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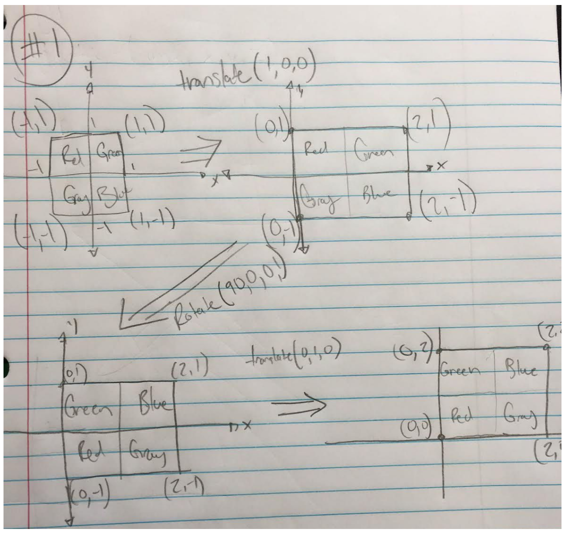
CS 112

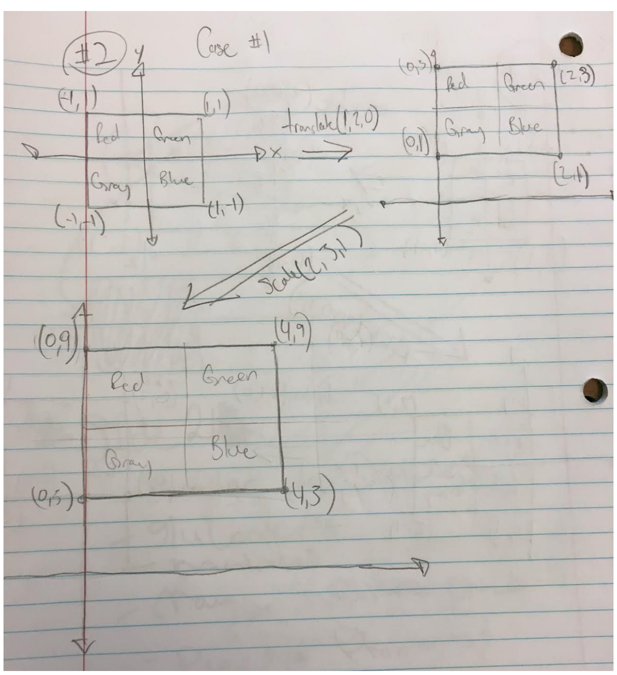
Written Assignment 2

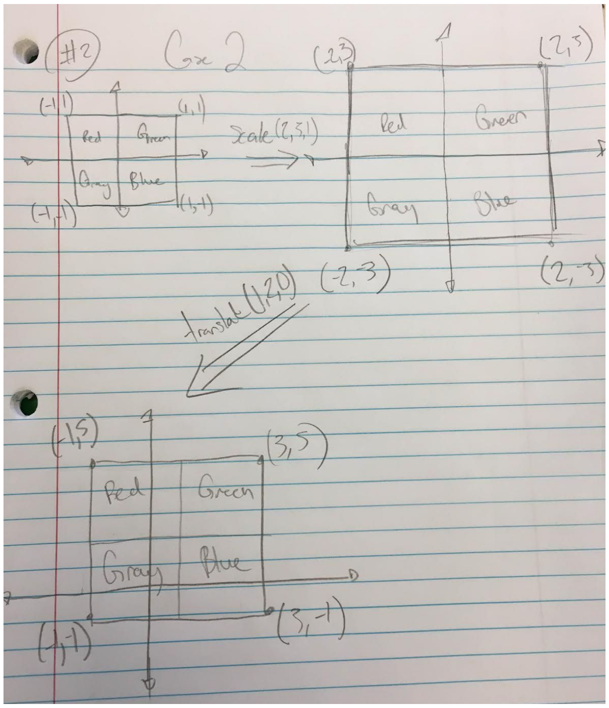
1.



You can first rotate 90° on the z-axis and then combine the two translations to (1,1,0), thus making it two operations instead of three.

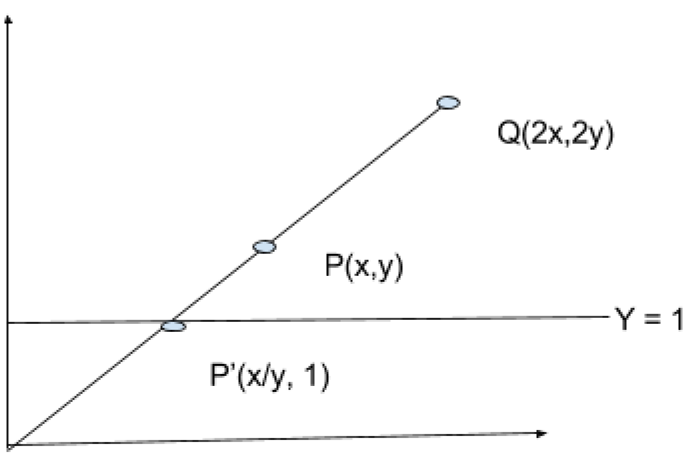
2.





In order for the squares in Case 1 and Case 2 to be the same, the square in Case 2 would need to be translated an additional (1,4,0). Meaning that the parameters for glScalef would remain the same at (2,3,1) and the new parameters of glTranslatef would be (2,6,0). As one can see, the translation parameters have to be scaled by the scaling parameters in order to make up for the difference in which the operations are ordered.

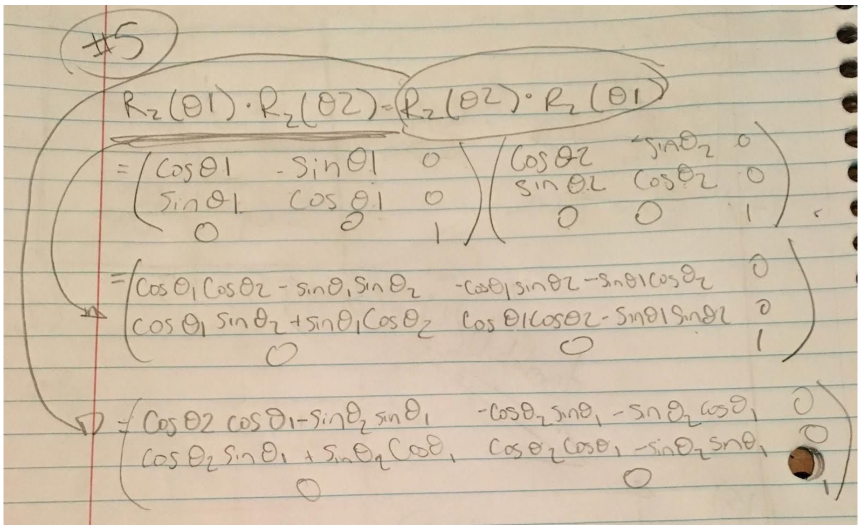
3. This can happen because all points that lie on the same vector will share homogeneous coordinates.



In the figure both P and Q would both map to P’, as would any other point that existed on the line

4. Row vectors and column vectors are unit long and perpendicular among themselves, meaning they’re orthogonal. The dot product of a row with itself is equal to 1, but the dot product of a row with another row is 0. The product of a rotation matrix R and its inverse R^t would result in the identity matrix.

5.



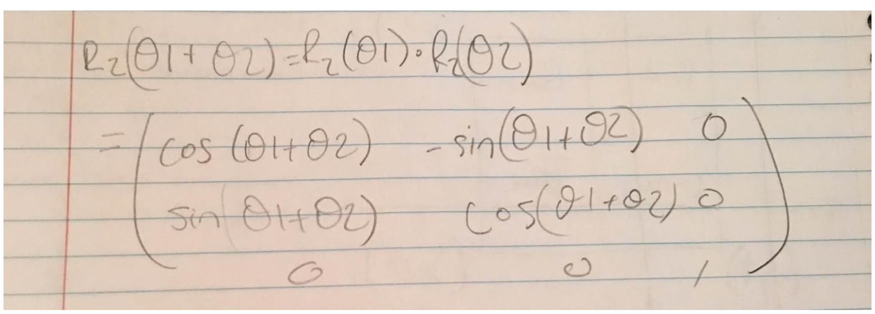
#5

$$R_z(\theta 1)\cdot R_z(\theta 2) = R_z(\theta 2)\cdot R_z(\theta 1)$$

$$= \begin{pmatrix} \cos\theta 1 & -\sin\theta 1 & 0 \\ \sin\theta 1 & \cos\theta 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta 2 & -\sin\theta_2 & 0 \\ \sin\theta 2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta_1\cos\theta 2 - \sin\theta_1\sin\theta_2 & -\cos\theta_1\sin\theta 2 - \sin\theta 1\cos\theta_2 & 0 \\ \cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2 & \cos\theta 1\cos\theta 2 - \sin\theta 1\sin\theta 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta 2\cos\theta_1 - \sin\theta_2\sin\theta_1 & -\cos\theta_2\sin\theta_1 - \sin\theta_2\cos\theta_1 & 0 \\ \cos\theta_2\sin\theta_1 + \sin\theta_2\cos\theta_1 & \cos\theta_2\cos\theta_1 - \sin\theta_2\sin\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$R_z(\theta 1 + \theta 2) = R_z(\theta 1)\cdot R_z(\theta 2)$$

$$= \begin{pmatrix} \cos(\theta 1 + \theta 2) & -\sin(\theta 1 + \theta 2) & 0 \\ \sin(\theta 1 + \theta 2) & \cos(\theta 1 + \theta 2) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Decomposing the rotation matrix into simpler ones and plugging in arbitrary values will show that $R(\Theta 1) \bullet R(\Theta 2) = R(\Theta 2) \bullet R(\Theta 1) = R(\Theta 1 + \Theta 2)$

6.

#6

$u = (2,1,2)$ rooted at $(2,3,4)$

$$T(2,3,4)\,R^T\,S\,R\,T(-2,-3,-4)$$

$$= \overset{T}{\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \cdot \overset{R}{\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \cdot$$

$$\overset{S}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \cdot \overset{R^T}{\begin{bmatrix} -\cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & -\cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \cdot$$

$$\overset{T}{\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

7.

#7

Eye Position : (0,0,0)
View Up : (0,2,0)
Equation of the image plane : $x+y+z=6$

Center Position = $(0,0,0) + k(1,1,1) = (2,2,2)$
Near Clipping Plane = $\sqrt{2^2+2^2+2^2}$
$= \sqrt{12} = 2\sqrt{3}$

gluLookAt:

Normalized vector = $f \Rightarrow \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

$$s = \frac{f \times up}{|f \times up|}, \quad u = s \times f$$

$$M = \begin{pmatrix} s0 & s1 & s2 & 0 \\ u0 & u1 & u2 & 0 \\ -f0 & -f1 & -f2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -.41 & .82 & -.41 & 0 \\ -1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gl frustrum:

$n = 2\sqrt{3}$, $f = 10$, $l = -2$, $r = 2$, $t = 4$, $b = 8$

$A = r+l / r-l = 0$
$B = t+b / t-b = -3$
$C = -(f+n) / f-n = -2.06$
$D = -(2*f*n) / f-n = -10.6$
$E = 2*n / r-l = 1.73$
$F = 2*n / t-b = -1.73$

$$P = \begin{pmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1.73 & 0 & 0 & 0 \\ 0 & -1.73 & -3 & 0 \\ 0 & 0 & -2.06 & -10.6 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$