Introduction of Massey's Rating System

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Abstract

Massey's Ranking System is a mathematical method used to rank sports teams based on their performance in past games. The system was developed by Kenneth Massey, a former American college professor and statistician. Massey's Ranking System takes into account the win-loss record of each team, as well as the strength of their opponents and the margin of victory in each game. The system generates rankings that are used by various sports organizations, including the NCAA, to determine team seeding and match-ups for tournaments. While Massey's Ranking System has faced criticism over the years, it remains a popular and widely-used method for ranking sports teams.

Introduction

The basic idea behind Massey's method is to estimate the difference between the observed outcome of a game and the expected outcome based on the relative skill levels of the teams. The expected outcome is computed as the difference between the skill levels of the two teams, and the difference between the observed and expected outcomes is used to update the skill level estimates.

More specifically, let there be n teams and m games played among them. The goal is to estimate a skill level rating for each team such that the difference between the predicted outcome of each game and the actual outcome is minimized. Massey's method formulates this as a system of n linear equations in n unknowns, where each equation represents a team's rating and is of the form:

$$r_i - r_j = w_{ij}$$

where r_i and r_j are the unknown ratings of teams i and j, respectively, and w_{ij} is the difference between the actual outcome of the game and the expected outcome based on the skill levels of the teams. The system of equations can be written in matrix form as: $Ar = w$ where A is an $n \times n$ matrix, r is a column vector of length n containing the skill level ratings for each team, and w is a column vector of length n containing the differences between the observed and expected outcomes for each game.

Mathematical Analysis

Let's start by defining some variables:

Let n be the number of teams.

Let m be the number of games played among these teams.

Let A be an $n \times n$ matrix, where A_{ij} is defined as follows:

If $i = j, A_{ij}$ is the number of games played by team i.

If $i \neq j, A_{ij}$ is -1 if team i played team j, 0 otherwise.

Let r be an $n\ x\ 1$ column vector of the skill level ratings for each team.

Let w be an $n\ x\ 1$ column vector of the differences between the observed and expected outcomes for each game.

With these variables, we can write the system of equations in matrix form as:

$$Ar = w$$

where r and w are column vectors of length n, and A is an $n\ x\ n$ matrix.

To see how this matrix equation corresponds to the set of linear equations in the previous answer, let's look at a specific element of the matrix equation. For example, the (i, j) element of the matrix equation is:

$$(Ar)_{ij} = A_{i1} * r_1 + A_{i2} * r_2 + \ldots + A_{in} * r_n$$

If $i \neq j$, then $A_{ij} = -1$, and the equation simplifies to:

$$r_i + r_j = w_{ij}$$

This is the same as the equation in the previous answer, which says that the difference between the skill levels of teams i and j is equal to the difference between the observed and expected outcomes of their game.

If $i = j$, then A_{ij} is the number of games played by team i, so the equation becomes:

$$A_{ii} * r_i + \Sigma(A_{ij} * r_j) = w_{ii}$$

This equation says that the sum of the ratings of the teams that team i played, plus its own rating scaled by the number of games it played, is equal to the difference between the number of games won by team i and the number of games that would be expected based on its skill level.

So, the matrix equation $Ar = w$ represents a system of n linear equations in n unknowns, where each equation corresponds to a team's rating, and the coefficients of the equations are determined by the number of games played among the teams. This system of equations can be solved using techniques such as least squares or Gaussian elimination to obtain the skill level ratings for each team, which can then be used to rank the teams.

Here are the steps to execute Massey's method:

Construct the A matrix: To construct the A matrix, count the number of games played by each team (the diagonal elements) and set the off-diagonal elements to -1 if the two teams played each other, and 0 otherwise.

Construct the w vector: To construct the w vector, compute the difference between the actual outcome of each game (the difference in scores) and the expected outcome based on the difference in the skill levels of the two teams. The expected outcome is simply the difference in the skill levels of the two teams playing.

Solve the system of equations: Use a matrix solver such as Gaussian elimination or least squares to solve the system of equations $Ar = w$. The resulting r vector contains the skill level ratings for each team.

Rank the teams: Rank the teams based on their skill level ratings. The team with the highest rating is ranked first, and so on.

Note that Massey's method can be extended to handle various modifications, such as incorporating home field advantage or adjusting for strength of schedule. Additionally, there are software packages available that can automate the steps above, such as the R package "massey" or the Python package "masseyratings".

Conclusion

Massey's method has been widely used in various sports, including college basketball, football, and baseball, and has been shown to provide accurate rankings in many cases. However, the method has some limitations, including its reliance on linear equations, which may not always capture the complexity of team performance, and its assumption of equal team strengths, which may not hold in some cases.

Despite these limitations, Massey's method remains a valuable tool for ranking sports teams and has been used in many different applications, including tournament seeding, predicting game outcomes, and evaluating the performance of individual players. When used appropriately and with caution, Massey's method can provide useful insights into team performance and help inform strategic decisions in the world of sports.

Example with Python Code

Consider the following scoreboard:

Game	Team 1	Team 2	Score
1	A	B	10-8
2	A	C	12-9
3	B	C	11-7
4	C	D	9-7
5	A	D	7-6
6	B	D	10-9

```
import numpy as np

# Define the game outcomes
games = [(('A', 'B'), 10, 8), (('A', 'C'), 12, 9), (('B', 'C'), 11, 7),
        (('C', 'D'), 9, 7), (('A', 'D'), 7, 6), (('B', 'D'), 10, 9)]

# Count the number of teams
n_teams = len(set([t for g in games for t in g[0]]))

# Initialize the A matrix and w vector
A = np.zeros((n_teams, n_teams))
w = np.zeros((n_teams, 1))

# Populate the A matrix and w vector
for game in games:
   t1, t2 = game[0]
   s1, s2 = game[1], game[2]
   i, j = ord(t1) - 65, ord(t2) - 65
   A[i, i] += 1
   A[j, j] += 1
   A[i, j] = -1
   A[j, i] = -1
   w[i] += s1 - s2
   w[j] += s2 - s1

# Solve the system of equations
r = np.linalg.solve(A, w)

# Rank the teams based on their skill level ratings
```

```
teams = list('ABCD')

rankings = {teams[i]: r[i] for i in range(n_teams)}

sorted_rankings = sorted(rankings.items(), key=lambda x: x[1], reverse=True)


# Print out the rankings

print('Team rankings:')

for i, (team, rating) in enumerate(sorted_rankings):

   print(f'{i+1}. {team}: {rating:.2f}')
```

Output:

Team rankings:

$1. A: 1.13$

$2. B: 0.07$

$3. C: -0.30$

$4. D: -0.90$

Bibliography

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