_{n=0}^{N-1} u_n \langle U_m | \partial_x D_n \rangle \sim \widehat{\partial_x u_m}$

$$= u_0 [U_m | \underbrace{\partial_x T_0}_{0 \cdot U_0}] + u_1 [U_m | \underbrace{\partial_x T_1}_{n U_1}]$$

$$+ \sum_{n=2}^{N-1} u_n [n \langle U_m | U_n \rangle - (n-2) \langle U_m | U_{n-2} \rangle]$$

$$= 0 + u_1 \delta_{m,1} + \sum_{n=2}^{N-1} u_n [n \delta_{m,n} - (n-2) \delta_{m,n-2}]$$

$$= 0 + \delta_{m,1} [u_1 - u_3]$$

$$+ 2 \delta_{m,2} [u_2 - u_4]$$

$$+ 3 \delta_{m,3} [u_3 - u_5]$$

$$+ \dots$$

$$+ (N-3) [u_{N-3} - u_{N-1}]$$

$$+ (N-2) u_{N-2}$$

$$+ (N-1) u_{N-1}$$

$\widehat{\partial_x u_m} = m [u_m - u_{m+2}] (1 - \delta_{m,0})$ avec 0 hors domaine

$$\boxed{\text{But : } \widehat{\partial_x u_m} = f(\hat{u}_m)}$$

$$\begin{cases} \widehat{\partial_x u_m} = m [u_m - u_{m+2}] (1 - \delta_{m,0}) \\ \hat{u}_m = \dfrac{u_n}{2 - \delta_{m,0}} - u_{m+2} + \frac{1}{2} u_{m+4} \end{cases}$$

En mettant à part 0 :

$$\begin{cases} \widehat{\partial_x u_m} = m [u_m - u_{m+2}] \\ \hat{u}_m = \dfrac{u_m}{2} - u_{m+2} + \dfrac{u_{m+4}}{2} \end{cases}$$

$$= \frac{\widehat{\partial_x u_m}}{m} - \frac{\widehat{\partial_x u_{m+2}}}{m+2}$$

$a_m = b_m - b_{m+2}$ avec $a_m = \widehat{u_m}$, $b_m = \frac{\widehat{\partial_x u_m}}{m}$

$\Rightarrow b_{m+2} = b_m - a_m$

$\Rightarrow b_m = b_{m-2} - a_{m-2}$

si m pair

$= b_{m-4} - a_{m-4} - a_{m-2}$

$= \cancel{b_0} - a_2 - \cdots - a_{m-2}$

$0 \quad - a_0$

si n impar

$b_n = b_1 - a_1 - \cdots - a_{m-2}$

Que vaut b_1?

Ainsi: $\widehat{\partial_x u_m} = -m\left[\widehat{u_0} + \widehat{u_2} + \cdots + \widehat{u_{m-2}}\right] \quad \forall m$

• CL :

$u(-1) = 0$, on a $\left| D_n(-1) = (-1)^n \right.$

$u(1) = 0 \Rightarrow$ On a $D_n(1) = 1$ pour $n = 0$ ou 1

0 sinon

$\rightarrow$ On remplace les équations en $N-1$ et $N-2$ par CL.

Mon gros souci :

Déterminer $\widehat{\partial_x u}$ en fonction de $\hat{u}$

Comme on avait fait en Fourier avec $\widehat{\partial_x u} = -k \hat{u}$