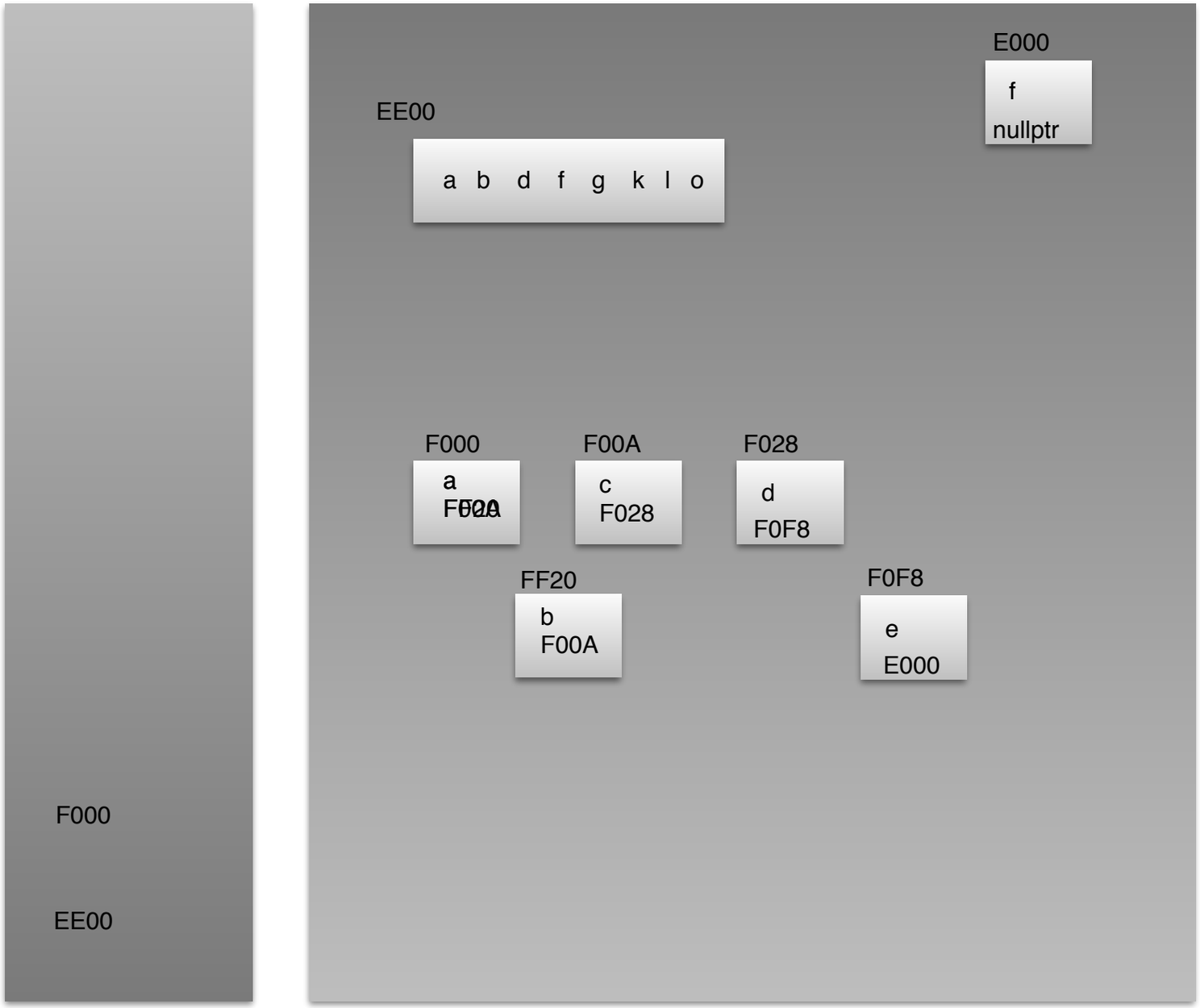
Lecture 15

Trees

CS2134

Thinking about storing items…
What could we do differently?

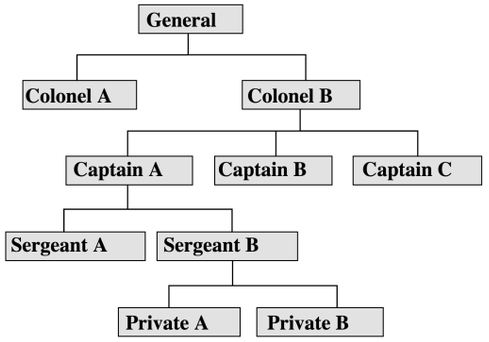


The best time to plant a tree was 20 years ago.
The next best time is now. ~Chinese Proverb

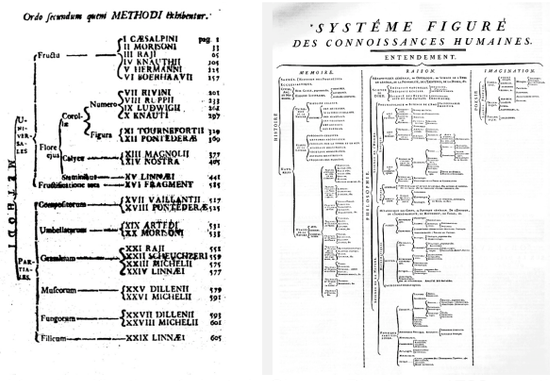
I willingly confess to so great a partiality for trees as tempts me to respect a man in exact proportion to his respect for them.
~James Russell Lowell

Trees

- Up to now, the data structures we've studied (vectors, stacks queues, lists) have stored sequences of data
  - each item has a single successor
- Often, data is organized hierarchically, not sequentially
  - organization chart for a company
  - classification of the animal kingdom
  - file system
- Trees can be used to characterize such relationships:
  - items may have several successors, called children
- We'll focus on binary trees, in which each node has at most two children
  - in addition to applications to hierarchical data, these have applications to searching and sorting.

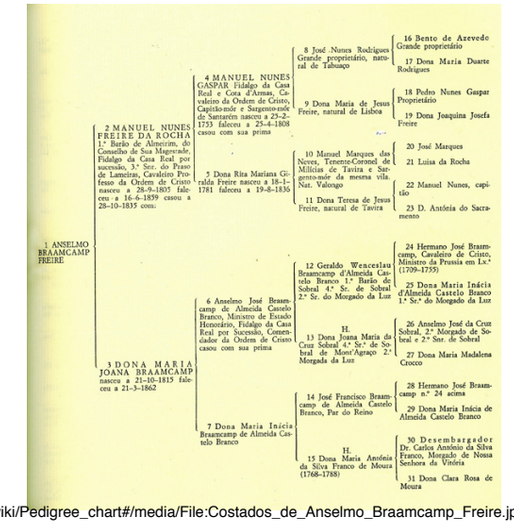


https://en.wikipedia.org/wiki/Organizational_chart#/media/File:Organizational_chart.svg



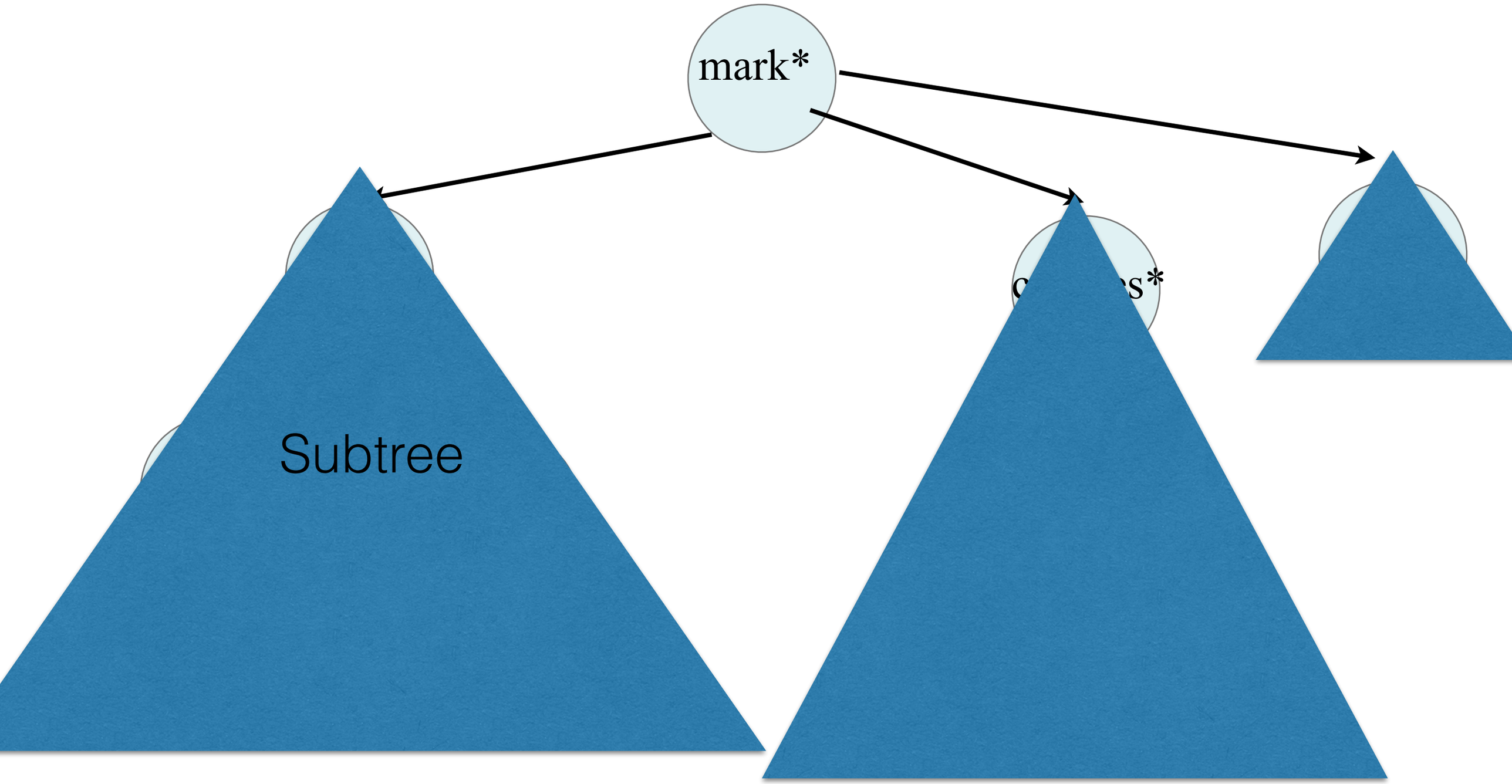
"Linné's method for classification of plants in Classes Plantarum 1738, and the Figurative system of human knowledge from Diderot's Encyclopédie , 1752."

https://upload.wikimedia.org/wikipedia/commons/7/70/Taxonomy_Linn%C3%A9_%26_Diderot.jpg



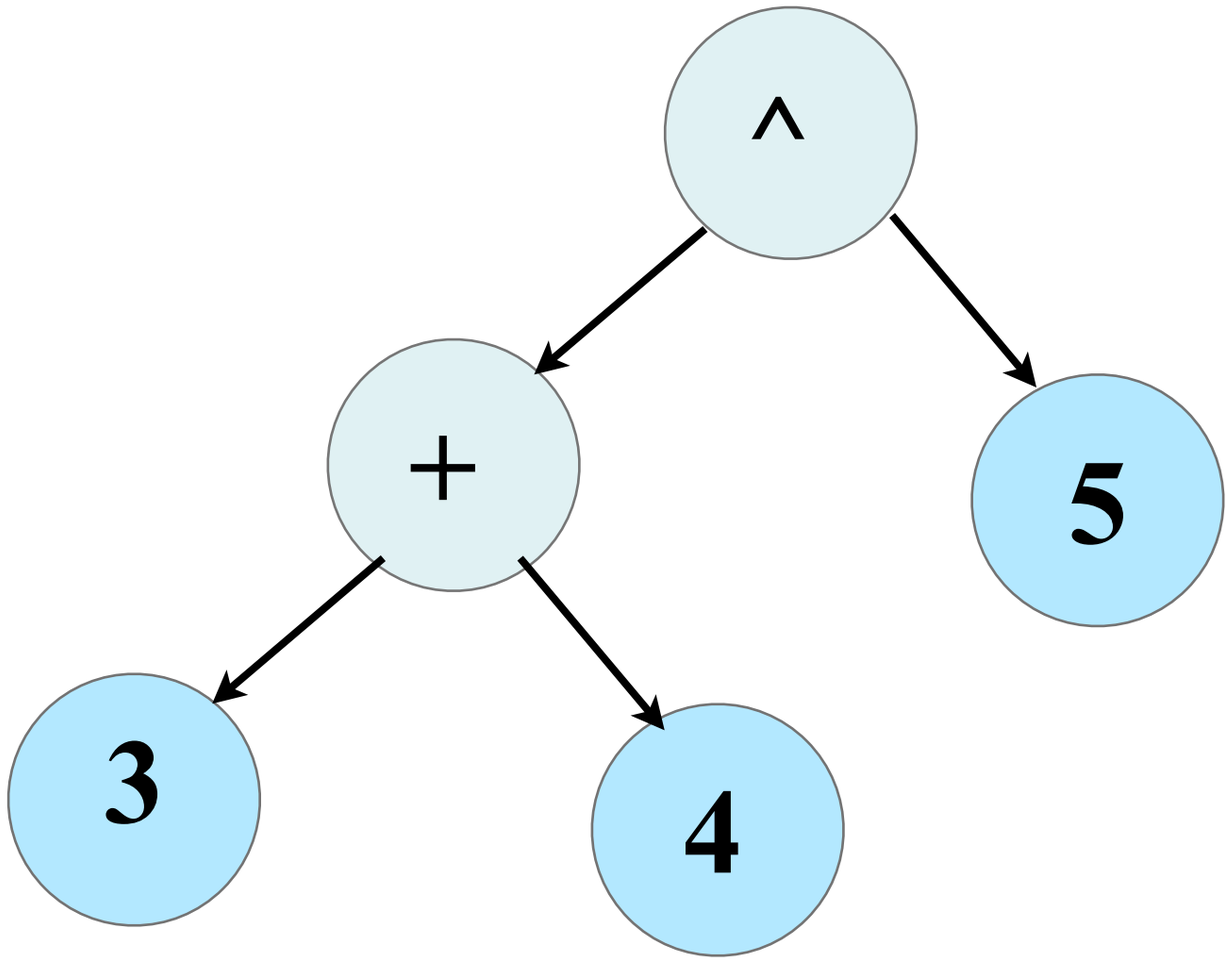
https://en.wikipedia.org/wiki/Pedigree_chart#/media/File:Costados_de_Anselmo_Braamcamp_Freire.jpg

Unix Directory Tree

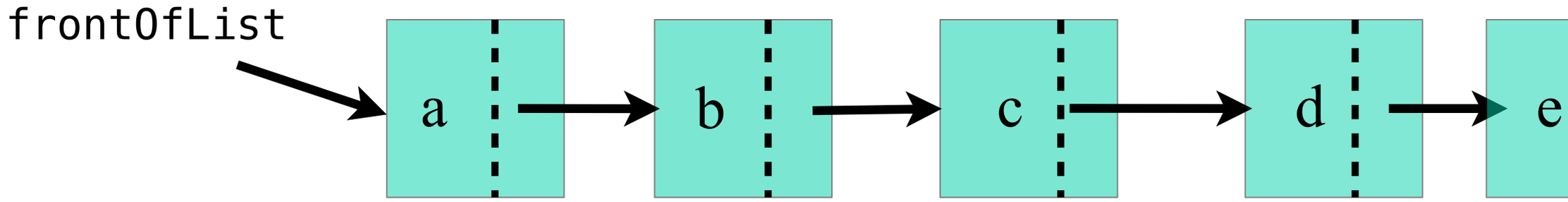


Expression Tree

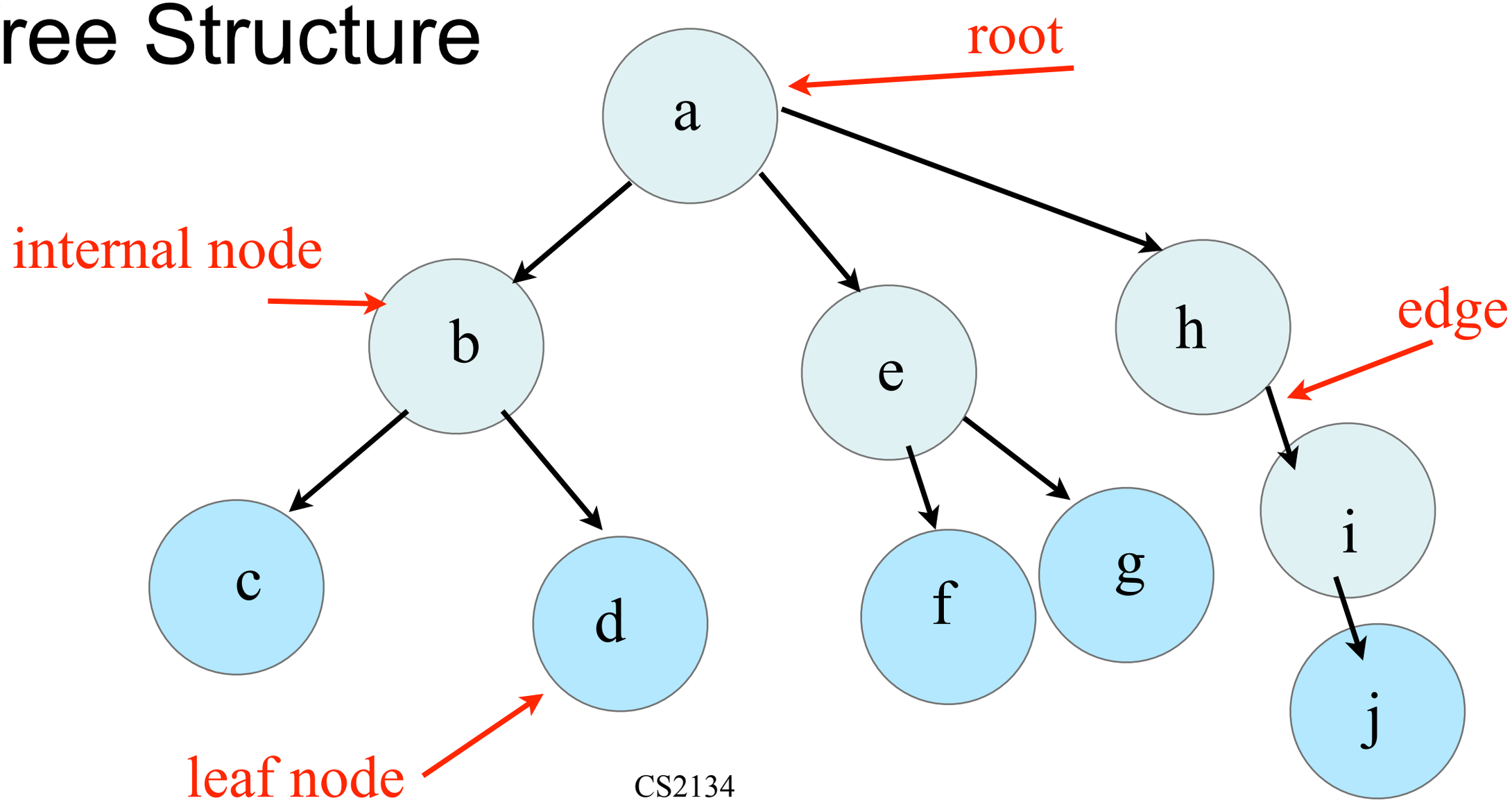
(3 + 4)^5



List Structure



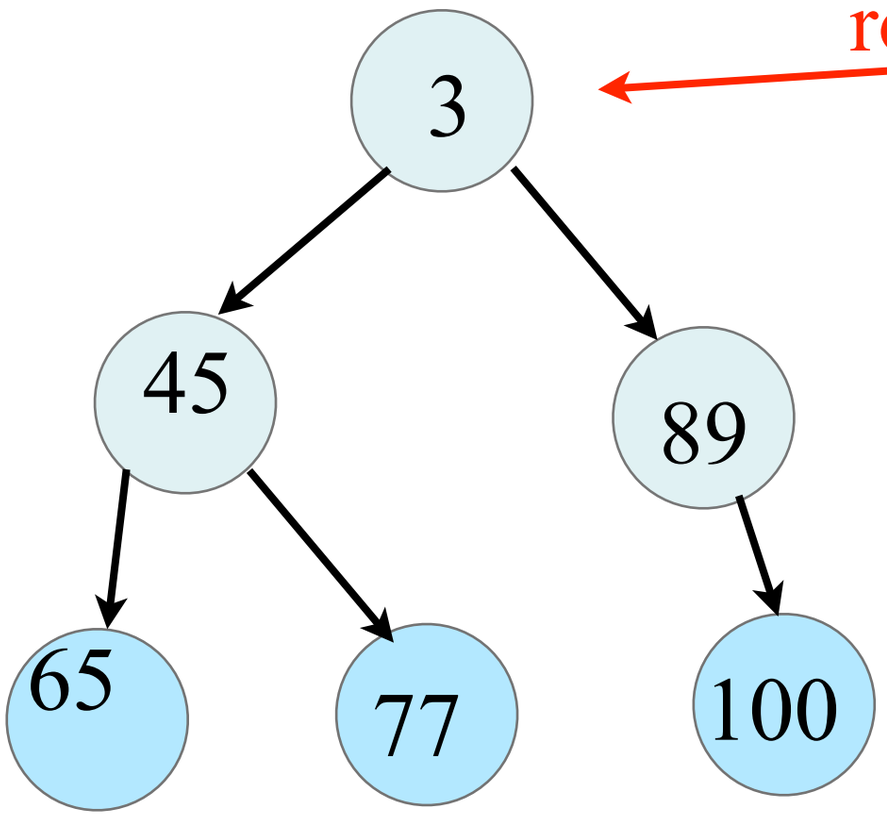
Tree Structure



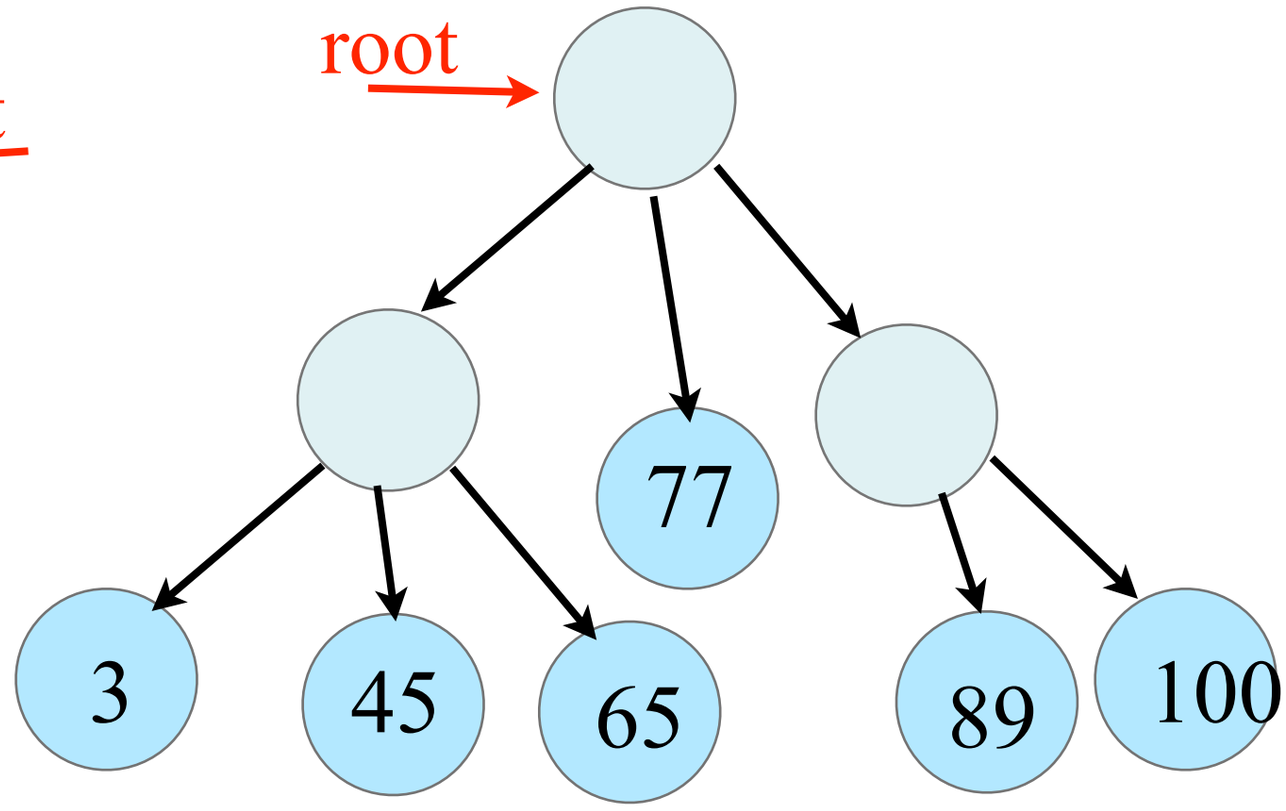
Family Tree, Biological classifications, Organization chart

- parent
- child
- sibling
- ancestor
- descendent
- root
- leaf
- internal node
- size
  - of node
  - of tree
- height

Storing #'s 3, 45, 65, 77, 89, 100

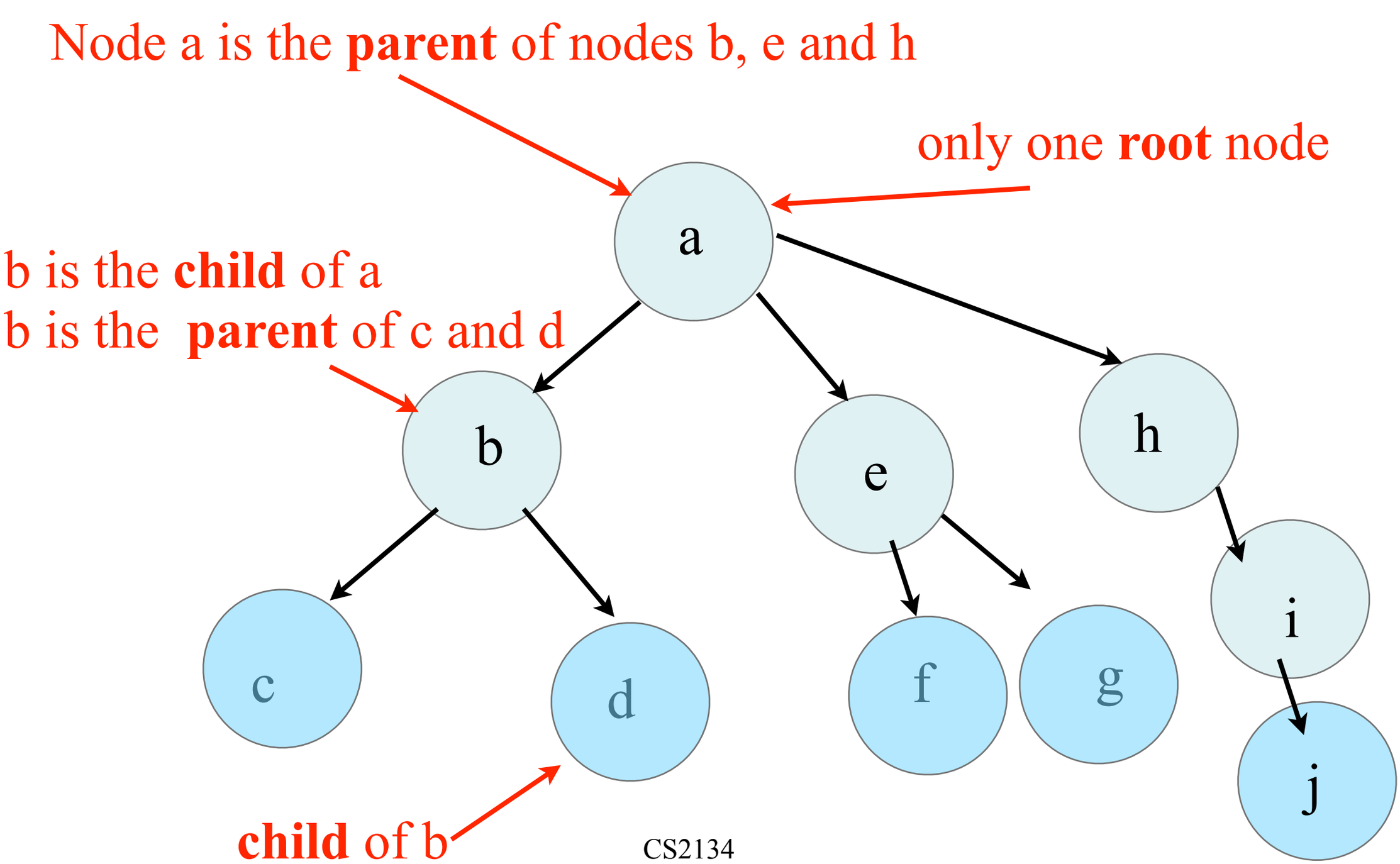


Information is stored in every node

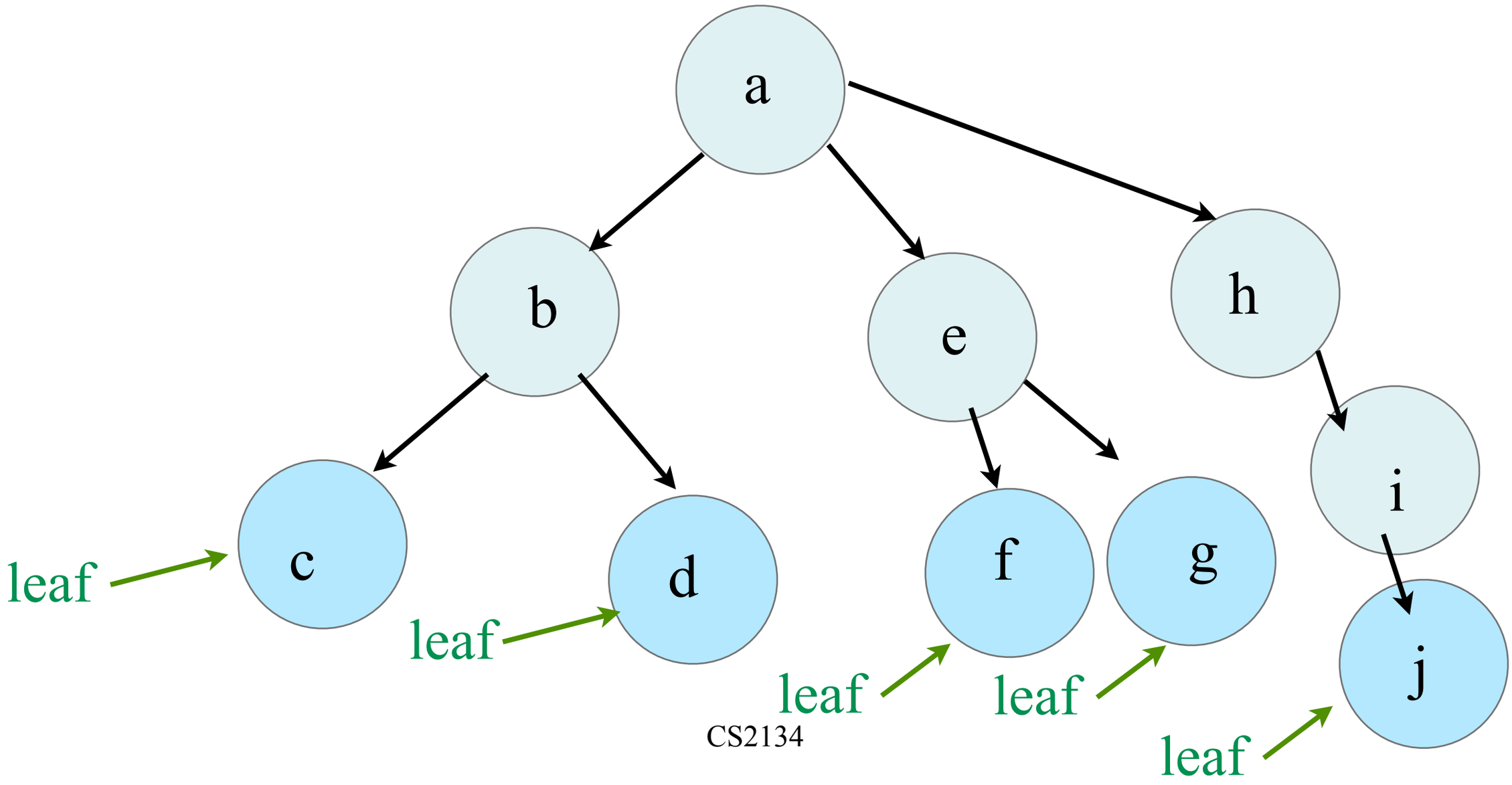


Information is stored only in the leaf nodes

Definitions:



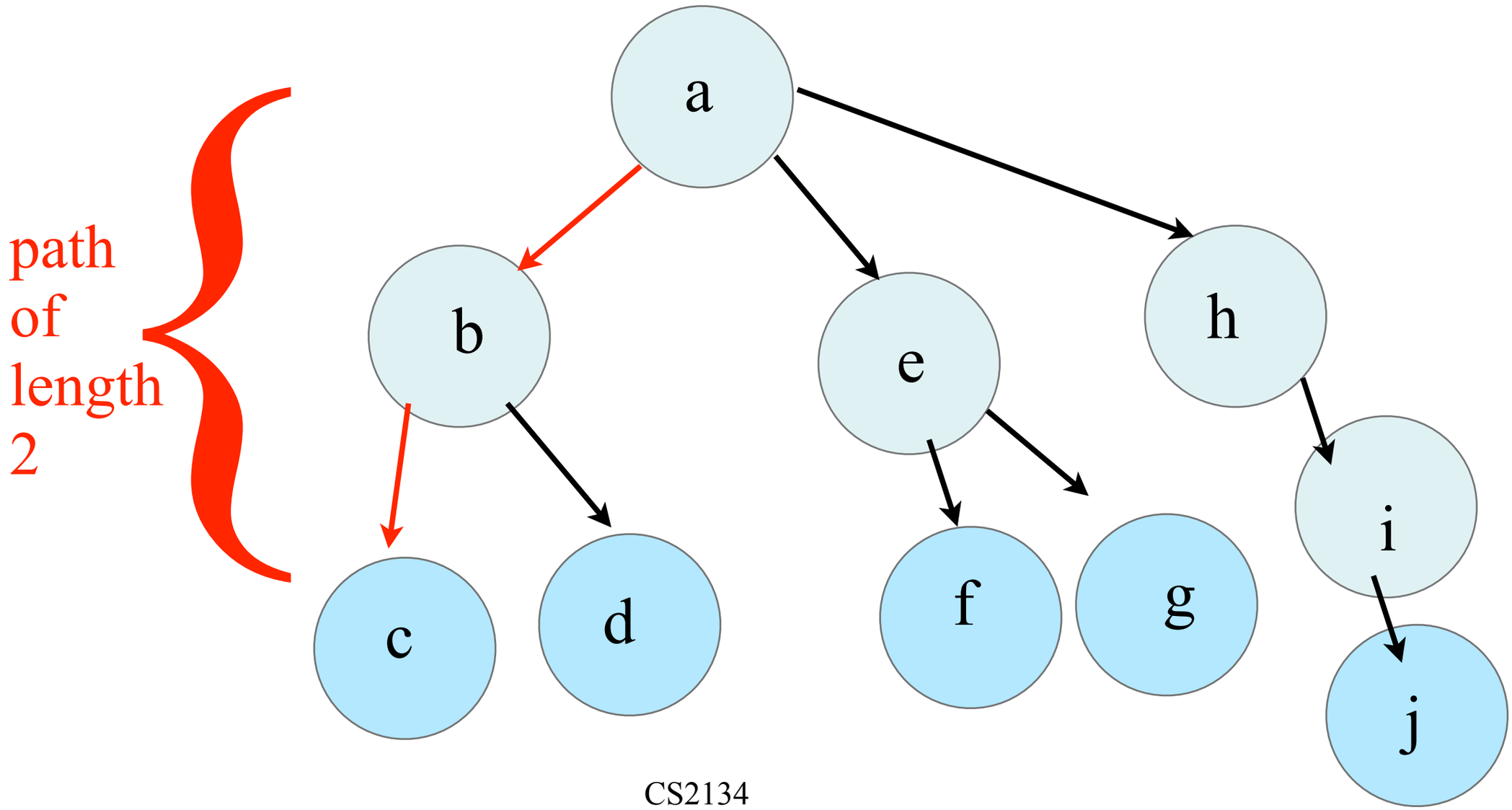
What is the definition of a leaf?



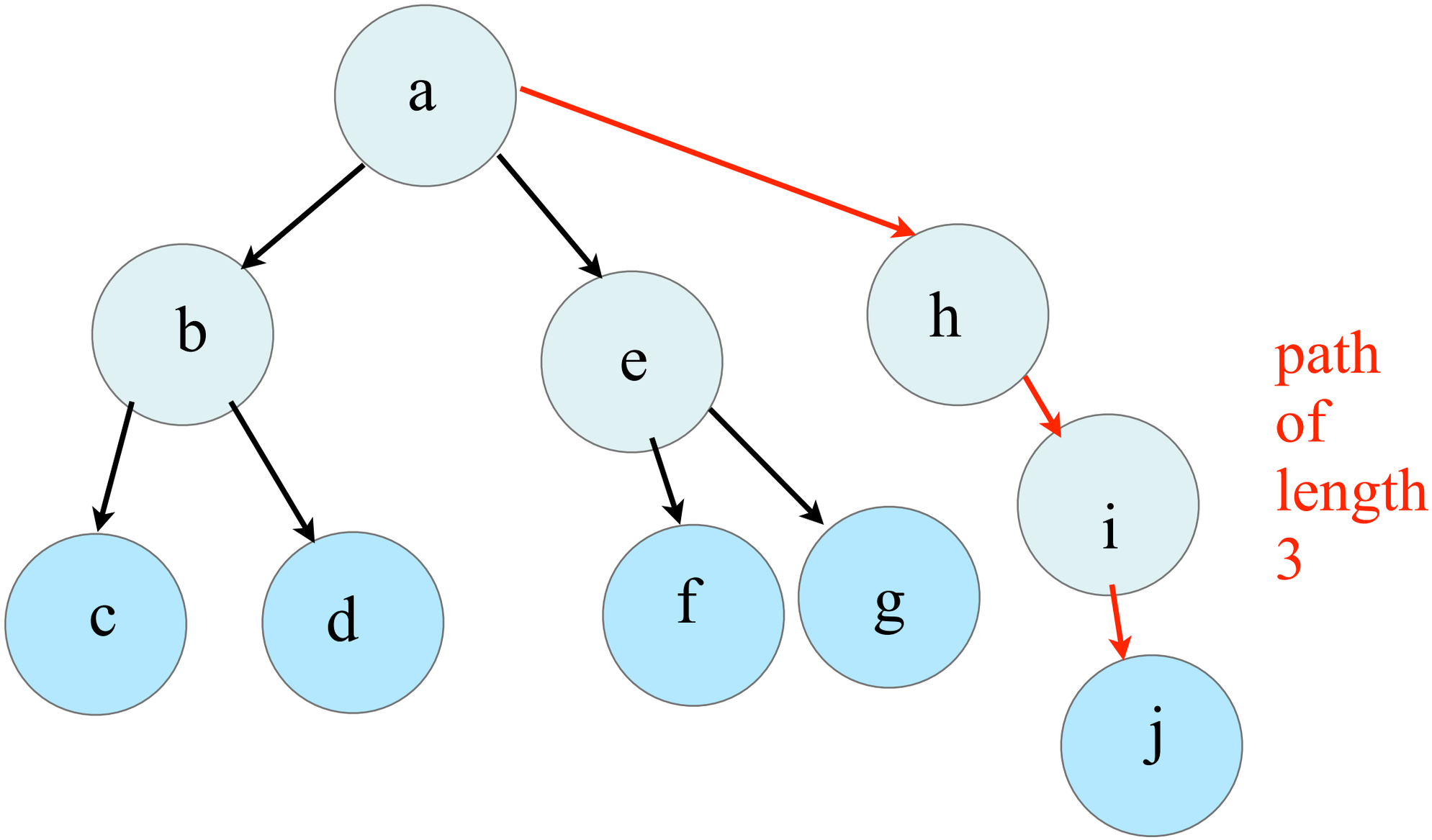
What is the definition of the length of a path between two nodes?

There is one unique path from the root to any node in the tree.

The number of edges along the path is the path length.

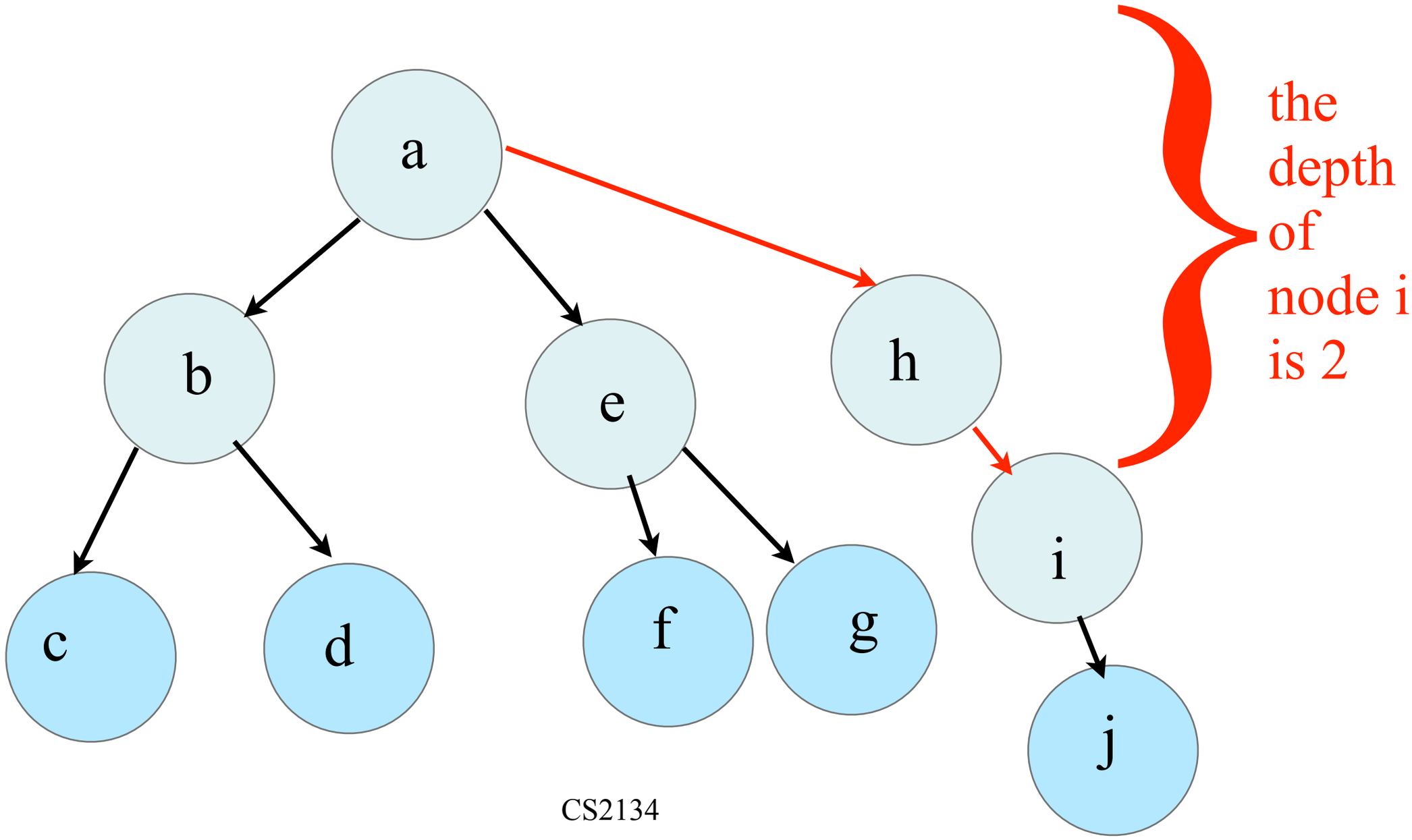


What is the path length from a to j?



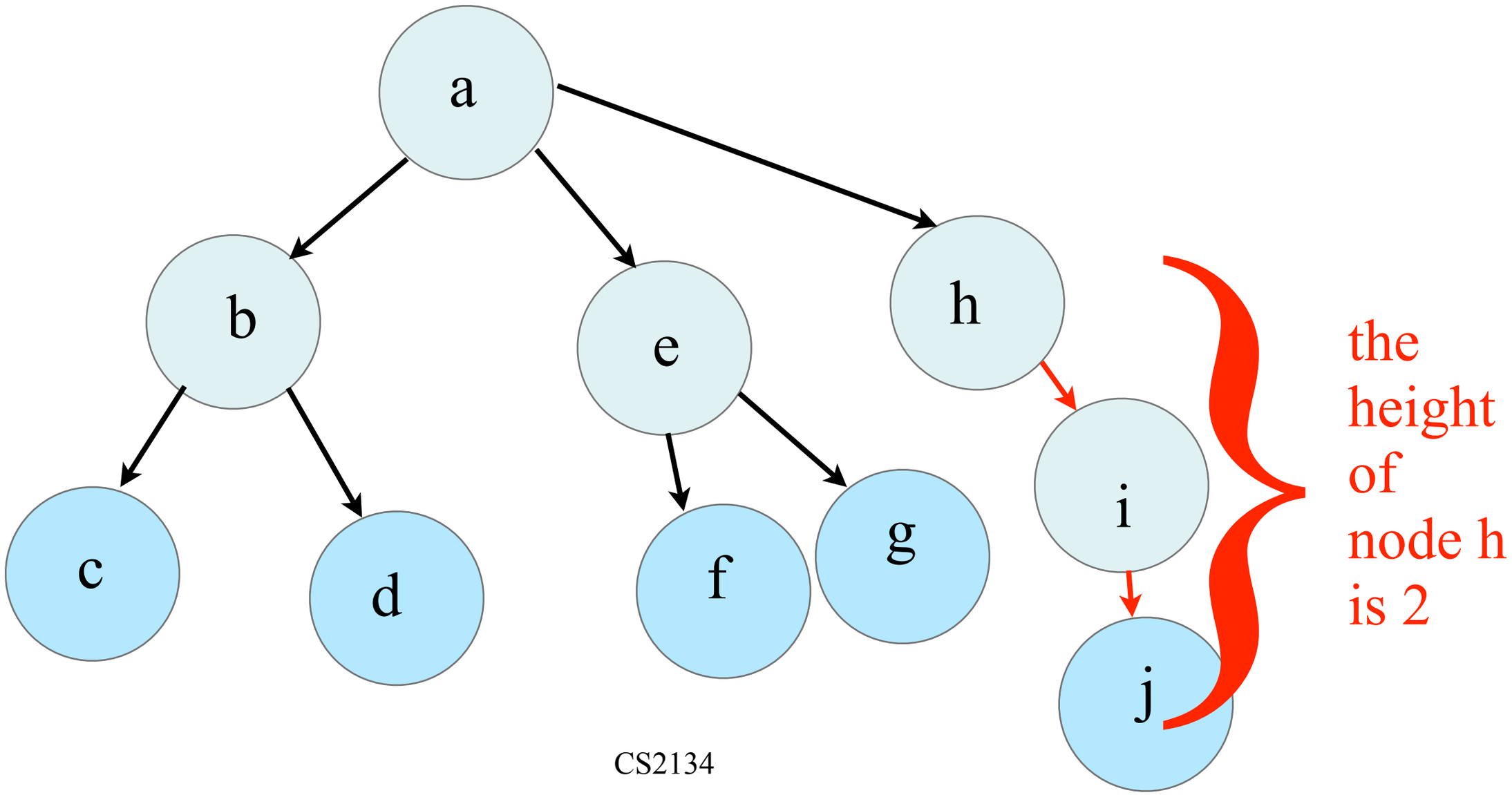
What is the definition of the depth of a node?

The *depth* of a node is the number of edges from the root to the node

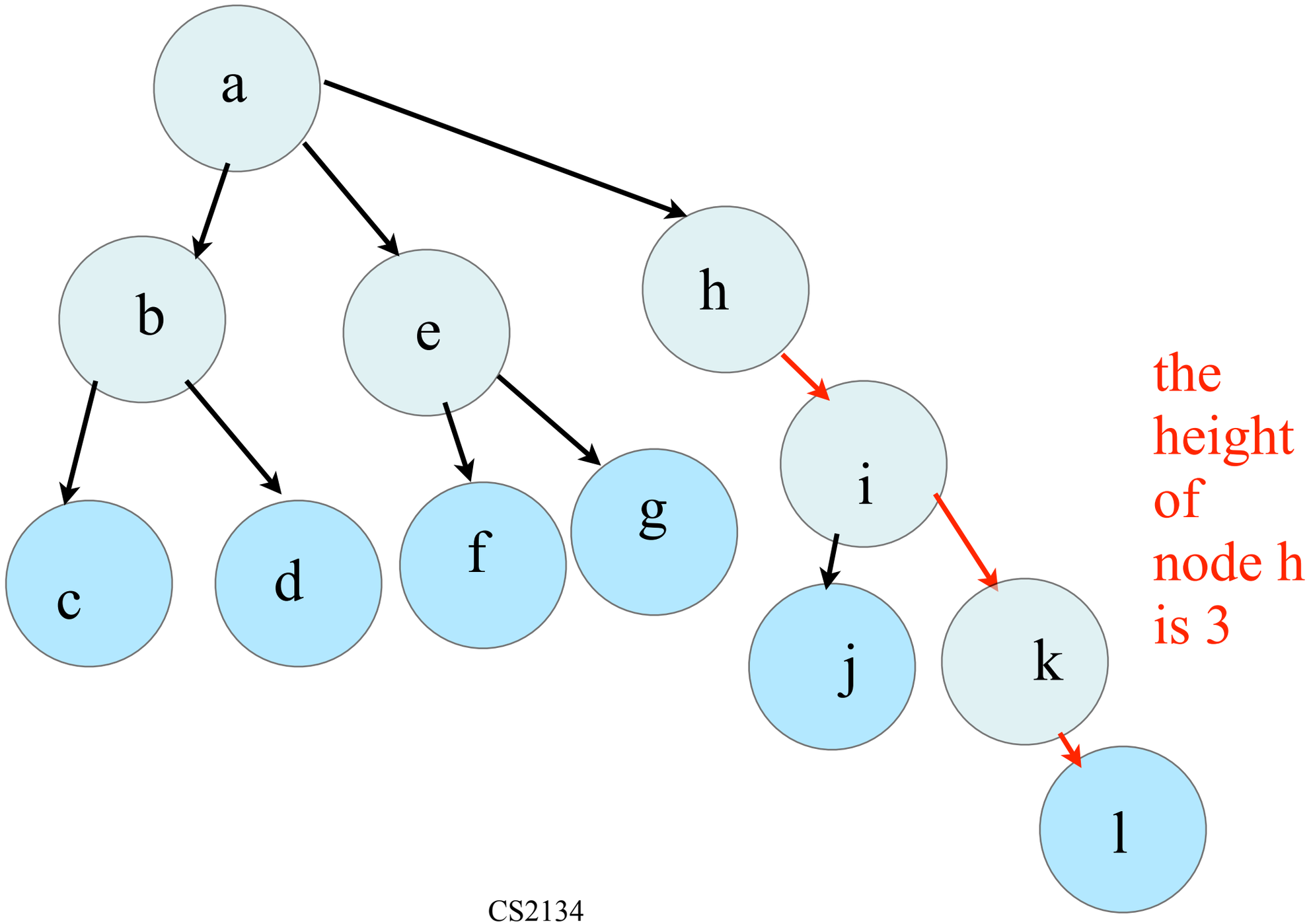


What is the definition of the height of a node?

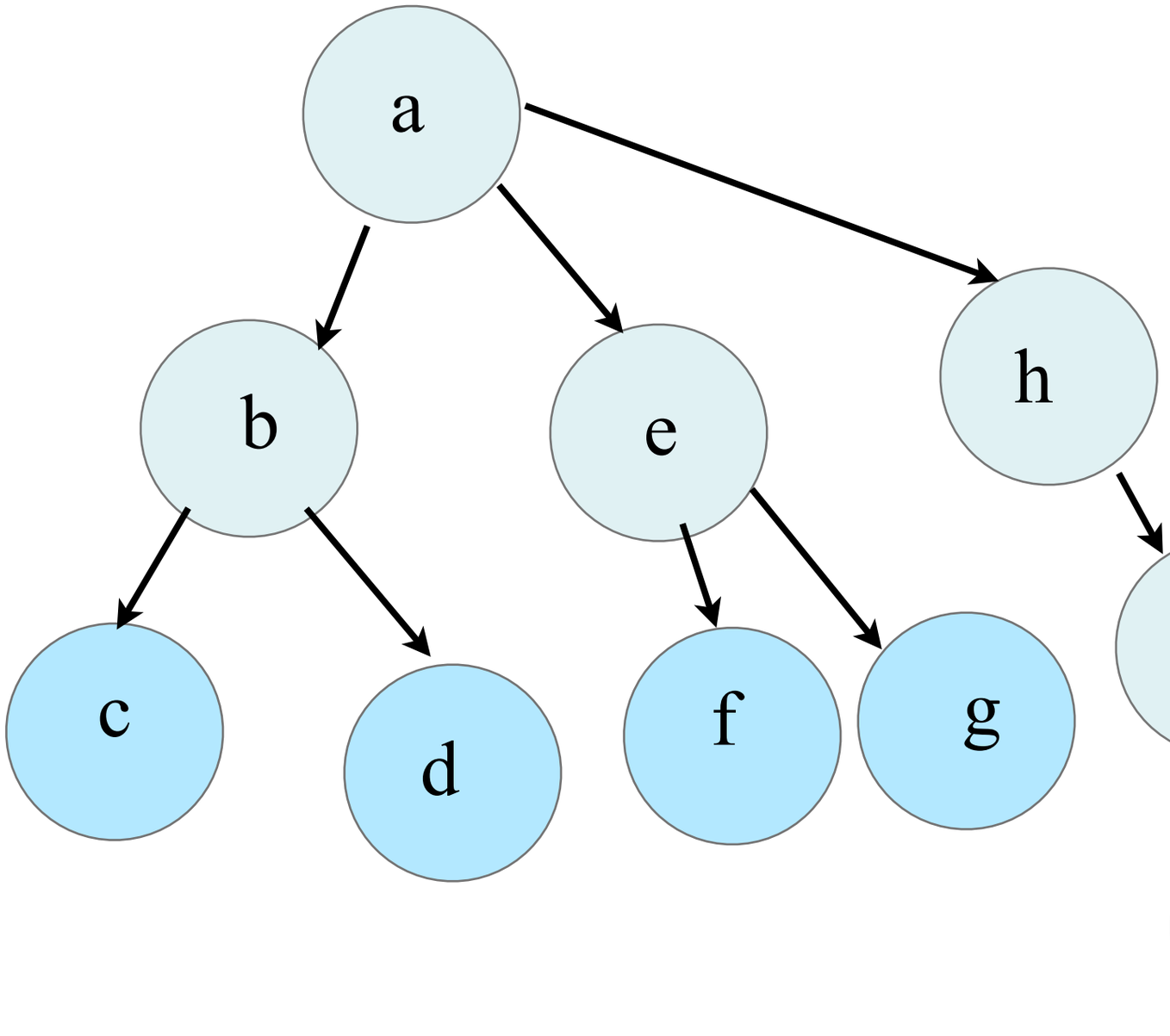
The *height* of a node is the number of edges from the node to the deepest leaf.



What is the height of node h?



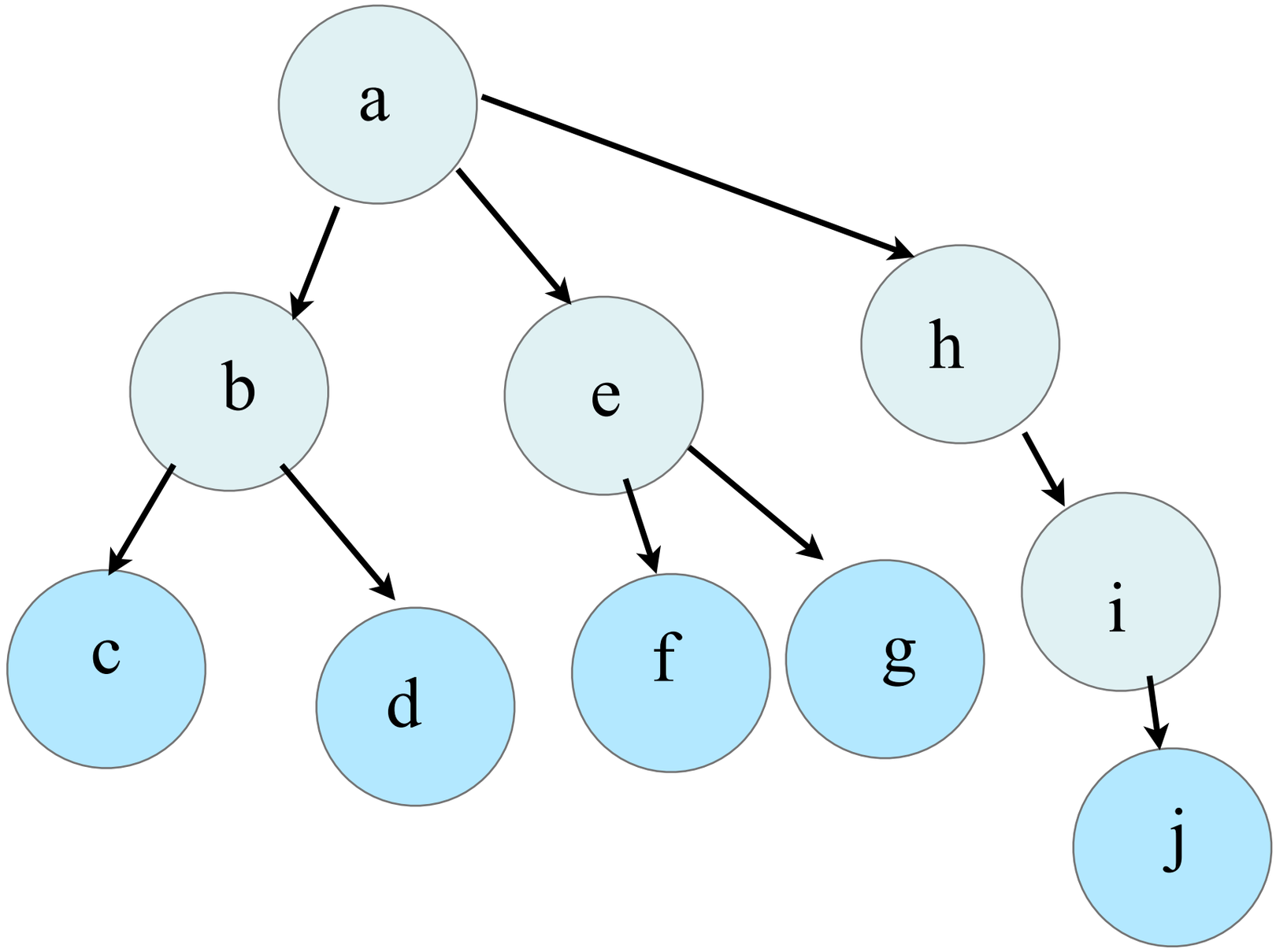
What are the height and depth of nodes a, b and c?



Node	Height	Depth
a	3	0
b	1	1
c	0	2
d	0	2
e	1	1
f	0	2
g	0	2
h	2	1
i	1	2
j	0	3

What is the definition of the size of a node?

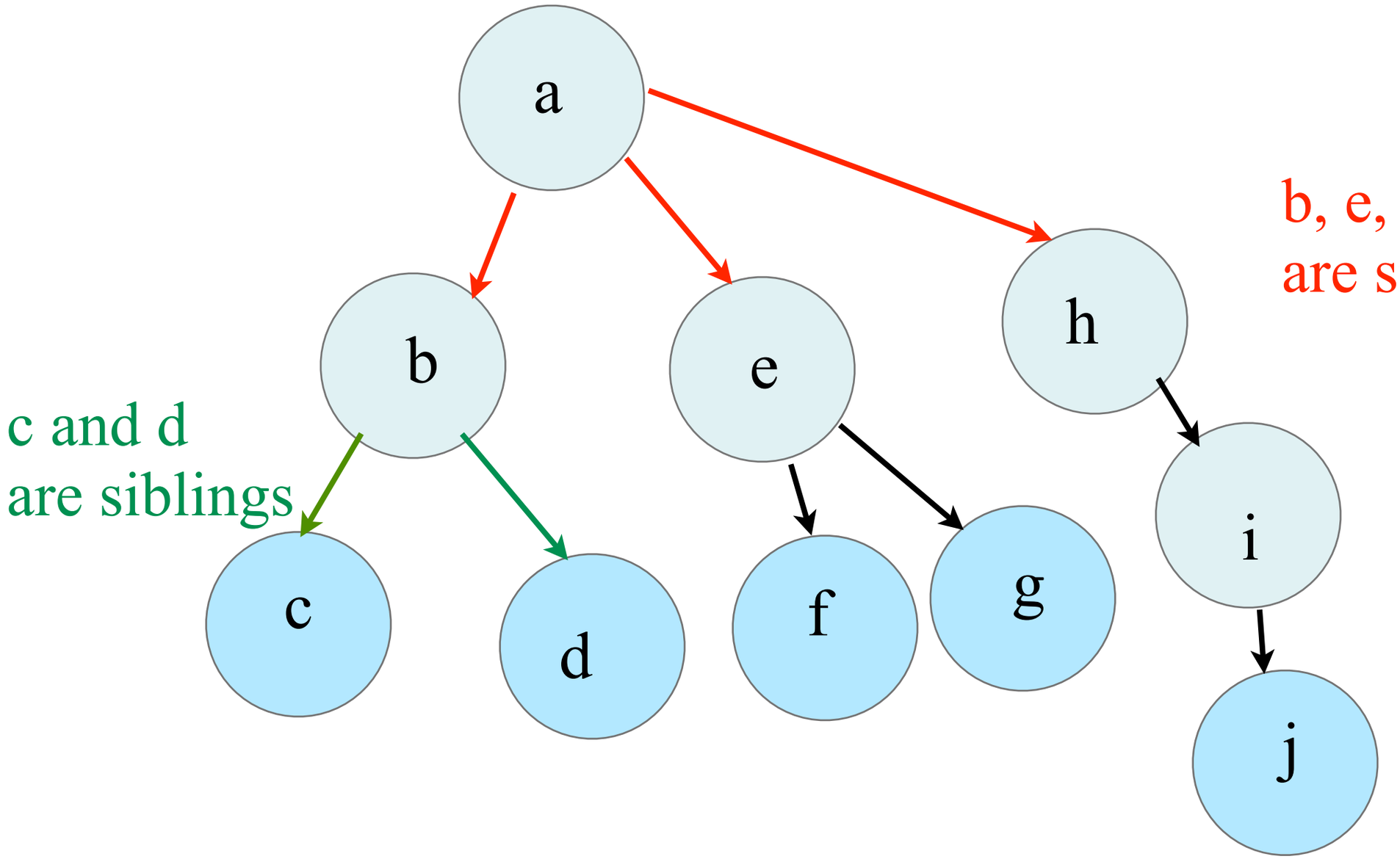
The *size* of a node is the number of descendants the node has (including the node itself.)



Node	Size
a	10
b	3
c	1
d	1
e	3
f	1
g	1
h	3
i	2
j	1

The *size* of a tree is the size of the root node.

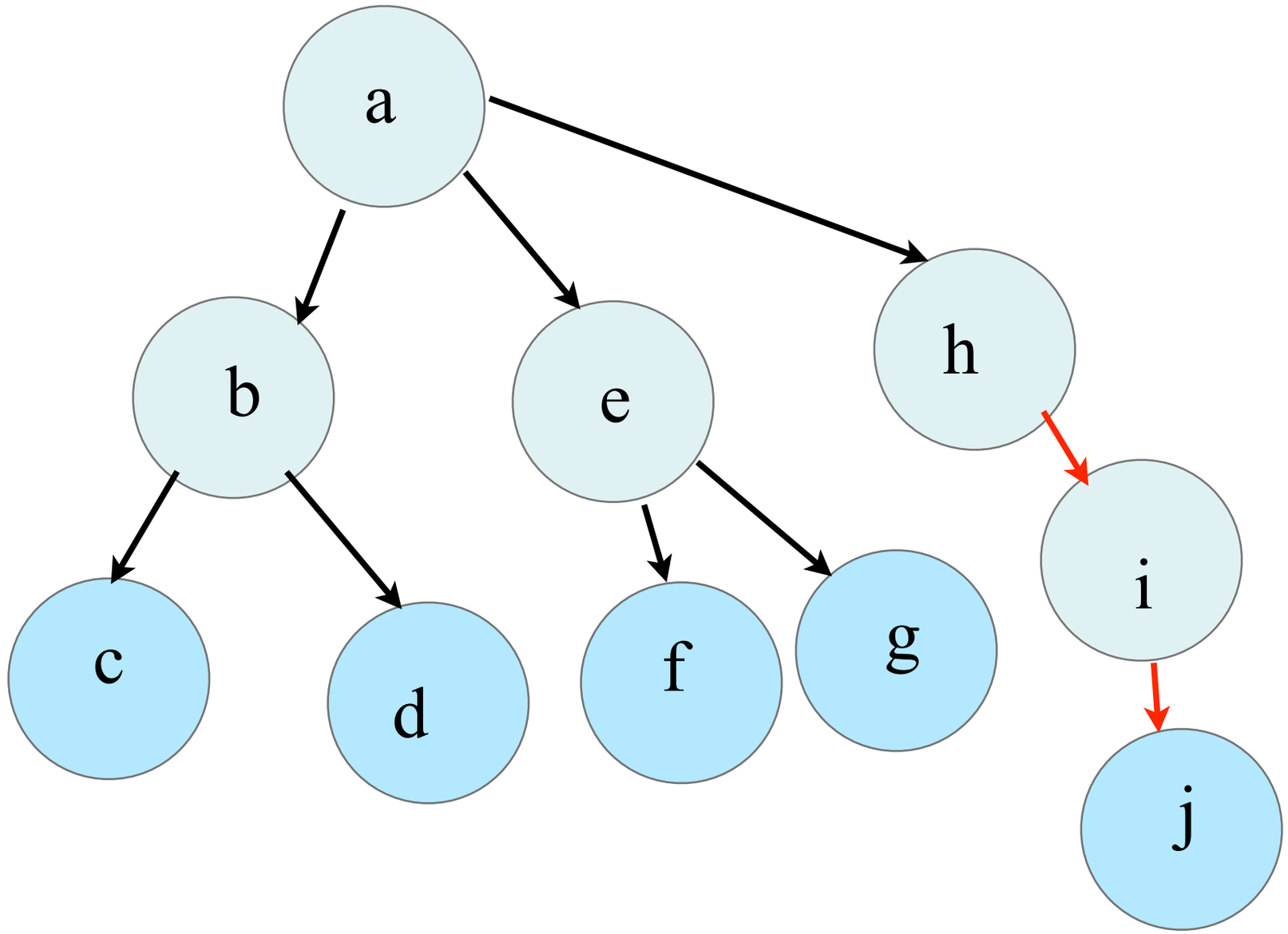
Nodes with the same parent are siblings



b, e, and h are siblings

c and d are siblings

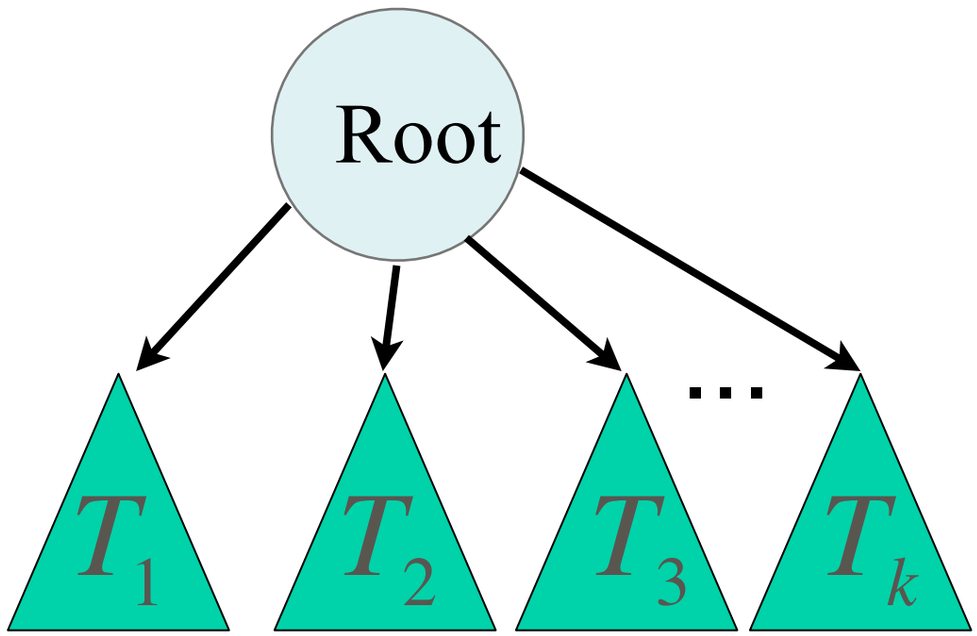
If there is a path from node h to node j, then
h is the *ancestor* of node j, and j is the *descendent* of node h



Recursive Definition of a Tree

- A tree is empty
- or it consists of a root and zero or more nonempty subtrees T_1 , T_2,..., T_k .

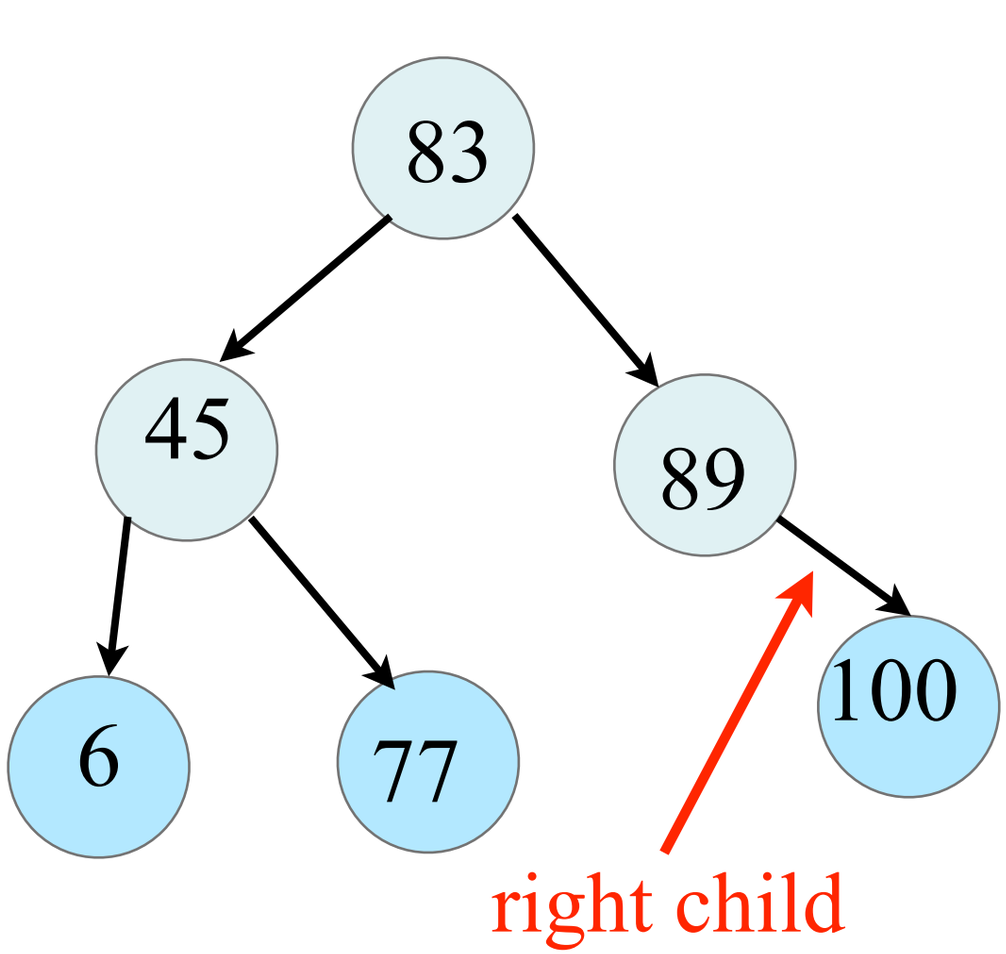
Each subtree is connected by an edge from the root.

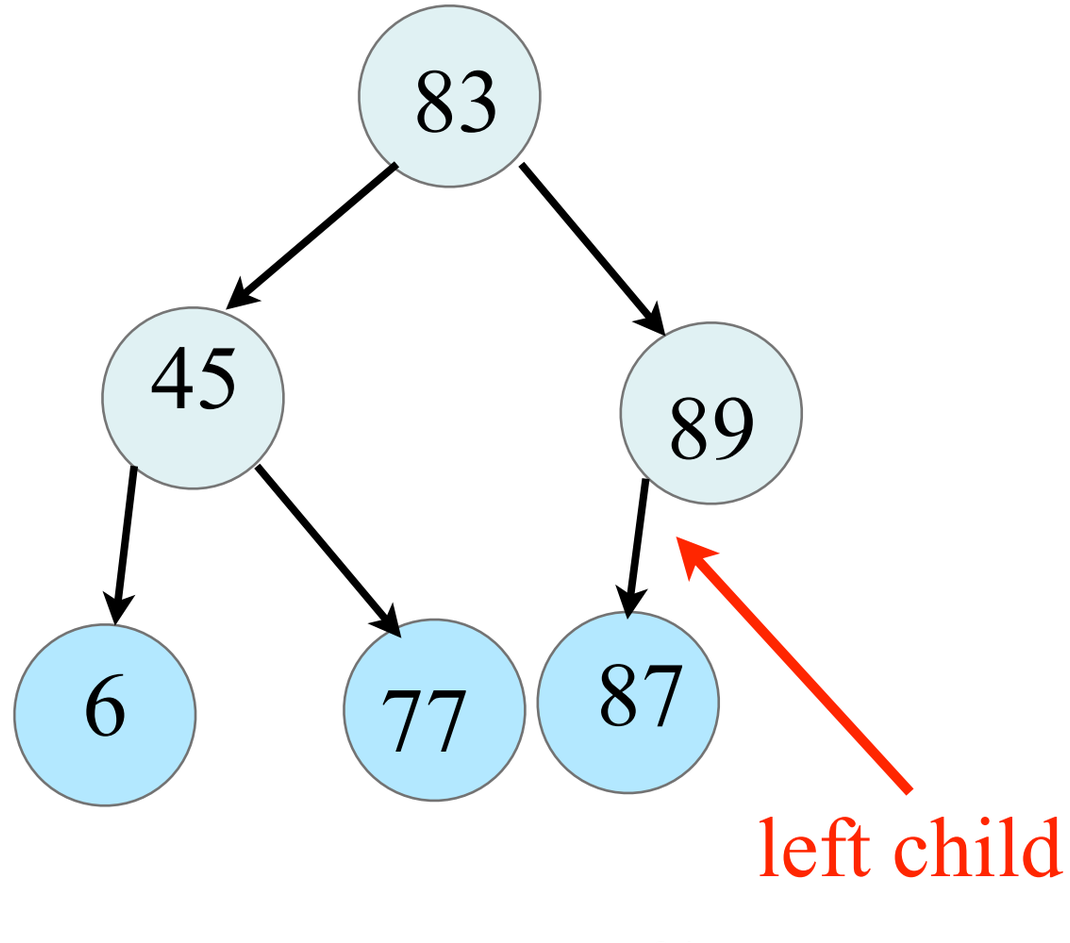


If a tree has n nodes how many edges does it have? n-1

Binary Trees

- We will focus on binary trees.
- 0, 1 or 2 children per node.
- Sometimes an “order” is imposed on the tree in the tree below. Can you see what it is?

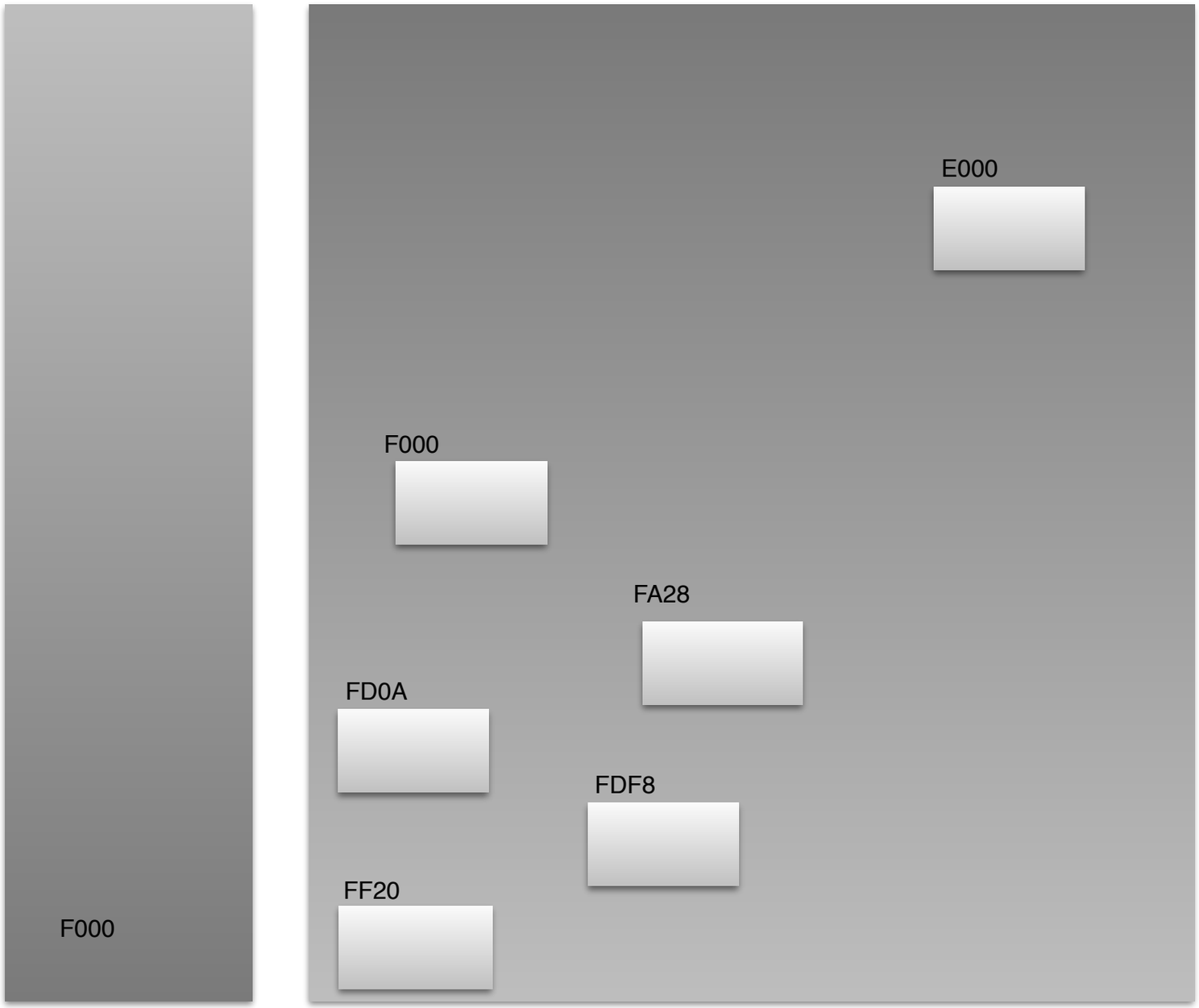




Binary tree Definition

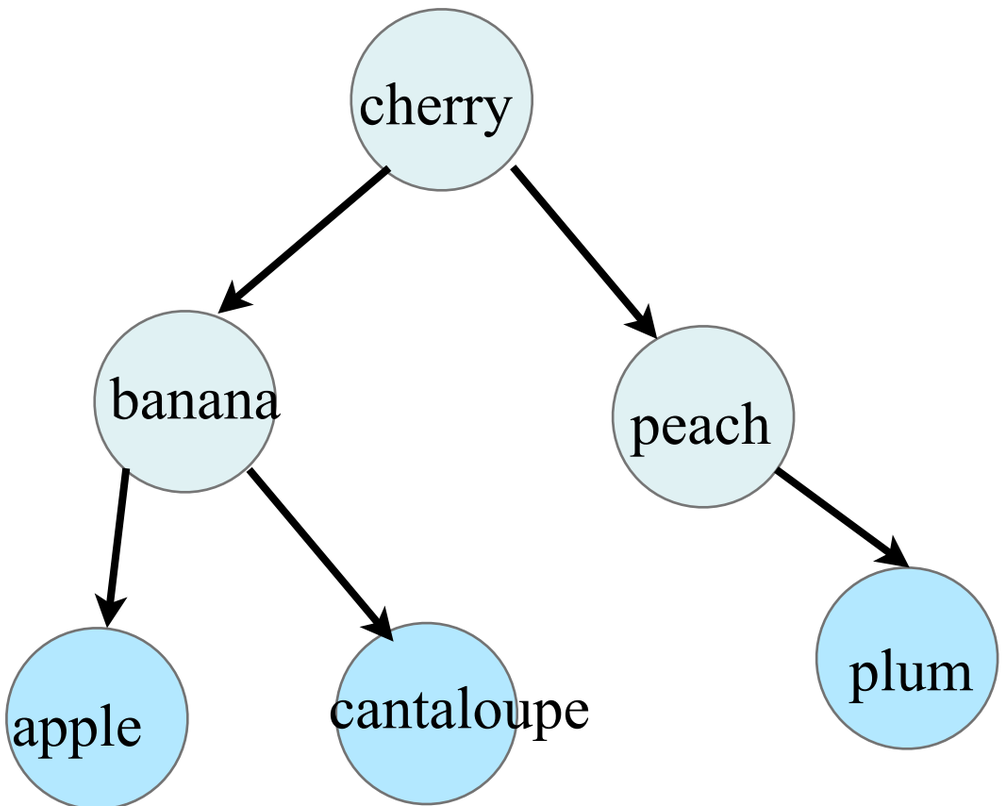
- Def 1:
  - empty, or
  - unique root node, and
  - each node has at most two children, the left child and the right child, and
  - for each node there a unique path from the root "down" to the node.
- Def 2:
  - empty, or
  - a node, or
  - a node along with two trees, the right subtree and the left subtree

Thinking about storing items as a tree
How could we implement this?

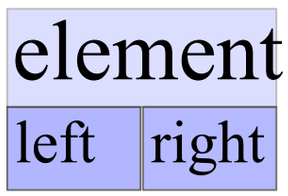


Implementing a node

```
template <class Object>
class BinaryNode {

public:
  BinaryNode( const Object & theElement = Object( ),
              BinaryNode *lt = nullptr, BinaryNode *rt = nullptr );
    BinaryNode( Object && theElement, BinaryNode *lt = nullptr, BinaryNode *rt = nullptr );

  // Some Methods

public: // To keep things simple
  Object   element;
  BinaryNode *left;
  BinaryNode *right;
};
```



A node in the tree



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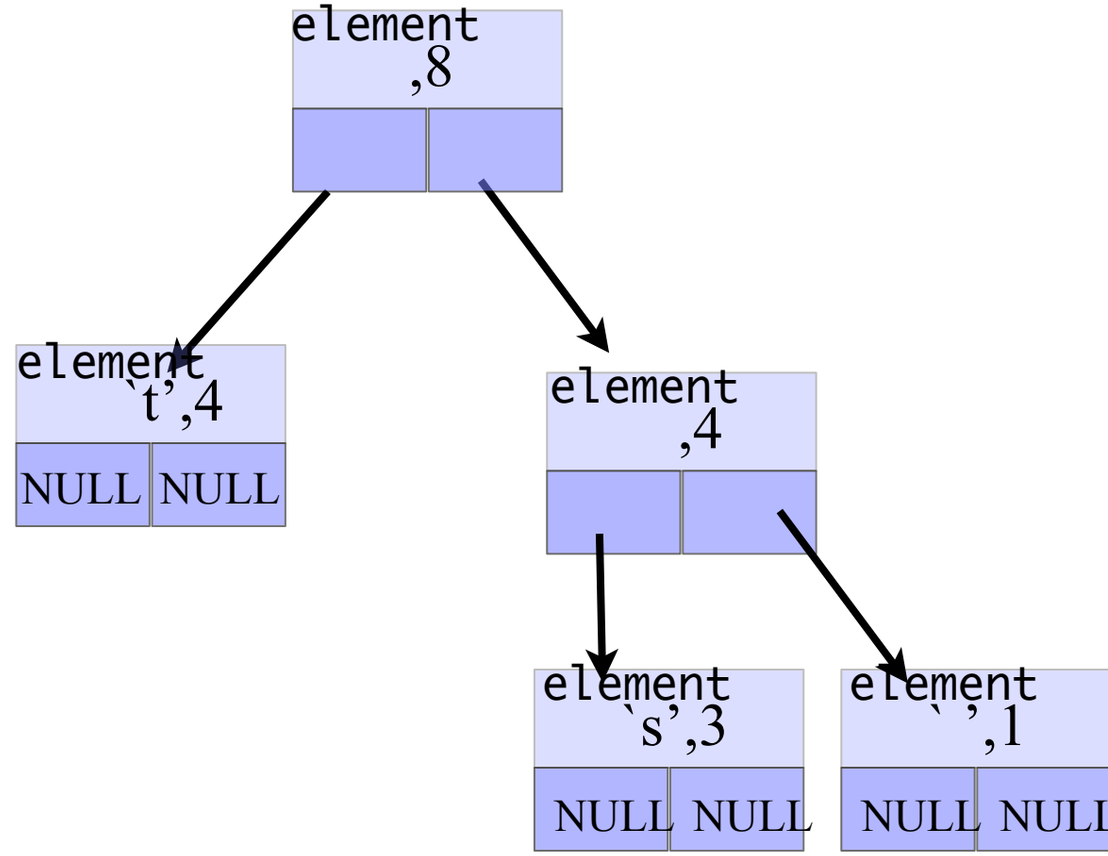
  BinaryNode( Object && theElement, BinaryNode *lt = nullptr, BinaryNode *rt = nullptr );

  // Some Methods

public: // To keep things simple
  Object   element;
  BinaryNode *left;
  BinaryNode *right;
 };
```

- pointer structure for tree
- dynamic growing and shrinking
- resides in heap

Tree example

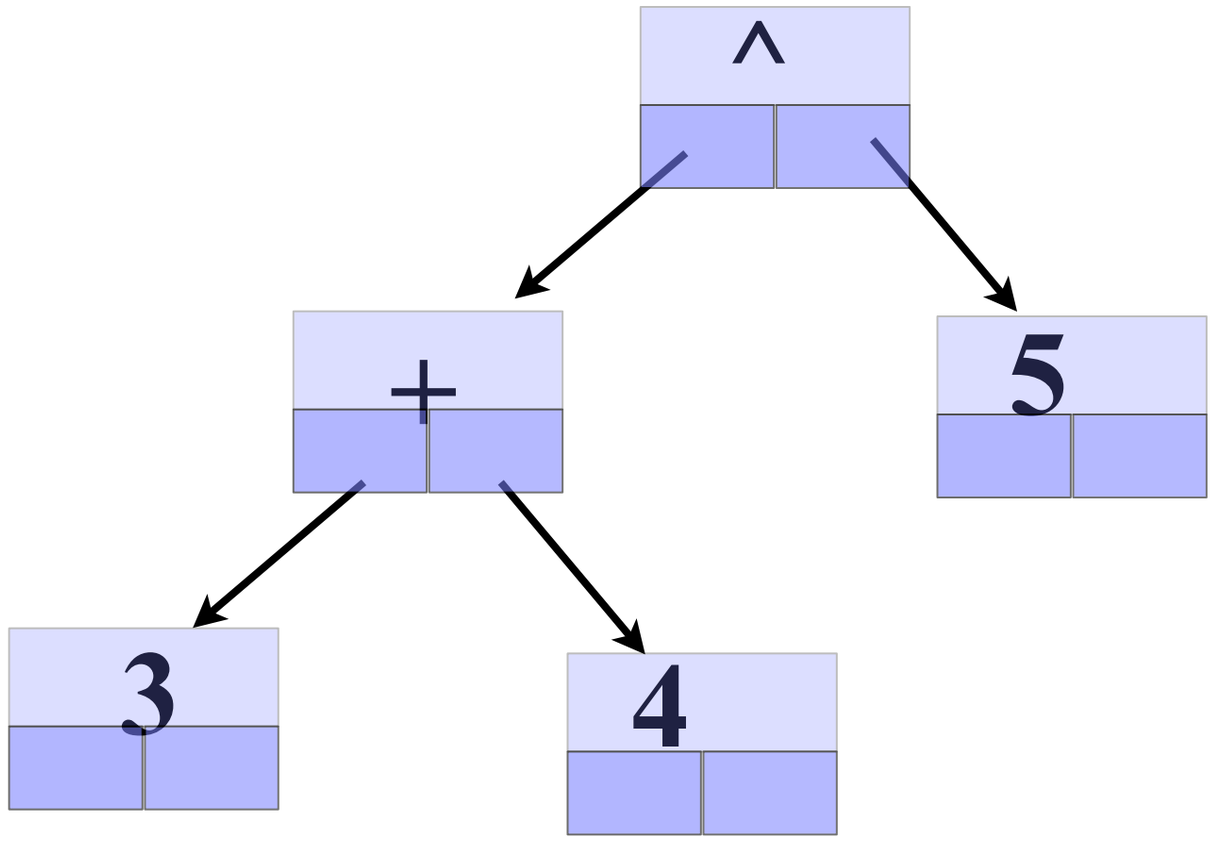


Example: Expression Trees

- Internal nodes: operator symbols
- Leaves: numbers (or other operand tokens)
- Left and right subtrees of a node represent the left and right operands for the operator stored in that node

Expression Tree

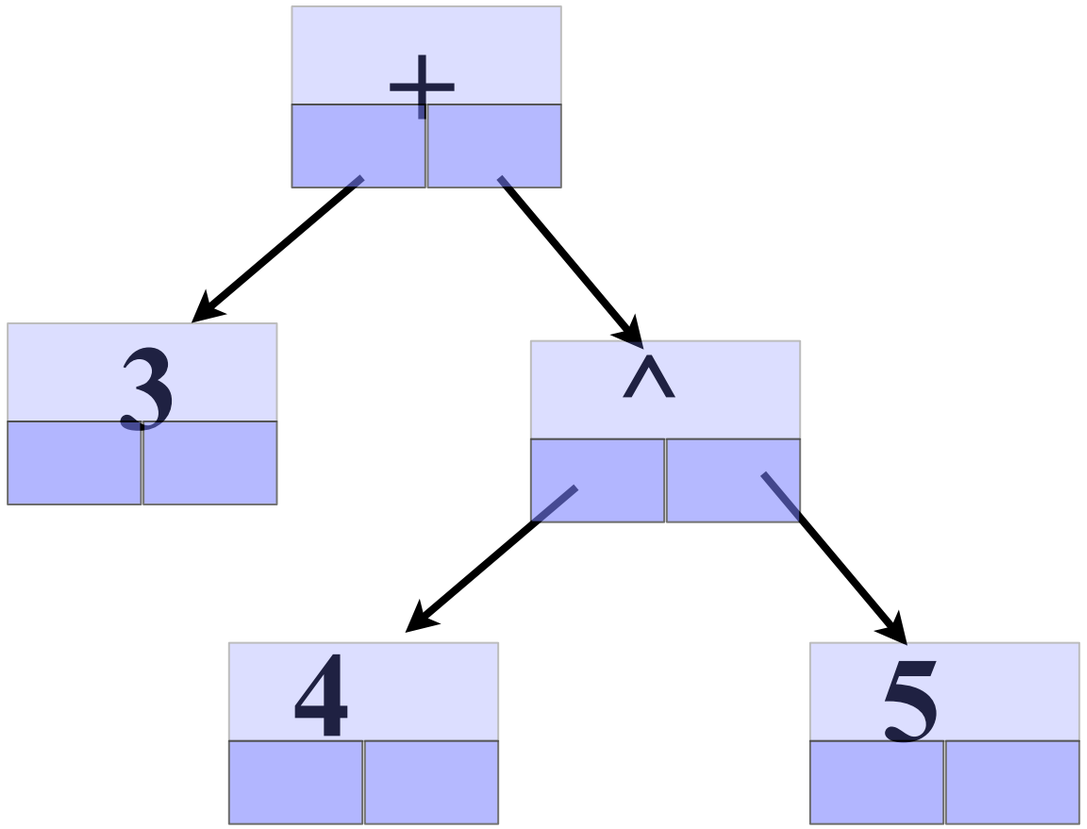
(3 + 4)^5



inorder: (3 + 4) ^ (5)
preorder: ^ + 3 4 5
postorder: 3 4 + 5 ^

Expression Tree

3 + 4^5



inorder: (3 + (4 ^ 5))
preorder: + 3 ^ 4 5
postorder: 3 4 5 ^ +

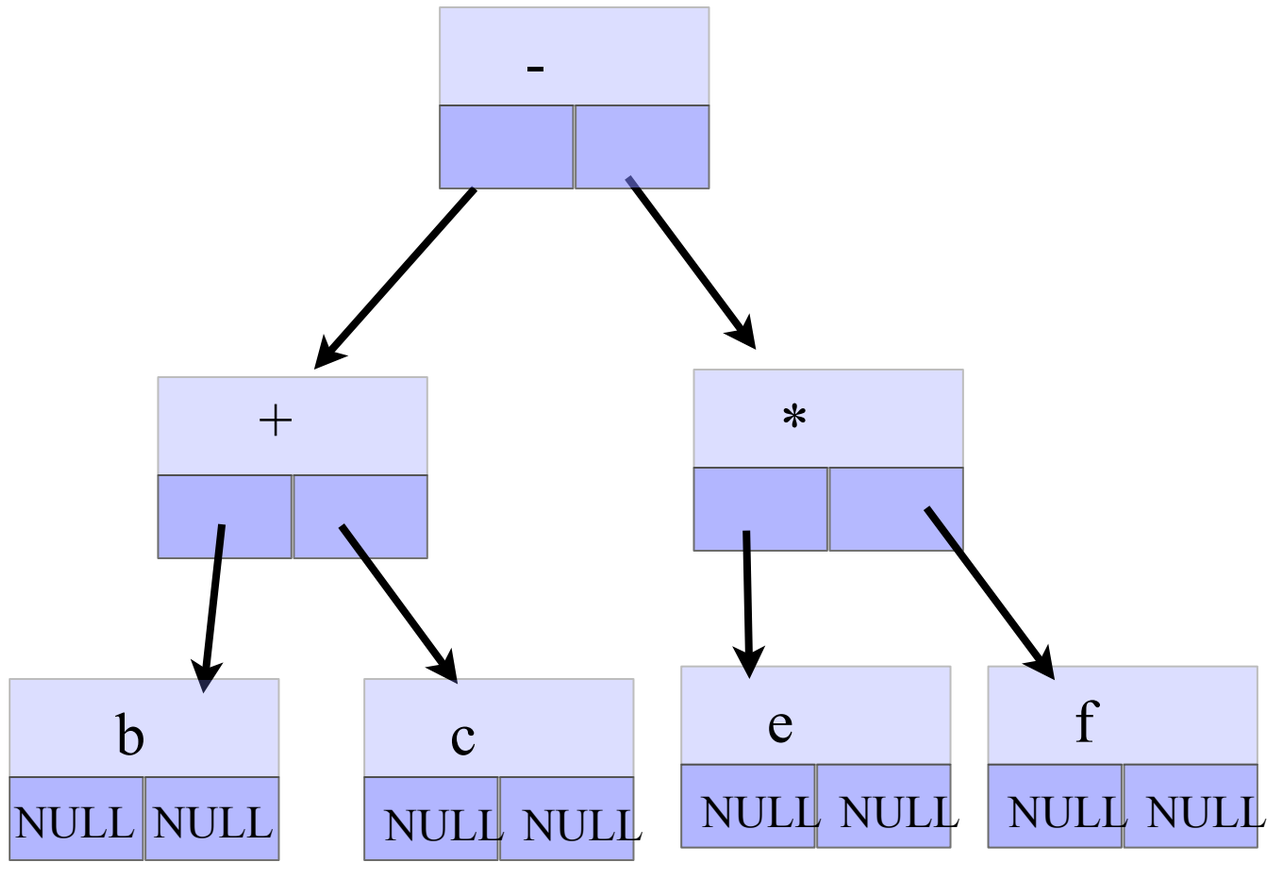
Expression Trees

- leaves are operands
- other nodes are operators
- binary operators implies binary tree
- a node would have only one child if unary (e.g. -)
- evaluate by applying the operator at the root and recursively evaluating the left and right subtrees

Expression Trees

Construct an expression tree from a postfix expression. Create a stack<pointer> see an operand create a single node tree and push a pointer to it on our stack, when we seen an operator, pop and merge the two top trees on the stack

Given a postfix expression, b c + e f * -,
we can represent it as a tree

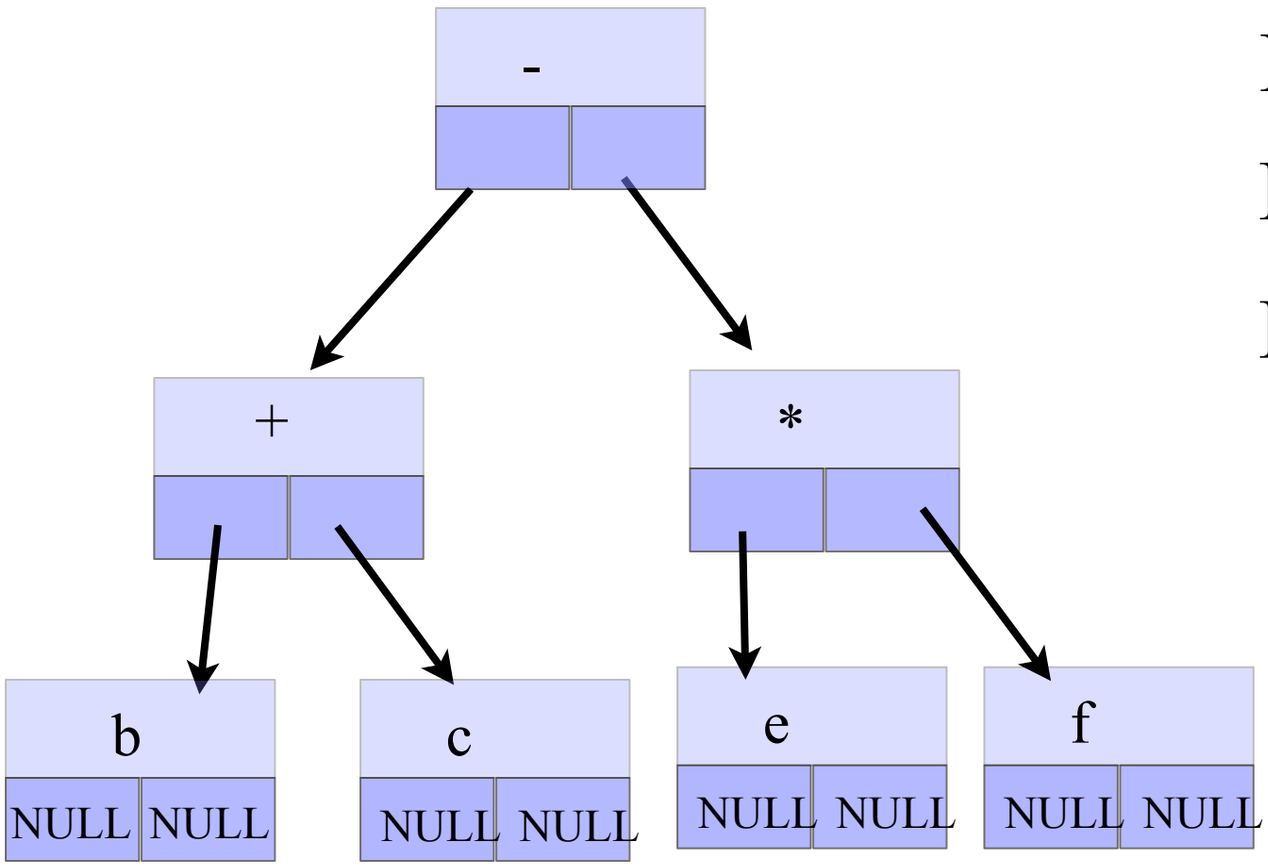


Tree Traversals

- pre-order
  - –visit root
  - –traverse left subtree
  - –traverse right subtree
- post-order
  - –traverse left subtree
  - –traverse right subtree
  - –visit root
- in-order
  - –traverse left subtree
  - –visit root
  - –traverse right subtree

Iteration: visit/process each node exactly one time. We have many choices in what order we visit (aka process the information at that node). Our application dictates the order we traverse the tree

The expression tree for b + c - e * f



Inorder (b + c) - (e * f)

Postorder b c + e f * -

Preorder - + b c * e f

The node class

```
template <class Object>
class BinaryNode
{
public:
  BinaryNode( const Object & theElement = Object( ),
              BinaryNode *lt = NULL, BinaryNode *rt = NULL )
    : element( theElement ), left( lt ), right( rt ){ }
  BinaryNode( Object && theElement,
              BinaryNode *lt = NULL, BinaryNode *rt = NULL )
    : element( std::move(theElement) ), left( lt ), right( rt ){ }

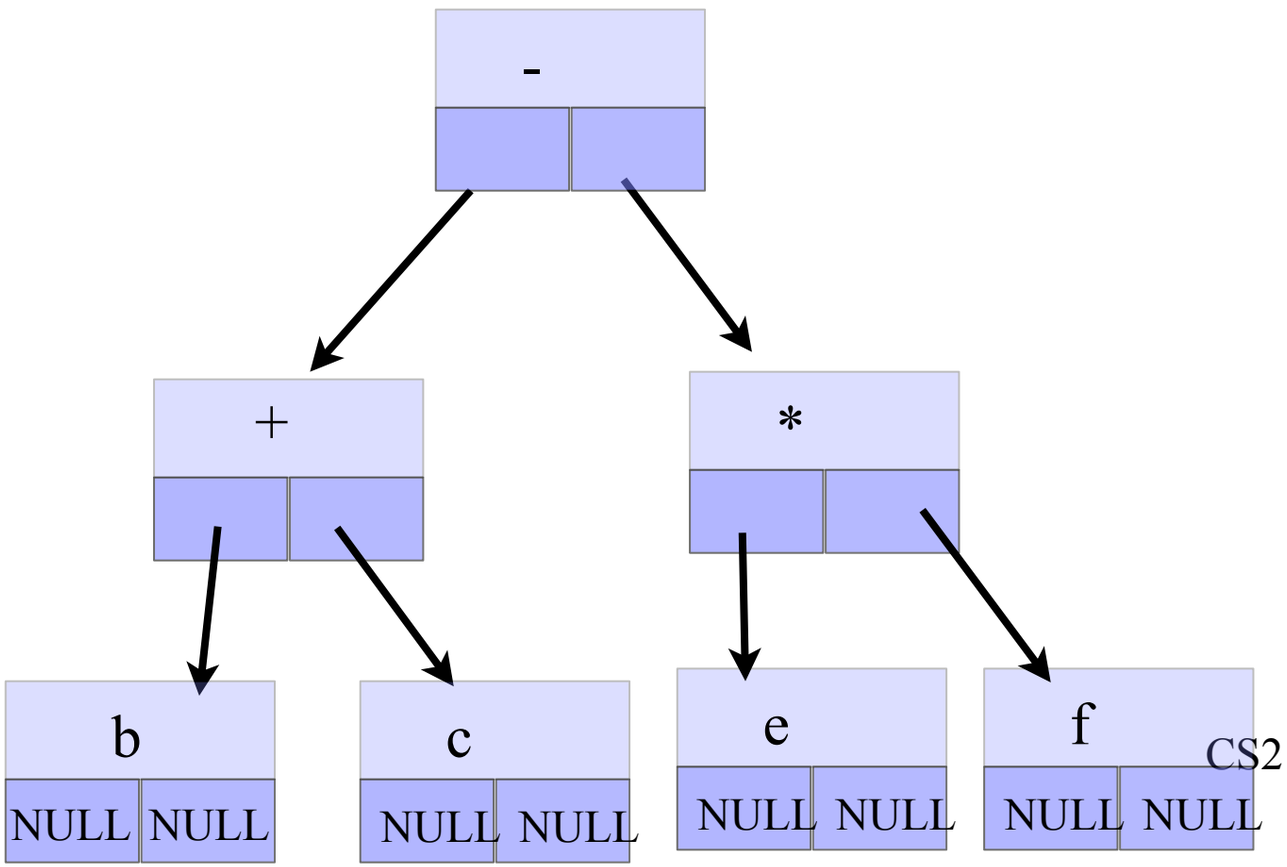
    static int size( BinaryNode *t );
    static int height( BinaryNode *t );
    BinaryNode *duplicate( ) const;

    void printPreOrder( ) const;
    void printPostOrder( ) const;
    void printInOrder( ) const;

  public:   // To keep things simple
    Object      element;
    BinaryNode *left;
    BinaryNode *right;
};
```

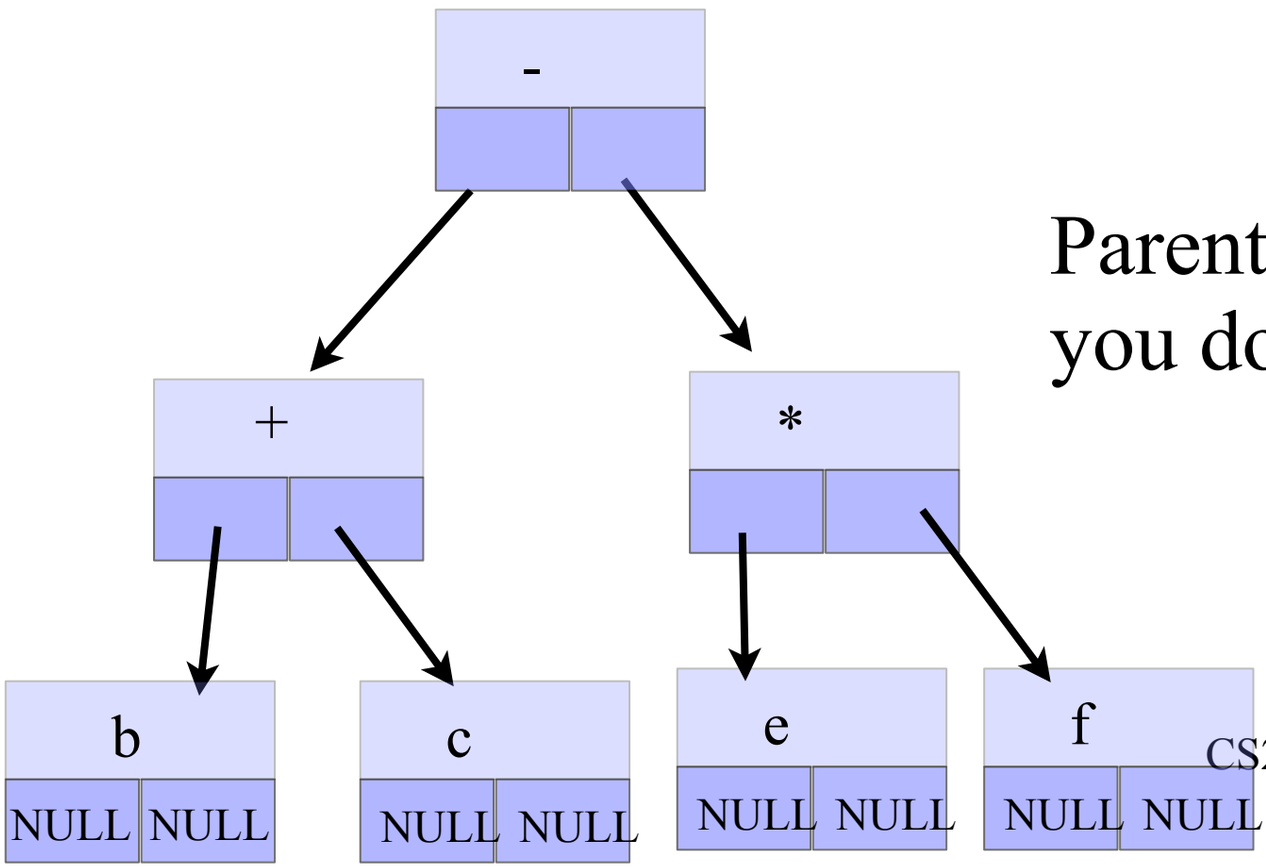
Code to print a tree in preorder

```
// Print the tree rooted at current node using preorder traversal.
template <class Object>
void BinaryNode<Object>::printPreOrder( ) const
{
        cout << element << " ";              // Node
        if( left != NULL )
            left->printPreOrder( );          // Left
        if( right != NULL )
            right->printPreOrder( );         // Right
}
```



Code to print a tree in inorder

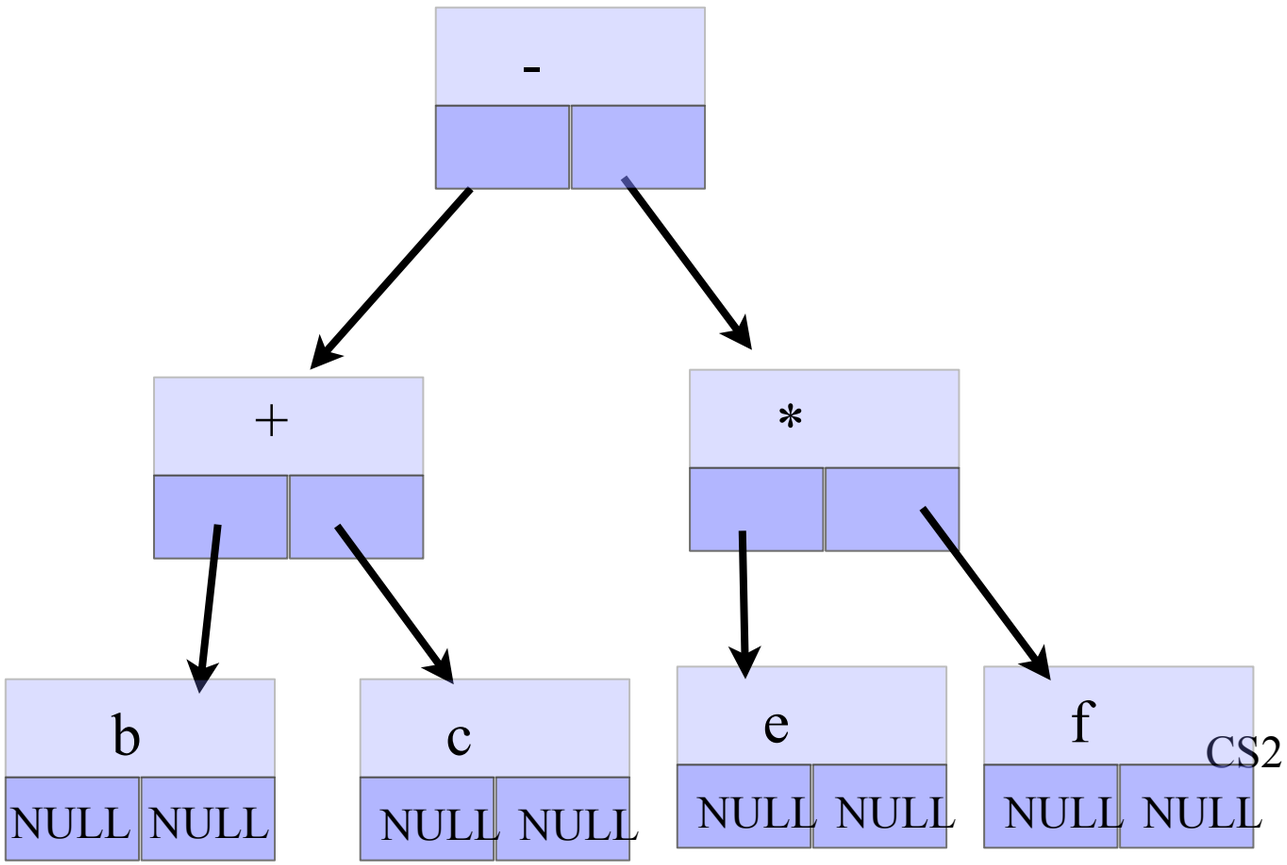
```
// Print the tree rooted at current node using inorder traversal.
template <class Object>
void BinaryNode<Object>::printInOrder( ) const
{
        if( left != NULL )                  // Left
            left->printInOrder( );
        cout << element << " ";              // Node
        if( right != NULL )
            right->printInOrder( );         // Right
}
```



Parenthesis is not written here how would you do it?

Code to print a tree in postorder

```
// Print the tree rooted at current node using postorder traversal.
template <class Object>
void BinaryNode<Object>::printPostOrder( ) const
{
    if( left != NULL )                  // Left
        left->printPostOrder( );
     if( right != NULL )                  // Right
        right->printPostOrder( );
     cout << element << " ";              // Node
}
```



The node class

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    BinaryNode( const Object & theElement = Object( ),
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      : element( theElement ), left( lt ), right( rt ){ }

    static int size( BinaryNode *t );
    static int height( BinaryNode *t );
    BinaryNode *duplicate( ) const;

    void printPreOrder( ) const;
    void printPostOrder( ) const;
    void printInOrder( ) const;


public:   // To keep things simple
    Object      element;
    BinaryNode *left;
    BinaryNode *right;
};
```
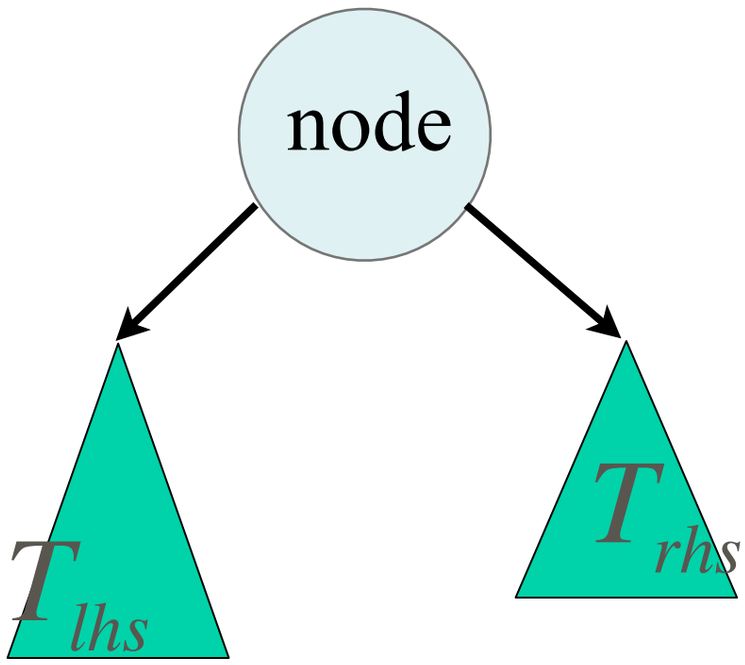
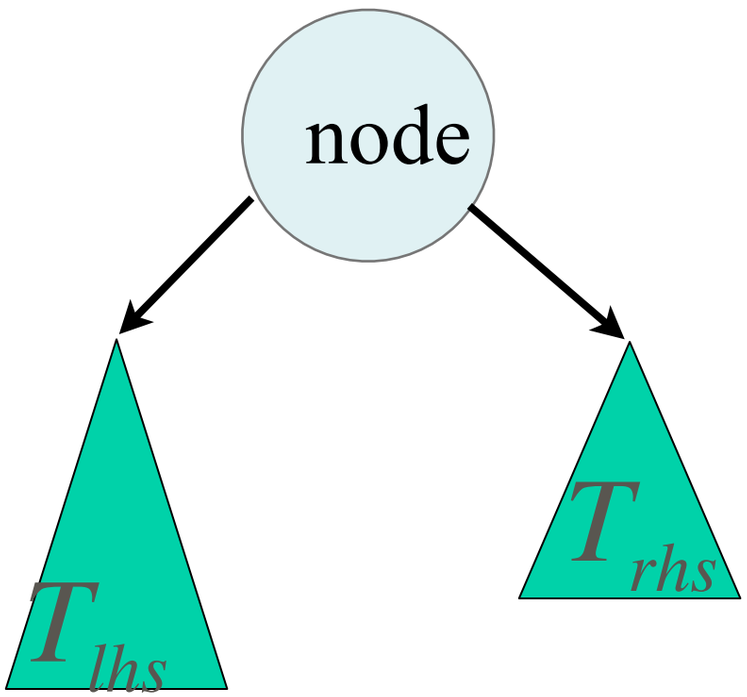

Code to compute the size of a tree

```
// Return size of tree rooted at t.
template <class Object>
int BinaryNode<Object>::size( BinaryNode<Object> * t )
{
    if( t == NULL )
        return 0;
    else
        return 1 + size( t->left ) + size( t->right );
}
```



Code to compute the height of a tree

```
// Return height of tree rooted at t.
template <class Object>
int BinaryNode<Object>::height( BinaryNode<Object> * t )
{

    if( t == NULL )
        return -1;
    else
        return 1 + max( height( t->left ),
                        height( t->right ) );
}
```



Code to recursively copy a tree

```
// Return a pointer to a node that is the root of a
// duplicate of the tree rooted at the current node.
template <class Object>
BinaryNode<Object> * BinaryNode<Object>::duplicate( ) const
{
        BinaryNode<Object> *root = new BinaryNode<Object>( element );

        if( left != NULL )        // If there's a left subtree
            root->left = left->duplicate( );// Duplicate; attach
        if( right != NULL )       // If there's a right subtree
            root->right = right->duplicate( );// Duplicate; attach

        return root;              // Return resulting tree
}
```