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CS 171 - Homework 4

Problem 7.4

c) Correct

e) Correct

g) Correct

h) Correct

k) Correct

Problem 7.7

a. 3

b. 15

c. 0

Problem 7.12

S1: A ⇔ (B ∨ E)

$$(\neg A \vee B \vee E) \wedge (\neg B \vee A) \wedge (\neg E \vee A)$$

S2: E ⇒ D

$$\neg E \vee D$$

S3: C ∧ F ⇒ ¬B

$$(\neg C \vee \neg F \vee \neg B)$$

S4: E ⇒ B

$$\neg E \vee B$$

S5: B ⇒ F

$$\neg B \vee F$$

S6: B ⇒ C

$\neg B \vee C$

S7: To prove $\neg B$, negate it

B

S8: Resolve S7 with S5

F

S9: Resolve S7 with S6

C

S10: Resolve S8 with S3

$(\neg C \vee \neg B)$

S11: Resolve S9 with S10

$\neg B$

S12: To prove $\neg A$, negate it

A

S13: Resolve S4 with the first clause of S1 $(\neg A \vee B \vee E)$

$\neg A \vee B$

S14: Resolve S12 with S11

$\neg A$

Resolve S7 with S11 and S12 with S14 which will return the empty set for both. Meaning that $\neg A \wedge \neg B$ is proven.

Problem 7.17

a. (A ∨ B) ∧ (¬A ∨ C) ∧ (¬B ∨ D) ∧ (¬C ∨ G) ∧ (¬D ∨ G)

Resolve (A ∨ B) ∧ (¬A ∨ C) to get:

(B ∨ C) ∧ (¬B ∨ D) ∧ (¬C ∨ G) ∧ (¬D ∨ G)

Resolve (B ∨ C) ∧ (¬B ∨ D) to get:

(C ∨ D) ∧ (¬C ∨ G) ∧ (¬D ∨ G)

Resolve (C ∨ D) ∧ (¬C ∨ G) to get:

(D ∨ G) ∧ (¬D ∨ G)

Resolve (D ∨ G) ∧ (¬D ∨ G) to get:

(G ∨ G)

G

b. $4\binom{n}{2} + 2n + 1$

$= 2n(n-1) + 2n + 1$

$= 2n^2 + 1$

c. Since there are at most f(n) clauses for a 2-CNF expression, any resolution step will take at most f(n) time.

d. The argument for (c) does not apply to 3-CNF because each clause will have an additional literal which means that it has a different termination for 2-CNF, meaning that the same formula can not be used.

Problem 7.20

S1: A ⇔ (B ∨ E)

$(\neg A \vee B \vee E) \wedge (\neg B \vee A) \wedge (\neg E \vee A)$

S2: E ⇒ D

$\neg E \vee D$

S3: C ∧ F ⇒ ¬B

$(\neg C \vee \neg F \vee \neg B)$

S4: E ⇒ B

$\neg E \vee B$

S5: B ⇒ F

$\neg B \vee F$

S6: B ⇒ C

$\neg B \vee C$

(¬A ∨ B ∨ E) ∧ (¬B ∨ A) ∧ (¬E ∨ A) ∧ (¬E ∨ D) ∧(¬C ∨ ¬F ∨ ¬B) ∧ (¬E ∨ B) ∧ (¬B ∨ F) ∧ (¬B ∨ C)