

# Field-level inference of growth rate

*Corentin Ravoux*

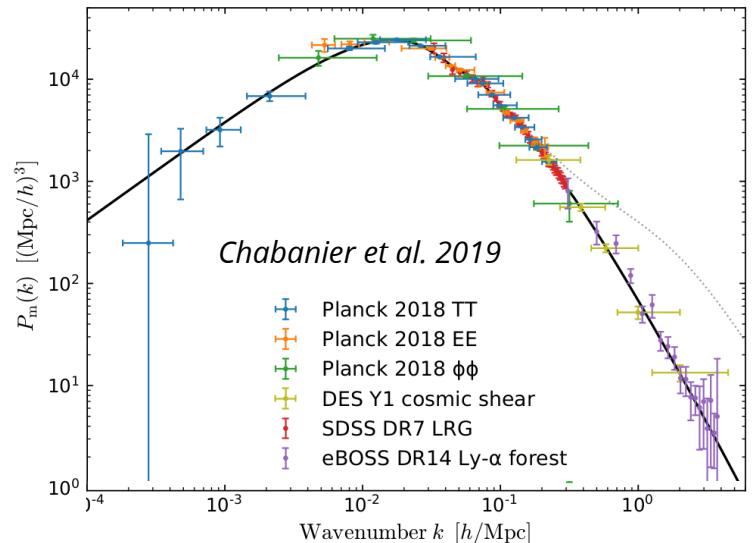
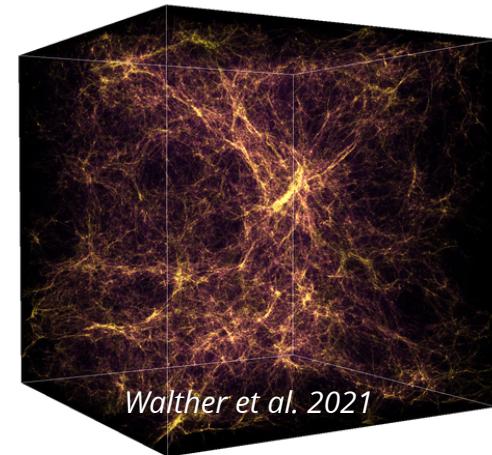
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# Velocities in the cosmic web

# Cosmic web

- Matter perturbations in the primordial Universe grew to form the cosmic web
- **Static description:** matter density contrast ( $\delta$ ) follows linear power spectrum at large-scales
- **Dynamic description:** Velocity of the cosmic web in linear perturbation theory:

$$\nabla \cdot \mathbf{v} \propto -aHf\delta$$



# Growth rate

$$\nabla \cdot \mathbf{v} \propto -aHf\delta$$



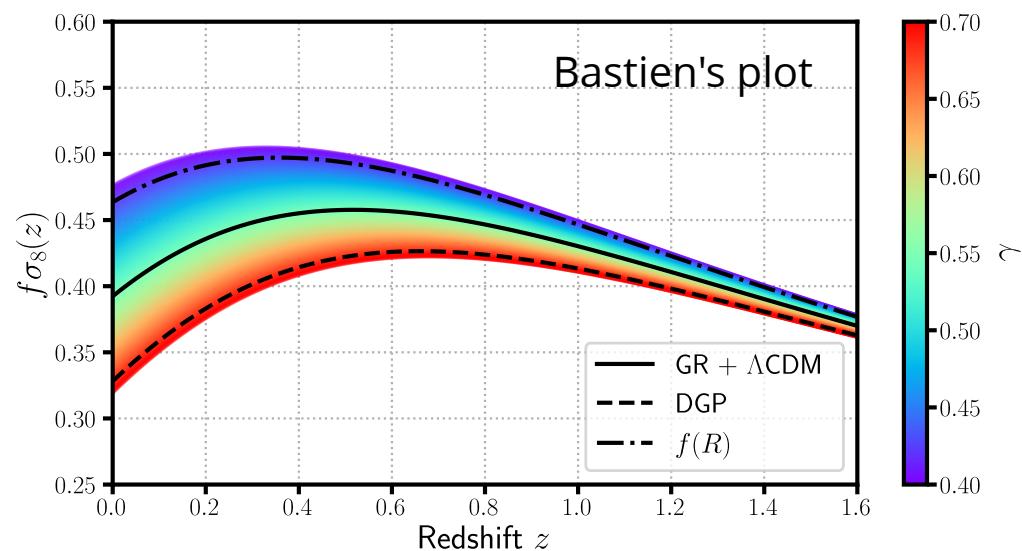
$$f = \frac{d \ln \delta_+(a)}{d \ln a}$$

Logarithmic growth rate of linear perturbations

- degeneracy with the amplitude of the linear power spectrum

$$P_m \propto \sigma_8^2$$

- **Applications:** Test of general relativity, constraints on dark energy properties

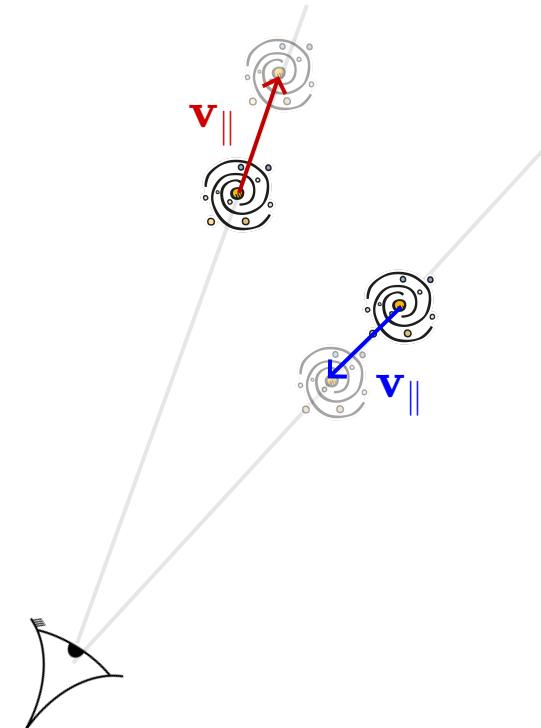


$$f = \Omega_m^\gamma$$

# Redshift Space Distortion

- Redshifts precisely measured by spectroscopic surveys:
  - Universe expansion + peculiar velocity
  - Position shifted by the line-of-sight peculiar velocity
- **Redshift Space Distortions (RSD):** Distortion of cosmological observable due to peculiar velocities

$$(1 + z_{\text{obs}}) = (1 + z_{\text{cosmo}})(1 + z_{\text{peculiar}})$$

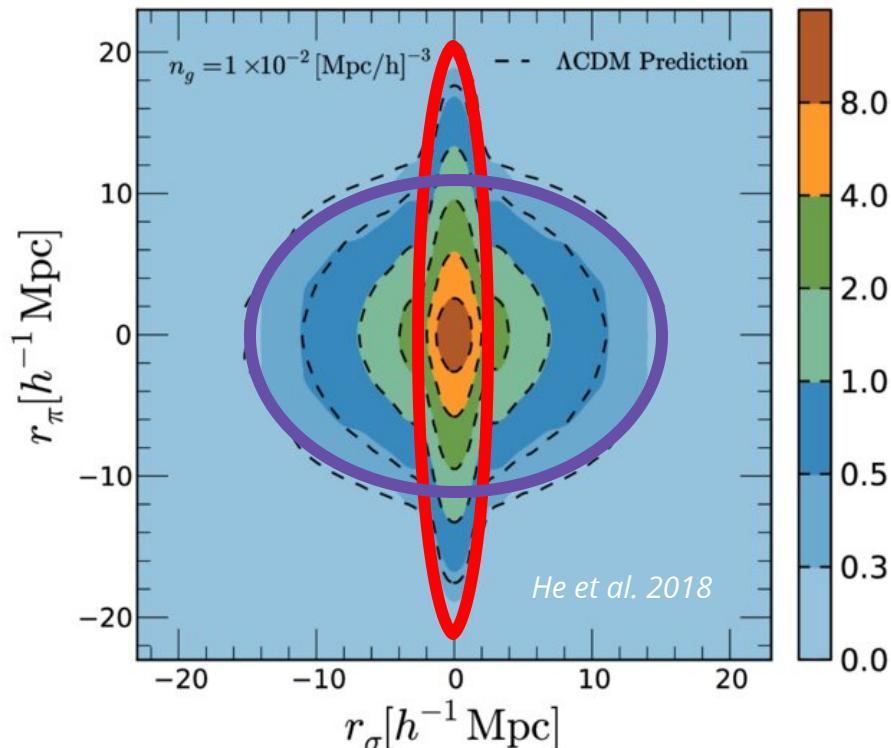


# RSD example

- Galaxy-galaxy auto-correlation

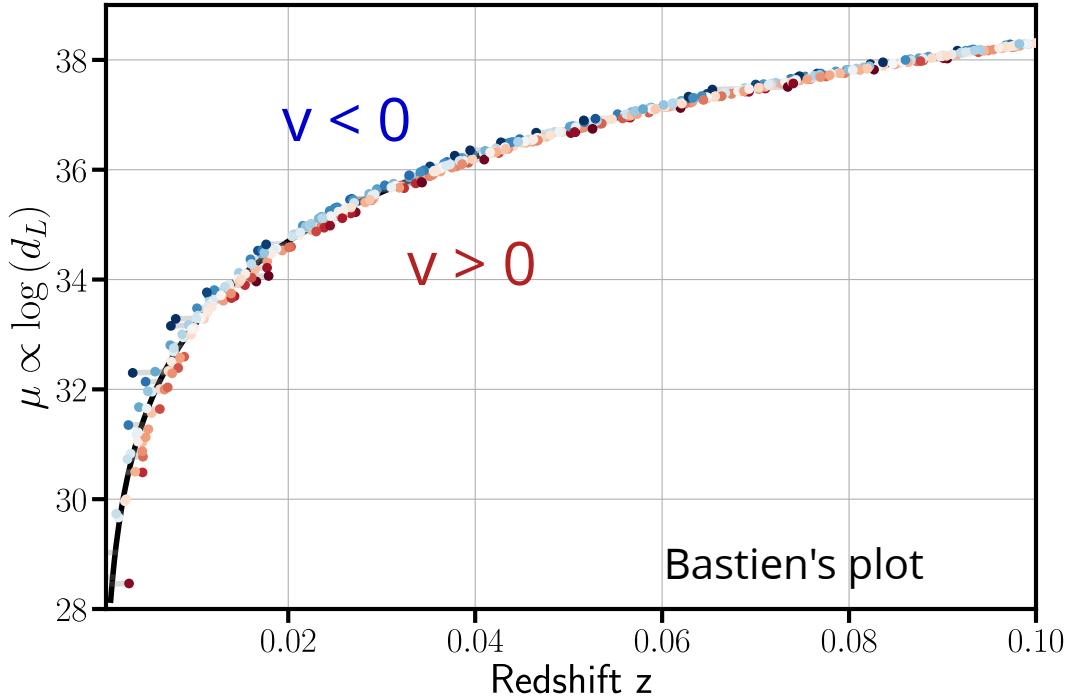
- **Two effects:**

- **Linear RSD (or Kaiser):**  
cohesive movement of  
galaxies around over densities
- **Non-linear RSD (or Finger-of-god):** small-scale random  
velocities due to collapse



# Peculiar velocities

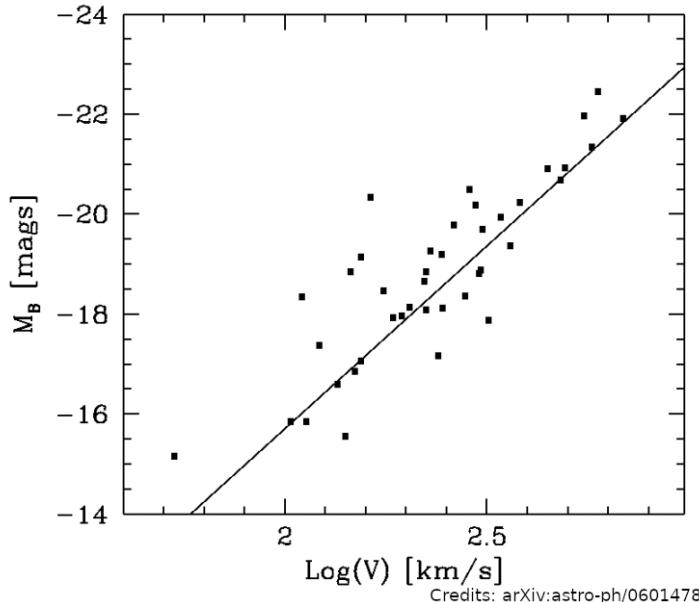
- Direct estimation of the galaxy peculiar velocity.
- Distance estimated with Type Ia supernovae (ZTF, LSST):  
 $\sigma_D/D \sim 7\%$
- Comparison between distance and redshift gives velocity



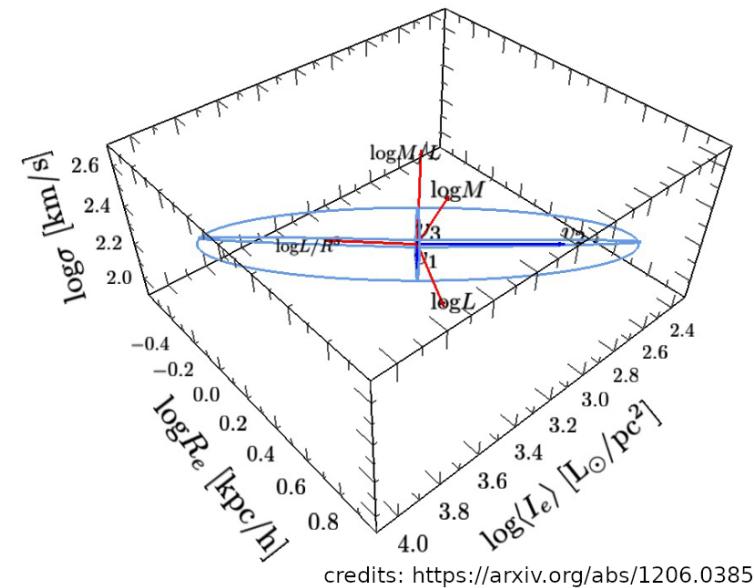
# Peculiar velocities

- Other estimators: empirical relations

$$\sigma_D / D \sim 20\%$$



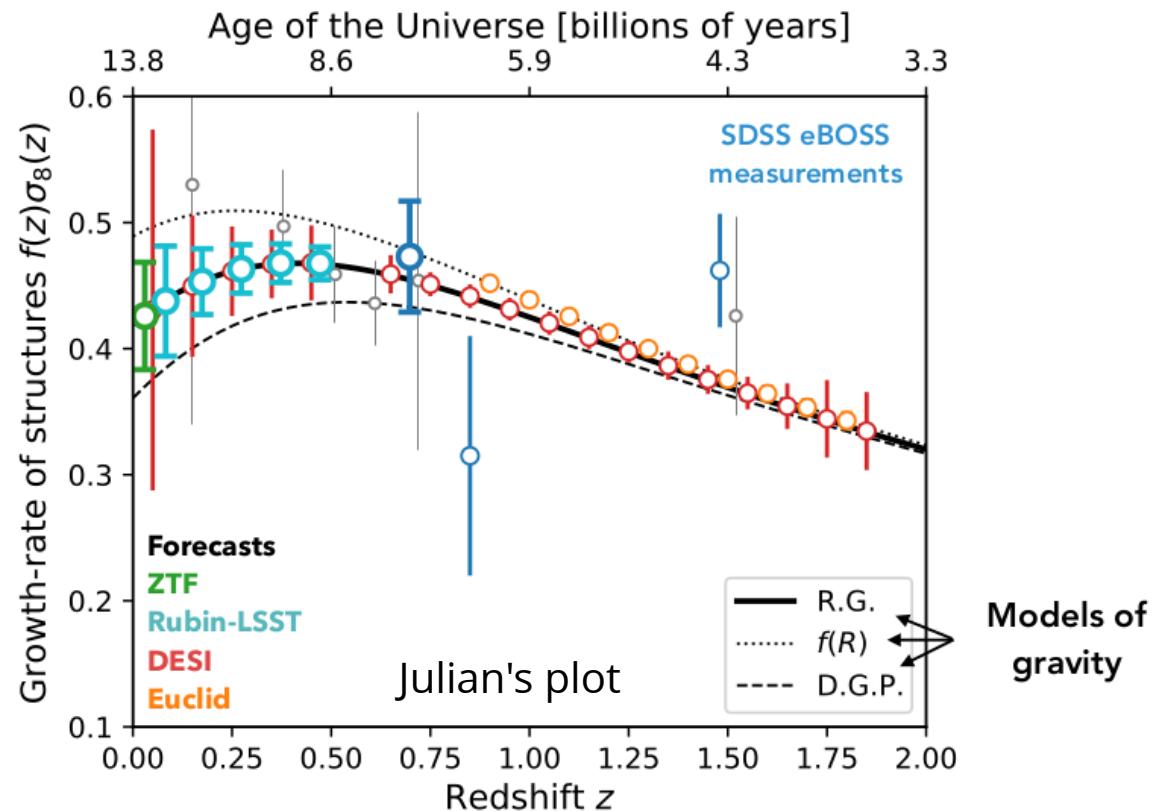
Tully-Fisher for spiral galaxies  
(luminosity/mass vs rotational velocity)



Fundamental plan for elliptical galaxies  
(effective angular radius vs velocity  
dispersion vs luminosity)

# Forecasts

- **RSD** very effective for high-z
- **Peculiar velocities** for low-z
- Constraint improvement with combination of methods



# Methods

- Growth rate measurement methods with peculiar velocities:

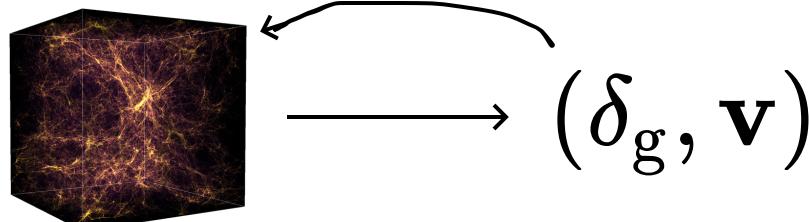
Density/momentum  
power spectra

$$\langle \delta_g(k) \delta_g(k) \rangle \langle \delta_g(k) \mathbf{v}(k) \rangle \langle \mathbf{v}(k) \mathbf{v}(k) \rangle$$

Density-velocity  
comparison

$$\mathbf{v}_{\text{measured}} \longleftrightarrow \nabla \cdot \mathbf{v}_{\text{pred}} \propto -aHf\delta$$

Forward modeling



Maximum likelihood

# Maximum likelihood estimator

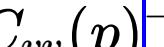
# Maximal likelihood estimator

- **Purpose:** Minimize a likelihood computed from all coordinates of the data
- **p = parameters:** contains cosmology and Hubble diagram parameters

$$\mathcal{L}(p) \propto \exp \left[ -\frac{1}{2} v^T(p) C_{vv}(p)^{-1} v(p) \right]$$

$\mathcal{L}(p)$     $f\sigma_8 \in p$

$v^T(p)$    peculiar velocities

$C_{vv}(p)$    Covariance matrix computed from theory and SNIa coordinates

Carreres et al. 2023

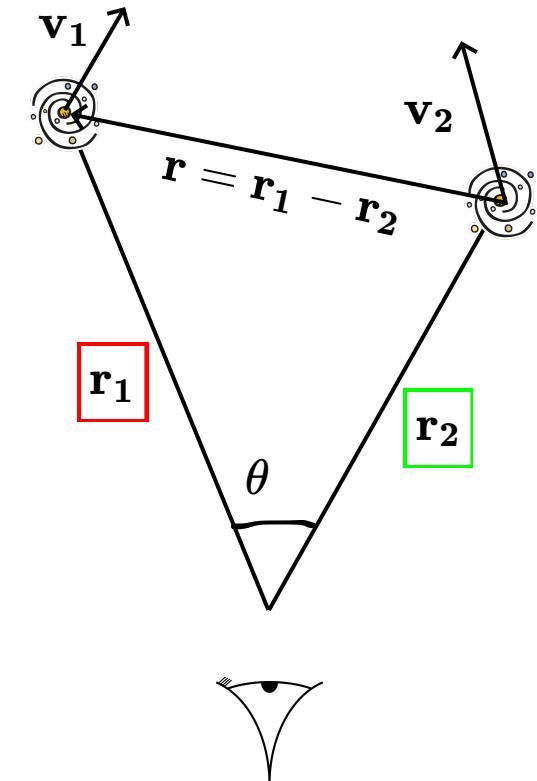
# Computation of covariance matrix

$$\mu_i = \mathbf{k} \cdot \mathbf{r}_i$$
$$C_{vv}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{(2\pi)^3} \int_{\mathbf{k}} P_{vv}(k, \mu_1, \mu_2) e^{i\mathbf{k} \cdot \mathbf{r}} d^3 \mathbf{k}$$

$$P_{vv} = (aHf)^2 \frac{\mu_1 \mu_2}{k^2} P_{\theta\theta}(k) D_u^2(k, \sigma_u)$$

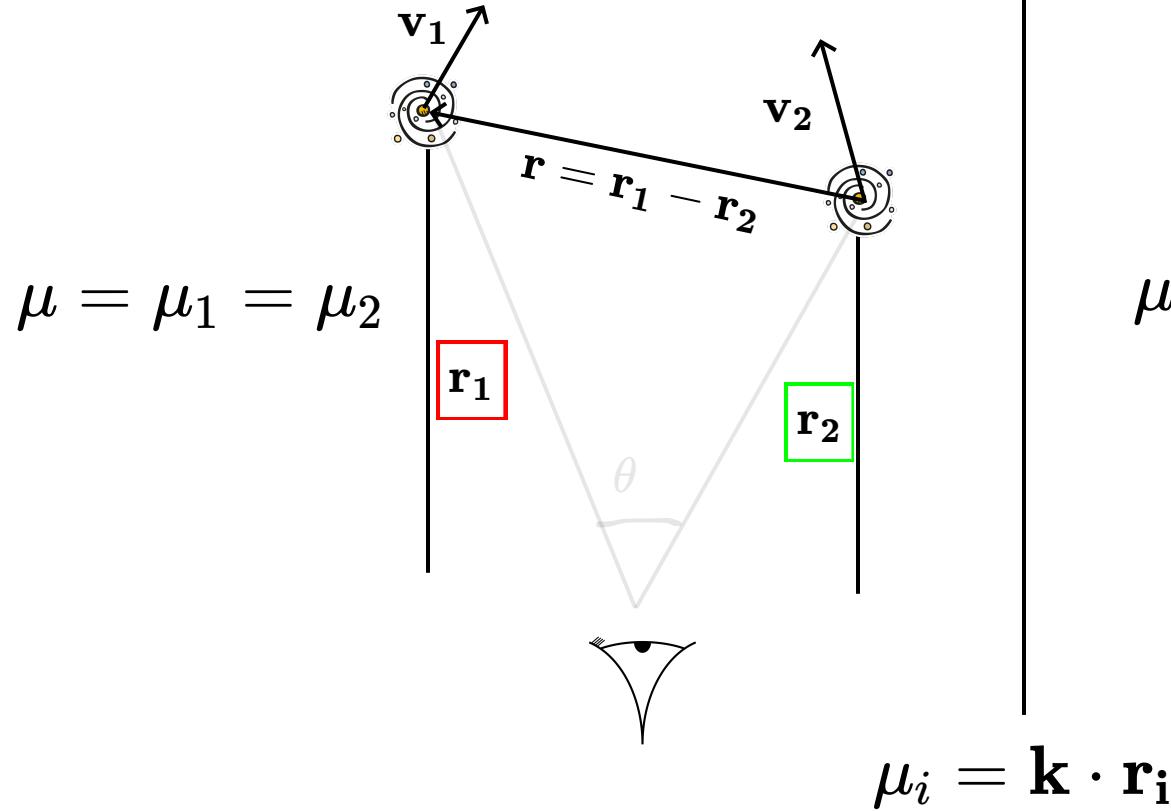
linear perturbation theory

Estimator with wide-angle effects

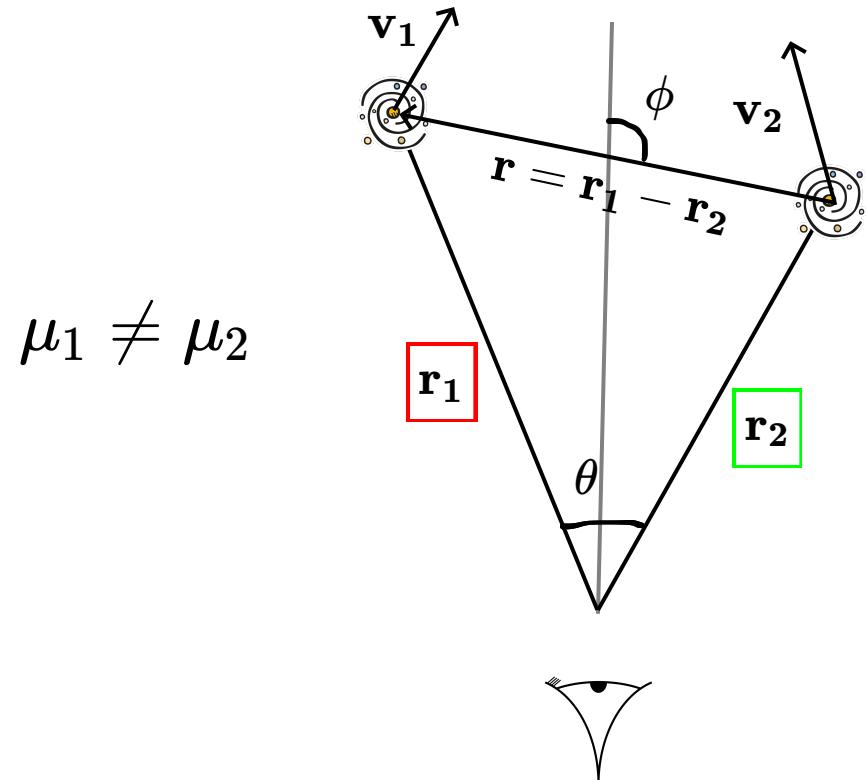


# A word on wide-angle in clustering

Plane parallel models



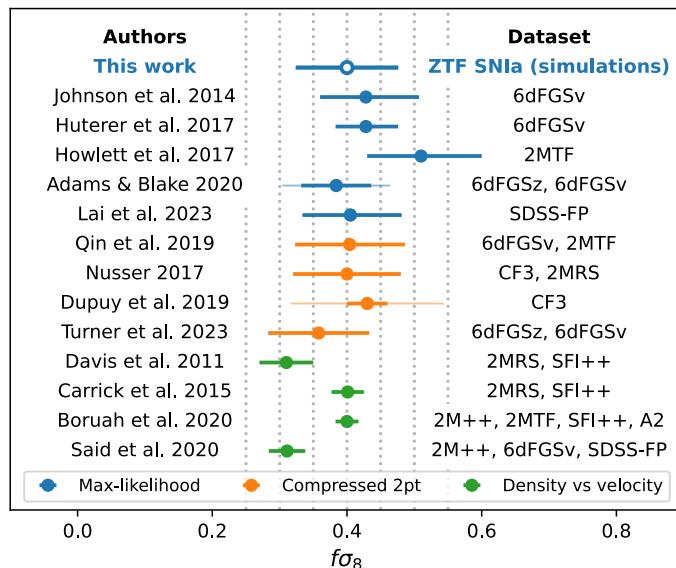
Wide angle models



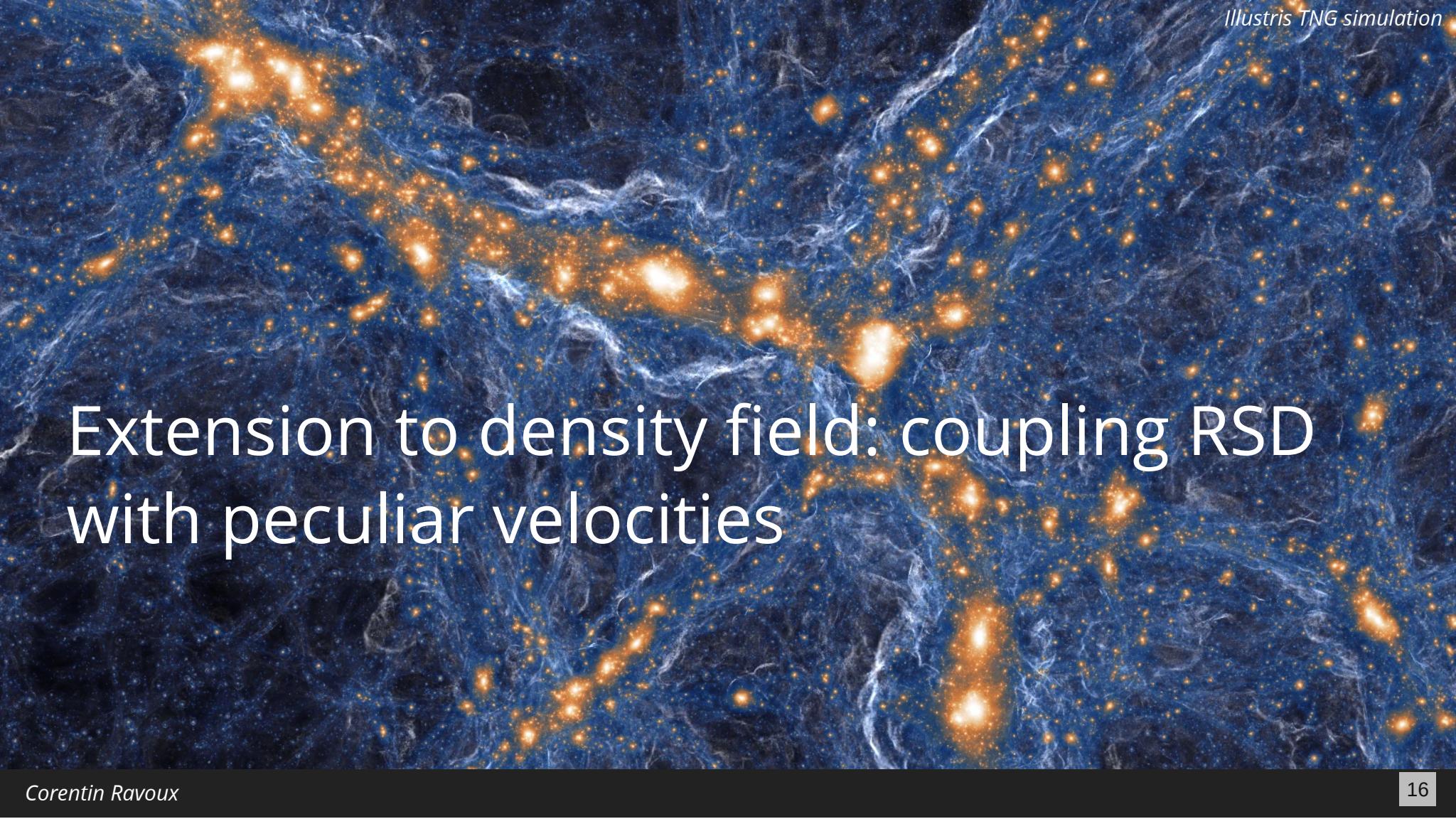
# Result for ZTF simulation

- Use of SNsim package based on SNcosmo
- ZTF survey parameters, and SNIa model

Carreres et al. 2023



- Other measurements performed with FP and TF
- ZTF will be competitive with only 1600 SNIa (compare to ~10k TF/FP)

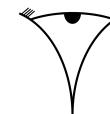
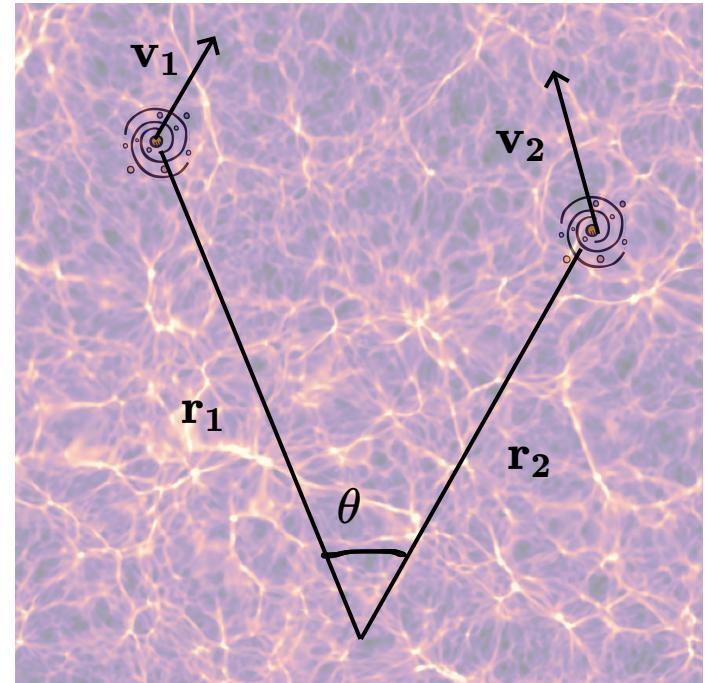


Extension to density field: coupling RSD  
with peculiar velocities

# Objective

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- Improvement of growth rate measurement using simultaneously velocities and densities
- Improvement of the python package for maximum likelihood fitting



# Covariance matrix

$$\mathcal{L}(p) \propto \exp \left[ -\frac{1}{2} \begin{bmatrix} \delta \\ v \end{bmatrix}^T \begin{bmatrix} (C_{gg}) & (C_{gv}) \\ (C_{gv}) & (C_{vv}) \end{bmatrix}^{-1} \begin{bmatrix} \delta \\ v \end{bmatrix} \right]$$

Density field from galaxy surveys (DESI, Euclid)

Velocities from SNIa (ZTF, LSST)

Covariance matrix computed from theory and coordinates

# Linear power spectrum model with wide-angle

$$P_{\text{gg}} = [b^2 P_{\text{mm}}(k) + bf(\mu_1^2 + \mu_2^2)P_{\text{m}\theta}(k) + f^2\mu_1^2\mu_2^2P_{\theta\theta}(k)] \exp\left[\frac{-k^2(\mu_1^2 + \mu_2^2)\sigma_g^2}{2}\right]$$

$$P_{\text{gv}} = iaH\frac{\mu_2}{k^2} (bfP_{\text{m}\theta}(k) + f^2\mu_1^2P_{\theta\theta}(k)) \exp\left[\frac{-k^2\mu_1^2\sigma_g^2}{2}\right] D_u(k, \sigma_u)$$

$$P_{\text{vv}} = (aHf)^2 \frac{\mu_1\mu_2}{k^2} P_{\theta\theta}(k) D_u^2(k, \sigma_u)$$

Very complicated (especially  
with the exponential term)

$$\rightarrow C_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{(2\pi)^3} \int_{\mathbf{k}} P_{ab}(k, \mu_1, \mu_2) e^{i\mathbf{k}\cdot\mathbf{r}} d^3\mathbf{k} \longrightarrow C_{\text{gg}}, C_{\text{gv}}, C_{\text{vv}}$$

# Lai et al. 2022 method

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- Taylor expansion of the finger-of-god term
- Complete equations:

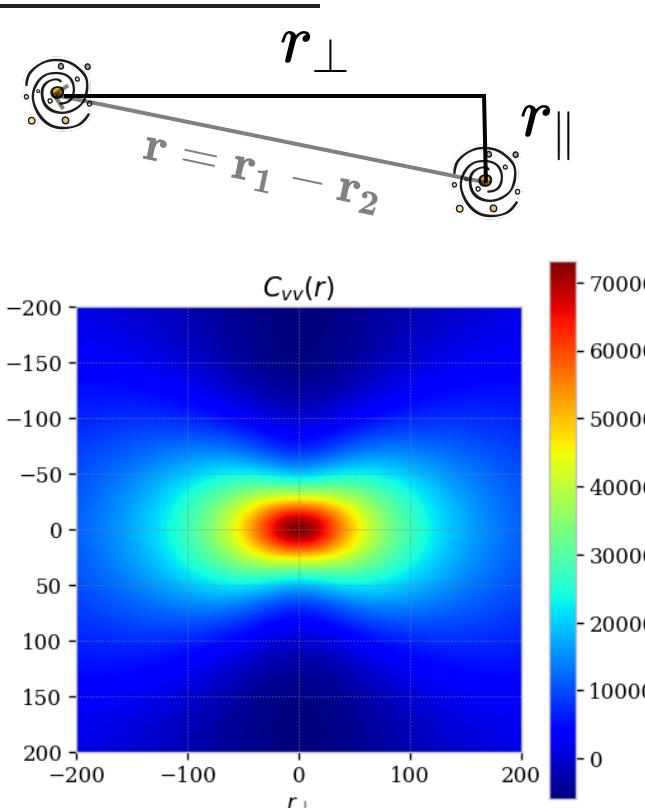
$$\exp \left[ \frac{-k^2 \mu^2 \sigma_g^2}{2} \right] = \sum_{i=0}^{\infty} \frac{(-1)^i (k\sigma_g)^{2i}}{2^i i!} \mu^{2i}$$

$$\begin{aligned} \mathbf{C}_{gg}(\mathbf{s}_1, \mathbf{s}_2) &= \sum_{pq} \frac{(-1)^{p+q}}{2^{p+q} p! q!} \sigma_g^{2(p+q)} \sum_l i^l \left[ b^2 \xi_{mm,l}^{p,q,0}(s, 0) H_{p,q}^l(\mathbf{s}_1, \mathbf{s}_2) \right. \\ &\quad \left. + f^2 \xi_{\theta\theta,l}^{p,q,0}(s, 0) H_{p+1,q+1}^l(\mathbf{s}_1, \mathbf{s}_2) + bf \xi_{m\theta,l}^{p,q,0}(s, 0) (H_{p+1,q}^l(\mathbf{s}_1, \mathbf{s}_2) + H_{p,q+1}^l(\mathbf{s}_1, \mathbf{s}_2)) \right] \\ \mathbf{C}_{gv}(s, \sigma_u) &= (aHf) \sum_p \frac{(-1)^p}{2^p p!} \sigma_g^{2p} \sum_l i^{l+1} \left( b \xi_{m\theta,l}^{p,-0.5,1}(s, \sigma_u) H_{p,0.5}^l(\mathbf{s}_1, \mathbf{s}_2) + f \xi_{\theta\theta,l}^{p,-0.5,1}(s, \sigma_u) H_{p+1,0.5}^l(\mathbf{s}_1, \mathbf{s}_2) \right) \end{aligned}$$

$$\mathbf{C}_{vv}(s, \sigma_u) = (aHf)^2 \sum_l i^{l+2} \xi_{\theta\theta,l}^{-0.5,-0.5,2}(s, \sigma_u) H_{0.5,0.5}^l(\mathbf{s}_1, \mathbf{s}_2)$$

# Model evaluation

- **Test:** expressing covariance matrix as a function of parallel and transverse separation



Shape similar to correlation function models

# More general framework

# Idea: general covariance calculation

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- **Objectives:**

- Test more density models
- Possible extension to non linear corrections

# Plane parallel model

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- Let's start "simple":

$$C_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{(2\pi)^3} \int_{\mathbf{k}} P_{ab}(k, \mu) e^{i\mathbf{k}\cdot\mathbf{r}} d^3\mathbf{k}$$

↓  
Using some spherical harmonic  
definition and multipole theorems

$$C_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{\ell} i^{\ell} \int_k \frac{k^2 dk}{2\pi^2} j_{\ell}(kr) L_{\ell}(\hat{\mathbf{d}} \cdot \hat{\mathbf{r}}) \int_{\mu=-1}^1 \frac{2\ell+1}{2} L_{\ell}(\mu) P_{ab}(k, \mu) d\mu$$

# Plane parallel model

- Express power spectrum model in a general way:

$$P_{ab}(k, \mu) = \sum_i A_{ab,i} B_{ab,i}(k, \mu) P_{ab,i}(k)$$

Parameters  
to fit

Terms to compute  
analytically

Analytical power  
spectra terms

- vv example  
(Bastien model):

$$P_{vv} = (f^2) \times \left( a^2 H^2 \frac{\mu^2}{k^2} \right) \times \left( P_{\theta\theta}(k) D_u^2(k, \sigma_u) \right)$$

# Plane parallel model

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- Without loosing generality we can express the covariance:

$$C_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \sum_i A_{ab,i} \sum_l \xi_{ab,\ell}^i(r)$$

$$\xi_{ab,\ell}^i(r) = L_\ell(\hat{\mathbf{d}} \cdot \hat{\mathbf{r}}) \left[ i^\ell \int_k \frac{k^2 dk}{2\pi^2} P_{ab,i}(k) K_{ab,\ell}^i(k) j_\ell(kr) \right]$$

$$K_{ab,\ell}^i(k, \theta, \phi) = \int_{\mu=-1}^1 \frac{2\ell+1}{2} L_\ell(\mu) B_{ab,i}(k, \mu) d\mu$$

# Plane parallel model

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- Some insights:

$$C_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \sum_i A_{ab,i} \sum_l \xi_{ab,l}^i(r)$$

Parameters  
to fit

linear sum of matrix

# Plane parallel model

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- **Some insights:**

$$C_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \sum_i A_{ab,i} \sum_l \xi_{ab,\ell}^i(r)$$

Hankel transform

$$\xi_{ab,\ell}^i(r) = L_\ell(\hat{\mathbf{d}} \cdot \hat{\mathbf{r}}) \left[ i^\ell \int_k \frac{k^2 dk}{2\pi^2} P_{ab,i}(k) K_{ab,\ell}^i(k) j_\ell(kr) \right]$$

- **Most important:** Algorithmically optimized way to compute power spectrum integral, with FFTLog

# Wide angle model

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- More complicated formalism derived and implemented for wide-angle
- **Conclusion:**
  - Formalism can reproduce all recent covariance models (*Adams & Blake 2017, Adams & Blake 2020, Lai et al. 2022, Carreres et al. 2023*)
  - Start extension to other models
  - Fast with FFTLog (Hankel transform)

# Wide angle model

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$$P_{ab}(k, \mu_1, \mu_2) = \sum_i A_{ab,i} B_{ab,i}(k, \mu_1, \mu_2) P_{ab,i}(k)$$

$$C_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \sum_i A_{ab,i} \sum_l \xi_{ab,\ell}^i(r)$$

$$\xi_{ab,\ell}^i(r) = \sum_j N_{ab,\ell}^{i,j}(\theta, \phi) \left[ i^\ell \int_k \frac{k^2 dk}{2\pi^2} P_{ab,i}(k) M_{ab,\ell}^{i,j}(k) j_\ell(kr) \right]$$

$$M_{ab,\ell}^{i,j=(\ell_1,\ell_2)}(k) = \int_{\mu_1=-1}^1 \int_{\mu_2=-1}^1 \frac{1}{4} L_{\ell_1}(\mu_1) L_{\ell_2}(\mu_2) B_{ab,i}(k, \mu_1, \mu_2) d\mu_1 d\mu_2$$

$$N_{ab,\ell}^{i,j=(\ell_1,\ell_2)}(\theta, \phi) = (4\pi)^2 \sum_{m,m_1,m_2} G_{l,l_1,l_2}^{m,m_1,m_2} Y_{\ell m}(\hat{\mathbf{r}})^* Y_{\ell_1 m_1}(\hat{\mathbf{r}}_1)^* Y_{\ell_2 m_2}(\hat{\mathbf{r}}_2)^*$$

$$G_{m_1,m_2,m_3}^{l_1,l_2,l_3} = \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

# **flip** Field Level Inference Package

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- **Currently:** python package for this new formalism

*<https://github.com/corentinravoux/flip>*

- **Contains:**

- Symbolic code generation for covariance model
- Fast generation of covariance
- Minuit fitter (MCMC in progress)
- Gridding routines
- Covariance model evaluation

# A word on symbolic calculation

$$K_{ab,\ell}^i(k, \theta, \phi) = \int_{\mu=-1}^1 \frac{2\ell+1}{2} L_\ell(\mu) B_{ab,i}(k, \mu) d\mu$$

Computed analytically

- With **Sympy**, similar to mathematica (but free)

$$P_{gg} = \exp(-(k\mu\sigma_g)^2)$$

$$\begin{aligned} & [b^2 P_{mm}(k) \\ & + 2bf\mu^2 P_{m\theta}(k) \\ & + f^2\mu^4 P_{\theta\theta}(k)] \end{aligned}$$

pycode generator

```
def N_gg_0_4_0(theta, phi):
    return (5 / 16) * np.cos(2 * phi) + (35 / 64) * np.cos(4 * phi) + 9 / 64

def M_gg_1_0_0(sig_g):
    def func(k):
        return -np.exp(-(k**2) * sig_g**2) / (k**2 * sig_g**2) + (
            1 / 2
        ) * np.sqrt(np.pi) * scipy.special.erf(k * sig_g) / (k**3 * sig_g**3)

    return func

def N_gg_1_0_0(theta, phi):
    return 1

def M_gg_1_2_0(sig_g):
    def func(k):
        return (
            -5 * np.exp(-(k**2) * sig_g**2) / (k**2 * sig_g**2)
            - 5
            / 4
            * np.sqrt(np.pi)
            * scipy.special.erf(k * sig_g)
            / (k**3 * sig_g**3)
            - 45 / 4 * np.exp(-(k**2) * sig_g**2) / (k**4 * sig_g**4)
            + (45 / 8)
            * np.sqrt(np.pi)
            * scipy.special.erf(k * sig_g)
            / (k**5 * sig_g**5)
        )

    return func

def N_gg_1_2_0(theta, phi):
    return (3 / 4) * np.cos(2 * phi) + 1 / 4
```

# Conclusion

- Density/velocity maximum likelihood method
- More general framework with flip