Models for Bayesian Final

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Part 1 and 2 Model

Basic Bayesian Linear Regression Models with Zellner's g-priors.

Part 3 Model

We will use a hierarchical linear regression model to describe daily consumption methods.

$$Y_i^j|\underline{\beta}^j, \tau^j \sim N(X_i\underline{\beta}^j, \tau^j)$$

where i= 1:151, j=1:121, X_i is the row of the predictor matrix corresponding to the i^{th} house, $\underline{\beta}^j$ is the vector of coeffeicents for the j^{th} day, and τ^j is the precision for the j^{th} day.

We choose to use g-priors, with $g = n$, on $\underline{\beta}^j$ and vague gamma priors on τ^j. This allows us to obtain a perpetrate regression line for each day, with its own precision. The following are the distributions:

$$\underline{\beta}^j|\tau^j \sim N\left(\underline{\beta}_p, \frac{n}{\tau^j}CO^{-1}\right) \qquad and \qquad \tau^j \sim Gamma(0.001, 0.001)$$

where $\underline{\beta}_p$ is a population level regression line and CO^{-1} is the covariance matrix for the predictors.

The model contains a separate regression line for each day and a regression line for the population. The last step is to put priors on the population level regression line. We again use g-priors for the population level regression coeffeicents and a vague prior on the precision at the population level.

$$\underline{\beta}_p|\tau_p \sim N\left(\underline{0}, \frac{n}{\tau_p}CO^{-1}\right) \qquad and \qquad \tau_p \sim Gamma(0.001, 0.001)$$

This allows for population level inference as well as daily inference. We will be able to obtain predictive posterior distributions for each household at both the daily level and the population level (overall).

Part 4 Model

We will use a hierarchical linear regression model to describe daily consumption methods before and after January 1 when the new tariffs were implemented.

The Y's (consumption) follow the distribution:

$$Y_i^j|\underline{\beta}^j, \tau^j \sim N(X_i\underline{\beta}^j, \tau^j)$$

where i= 1:151, j=1:121, X_i is the row of the predictor matrix corresponding to the i^{th} house, $\underline{\beta}^j$ is the vector of coeffeicents for the j^{th} day, and τ^j is the precision for the j^{th} day.

Now we need prior distributions on $\underline{\beta}^j$ and τ^j. We choose to use g-priors, with $g = n$, on the $\underline{\beta}^j$ and vague gamma priors on τ^j. This allows us to obtain a separate regression line for each day, with its own precision. This time we break up the days into two time intervals; before and after January 1st. The following are the distributions for the day level β's and the day level τ's:

For j=1:47:

$$\underline{\beta}^j|\tau^j \sim N(\underline{\beta}_{BJ1}, \frac{n}{\tau^j}CO^{-1}) \qquad and \qquad \tau^j \sim Gamma(0.001, 0.001)$$

where $\underline{\beta}_{BJ1}$ is a regression line for days before January 1st and CO^{-1} is the covariance matrix for the predictors.

For j=48:121:

$$\underline{\beta}^j|\tau^j \sim N(\underline{\beta}_{AJ1}, \frac{n}{\tau^j}CO^{-1}) \qquad and \qquad \tau^j \sim Gamma(0.001, 0.001)$$

where $\underline{\beta}_{AJ1}$ is a regression line for days after January 1st and CO^{-1} is the covariance matrix for the predictors.

Now we have a model with a separate regression line for each day, and a regression line for each time period. Now each time period should come from an overall population level β. This allows us to connect the two time periods together. We also consider different precisions for each time period. The following are the distributions on the two sets of parameters for before and after January 1st:

$$\underline{\beta}_{BJ1}|\tau_{BJ1} \sim N\left(\underline{\beta}_p, \frac{n}{\tau^{BJ1}}CO^{-1}\right) \qquad and \qquad \tau_{BJ1} \sim Gamma(0.001, 0.001)$$

where $\underline{\beta}_p$ is a regression line for all days and CO^{-1} is the covariance matrix for the predictors.

and

$$\underline{\beta}_{AJ1}|\tau_{AJ1} \sim N\left(\underline{\beta}_p, \frac{n}{\tau_{AJ1}}CO^{-1}\right) \qquad and \qquad \tau_{AJ1} \sim Gamma(0.001, 0.001)$$

where $\underline{\beta}_p$ is a regression line for all days and CO^{-1} is the covariance matrix for the predictors.

Finally, we put priors on the population level regression line. We again use g-priors for the population level regression coeffeicents and a vague prior on the precision at the population level.

$$\underline{\beta}_p|\tau_p \sim N\left(\underline{0}, \frac{n}{\tau_p}CO^{-1}\right) \qquad and \qquad \tau_p \sim Gamma(0.001, 0.001)$$

This allows for population level inference as well as daily inference. We now can investigate differences in regression lines for the two time periods. We are able to obtain predictive posterior distributions for each household as well at the daily level, time period level, and the population level (overall).