

# Stochastic Process for European Options

Corey Zhang

West Chester University of Pennsylvania

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# 1 Intro

In this project, we will evaluate three different stocks from three different companies (Ford, Microsoft, and MGM). The things we mainly force on are calculating the value of the option by using Black-Scholes equation. And run the simulations to predict the stock price for April 23 in both binomial model and continuous model. The data we are using are from Yahoo Finance, time periods from Jan.1 to March 31. [1]

## 2 Black-Sholes Equation

Black-Scholes equation is a partial differential equation (PDE) governing the price evolution of a European call or European put under the Black-Scholes model. This is a solved version. [2]

$$F(S, t) = S(0) \phi(w) - Ke^{-r(T-t)} \phi(w - \sigma\sqrt{T-t})$$
$$w = \frac{1}{\sqrt{(T-t)\sigma^2}} \left( \ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t) \right) \quad (1)$$

K: Strike price of an option

T: exercise time or expiry of an option

t: current time

$\sigma$ : volatility

S: price of a share of a security

r: risk-free interest rate

We will use this formula to solve the option price listed on March 31 and Strike time is April 23. Therefore  $T - t = 16/252$  and  $r = 0.016$

### 2.1 Ford Motor Company(F)

Ford Motor Company designs, manufactures, markets, and services a range of Ford trucks, cars, sport utility vehicles, electrified vehicles, and Lincoln luxury vehicles worldwide.

The stock closing price on 3/31/2021 is \$12.25,  $\sigma = 0.459$ ,  $K = 13$ .

We now apply Black-Sholes equation: we calculate  $w = -0.4472$  and calculated price for call is \$0.286. The last trading price is \$0.13. The actual price as 4/23 is \$11.97.

## 2.2 Microsoft (MSFT)

Microsoft Corporation develops, licenses, and supports software, services, devices, and solutions worldwide. It has many popular products like Microsoft Office, Outlook, Skype, LinkedIn, etc.

The stock closing price on 3/31/2021 is \$235.77,  $\sigma = 0.2691$ ,  $K = 240$ .

We now apply Black-Sholes equation: we calculate  $w = -0.2134$ , calculated price for call is \$4.63. The last trading price is \$3.8. The actual price as 4/23 is \$237.49.

## 2.3 MGM Resorts International (MGM)

MGM Resorts International, through its subsidiaries, owns and operates casino, hotel, and entertainment resorts in the United States and Macau. The company operates through three segments: Las Vegas Strip Resorts, Regional Operations, and MGM China. Its casino resorts offer gaming, hotel, convention, dining, entertainment, retail, and other resort amenities. The company's casino operations include slots and table games, as well as online sports betting and iGaming through BetMGM.

The stock closing price on 3/31/2021 is \$37.99,  $\sigma = 0.5005$ ,  $K = 40$ .

We now apply Black-Sholes equation: we calculate  $w = -0.337696$  and calculated price for call is \$1.13. The last trading price is \$0.96. The actual price as 4/23 is \$41.05.

# 3 Models

Use discrete binomial models and continuous geometric Brownian motion to simulate the trend for these three stocks in April.

## 3.1 Ford

Start price: 12.46

Base on the stats from Jan. 1 to Mar. 31



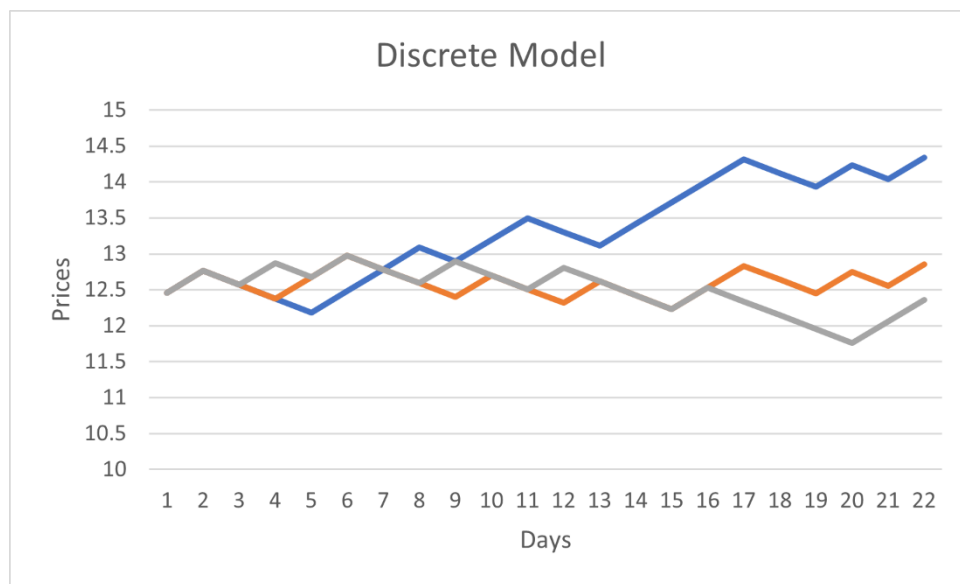
For discrete model:

The chance of increase: .5254

The chance of decrease: .4746

Avg increase: 0.30129

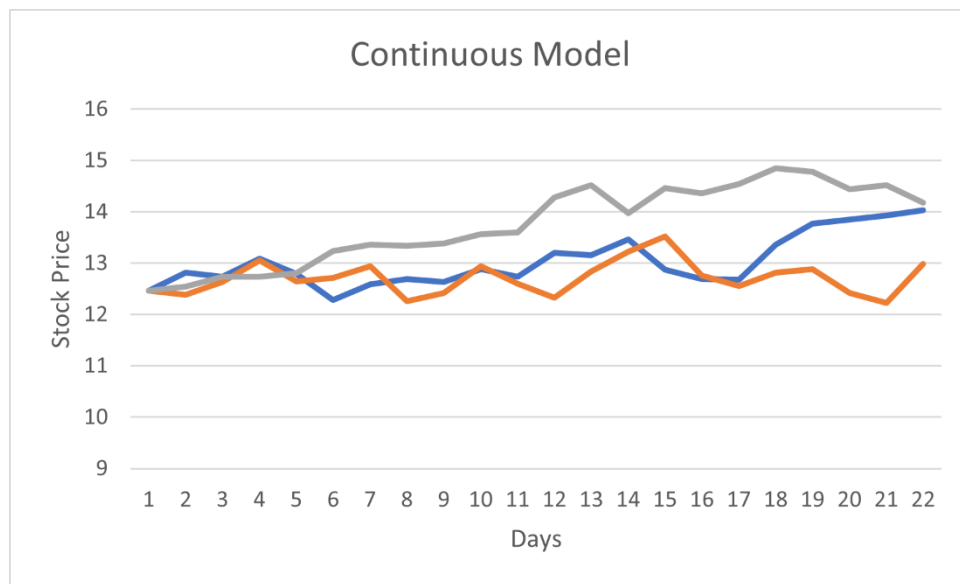
Avg decrease: 0.19286



For continuous models:

$$\mu = 0.006442$$

$$\sigma = 0.027417$$



After 1,000 runs for each model to predict the price on 4/23/21.

The mean for discrete model is \$13.55 with one standard deviation of \$0.95.

The mean for continuous model is \$13.05 with one standard deviation of \$1.27.

The actual price as 4/23/21 is \$11.97.

Call price analysis:

$$\text{Expected profit(discrete)} = 13.55 - 13 - 0.13e^{0.016 \cdot 16/252} = \$0.41$$

$$\text{Expected profit(continuous)} = 13.05 - 13 - 0.13e^{0.016 \cdot 16/252} = -\$0.08$$

$$\text{Actual return} = -0.13 e^{0.016 \cdot 16/252} = -\$0.13$$

## 3.2 Microsoft Corporation

Start price: 231.85

Base on the stats from Jan. 1 thru Mar. 31.

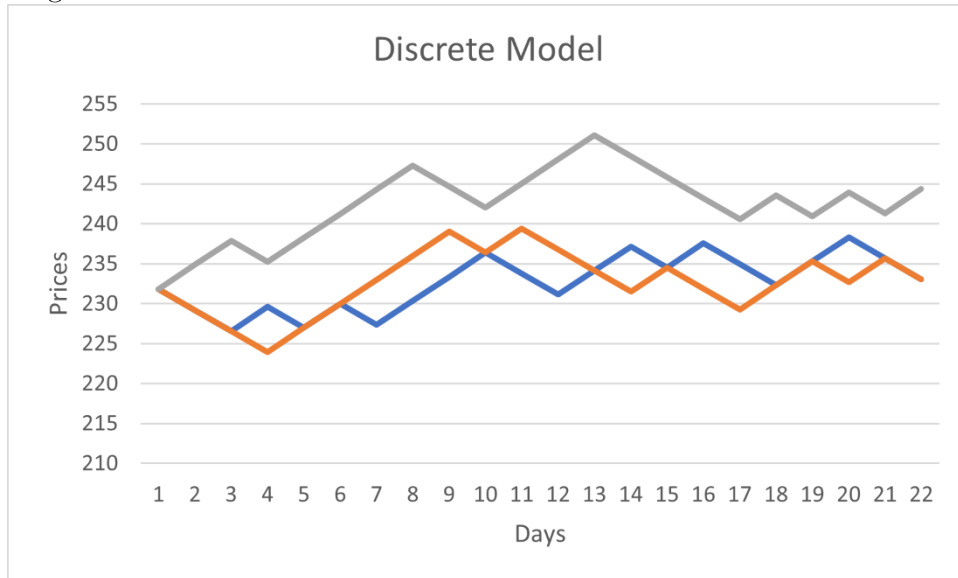
For discrete model:

The chance of increase: .5085

The chance of decrease: .4915

Avg increase: 3.011667

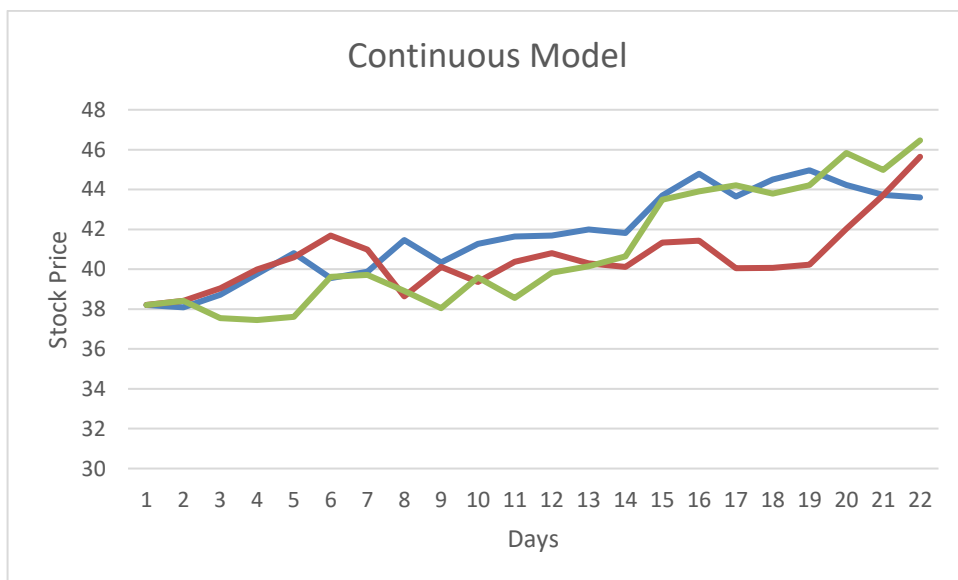
Avg decrease: 2.62724



For continuous models:

$$\mu = 0.00107$$

$$\sigma = 0.0157$$



After 1,000 runs for each model to predict the price on 4/23/21.

The mean for discrete model is \$240.27 with one standard deviation of \$11.39.

The mean for continuous model is \$248.28 with one standard deviation of \$28.36.

The actual price as 4/23/21 is \$237.49.

Call price analysis:

$$\text{Expected profit(discrete)} = 240.27 - 240 - 3.8e^{0.016*16/252} = -\$3.53$$

The expected profit =  $248.28 - 240 - 3.8$

$$\text{Expected profit(continuous)} = 248.28 - 240 - 3.8e^{0.016*16/252} = \$4.48$$

$$\text{Actual return} = -3.8e^{0.016*16/252} = -3.80$$

### 3.3 MGM International Resort

Start price: 38.21

Base on the stats from Jan. 1 thru Mar. 31.



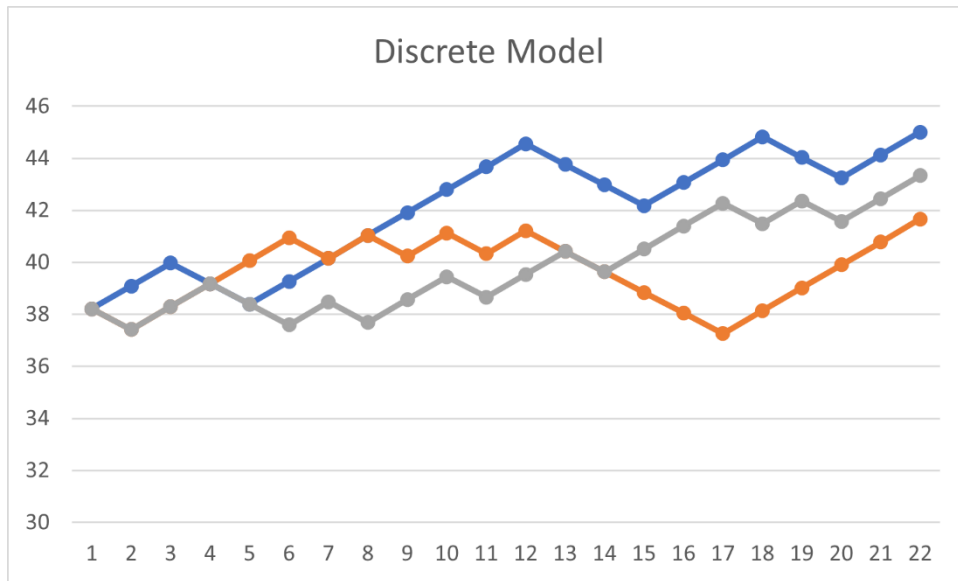
For discrete model:

The chance of increase: .5593

The chance of decrease: .4407

Avg increase: 0.880606

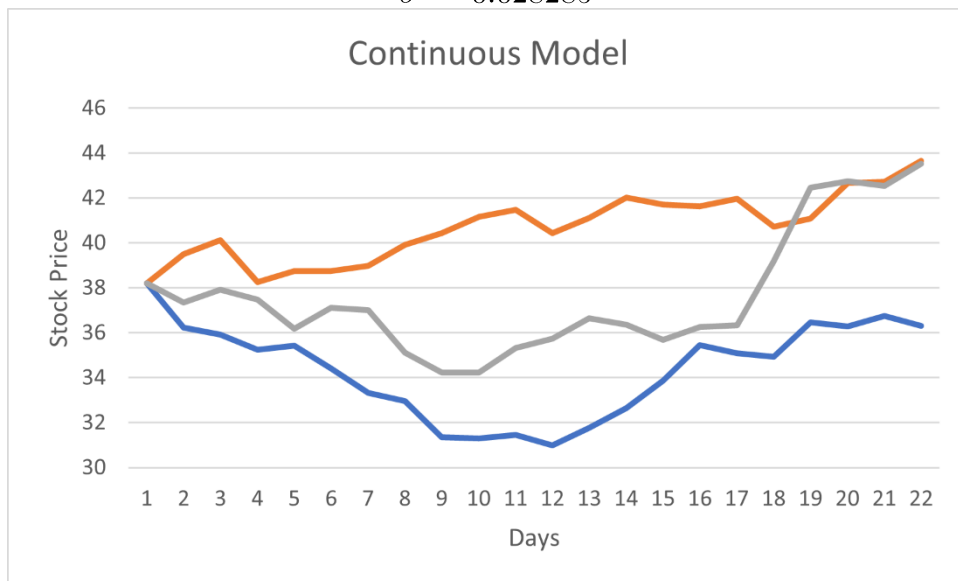
Avg decrease: 0.79038



For continuous models:

$$\mu = 0.00427$$

$$\sigma = 0.028285$$



After 1,000 runs for each model to predict the price on 4/23/21.

The mean for discrete model is \$40.49 with one standard deviation of \$3.31.

The mean for continuous model is \$41.1 with one standard deviation of \$4.75.

The actual price as 4/23/21 is \$41.05.



Call price analysis:

$$\text{Expected profit(discrete)} = 40.49 - 40 - 0.96e^{0.016*16/252} = -\$0.47$$

$$\text{Expected profit(continuous)} = 41.1 - 40 - 0.96e^{0.016*16/252} = \$0.14$$

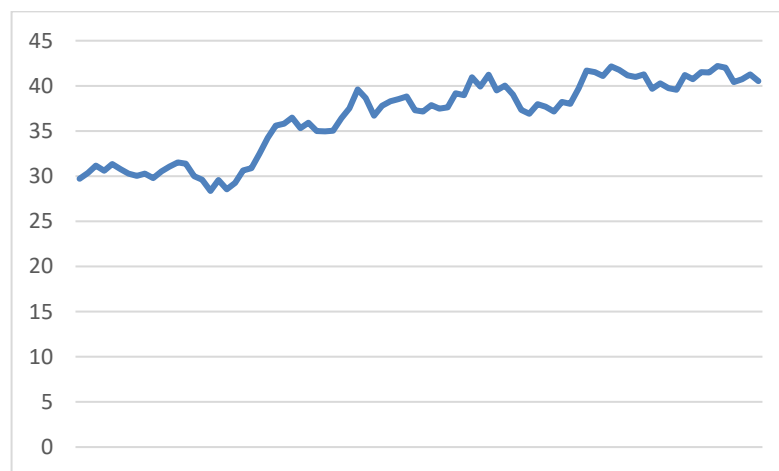
$$\text{Actual return} = 41.05 - 40 - 0.96e^{0.016*16/252} = \$0.09$$

## 4 Stock Recommendation

Among the three stocks we've been analyzed. MGM is a good option. The news is Las Vegas is about to open capacity up to 90% and reducing social distancing by 3 ft. The business might have a good chance to return in this situation.

2021-05-05 11:55AM EDT	43.00	0.95	0.94	0.98	+0.48	+102.13%	815	1,164	39.11%
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First, we use the data from 1/1/21 to 5/4/21.



Starting price: \$41.81(the moment of screenshot)

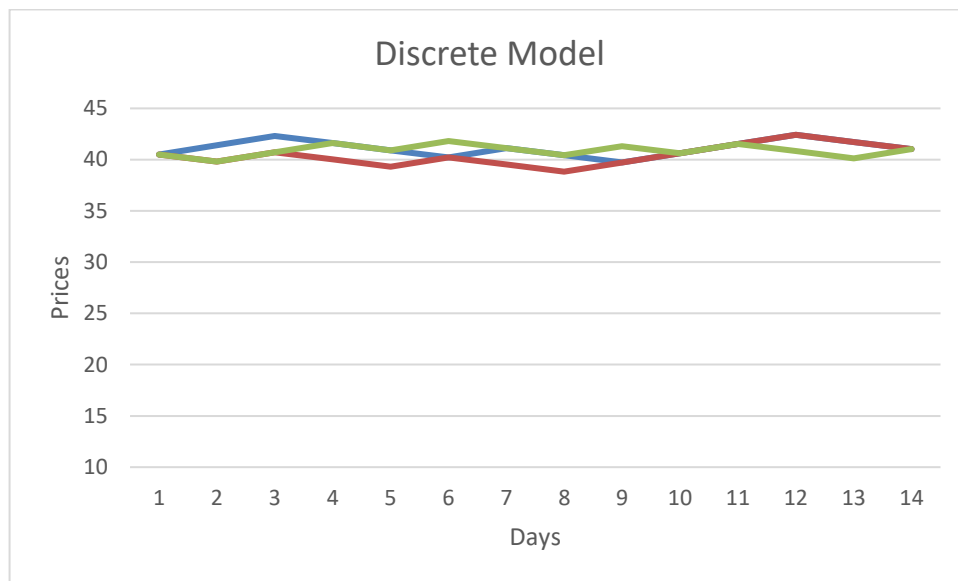
For discrete model:

The chance of increase: .5181

The chance of decrease: .4819

Avg increase: 0.89604

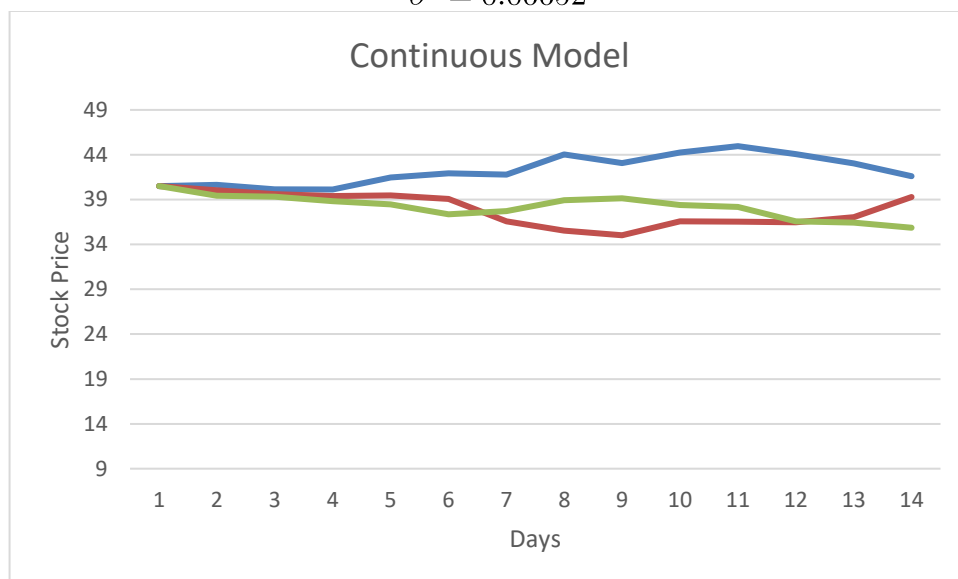
Avg decrease: 0.69575



For continuous model

$$\mu = 0.00374$$

$$\sigma = 0.06652$$



After 1,000 runs for each model to predict the price on 5/21/21.

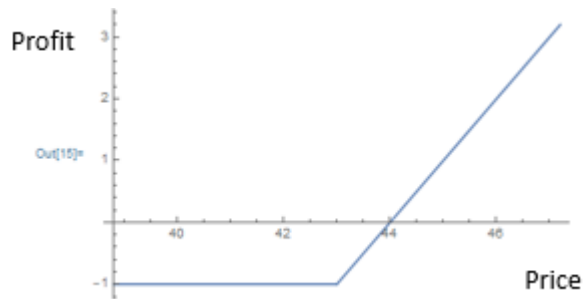
The mean for discrete model is \$43.54 with one standard deviation of \$2.82.

The mean for continuous model is \$43.28 with one standard deviation of \$3.75. Using Black-Scholes formula,  $K = 43$ ,  $t = 13/252$ ,  $\sigma = 0.03911$ .

We calculate that  $w = -0.262229$ , and calculated call price is 0.99. The last trade price is 0.95.

$$\text{Expected profit(discrete)} = 43.54 - 43 - 0.95e^{0.016*13/252} = -\$0.42$$

$$\text{Expected profit(continuous)} = 43.28 - 43 - 0.96e^{0.016*13/252} = -\$0.67$$



Although that seems expected return is negative, but nevertheless, the good news seems in favor of the trending of MGM stock.

## 5 References

- [1] »Yahoo Finance,« Yahoo, 2021. [Online]. Available:  
<https://finance.yahoo.com/>.
- [2] J. R. Buchanan, An Undergraduate Introduction to Financial  
Mathematics, World Scientific Publishing Co., 2012.