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CS 171 Homework 5

**Problem 8.8**

No, ¬Spouse(George, Laura) does not follow from the facts Jim != George and Spouse(Jim, Laura). The additional axiom needed to prove it would be: Spouse(x,y) and (z != x) => ¬Spouse(z, y). If we use Spouse as a unary function symbol instead of a binary predicate then no additional axiom is needed.

**Problem 8.10**

a. $Occupation(Emily, Surgeon) \vee Occupation(Emily, Lawyer)$

b. $\exists\ o \neq actor \wedge Occupation(Joe, Actor) \wedge Occupation(Joe, o)$

c. $\forall p\ Occupation(p, Surgeon) \rightarrow Occupation(p, Doctor)$

d. $\neg \exists p\ Occupation(p, Lawyer) \wedge Customer(Joe, p)$

e. $\exists p\ Occupation(p, Lawyer) \wedge Boss(p, Emily)$

f. $\exists p1\ Occupation(p1, Lawyer) \wedge \forall p2\ Customer(p2, p1) \rightarrow Occupation(p2, Doctor)$

g. $\forall p1\ Occupation(p1, Surgeon) \rightarrow \exists p2\ Occupation(p2, Lawyer) \wedge Customer(p1, p2)$

**Problem 8.28**

b. $\neg Wrote(Gershwin, Eleanor\ Rigby)$

d. $\exists s\ Wrote(Joe, s)$

g. $\neg[\exists s\ Wrote(Gershwin, s) \wedge \exists\ p\ Sings(p, s, Revolver)]$

h. $\forall s\ Wrote(Gershwin, s) \rightarrow \exists\ p, a\ \ Sings(p, s, a)$

k. $\forall a\ [\exists s\ Sings(McCartney, s, a)] \rightarrow \exists d\ CopyOf(d, a) \wedge Owns(Joe, d)$

**Problem 9.9**

1. $0 \leq 3$.

2. $7 \leq 9$.

3. $\forall x \; x \leq x$.

4. $\forall x \; x \leq x + 0$.

5. $\forall x \; x + 0 \leq x$.

6. $\forall \; x, y \; x + y \leq y + x$.

7. $\forall \; w, x, y, z \; w \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$.

8. $\forall \; x, y, z \; x \leq y \wedge y \leq z \Rightarrow x \leq z$

a. Start with $7 \leq 3 + 9$ using Rule 8 you're left with $7 + 0 \leq 3+9$

   Using Rule 6 gives $9+3 \leq 3+9$

   Using Rule 7 gives $7 + 0 \leq 9+3$

   Using Rule 1 gives $0 \leq 3$

   Using Rule 2 gives $7 \leq 9$

b. Using Rule 7, and w = 0, x =7, y=3, z=9 gives $0 + 7 \leq 3 + 9$

   Rule 6 gives $9 + 3 \leq 3 + 9$

   Rule 4 gives $7 \leq 7+0$

   Rule 8 and x = 0 + 7, y = 7+0, z=3+9 gives $7+0 \leq 3+9$

   Strip away the 0 with Rules 5 and 8 and you're left with $7 \leq 3+9$

**Problem 9.20**

a. $\exists p \; \forall q \; person(p) \wedge person(q) \wedge ((\neg S(q,q) \leftrightarrow S(p,q))$

b. $person(p) \wedge person(q) \wedge (S(q,q) \vee S(p,q)) \wedge (\neg S(p,q) \vee \neg S(q,q))$

c. The clausal form resolves to empty clause meaning the logic is not satisfiable because it is false.

**Problem 13.8**

| | *toothache* | | *¬toothache* | |
|---|---|---|---|---|
| | *catch* | *¬catch* | *catch* | *¬catch* |
| *cavity* | 0.108 | 0.012 | 0.072 | 0.008 |
| *¬cavity* | 0.016 | 0.064 | 0.144 | 0.576 |

**Figure 13.3** A full joint distribution for the *Toothache*, *Cavity*, *Catch* world.

a. P(Toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2
b. P(Cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2
c. P(Toothache | Cavity)

   $= \frac{P(toothache \wedge cavity)}{p(cavity)} = \frac{0.108 + 0.012}{0.2} = 0.6$
d. P(Cavity | toothache v catch)

   $= \frac{P(\ cavity \wedge (toothache \vee catch))}{P(\ toothache \vee catch\ )} = \frac{0.108 + 0.012 + 0.072}{0.108 + 0.012 + 0.072 + 0.016 + 0.064 + 0.144} = 0.46$

**Problem 13.13**

$P(Test\ A\ positive \mid virus\ is\ present) = 0.95$

$P(Test\ A\ positive \mid virus\ is\ not\ present) = 0.1$

$P(Test\ B\ positive \mid virus\ is\ present) = 0.9$

$P(Test\ B\ positive \mid virus\ is\ not\ present) = 0.05$

$P(Virus\ is\ present) = 0.01$

$P(Virus\ is\ absent) = 0.99$

$P(Virus\ is\ present \mid Test\ A\ positive)$

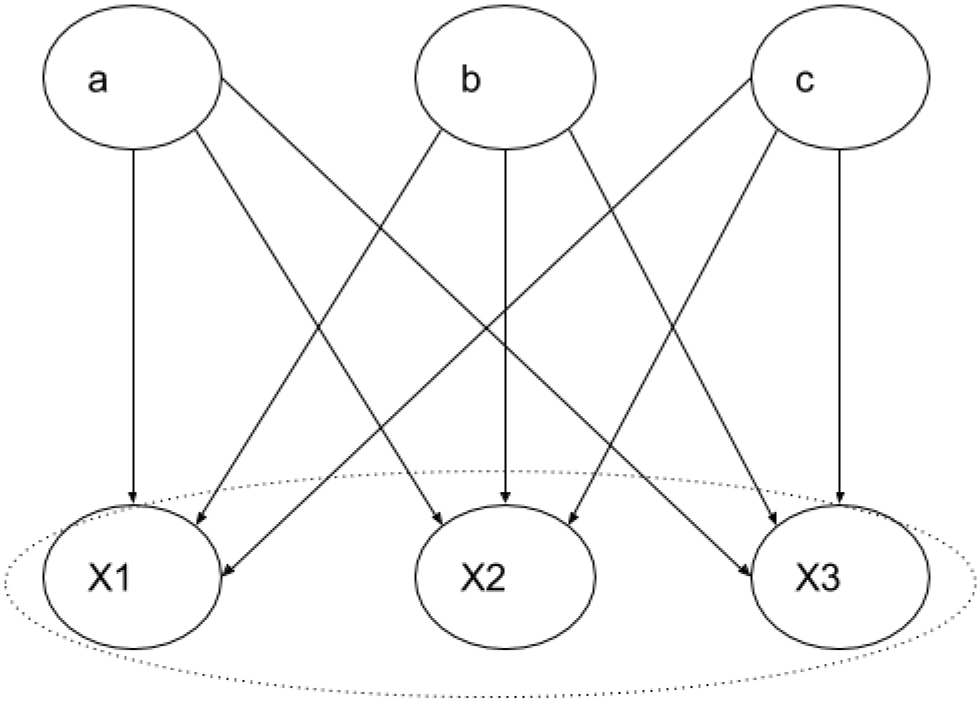
$$= \frac{0.95 * 0.01}{0.95*0.01 + 0.1*0.99} = 0.088$$

$P(Virus\ is\ present \mid Test\ B\ positive)$

$$= \frac{0.9*0.01}{0.9*0.01 + 0.05*0.99} = 0.15$$

Test B is more indicative of someone really carrying the virus.

**Problem 14.1**




a.

b. P(a | H, H, T) = 0.2 * 0.2 * 0.8 * 0.333 = 0.0107

P(b | H, H, T) = 0.6 * 0.6 * 0.4 * 0.333 = 0.048

P(c | H, H, T) = 0.8 * 0.8 * 0.2 * 0.333 = 0.0427

Therefor coin b is the mostly like to be drawn from the bag.

**Problem 14.15**

**function** ELIMINATION-ASK($X$, **e**, $bn$) **returns** a distribution over $X$
**inputs**: $X$, the query variable
**e**, observed values for variables **E**
$bn$, a Bayesian network specifying joint distribution $\mathbf{P}(X_1, \ldots, X_n)$

$factors \leftarrow [\,]$
**for each** $var$ **in** ORDER($bn$.VARS) **do**
$factors \leftarrow$ [MAKE-FACTOR($var$, **e**)|$factors$]
**if** $var$ is a hidden variable **then** $factors \leftarrow$ SUM-OUT($var$, $factors$)
**return** NORMALIZE(POINTWISE-PRODUCT($factors$))

**Figure 14.11** The variable elimination algorithm for inference in Bayesian networks.

a. P(Burglary | JohnCalls =true,MaryCalls =true) = <0.284, 0.716>

b. The number of arithmetic operations was 2 * 11 + 3 = 25. While using the enumeration algorithm there are 2 * (16 + 3) + 3 operations giving a total of 41.

c. The enumeration algorithm would take $O(n * 2^{n-2})$ time. Whereas variable elimination would have a running time of $O(n)$.

d. If one were to ignore the the direction of the edges in the polytree network, then we would be left with an undirected graph G with V being our variable elimination ordering. When a node is eliminated from the graph, it has no children left in the ordering V. Therefore the only remaining neighbor is the parent of the node. Thus the factor corresponding to that node will be a function of the parent alone and can be computed in linear time.