A Study on Competition Model

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1 Lotka Volterra Competition Introduction

Competition model are the model between two or more species with the same limiting resource. This limiting resource can be food, space, etc.When one species is a better competitor, it negatively influences the other species by reducing population sizes and/or growth rates. There is a classical model of competition due to Lotka (1932) and Volterra (1926).
Future work: Going to read more competition models, they might be included in the paper

2 Derivation of Lotka Volterra Competition

Let N_1 and N_2 denote the population of these 2 species. We first assume that if the other species is absent, the population of the given species obeys the limited-growth population.

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN(1 - \frac{N}{k})$$

r: the inherent per-capita growth rate
k: carrying capacity

Our second assumption is that if the N_2 population increases, it should have a negative effect on the N_1 population, and vice versa. So if N_2 increases, N_1' should decrease.

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = r_1 N_1(1 - \frac{N_1 + \alpha_{12} N_2}{k_1})$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = r_2 N_2 (1 - \frac{N_2 + \alpha_{21} N_1}{k_2})$$

Where $\alpha_{ij} \geq 0$ represents the effect species j has on the population of species i.

After Simplify we can get:

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = \frac{r_1}{k_1} N_1 (k_1 - N_1 - \alpha_{12} N_2)$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = \frac{r_2}{k_2} N_2 (k_2 - N_2 - \alpha_{21} N_1)$$

We have 6 parameters $r_1, r_2, k_1, k_2, \alpha_{12}, \alpha_{21}$.

future work: Using nondimensionlization to reduce parameters.

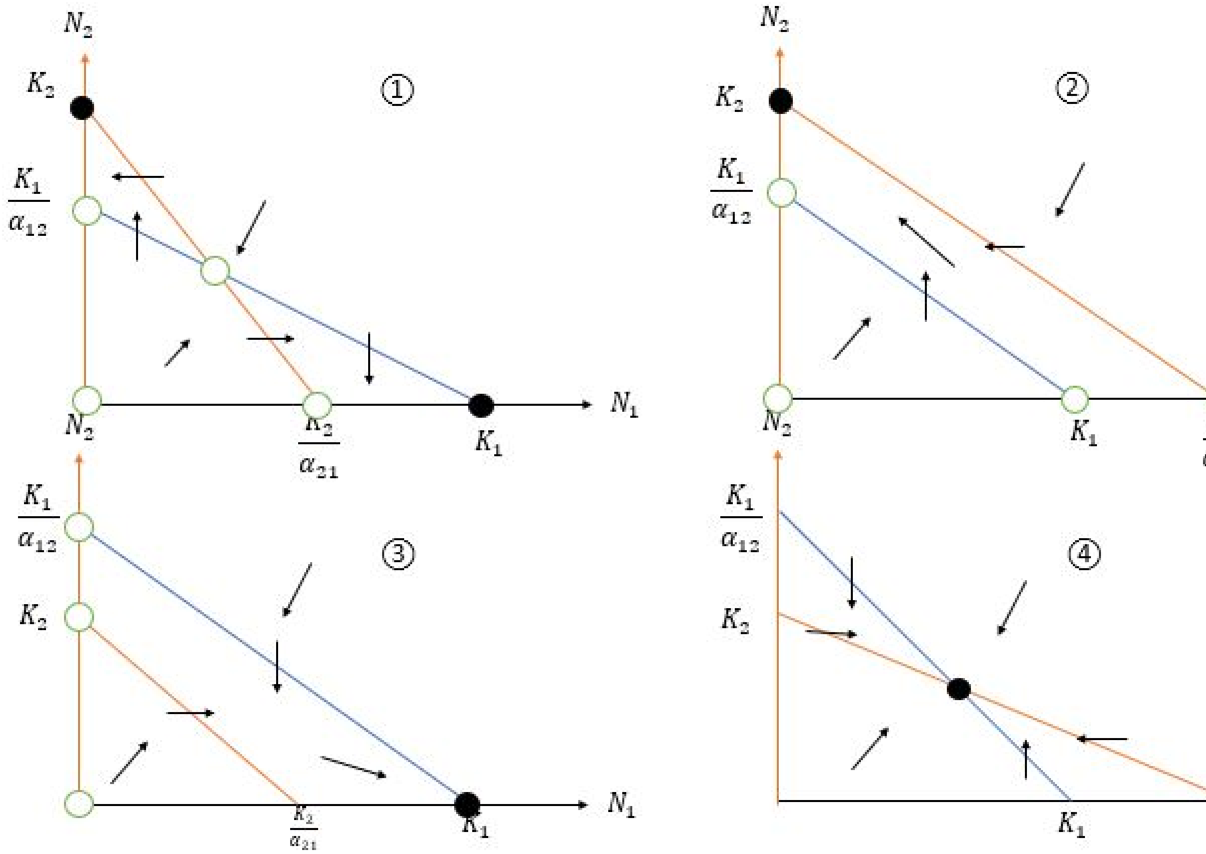
3 Model Analysis

3.1 Nullclines

N_1 - Nullclines: $N_1 = 0$, $N_1 = \frac{k_2}{\alpha_{12}}$.

N_2 - Nullclines: $N_2 = 0$, $N_2 = \frac{k_1}{\alpha_{21}}$.

future work: Linearization and bifurcation analysis and gets better picture by using software like Matlab and Mathematica

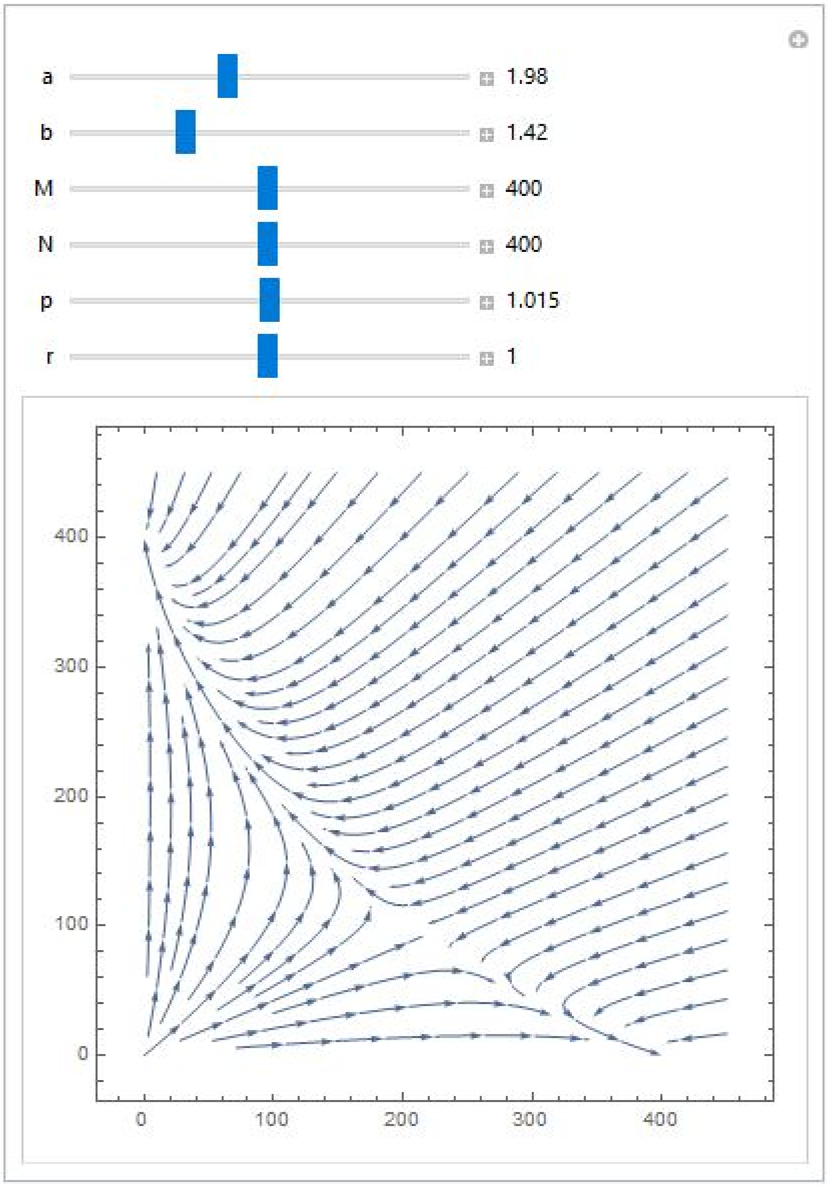


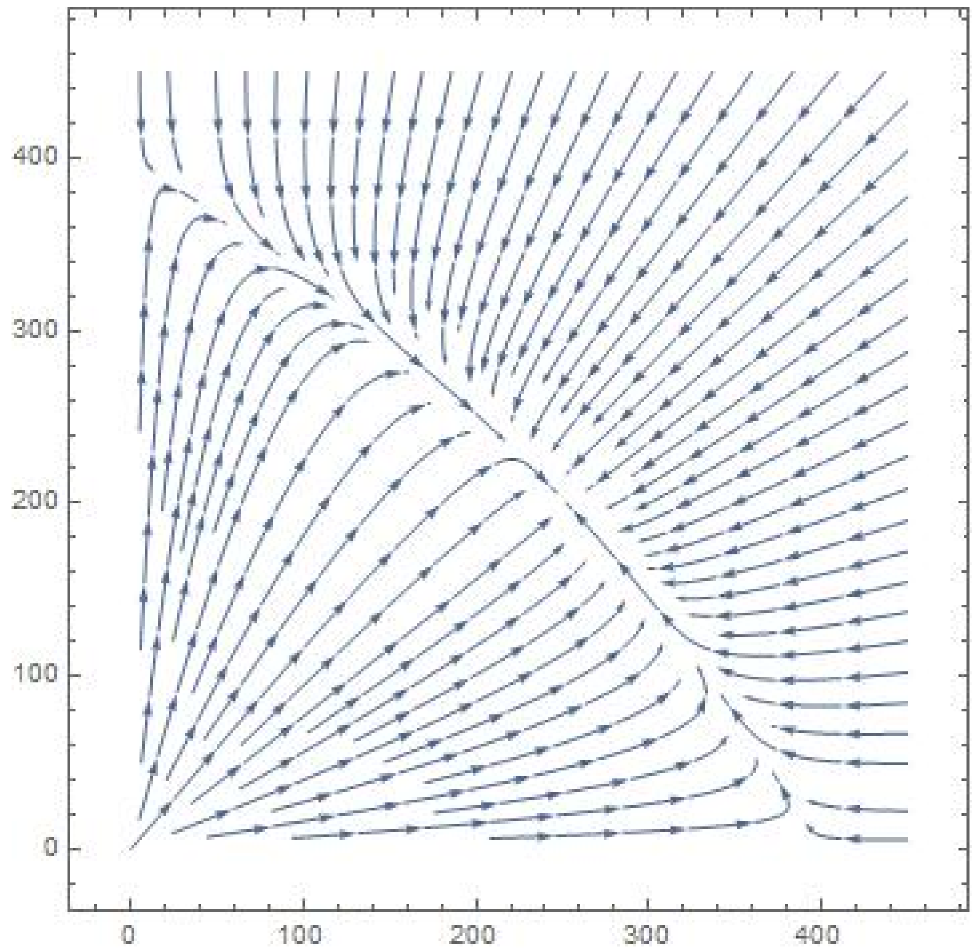
3.2 Phase Potrait (Mathematica)

For the convenience, we set the carry capacity k_1, k_2 to 400 for now. We will be interested in seeing all different resulting cases when we manipulate each parameter of $r1, r2, k1, k2$. Especially the co-exist case.

```
In[1]:= Manipulate[StreamPlot[{r * x * (1 - (x + a * y) / M), p * y * (1 - (y + b * x) / N)},
         {x, 0, 450}, {y, 0, 450}], {{a, 1}, 0, 5, Appearance -> "Labeled"},
        {{b, 1}, 0, 5, Appearance -> "Labeled"}, {{M, 400}, 0, 800, Appearance -> "Labele
        {{N, 400}, 0, 800, Appearance -> "Labeled"}, {{p, 1}, 0, 2, Appearance -> "Labele
        {{r, 1}, 0, 2, Appearance -> "Labeled"}]
```

Out[1]=





future work: reducing parameters, explanations of images, getting better plots of vectors

4 future work

So far I have read/watched only 3 recourses. More books, paper or videos will be introduced.

References

[1] Kot, Mark. *Elements of Mathematical Ecology.* Cambridge University Press, 2001, pp. 198–219..

[2] German A. Enciso, Ph.D.*Mathematatial Biology. 13: Lotka Volterra Competiton*

https://www.youtube.com/watch?v=p4Y9b8sgnOU.

[3] Wikipedia *Competitive Lotka–Volterra Equations* ,Wikimedia Foundation, Inc., 17 Dec. 2004,

https://en.wikipedia.org/wiki/Competitive_Lotka%E2%80%93Volterra_equations