Distributionally Robust Dispatch of Distributed Energy Resources

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1 Introduction

Currently, the European electricity market operators use a zonal model to dispatch electricity generation. The zonal market-clearing ignores the underlying grid constraints of the network, bringing challenges to both the Transmission System Operators (TSOs) and Distribution System Operators (DSOs).

Various types of flexibility exist throughout the transmission and distribution networks. The more conventional sources of flexibility found at the transmission level with, for example, industrial load that may be shed or curtailable power generation. At the distribution level thermostatically controlled loads (TCL) which includes heat-pumps, water heaters, and refrigeration units, may be controlled remotely by aggregators while ensuring they limit the discomfort to the end consumer. The benefit of utilising the flexibility of the aggregation thermostatically controlled loads (TCLs) is highlighted in [1] and [2]. Electric cars and household batteries provide flexibility by having the property of being a deferrable load (DL). Instead of a specific power requirement that must be immediately satisfied, DLs have a minimum energy requirement that must be completely satisfied by some expiration time. The benefit of utilising the flexibility of DL is highlighted in [3]. Flexibility markets [4] as part of ensuring grid security have recently been proposed. This provides opportunities for TSOs and DSOs to lower their grid costs and consumers to lower their electricity consumption costs.

Many flexibility dispatch models have recently been proposed, with each model making different assumptions resulting in each model having different advantages and disadvantages. In [5], flexibility is made available and bid into the market through 'so-called' asymmetric block offers. The market aggregators to define how they can offer demand response to the market and assumes they follow a particular shape. These block offers have the advantage of a relatively short dispatch time as all of the more difficult modelling and solving steps is given to the aggregators. However, this requires that aggregators both have a good model of their flexibility, of future spot market prices, and can take into consideration their uncertainty.

In contrast, [6] models aggregators that offering their cost functions explicitly. However, it models a single time step, limiting the types of flexibility considered. More sophisticated models are applied in the context of unit commitment. For example, [7] considers the multistage uncertainty inherent to the electricity market. [8] expands upon this to construct a model to dispatch units (specifically hydroelectric power) in a 'distributionally robust' manner.

In this paper, taking inspiration from these models applied in unit commitment, develop a model to dispatch and redispatch flexibility in the electricity market optimally. In this model, we consider a system operator that takes in cost functions and constraints from units (or aggregators of units) to optimally dispatch their flexibility.

Utilising the flexibility of both TCLs and DLs involves making decisions now while there is uncertainty about the future. With the constraints and costs of components in the network continually changing, it is also a multistage problem that we are solving. Solving these types of problems of any reasonably large size becomes intractable quickly if we try to solve the full problem as a single optimisation problem from scratch. Stochastic Dual Dynamic Programming (SDDP) is a widely used decomposition method used to build policies for these multistage problems with uncertainty.

Quantifying a distribution to represent the uncertainty is an essential step in stochastic optimisation. However, with only partial information available (typically in the form of historical data) obtaining truly representative distributions is especially tricky. Thus, if we were attempting to apply policies obtained from a classic stochastic risk-neutral optimisation model in the real world, we may obtain policies that are highly sub-optimal because of this ambiguity in what truly representative distribution is. To address this distributional ambiguity, we can create risk-averse models, which place a higher weight on higher cost scenarios when finding the solution that minimises the risk-adjusted cost. Alternatively, we can first define an ambiguity set, which assumes that the actual distribution is unknown but lies within some range of the fitted model, then find the probability measure in this ambiguity set that defines the highest expected cost (where our flexibility decisions would attempt to minimise this cost). The benefit in each of these methods may lead to solutions that work well for a broad range of possible outcomes, instead of working great for a narrow range of outcomes.

The goal of this work is to create a tool to dispatch flexibility that performs well across a variety of use cases, where the available information and the accuracy of this information may vary. In this context, this paper develops a formulation optimising the dispatch of demand response flexibility, considering both the uncertainty and the multistage nature of the system. We also consider versions of this model that are risk-averse or distributionally robust, making direct comparisons in the types of solutions that we obtain from each of them.

The paper is laid out as follows. In Section 2 background information relevant to the formulation of the model is given. In Section 3 a symbolic version of the flexibility model is formulated (with a more complete version given in Appendix A). In Section 4, we use a small case study to verify the implementation of the model. In Section 4.1, we compare the solutions given by the distributionally robust model to other models. Finally, we draw conclusions in 5.

2 Preliminaries

This paper focuses on the application of models that involve making many sequential decisions while having uncertainty about the future, requiring the introduction of how time and uncertainty are modelled.

2.1 Multistage Stochastic problems

We consider a discrete problem with time broken down into T stages, with $t = 1 \dots T$ indexing the times when we can make flexibility decisions. At each stage, there is uncertainty about the future states of the network, which we also discretise into Ω scenarios at each stage t. In each stage, the random vector is $b_t(\omega_t) \in \mathbb{R}^{\Omega}$, used to model our uncertainty about the future, is realised. We assume we have already cleared the day ahead market, and are now utilising the capacity for flexibility in the real time market, optimally balancing supply and demand. In Figure 1, we show a full scenario tree representation of this stochastic process. In this model, the exact sequence of all prior events (within the considered horizon) influences the probability of future outcomes.

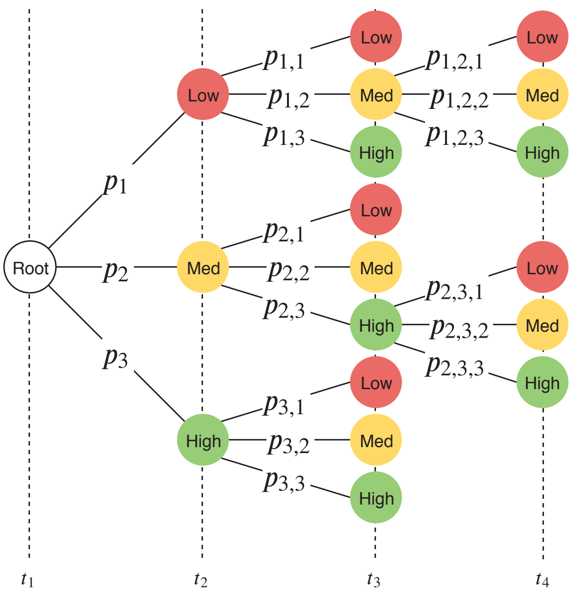

Figure 1: Full scenario tree with full conditional dependence

In Figure 2, we present a simplification to a Markov process, where each scenario ω_t only depends on the state preceding it (and not the sequence of events leading up to the prior state). Examples of Markov processes include random walks, sampling from a limited population without replacement, etc.

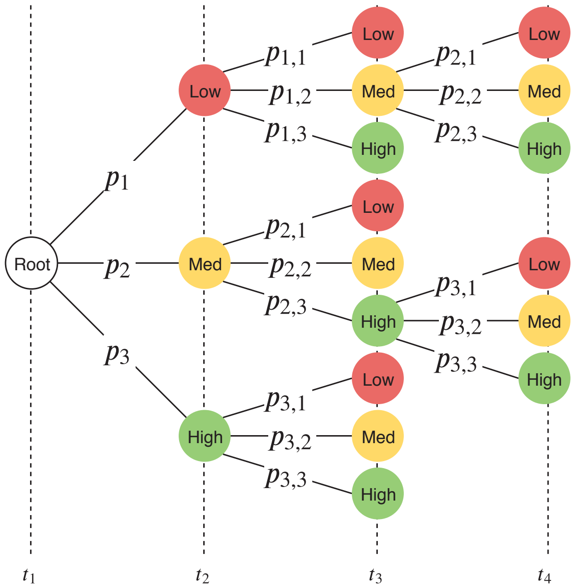

Figure 2: Scenario tree with events following a Markov process.

Finally in Figure 3, we present a further simplification which assumes that all observations are independent from prior observations and follow the same distribution. Examples of idependent and identically distributed processes include random sampling with replacement, rolling dice, flipping a coin etc.

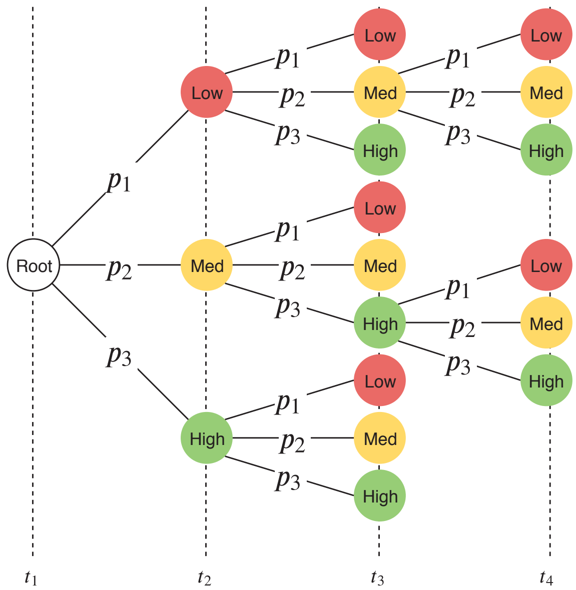

Figure 3: Scenario tree with events independent and identically distributed (i.i.d.).

The model that gives the best trade-off between computational complexity and usefulness depend on many factors including the processes being modelled, the scale of the model (number of stages, scenarios and units being modelled), and the accuracy and availability of data representing the concerned process.

2.2 Risk Aversion

In portfolios with random costs, let ρ represent a risk measure that maps these random costs to real numbers. For this to be a coherent risk measure, it must satisfy the axioms, defined in [9]. Consider a portfolio, $\boldsymbol{Z}$, with discrete scenarios $\omega \in \Omega$, occurring with probability $\mathbb{P}_\omega$.

Expectation is a simple example of a coherent risk measure, defined as follows

$$\rho_{\mathrm{E}}(\boldsymbol{Z}) = \sum_{\omega \in \Omega} \mathbb{P}_\omega Z(\omega) \tag{1}$$

Another simple example is the 'Worst Case' risk measure (the cost in the worst possible outcome). This is calculated as part of solving an LP using the following

$$\rho_{\mathrm{WC}}(\boldsymbol{Z}) = \min_{\theta} \theta \tag{2}$$

$$\text{s.t.} \qquad \theta \geq Z(\omega) \qquad \forall \omega \in \Omega \tag{3}$$

The formula 4 by [10] presents a formula that allows us to apply the *Conditional Value at Risk* in mathematical models. $\text{CVaR}_\alpha(\boldsymbol{Z})$ is the expected cost of $\boldsymbol{Z}$ in the worst α scenarios

$$\text{CVaR}_\alpha(\boldsymbol{Z}) = \inf_{\xi} \left[\xi + \frac{1}{\alpha} \mathbb{E}(Z - \xi)_+ \right]. \tag{4}$$

With discrete scenarios ($\omega \in \Omega$), and probabilities of each disbenefit, $Z(\omega)$, occurring with probability $\mathbb{P}_\omega$, we can calculate $\text{CVaR}_\alpha(Z)$ as part of an optimisation problem using the following

$$\rho_{\text{CVaR}}(\boldsymbol{Z}) = \min_{\xi, \Theta_\omega} \xi + \frac{1}{\alpha} \sum_{\omega \in \Omega} \mathbb{P}_\omega \Theta_\omega \tag{5}$$

$$\text{s.t.} \qquad \Theta_\omega \geq Z_\omega - \xi \qquad \forall \omega \in \Omega \tag{6}$$

$$\Theta_\omega \geq 0 \qquad \forall \omega \in \Omega \tag{7}$$

Taking convex combinations of coherent risk measures, we can create another coherent risk measure, satisfying all of the required properties stated in [9]. In our case we use the E-CVaR risk measure, which is a convex combination of the Expectation and CVaR risk measure.

2.3 Distributional robustness

Let the ambiguity set, $\mathcal{P}$, define the range of probability measures, $\boldsymbol{p} \in \mathbb{R}^\Omega$, that we believe can reasonably represent a random outcome $\boldsymbol{Z}$. A method of defining this set involves first defining a reference probability measure and allowed difference from this measure. Specifically, we need a reference vector of probabilities $\boldsymbol{q}$ (typically derived using historical data), a function to define the *distance* from this reference probability, and a maximum distance from this probability measure that we allow, r. There are many functions that can be used to define the distance between any two probability measures. A broad class of ϕ-divergences (χ^2-distance, Kullback-Leibler (KL) divergence etc.), and their use in in stochastic programming is discussed in [11]. Consider two probability measures $\boldsymbol{p}$ and $\boldsymbol{q}$ of dimension Ω. $\phi : [0, \infty) \to \mathbb{R}$ is a strictly convex function, twice continuously differentiable in $(0, \infty)$. The ϕ-divergence between $\boldsymbol{p}$ and $\boldsymbol{q}$ is defined by [12]

$$d_\phi(\boldsymbol{p}, \boldsymbol{q}) = \sum_{\omega=1}^{\Omega} q_\omega \phi \left(\frac{p_\omega}{q_\omega} \right) \tag{8}$$

The χ^2-distance function is defined as $\phi_{\chi^2}(t) = (1 - t)^2$, leading to the following definition for its ϕ-divergence

$$d_{\phi_{\chi^2}}(\boldsymbol{p},\boldsymbol{q}) = \sum_{\omega=1}^{\Omega} q_\omega \left(1 - \frac{p_\omega}{q_\omega}\right)^2 \tag{9}$$

Similarly, the KL-divergence function is defined as $\phi_{\mathrm{KL}}(t) = -\log(t)$, leading to the following definition for its ϕ-divergence

$$d_{\phi_{\mathrm{KL}}}(\boldsymbol{p},\boldsymbol{q}) = -\sum_{\omega=1}^{\Omega} q_\omega \log\left(\frac{p_\omega}{q_\omega}\right) \tag{10}$$

The Wasserstein distance, discussed in [13, 14], is another distance measure and can be thought of as the minimum cost of transforming one distribution, $\boldsymbol{p}$, into another distribution, $\boldsymbol{q}$. This cost is defined as the as the amount of probability mass that needs to be moved multiplied by the mean distance it's moved.

The advantage of ϕ-divergences, is that many of the common ones (including the χ^2-distance and the KL divergence) have an established use in statistics (especially in goodness-of-fit tests). However, in contrast to the Wasserstein distance, many ϕ-divergences (one exception being the L_1 norm) have the disadvantage of not being a proper distance metric, with $d_\phi(\boldsymbol{p},\boldsymbol{q}) \neq d_\phi(\boldsymbol{q},\boldsymbol{p})$. Also, again in contrast to the Wasserstein distance, the two distributions have the same support (i.e. $\frac{q_\omega}{p_\omega} \in (0,\infty), \quad \forall \omega = 1,\ldots,\Omega$).

We use the L_2-norm (which can be interpreted as a modified χ^2 distance) throughout this paper, due to its broad use in goodness-of-fit tests. The closed form representation of the modified χ^2-distance is given in [11].

$$d_{\mathrm{m}\chi^2}(\boldsymbol{p},\boldsymbol{q}) = ||\boldsymbol{p}-\boldsymbol{q}|| = \sqrt{\sum_{\omega=1}^{\Omega} (p_\omega - q_\omega)^2} \tag{11}$$

Thus, for a given set of outcomes $\omega \in \Omega$ we can find the values for $\boldsymbol{p}$ that give the probability measure that gives the worst expected value, while still satisfying the goodness of fit requirement.

$$Q_{\mathrm{m}\chi^2} = \max_{p_\omega} \sum_{\omega\in\Omega} p_\omega Z_\omega \tag{12}$$

$$\text{s.t.} \qquad \sqrt{\sum_{\omega=1}^{\Omega} (p_\omega - q_\omega)^2} \leq \mathrm{r} \tag{13}$$

$$\sum_{\omega\in\Omega} p_\omega = 1 \tag{14}$$

$$p_\omega \geq 0 \qquad \forall \omega = 1, \ldots, \Omega \tag{15}$$

A closed-form algorithm to find the worst case probability distribution $\boldsymbol{p}$ that maximises $Q_{\mathrm{m}\chi^2}$ is given in page 10 of [15]

3 Model Formulation

We consider a Transmission System Operator (TSO) that buys flexibility-based services from aggregators within a competitive market. This is a day ahead market that also considers the fact the TSO will be able to adjust their flexibility decisions in the real time market (although at a premium).

In Appendix A we build up the formulation optimising the operation of conventional units, deferrable loads (DL), and thermostatically controlled loads (TCL). Here, we symbolically present the stochastic dynamic problem (SDP) and equivalent the stochastic problem (SP).

Through appropriate rearranging and introducing slack variables, we can simplify the representation of our problem into a symbolic representation of our decisions, $\boldsymbol{x}$, with costs, $\boldsymbol{c}$. First consider a two stage rolling horizon optimisation problem. Following the decisions $\boldsymbol{x}_0$, made in the previous stage (stage 0), we realise which scenario (ω_1) has occurred. Given this outcome, we make flexibility decisions now ($\boldsymbol{x}_1$), to minimise the sum of the system cost in this stage (stage 1) and the following stage (stage 2). This leads to the following 2 stage stochastic optimisation problem.

$$\min_{\boldsymbol{x}_1, \boldsymbol{x}_2(\omega_2)} \left\{ \mathbf{c}_1^\intercal \boldsymbol{x}_1 + \mathbb{E}_{\mathbb{Q}}(\mathbf{c}_2^\intercal \boldsymbol{x}_2(\omega_2)) \right\} \tag{16}$$

$$\text{s.t.} \quad A_1 \boldsymbol{x}_1 \leq \boldsymbol{f}_1(\omega_1) - F_1 \boldsymbol{x}_0 \tag{17}$$

$$A_2 \boldsymbol{x}_2(\omega_2) \leq \boldsymbol{f}_2(\omega_2) - F_1 \boldsymbol{x}_1 \tag{18}$$

$$\boldsymbol{x}_1, \boldsymbol{x}_2(\omega_2) \geq 0 \tag{19}$$

In equation (16) we define the first stage objective of optimising the use demand response flexibility, $\boldsymbol{x}_1$ and $\boldsymbol{x}_2(\omega_1)$. This cost is defined as the sum of the cost of our flexibility decisions at $t = 1$ ($\mathrm{c}_1^\intercal \boldsymbol{x}_1$) and the expected cost of our flexibility decisions at time $t = 2$ $\mathbf{c}_1^\intercal \boldsymbol{x}_2(\omega_2)$. The feasible decisions in stage 2 are subject to our first stage decisions $\boldsymbol{x}_1$. In this formulation, the objective is to minimise the the expected cost, with $\mathbb{Q}$ defining the probability of each scenario ω_2. In equation (17) we define the inequality constraints that must be satisfied now, given our decisions in the prior stage. In (18) we define the inequality constraints that must be satisfied in the following stage given our decisions now. Finally, in equation (27) we define the non-negativity constraints.

Expanding this formulation to a 3 stage optimisation problem we have

$$\min_{\boldsymbol{x}_1,\boldsymbol{x}_2(\omega_2),\boldsymbol{x}_3(\omega_3)} \left\{\mathbf{c}_1^\intercal \boldsymbol{x}_1 + \mathbb{E}_{\mathbb{Q}}(\mathbf{c}_2^\intercal \boldsymbol{x}_2(\omega_2) + \mathbb{E}_{\mathbb{Q}}(\mathbf{c}_3^\intercal \boldsymbol{x}_3(\omega_3)))\right\} \tag{20}$$

$$\text{s.t.} \quad A_1\boldsymbol{x}_1 \leq \boldsymbol{f}_1(\omega_1) - F_1\boldsymbol{x}_0 \tag{21}$$

$$A_2\boldsymbol{x}_2(\omega_2) \leq \boldsymbol{f}_2(\omega_2) - F_1\boldsymbol{x}_1 \tag{22}$$

$$A_3\boldsymbol{x}_3(\omega_3) \leq \boldsymbol{f}_3(\omega_3) - F_2\boldsymbol{x}_2(\omega_2) \tag{23}$$

$$\boldsymbol{x}_1, \boldsymbol{x}_2(\omega_2), \boldsymbol{x}_3(\omega_3) \geq 0 \tag{24}$$

Alternatively, we can formulate this problem as a stochastic dynamic program with T stages. Now the first stage problem is

$$\min_{\boldsymbol{x}_1,\boldsymbol{x}_2(\omega_2)} \left\{\mathbf{c}_1^\intercal \boldsymbol{x}_1 + \mathbb{E}_{\mathbb{Q}}(Q_2(\boldsymbol{x}_1, \omega_2))\right\} \tag{25}$$

$$\text{s.t.} \quad A_1\boldsymbol{x}_1 \leq \boldsymbol{f}_1(\omega_1) - F_1\boldsymbol{x}_0 \tag{26}$$

$$\boldsymbol{x}_1 \geq 0 \tag{27}$$

Identical to equation (16), in equation (25) we define the first stage objective of optimising the use demand response flexibility, $\boldsymbol{x}_1$. This cost is defined as the sum of the cost of our flexibility decisions at $t = 1$ ($\mathrm{c}_1^\intercal \boldsymbol{x}_1$) and the expected cost of our flexibility decisions up to time $t = T$ given our decision for $\boldsymbol{x}_1$. The function $Q_2(\boldsymbol{x}_1, \omega_2)$, defines the second stage objective for each stage 2 scenario, ω_2, that can occur in stage 2. The feasible decisions in stage 2 are subject to our first stage decisions $\boldsymbol{x}_1$. In this formulation, the objective is to minimise the the expected cost, with $\mathbb{Q}$ defining the probability of each scenario ω_2. In equation (26) we define the inequality constraints that must be satisfied, given our decisions in the prior stage. Finally, in equation (27) we define the non-negativity constraints.

For $t = 2 \ldots T$

$$Q_t(\boldsymbol{x}_{t-1}, \omega_t) = \min_{\boldsymbol{x}_t} \left\{\mathbf{c}_t^\intercal \boldsymbol{x}_t + \mathbb{E}_{\mathbb{Q}}(Q_{t+1}(\boldsymbol{x}_t, \omega_{t+1}))\right\} \tag{28}$$

$$\text{s.t.} \quad A_t\boldsymbol{x}_t \leq \boldsymbol{f}_t(\omega_t) - F_t\boldsymbol{x}_{t-1} \tag{29}$$

$$\boldsymbol{x}_t \geq 0 \tag{30}$$

For simplicity, we assume that $Q_{T+1}(\boldsymbol{x}_T, \omega_{T+1})$ (the $T + 1$ stage objective value in each scenario ω_{T+1}) is 0, though this may mean our model will be utilising flexibility aggressively towards the end of the horizon.

3.1 Distributionally Robust Model

The set $\mathcal{P}_t$, defines the ambiguity set branching from each scenario ω_{t+1} ($\mathbb{P} \in \mathcal{P}_t$) for all $t = 1, \ldots, T$. Thus, our modified distributionally robust real time model is:

for $t = 1, \ldots, T$

$$\min_{\boldsymbol{x}_t} \left\{ \mathbf{c}_t^\intercal \boldsymbol{x}_t + \max_{\mathbb{P} \in \mathcal{P}_t} \mathbb{E}_\mathbb{P}(Q_{t+1}(\boldsymbol{x}_t, \omega_{t+1})) \right\} \tag{31}$$

$$\text{s.t.} \quad A_t \boldsymbol{x}_t \leq \boldsymbol{f}_t(\omega_t) - F_t \boldsymbol{x}_{t-1} \qquad [\pi_t(\omega_t)] \tag{32}$$

$$\boldsymbol{x}_t \geq 0. \tag{33}$$

Again, we assume that $Q_{T+1}(\boldsymbol{x}_T, \omega_{T+1}) = 0$.

The SDDP algorithm is implemented in Julia [16], utilising the package SDDP.jl [17]. Here, we give a brief overview of the main steps to solve this model utilising SDDP, with the overall process of using SDDP to solve these multistage, distributionally robust optimisation problems explained in detail in [8].

Starting without any cuts (K = 0), We define $\hat{Q}_t$ and $\pi_t(\omega_t)$ (the Lagrange multipliers of the constraints), approximating the stage problem Q_t

$$\hat{Q}_t(\boldsymbol{x}_{t-1}, \omega_t) = \min \boldsymbol{c}_t^\intercal \boldsymbol{x}_t + \theta_{t+1} \tag{34}$$

$$\text{s.t.} \quad A_t \boldsymbol{x}_t = \boldsymbol{f}_t(\omega_t) - F_t \boldsymbol{x}_{t-1} \qquad [\pi_t(\omega_t)] \tag{35}$$

$$\theta_{t+1} + \bar{\pi}_{t+1,k}^\intercal F_{t+1} \boldsymbol{x}_t \geq \boldsymbol{g}_{t+1,k} \qquad \forall \quad k = 1, \ldots, \mathrm{K} \tag{36}$$

$$\theta_{t+1} \geq -M \tag{37}$$

$$\boldsymbol{x}_t \geq 0, \tag{38}$$

with $\hat{Q}_t(\bar{\boldsymbol{x}}_{t-1}, \omega_t)$ providing a lower bound to $Q_t(\bar{\boldsymbol{x}}_{t-1}, \omega_t)$. It is necessary to add the constraint $\theta_{t+1} \geq -M$ (where M is sufficiently large enough for the problem to be feasible) to ensure that θ_{t+1} is bounded, before any cuts are made.

Given a candidate solution $\bar{\boldsymbol{x}}_t$, we calculate the probability measure that gives the worst-case expectation, $\mathbb{P}_t^*$, defined as $\mathbb{P}_t^* \in \arg\max_{\mathbb{P} \in \mathcal{P}_t} \mathbb{E}_\mathbb{P}[Q_{t+1}(\bar{\boldsymbol{x}}_t, \omega_{t+1})]$. We set $\bar{\pi}_{t+1,k}^\intercal = \mathbb{E}_{\mathbb{P}_t^*}[\pi_{t+1}(\omega_{t+1})]$, which defines the sub-gradient $-\bar{\pi}_{t+1,k}^\intercal F_{t+1}$ and the intercept $\boldsymbol{g}_{t+1,k}$ defined as

$$\boldsymbol{g}_{t+1,k} = \mathbb{E}_{\mathbb{P}_t^*}[\hat{Q}_{t+1}(\bar{\boldsymbol{x}}_t, \omega_{t+1})] \tag{39}$$

We replace $\max_{\mathbb{P} \in \mathcal{P}_t} \mathbb{E}_\mathbb{P}(Q_{t+1}(\boldsymbol{x}_t, \omega_{t+1}))$ with the variable θ_{t+1} which is constrained by the following set of linear inequalities

$$\theta_{t+1} + \bar{\pi}_{t+1,k}^\intercal F_{t+1} \boldsymbol{x}_t \geq \boldsymbol{g}_{t+1,k} \qquad \forall k = 1, \ldots, \mathrm{K} \tag{40}$$

Thus, if we denote $\Omega_{t+1} = \{\omega_{t+1}^1, \omega_{t+1}^2, \ldots, \omega_{t+1}^M\}$ and $\mathbb{P}_t^*(\omega_{t+1}^i) = p_i$ then the cut parameters are defined by

$$\bar{\pi}_{t+1,k}^\intercal = \sum_{i=1}^{\mathrm{M}} p_i \pi_{t+1,k}^\intercal(\omega_{t+1}^i) \tag{41}$$

$$\boldsymbol{g}_{t+1,k} = \sum_{i=1}^{\mathrm{M}} p_i \hat{Q}_{t+1}(\boldsymbol{x}_t, \omega_{t+1}^i) + \bar{\pi}_{t+1,k}^\intercal F_{t+1} \boldsymbol{x}_t \tag{42}$$

Defining a cut that improved upon our lower bound approximation of the function $Q_{t+1}(\boldsymbol{x}_t, \omega_{t+1})$. This process can be repeated until a satisfactory representation of $Q_{t+1}(\boldsymbol{x}_t, \omega_{t+1})$ is found. We can also calculate the upper bound of $Q_{t+1}(\boldsymbol{x}_t, \omega_{t+1})$ by enumerating the policy $\bar{\boldsymbol{x}}$, though it may be much more practical to simulate the policy $\bar{\boldsymbol{x}}$ and instead estimate the upper bound.

4 Verifying implementation of Models

Below we show results helping to verify that the Julia SDDP formulation and the GAMS implementation of a model are equivalent. These models optimise the dispatch of demand response under uncertainty over multiple stages. For our DRO, we constrain the ambiguity set, $\mathcal{P}$ to be within some 'L2-distance' of the input probabilities. Other risk measures can also be used. In our example, we have two DL units and three conventional units.

In the corresponding GAMS models, we define each stochastic optimisation problem explicitly, defining all possible trajectories. In versions of the model that use the *Expected*, *E-CVaR*, and *Worst* risk measures can all be solved as LPs. However when creating a distributionally robust model subject to an ambiguity set within some 'χ^2-distance', 'L_2-distance', or 'KL-divergence' from the input probabilities, leads to a mini-max problem. This can be thought of as one agent making flexibility decisions to minimise the expected cost given that they know that after they make these flexibility decisions another agent will set the probability of each outcome (constrained by non-negativity and the ϕ-divergence constraints) to maximise the expected cost. This bi-level optimisation problem can then be solved as an MPEC (Mathematical Program with Equilibrium Constraints).

In Figure 4, we verify the correctness of the implementation of each model by comparing their objective values in a simple 3 stage stochastic problem. The formulation and parameter values can be found in *ModChi_3Stage.gms*.

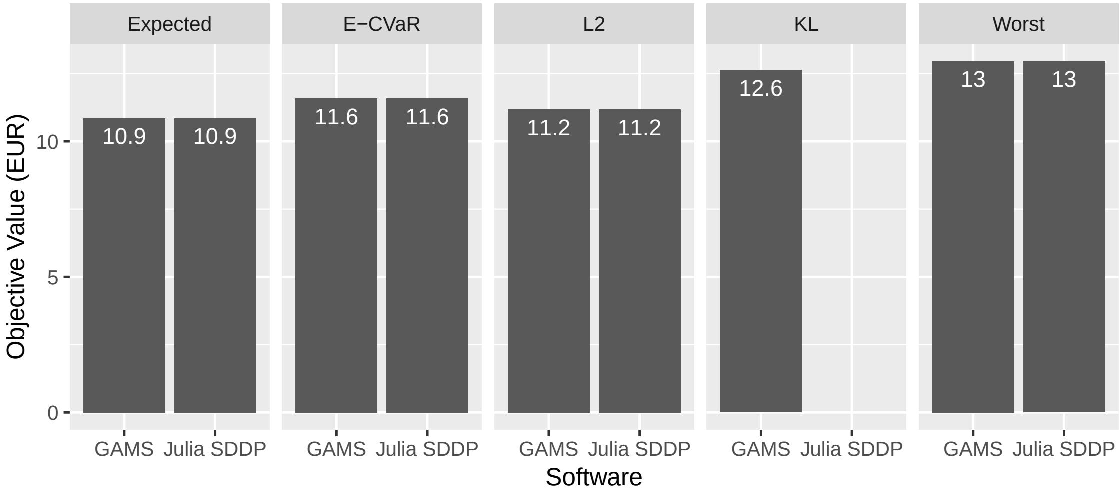

Figure 4: Objective value given each risk measure and each implementation of model. For the E-CVaR model: $\beta = \lambda = 0.5$. For the L2 model: $r = 0.2$. For the KL model: $r = 0.2$.

KL divergence is currently not implemented within *SDDP.jl*, so we cannot use it to verify our GAMS model; however, the objective value appears to be reasonable. In Figures 5, 6, 7, 8, and 9 we present the probability allocations that each of these models place on each scenario (the corresponding dual probability weights for the E-CVaR model) and the cost-to-go using the corresponding measure at each node. The red nodes in this tree represent low wind states, the yellow nodes correspond to moderate wind states, and the green nodes corresponding to high wind scenarios. As we arbitrarily chose the risk parameters, there are limited comparisons we can make between the advantages and disadvantages of each measure. One observation we can make is in how the distributionally robust models shift the probability weights. In the *E-CVaR* and *Worst-Case* models, there is a direct transfer of probability mass from the high wind scenario to the low wind scenario. Also, in the E-CVaR and Worst-Case models model, the shift is only impacted by where it ranks relative to the other scenarios (and not directly by how different the cost is compared to the other scenarios. The shift in probability in the *L2* and *KL* models follows a much more complicated relationship, where the relative difference in costs between scenarios impacts how the worst case probability shifts between the nodes.

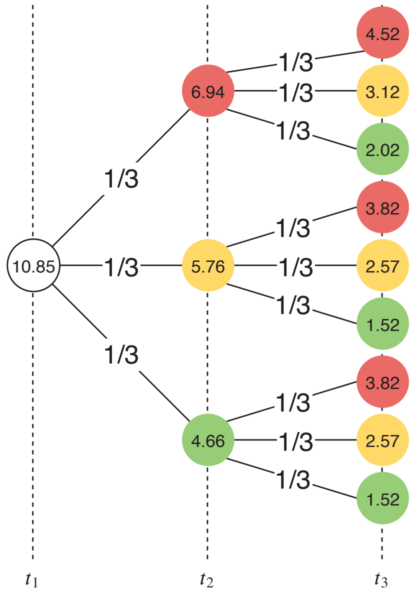

Figure 5: Actual probability of each scenario.

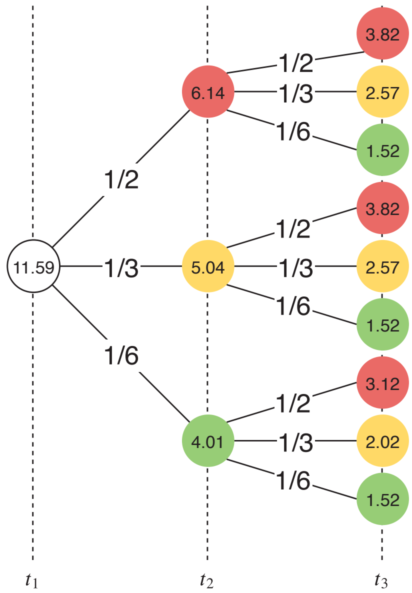

Figure 6: Risk adjusted probability weights of each scenario using **E-CVaR** risk measure ($\beta = \lambda = 0.5$).

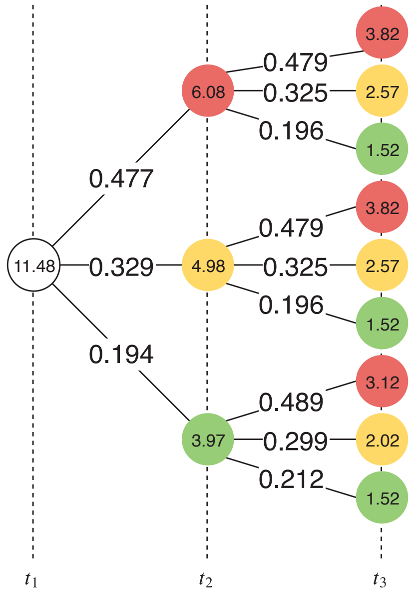

Figure 7: Worst case probability weights within specified **L2 distance** requirement.

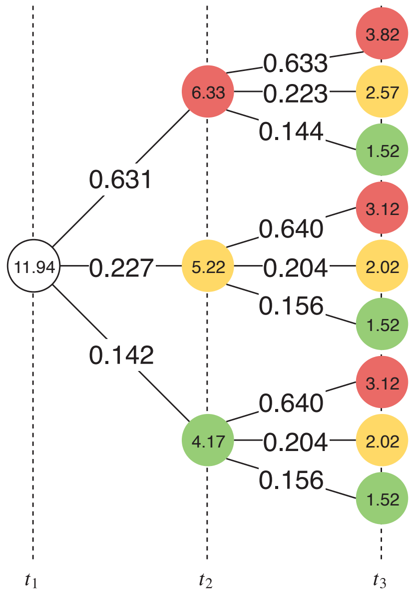

Figure 8: Worst case probability weights within specified **KL divergence** requirement.

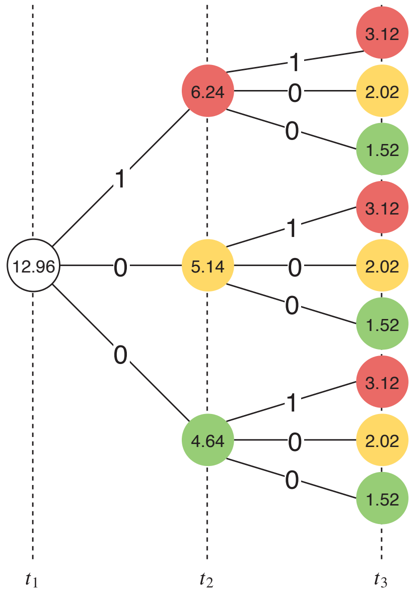

Figure 9: Risk adjusted probability weights of each scenario using the **Worst case** risk measure.

4.1 In sample verification

In this case study we expand our case study to model six stages and four possible wind realisations to occur at each stage. These wind outcomes are i.i.d. are stage-wise independent, allowing the use of the 'modified χ^2-distance' function included within the 'SDDP.jl' package.

4.2 Parameter values

Summarising the parameter values in our case study we have

$$\mathrm{C}_t^{\mathrm{Con,up}} := [0.2,\ 0.4,\ 0.6,\ 0.5,\ 0.4,\ 0.3],\ \forall t \in \mathcal{T}$$

$$\mathrm{C}_t^{\mathrm{Con,dn}} := 0.1,\ \forall t \in \mathcal{T}$$

$$\mathrm{C}_t^{\mathrm{DL,dn}} := 0.1,\ \forall t \in \mathcal{T}$$

$$\mathrm{C}_t^{\mathrm{DL,ct}} := 0.5,\ \forall t \in \mathcal{T}$$

$$\mathrm{R}_{j,t}^{\mathrm{up}} := 2.0,\ \forall j \in \mathcal{J}, t \in \mathcal{T}$$

$$\mathrm{R}_{j,t}^{\mathrm{dn}} := 2.0,\ \forall j \in \mathcal{J}, t \in \mathcal{T}$$

$$\bar{\mathrm{P}}_t^{\mathrm{DL,dn}} := 4.0, \ \forall t \in \mathcal{T}$$

$$\bar{\mathrm{P}}_t^{\mathrm{DL,ct}} := 4.0, \ \forall t \in \mathcal{T}$$

$$\mathrm{E}_t^{\mathrm{Req}} := [0, 0, 0, 0, 0, 5]$$

$$\mathrm{E}^{\mathrm{Cap}} := 12$$

$$\mathrm{d}_t(\omega_t) := [2.0, 3.0, 4.0, 3.0, 2.0, 2.0]$$

$$\mathrm{w}_t(\omega_t) := [2.0, 3.0, 4.0, 5.0] \ \forall t \in \mathcal{T}$$

$$\mathbb{P} := [0.25, 0.25, 0.25, 0.25] \ \forall t \in \mathcal{T}$$

As another sanity check, the box-plot in Figure 10 shows, as expected, that the model minimising the expected total cost, 'Expected', in fact has the lowest mean cost when the corresponding policy is simulated. Similarly, the model minimising the worst case cost, 'Worst', has the lowest maximum cost.

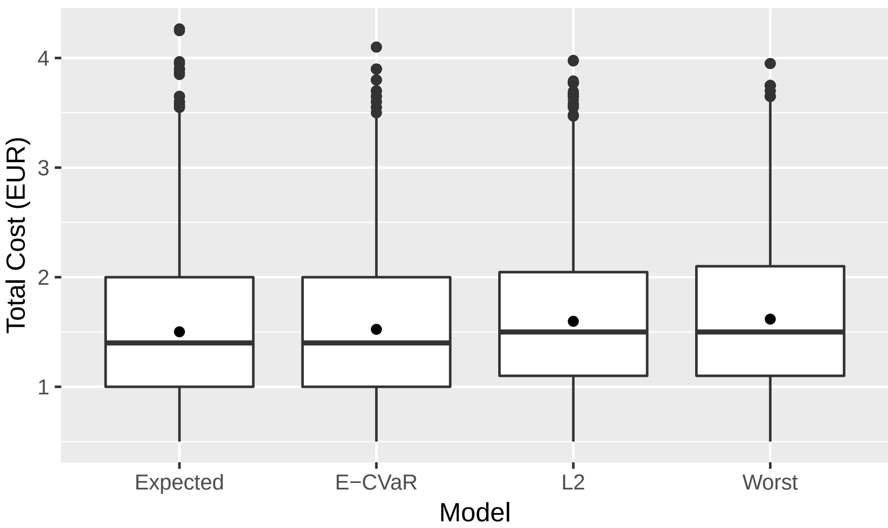

Figure 10: Box-plot comparing simulations with different risk measures.

Now, training each model from scratch 50 times, simulating each model 50 times once trained (for a total of 2500 simulations per model), we compare the mean cost, and worst case cost over the simulations. In Figure 11 and 12, we see the 'Expected' model has the lowest mean cost over the 50 models, and the 'Worst Case; model has the lowest maximum cost cost over the 50 models.

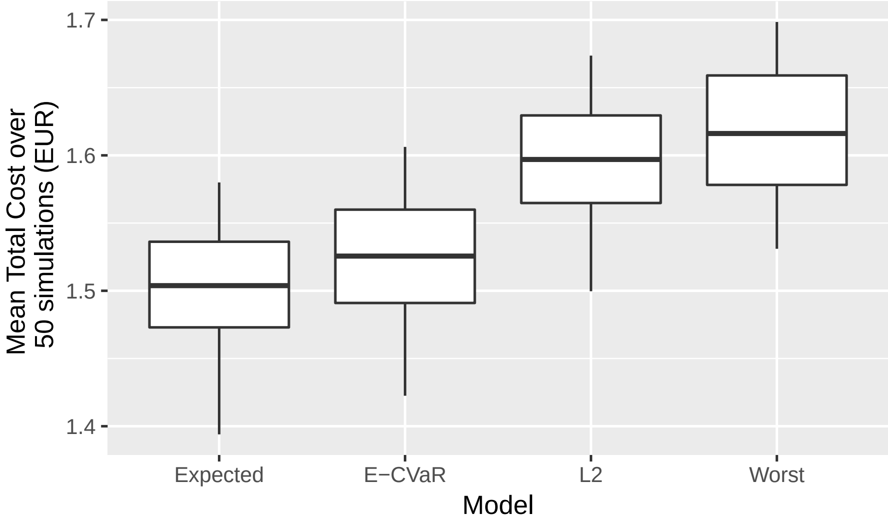

Figure 11: Box-plot of mean cost, training each model 50 times to convergence and simulating each of these trained models 50 times.

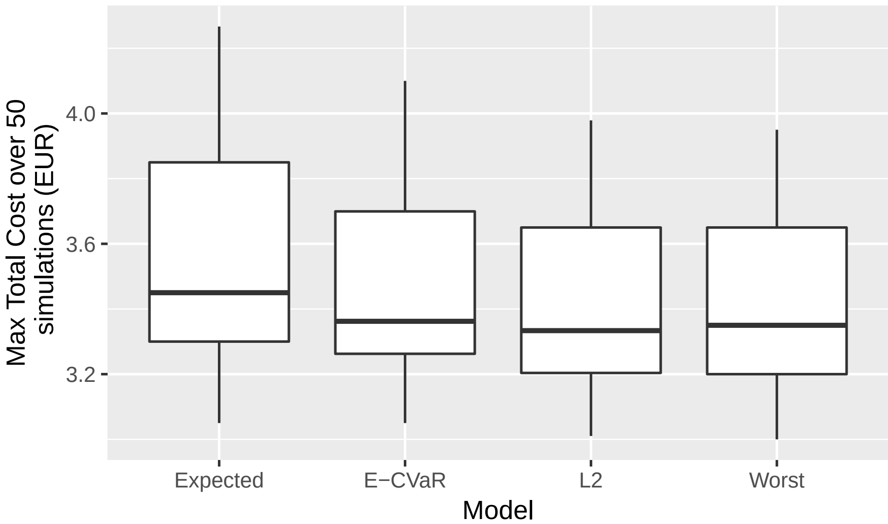

Figure 12: Box-plot of max cost over, training each model 50 times to convergence and simulating each of these trained models 50 times.

We can compare each risk measure directly by first ensuring that the same random seeds are used in the simulations and comparing the differences between the costs. In Figure 13, we see that within this case study the costs are the same across all models for a significant proportion of the samples (the full middle quantile for the 'EAVaR' and 'Modified Chi Squared' models are the same as the 'Expected' model' and a full quantile of the 'Worst Case' model).

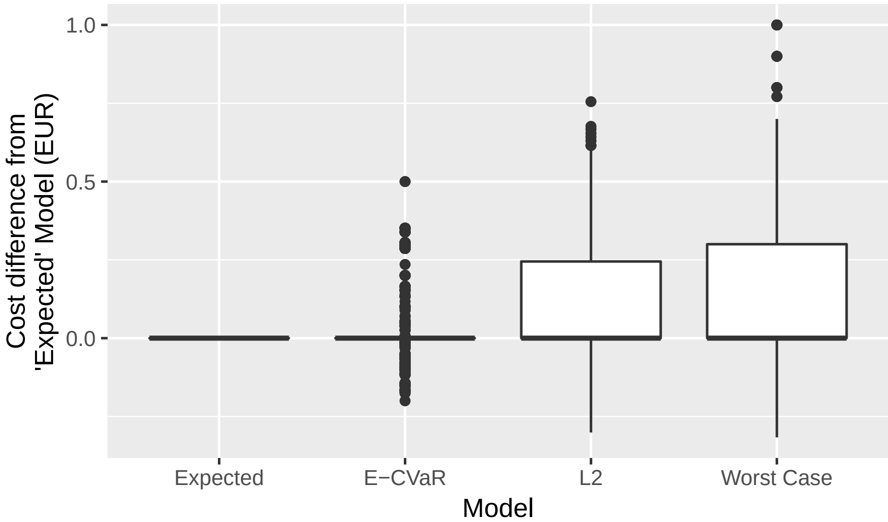

Figure 13: Box-plot comparing simulations with different risk measures.

In Figures 14 and 15, we more clearly show that the 'Expected' model is better at minimises the mean cost across scenarios and the 'Worst Case' model is better at minimising the maximum observed cost (with the 'EAVaR' and 'Modified Chi Squared' models somewhere in between the two).

There still appears to be some cases where the worst case model will lead to a policy that has a higher maximum cost than the policy generated from the 'Expected' model. This may be due insufficient training, insufficient simulations to find the worst case cost, or a combination of both.

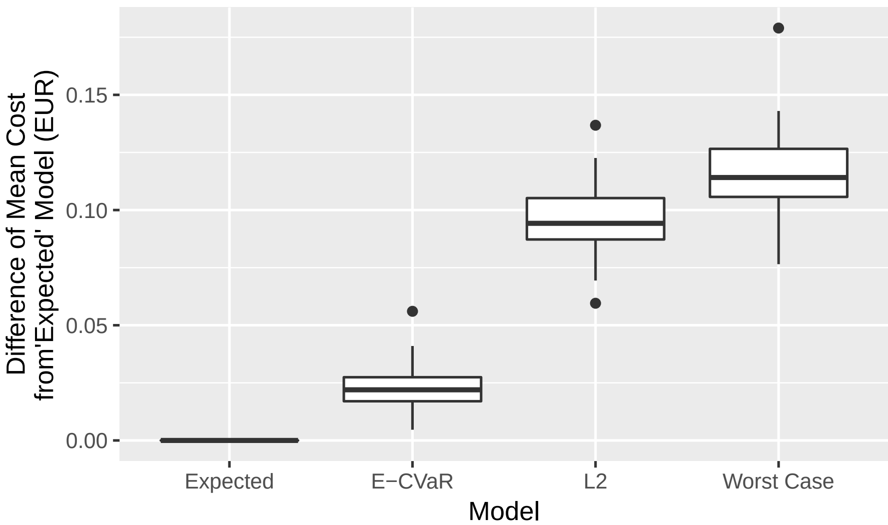

Figure 14: Mean cost difference from 'Expected' model, utilising each risk measure.

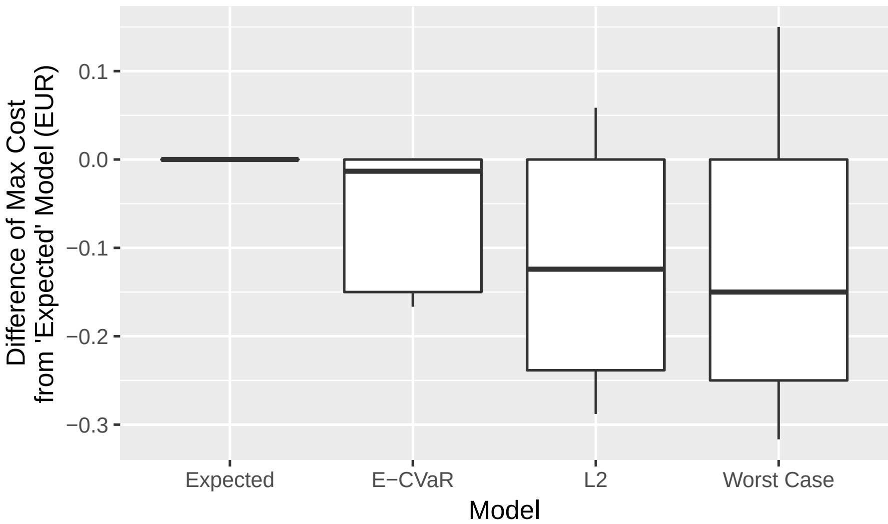

Figure 15: Maximum cost difference from 'Expected' model, utilising each risk measure.

4.3 Out of sample tests

Training each model with a larger scale model, we now simulate the performance of each model when only a sample of the real wind data is available. In the *real wind data*, there are 24 stages, with 10 possible wind outcomes at each stage, and again assume that wind is i.i.d.

4.3.1 Parameter values

Summarising the parameter values in our case study we have

$$\begin{aligned}
\mathrm{C}_t^{\mathrm{Con,up}} &:= [0.3, 0.2, 0.2, 0.1, 0.1, 0.2, 0.4, 0.3, 0.2, 0.2, 0.3, 0.3\ 0.2, 0.2, 0.2, 0.2, 0.4, 0.5, 0.6,\\
&\quad 0.5, 0.4, 0.3, 0.3, 0.3],\ \forall t \in \mathcal{T}\\
\mathrm{C}_t^{\mathrm{Con,dn}} &:= 0.1,\ \forall t \in \mathcal{T}\\
\mathrm{C}_t^{\mathrm{DL,dn}} &:= 0.1,\ \forall t \in \mathcal{T}\\
\mathrm{C}_t^{\mathrm{DL,ct}} &:= 0.5,\ \forall t \in \mathcal{T}\\
\mathrm{R}_{j,t}^{\mathrm{up}} &:= 2.0,\ \forall j \in \mathcal{J}, t \in \mathcal{T}\\
\mathrm{R}_{j,t}^{\mathrm{dn}} &:= 200.0,\ \forall j \in \mathcal{J}, t \in \mathcal{T}\\
\bar{\mathrm{P}}_t^{\mathrm{DL,dn}} &:= 4.0,\ \forall t \in \mathcal{T}
\end{aligned}$$

$$\bar{\mathrm{P}}_t^{\mathrm{DL,ct}} := 100.0,\ \forall t \in \mathcal{T}$$

$$\mathrm{E}_t^{\mathrm{Req}} := [0,\ 0,\ \ldots,\ 0,\ 5]$$

$$\mathrm{E}^{\mathrm{Cap}} := 100$$

$$\mathrm{d}_t(\omega_t) := [1.0,\ 1.0,\ 1.0,\ 1.0,\ 1.0,\ 4.0,\ 4.0,\ 4.0,\ 3.0,\ 3.0,\ 3.0,\ 3.0,\ 3.0,\ 3.0,\ 3.0,\ 3.0,\ 5.0,\ 5.0,\ 5.0,\ 3.0,\ 3.0,\ 3.0,\ 3.0,\ 3.0]$$

$$\mathrm{w}_t(\omega_t) := [0.5,\ 1.0,\ 1.5,\ 2.0,\ 2.5,\ 3.0,\ 3.5,\ 4.0,\ 4.5,\ 5.0,\ 5.5,\ 6.0,\ 6.5,\ 7.0,\ 7.5,\ 8.0,\ 8.5,\ 9.0,\ 9.5,\ 10.0]\ \forall t \in \mathcal{T}$$

$$\mathbb{P} := 0.05\ \forall t \in \mathcal{T}, \omega_t \in \Omega$$

We have used the same policies generated from the previous models simulating the resulting policy on the *real wind data* here. In figure 16, we plot a boxplot of the simulated cost of each model, showing that the Expected, E-CVaR, and L2 models give roughly equivalent policies. We see that, with limited information in the training of each model, there is no longer a trade-off between the expected cost and max cost (the *Expected* model performs the best in both). There appears to be limited benefit in distributionally robust or risk-averse models when we apply this 10 second time limit to solve the model.

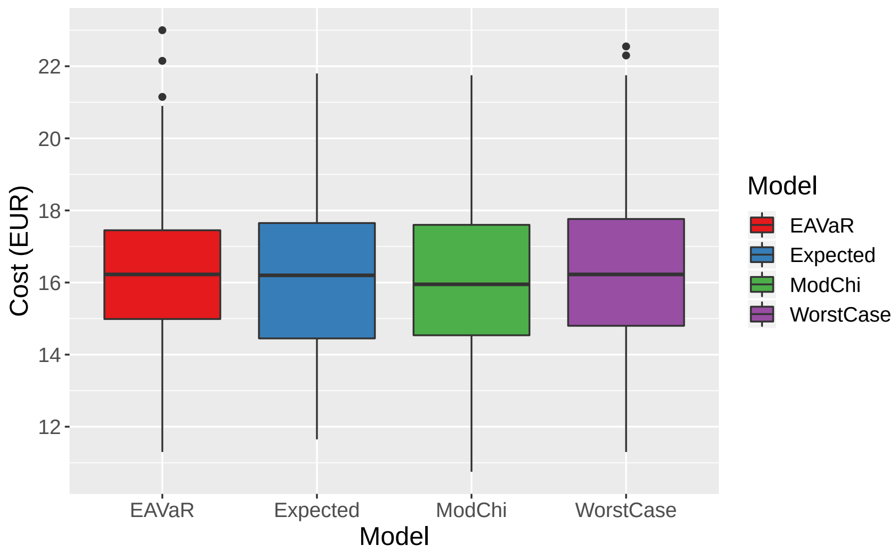

Figure 16: Box-plot comparing the total cost between models using out of sample simulations.

5 Conclusions

This paper proposes a tool to dispatch demand response in the real-time market optimally and includes versions of the tool that are risk-averse and distributionally robust. We have implemented this model within Julia using the SDDP.jl package, verifying it using a small case study against a stochastic optimisation model implemented in GAMS. With our case study, there was limited support for using the risk-averse and distributionally robust model (though it is very case dependent). In the future, we would also want to improve (possibly optimise) the day ahead decisions either by modifying our formulation or through heuristics that embed our model.

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Appendix A: Formulation of Real Time Balancing Market

We first describe the problem of optimising the operation of conventional units, deferrable loads (DL), and thermostatically controlled loads (TCL) individually in a real-time market with the objective of minimising the expected total cost of transmission balancing with uncertainty in wind. We then move onto the full model with all three types of units.

Now we define the relevant sets, parameters, and variables that we use in the formulation of this market clearing tool. We use the convention that calligraphic letters are sets, Roman type text denotes parameters, and math-type text denotes variables and indices, and bold is used to denote parameters and variables that are not fully indexed.

In our model of conventional units, we assume regulation decisions have no effect on decisions that follow. We can optimise each stage independently. However, as we also model deferrable load (DL) and thermostatically controlled load (TCL), we describe the problem as a dynamic program for consistency.

Conventional Units

Sets

$$i \in \mathcal{I} := \text{Set of conventional units}$$

$$t \in \mathcal{T} := \text{Set of stages in the optimisation horizon (Indexed from 1 to T)}$$

$$\omega_t \in \Omega_t := \text{Set of discrete and finite realisations that can occur at stage } t$$

Parameter definition

$$\mathrm{C}_{i,t}^{\mathrm{Con,up}}/\mathrm{C}_{i,t}^{\mathrm{Con,dn}} := \text{Real-time conventional unit up/down-regulation cost}$$

$$\mathrm{C}^{\mathrm{Sl,up}}/\mathrm{C}^{\mathrm{Sl,dn}} := \text{Real-time load shedding cost unit up / generation spillage cost}$$

$$\bar{\mathrm{P}}_{i,t}^{\mathrm{Con,up}}/\bar{\mathrm{P}}_{i,t}^{\mathrm{Con,dn}} := \text{Up/down-regulation limit}$$

$$\mathrm{d}_t(\omega_t) := \text{Baseline demand response requirement}$$

$$\mathrm{w}_t(\omega_t) := \text{Wind error relative to baseline}$$

Variable definition

$$c_{i,t}^{\mathrm{up}} := \text{Conventional unit up-regulation}$$

$$c_{i,t}^{\mathrm{dn}} := \text{Conventional unit down-regulation}$$

$$s_t^{\mathrm{up}} := \text{Load shedding}$$

$$s_t^{\mathrm{dn}} := \text{Generation spillage}$$

Problem formulation

Describing the problem as a dynamic program, the first stage problem is

$$\min_{\Xi_1}\Big\{\sum_{i\in\mathcal{I}}\mathrm{C}^{\mathrm{Con,up}}_{i,1}c^{\mathrm{up}}_{i,1}+\sum_{i\in\mathcal{I}}\mathrm{C}^{\mathrm{Con,dn}}_{i,1}c^{\mathrm{dn}}_{i,1} \tag{43}$$

$$+\mathrm{C}^{\mathrm{Sl,up}}s^{\mathrm{up}}_{1}+\mathrm{C}^{\mathrm{Sl,dn}}s^{\mathrm{dn}}_{1} \tag{44}$$

$$+\mathbb{E}_{\mathbb{P}}(Q_2(\Xi_1,\omega_2))\Big\} \tag{45}$$

$$\text{s.t.}\qquad c^{\mathrm{up}}_{i,1}-c^{\mathrm{dn}}_{i,1}+s^{\mathrm{up}}_{i,1}-s^{\mathrm{dn}}_{i,1}=\mathrm{d}_1-\mathrm{w}_1(\omega_1) \tag{46}$$

$$0\le c^{\mathrm{up}}_{i,1}\le\bar{\mathrm{P}}^{\mathrm{Con,up}}_{i,1}\forall i\in\mathcal{I} \tag{47}$$

$$0\le c^{\mathrm{dn}}_{i,1}\le\bar{\mathrm{P}}^{\mathrm{Con,dn}}_{i,1}\forall i\in\mathcal{I} \tag{48}$$

$$0\le s^{\mathrm{up}}_{1} \tag{49}$$

$$0\le s^{\mathrm{dn}}_{1}. \tag{50}$$

With Ξ_t including the set of variables $c^{\mathrm{up}}_{i,t}$, $c^{\mathrm{up}}_{i,t}$, $s^{\mathrm{up}}_{i,t}$, $s^{\mathrm{up}}_{i,t}$ decided at stage t ($\forall t\in\mathcal{T}$).

In the first term of equation (43), we define the cost from conventional load aggregators, where the cost per kW is given by $\mathrm{C}^{\mathrm{Con,up}}_{i,t}$ (conventional load aggregator i, time 1) and the amount of conventional load reduction (in kW) is given by $c_{i,t}$. In the second term of (44), we similarly define the cost of down regulation from conventional load. The first and second terms in equation (44) define the cost from load shedding and generation curtailment respectively. Finally, in equation (45) we have the expected cost in the following stages, given our regulation decision in this stage.

Equation (46) enforces the constraint that some combination of conventional units, load curtailment, and generation curtailment provide the necessary balancing.

Equations (47) through (50) give the bounds on the conventional units and shedding decisions.

For $t=2\dots T$, the problem is

$$Q_t(\Xi_{t-1},\omega_t)=\min_{\Xi_t}\Big\{\sum_{i\in\mathcal{I}}\mathrm{C}^{\mathrm{Con,up}}_{i,t}c^{\mathrm{up}}_{i,t}+\sum_{i\in\mathcal{I}}\mathrm{C}^{\mathrm{Con,dn}}_{i,t}c^{\mathrm{dn}}_{i,t} \tag{51}$$

$$+\mathrm{C}^{\mathrm{Sl,up}}s^{\mathrm{up}}_{t}+\mathrm{C}^{\mathrm{Sl,dn}}s^{\mathrm{dn}}_{t} \tag{52}$$

$$+\mathbb{E}_{\mathbb{P}}(Q_{t+1}(\Xi_t,\omega_{t+1}))\Big\} \tag{53}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{I}} c_{i,t}^{\text{up}} - \sum_{i\in\mathcal{I}} c_{i,t}^{\text{dn}} + s_t^{\text{up}} - s_t^{\text{dn}} = \mathrm{d}_t(\omega_t) - \mathrm{w}_t(\omega_t) \tag{54}$$

$$0 \le c_{i,t}^{\text{up}} \le \bar{\mathrm{P}}_{i,t}^{\text{Con,up}} \qquad \forall i \in \mathcal{I} \tag{55}$$

$$0 \le c_{i,t}^{\text{dn}} \le \bar{\mathrm{P}}_{i,t}^{\text{Con,dn}} \qquad \forall i \in \mathcal{I} \tag{56}$$

$$0 \le s_t^{\text{up}} \tag{57}$$

$$0 \le s_t^{\text{dn}} \tag{58}$$

For simplicity, we assume that $Q_{T+1}(\Xi_T, \omega_{T+1}) = 0$, though this can be converted to be some known polyhedral function of our regulation decisions Ξ_T.

Deferrable Load Units

These units have a minimum energy requirement that must be satisfied by some expiration time (or else there is a penalty cost). These units must also satisfy ramping and consumption limit constraints.

Sets

$j \in \mathcal{J}$:= Set of deferrable load units

$t \in \mathcal{T}$:= Set of stages in the optimisation horizon (Indexed from 1 to T)

$\omega_t \in \Omega_t$:= Set of discrete and finite realisations that can occur at stage t

Parameter definition

$\mathrm{C}_{j,t}^{\text{DL,dn}}/\mathrm{C}_{j,t}^{\text{DL,ct}}$:= Real-time deferrable load unit consumption/curtailment cost

$\mathrm{C}^{\text{Sl,up}}/\mathrm{C}^{\text{Sl,dn}}$:= Real-time load shedding cost unit up / generation spillage cost

$\mathrm{R}_{j,t}^{\text{up}}$:= Deferrable load ramping up limit

$\mathrm{R}_{j,t}^{\text{dn}}$:= Deferrable load ramping down limit

$\bar{\mathrm{P}}_{j,t}^{\text{DL,dn}}$:= Deferrable load consumption limit

$\mathrm{E}_{j,t}^{\text{Req}}$:= Minimum energy requirement at each stage

$\mathrm{E}_{j}^{\text{Cap}}$:= Unit capacity

$\mathrm{d}_t(\omega_t)$:= Baseline demand response requirement

$\mathrm{w}_t(\omega_t)$:= Wind error relative to baseline

Variable definition

$d_{j,t}^{\mathrm{dn}}$:= Deferrable load consumption

$d_{j,t}^{\mathrm{ct}}$:= Deferrable load curtailment

$e_{j,t}$:= Energy state of each unit

s_t^{up} := Load shedding

s_t^{up} := Generation spillage

Problem formulation

Describing the problem as a dynamic program, the first stage problem is

$$\min_{\Xi_1}\Big\{\sum_{j\in\mathcal{J}} \mathrm{C}^{\mathrm{DL,dn}}_{j,1} d^{\mathrm{dn}}_{j,1} + \sum_{j\in\mathcal{J}} \mathrm{C}^{\mathrm{DL,ct}}_{j,1} d^{\mathrm{ct}}_{j,1} \tag{59}$$

$$+ \mathrm{C}^{\mathrm{Sl,up}} s^{\mathrm{up}}_1 + \mathrm{C}^{\mathrm{Sl,dn}} s^{\mathrm{dn}}_1 \tag{60}$$

$$+ \mathbb{E}_{\mathbb{P}}(Q_2(\Xi_1, \omega_2))\Big\} \tag{61}$$

$$\text{s.t.} \qquad -\sum_{j\in\mathcal{J}} d^{\mathrm{dn}}_{j,1} + s^{\mathrm{up}}_1 - s^{\mathrm{dn}}_1 = \mathrm{d}_1 - \mathrm{w}_1(\omega_1) \tag{62}$$

$$e_{j,1} \geq \mathrm{E}^{\mathrm{Req}}_{j,1} \qquad \forall j \in \mathcal{J} \tag{63}$$

$$d^{\mathrm{dn}}_{j,1} \leq d^{\mathrm{dn}}_{j,0} + \mathrm{R}^{\mathrm{dn}}_{j,1} \qquad \forall j \in \mathcal{J} \tag{64}$$

$$d^{\mathrm{dn}}_{j,1} \geq d^{\mathrm{dn}}_{j,0} - \mathrm{R}^{\mathrm{up}}_{j,1} \qquad \forall j \in \mathcal{J} \tag{65}$$

$$e_{j,1} = e_{j,0} + d^{\mathrm{up}}_{j,1} + d^{\mathrm{ct}}_{j,1} \qquad \forall j \in \mathcal{J} \tag{66}$$

$$0 \leq d^{\mathrm{dn}}_{j,1} \leq \bar{\mathrm{P}}^{\mathrm{DL,dn}}_{j,1} \qquad \forall j \in \mathcal{J} \tag{67}$$

$$0 \leq d^{\mathrm{ct}}_{j,1} \leq \bar{\mathrm{P}}^{\mathrm{DL,ct}}_{j,1} \qquad \forall j \in \mathcal{J} \tag{68}$$

$$0 \leq e_{j,1} \leq \mathrm{E}^{\mathrm{Cap}}_{j} \qquad \forall j \in \mathcal{J} \tag{69}$$

$$0 \leq s^{\mathrm{up}}_1 \tag{70}$$

$$0 \leq s^{\mathrm{dn}}_1 \tag{71}$$

With Ξ_t including the set of variables $d^{\mathrm{dn}}_{j,t}$, $d^{\mathrm{ct}}_{j,t}$, $e_{j,t}$, s^{up}_t, s^{dn}_t, decided at stage t ($\forall t \in \mathcal{T}$). We may also want to introduce the ability for deferrable load to provide upregulation (introducing the variable $d^{\mathrm{up}}_{j,t}$), however, we would also need to model the resulting power losses.

In the first term of equation (59), we define the cost of satisfying the deferrable load, where the cost per kW is given by $\mathrm{C}^{\mathrm{DL,dn}}_{j,t}$ (deferrable load unit j, time 1) and the amount of load consumption (in kW) is given by $d_{j,t}$. In the second term of (59), we similarly define the cost of curtailing deferrable load (assuming there is some value to knowing that the energy requirement will not be satisfied some in advanced). The first and second terms in equation (60) define the cost from load shedding and generation curtailment respectively. Finally, in equation (61) we have the expected cost in the following stages, given our regulation decision in this stage.

Equation (62) enforces the constraint that some combination satisfying of deferrable load, load curtailment, and generation curtailment provide the necessary balancing. Equation

(63), ensures that we satisfy the energy level requirement for each unit. Equations (64) and (65) ensure that we satisfy the ramp-up and ramp-down constraints on each of the DL units. Constraint (66) defines the energy state of each unit.

Equations (67) through (71) give the bounds on the DL units and shedding decisions.

For $t = 2 \dots T$, the problem is

$$Q_t(\Xi_{t-1}, \omega_t) = \min_{\Xi_t} \Big\{ \sum_{j \in \mathcal{J}} \mathrm{C}^{\mathrm{DL,dn}}_{j,t} d^{\mathrm{dn}}_{j,t} + \sum_{j \in \mathcal{J}} \mathrm{C}^{\mathrm{DL,ct}}_{j,t} d^{\mathrm{ct}}_{j,t} \tag{72}$$

$$+ \mathrm{C}^{\mathrm{Sl,up}} s^{\mathrm{up}}_t + \mathrm{C}^{\mathrm{Sl,dn}} s^{\mathrm{dn}}_t \tag{73}$$

$$+ \mathbb{E}_{\mathbb{P}}(Q_{t+1}(\Xi_t, \omega_{t+1})) \Big\} \tag{74}$$

$$\text{s.t.} \quad - \sum_{j \in \mathcal{J}} d^{\mathrm{dn}}_{j,t} + s^{\mathrm{up}}_t - s^{\mathrm{dn}}_t = \mathrm{d}_t(\omega_t) - \mathrm{w}_t(\omega_t) \tag{75}$$

$$e_{j,t} \geq \mathrm{E}^{\mathrm{Req}}_{j,t} \qquad \forall j \in \mathcal{J} \tag{76}$$

$$d^{\mathrm{dn}}_{j,t} \leq d^{\mathrm{dn}}_{j,t-1} + \mathrm{R}^{\mathrm{up}}_{j,t} \qquad \forall j \in \mathcal{J} \tag{77}$$

$$d^{\mathrm{dn}}_{j,t} \geq d^{\mathrm{dn}}_{j,t-1} - \mathrm{R}^{\mathrm{dn}}_{j,t} \qquad \forall j \in \mathcal{J} \tag{78}$$

$$e_{j,t} = e_{j,t-1} + d^{\mathrm{up}}_{j,t} + d^{\mathrm{ct}}_{j,t} \qquad \forall j \in \mathcal{J} \tag{79}$$

$$0 \leq d^{\mathrm{dn}}_{j,t} \leq \bar{\mathrm{P}}^{\mathrm{DL,dn}}_{j,t} \qquad \forall j \in \mathcal{J} \tag{80}$$

$$0 \leq d^{\mathrm{ct}}_{j,t} \leq \bar{\mathrm{P}}^{\mathrm{DL,ct}}_{j,t} \qquad \forall j \in \mathcal{J} \tag{81}$$

$$0 \leq e_{j,1} \leq \mathrm{E}^{\mathrm{Cap}}_{j} \qquad \forall j \in \mathcal{J} \tag{82}$$

$$0 \leq s^{\mathrm{up}}_t \tag{83}$$

$$0 \leq s^{\mathrm{dn}}_t \tag{84}$$

For simplicity, we assume that $Q_{T+1}(\Xi_T, \omega_{T+1}) = 0$, though this can be converted to be some known polyhedral function of our regulation decisions Ξ_T.

Thermostatically Controlled Units

These units are used to keep an ideal set point temperature at each time step, or else there is an associated (assumed to be convex) cost.

Sets

$k \in \mathcal{K}$:= Set of conventional units

$t \in \mathcal{T}$:= Set of stages in the optimisation horizon (Indexed from 1 to T)

$\omega_t \in \Omega_t$:= Set of discrete and finite realisations that can occur at stage t

Parameter definition

$\mathrm{C}_{k,t}^{\mathrm{TCL,H}}/\mathrm{C}_{k,t}^{\mathrm{TCL,C}}$:= Real-time TCL unit heating/cooling cost

$\mathrm{C}^{\mathrm{Sl,up}}/\mathrm{C}^{\mathrm{Sl,dn}}$:= Real-time load shedding cost unit up / generation spillage cost

$\bar{\mathrm{P}}_{k,t}^{\mathrm{TCL,H}}/\bar{\mathrm{P}}_{k,t}^{\mathrm{TCL,C}}$:= Up/down-regulation limit

$\mathrm{T}_{k,t}^{\mathrm{I}}$:= Ideal operating temperature

$\mathrm{T}_{k,t}^{\mathrm{A}}$:= Ambient temperature

a_k := Relative weight of ambient temperature on room temperature (compared to previous temperature)

h_k := Temperature increase per kW of generation

c_k := Temperature decrease per kW of generation

$\mathbf{C}_{k,t}^{\mathbf{TCL,Temp}}(*)$:= Convex cost function for deviating from ideal operating temperature

$\mathrm{d}_t(\omega_t)$:= Baseline demand response requirement

$\mathrm{w}_t(\omega_t)$:= Wind error relative to baseline

Variable definition

$t_{k,t}^{\mathrm{H}}$:= Heating down-regulation

$t_{k,t}^{\mathrm{C}}$:= Cooling down-regulation

$T_{k,t}$:= Temperature of unit

s_t^{up} := Load shedding

s_t^{up} := Generation spillage

Problem formulation

Describing the problem as a dynamic program, the first stage problem is

$$\min_{\Xi_1} \Big\{ \sum_{k\in\mathcal{K}} \mathrm{C}^{\mathrm{TCL,H}}_{k,1} t^{\mathrm{H}}_{k,1} + \sum_{k\in\mathcal{K}} \mathrm{C}^{\mathrm{TCL,C}}_{k,1} t^{\mathrm{C}}_{k,1} + \sum_{k\in\mathcal{K}} \mathbf{C}^{\mathbf{TCL,Temp}}_{k,t} (T_{k,t} - \mathrm{T}^{\mathrm{I}}_{k,t}) \tag{85}$$

$$+ \mathrm{C}^{\mathrm{Sl,up}} s^{\mathrm{up}}_1 + \mathrm{C}^{\mathrm{Sl,dn}} s^{\mathrm{dn}}_1 \tag{86}$$

$$+ \mathbb{E}_{\mathbb{P}}(Q_2(\Xi_1, \omega_2)) \Big\} \tag{87}$$

$$\text{s.t.} \qquad - \sum_{k\in\mathcal{K}} t^{\mathrm{H}}_{k,1} - \sum_{k\in\mathcal{K}} t^{\mathrm{C}}_{k,1} + s^{\mathrm{up}}_1 - s^{\mathrm{dn}}_1 = \mathrm{d}_1 - \mathrm{w}_1(\omega_1) \tag{88}$$

$$T_{k,1} = (1 - \mathrm{a}_k) T_{k,0} + \mathrm{a}_k \cdot \mathrm{T}^{\mathrm{A}}_{k,1} + \mathrm{h}_k \cdot t^{\mathrm{H}}_{k,1} - \mathrm{c}_k \cdot t^{\mathrm{C}}_{k,1} \qquad \forall k \in \mathcal{K} \tag{89}$$

$$0 \le t^{\mathrm{H}}_{k,1} \le \bar{\mathrm{P}}^{\mathrm{TCL,H}}_{k,1} \qquad \forall k \in \mathcal{K} \tag{90}$$

$$0 \le t^{\mathrm{C}}_{k,1} \le \bar{\mathrm{P}}^{\mathrm{TCL,C}}_{k,1} \qquad \forall k \in \mathcal{K} \tag{91}$$

$$0 \le s^{\mathrm{up}}_1 \tag{92}$$

$$0 \le s^{\mathrm{dn}}_1 \tag{93}$$

With Ξ_t including the set of variables $t^{\mathrm{H}}_{k,t}$, $t^{\mathrm{C}}_{k,t}$, $T_{k,t}$, s^{up}_t, s^{dn}_t, decided at stage t ($\forall t \in \mathcal{T}$). For simplicity, we assume the baseline operation of each TCL unit is 0.

In the first term of equation (85), we define the cost from heating, where the cost per kW is given by $\mathrm{C}^{\mathrm{TCL,up}}_{k,t}$ (TCL unit k, time 1) and the amount of heating (in kW) is given by $t^H k, t$. In the second term of (85), we similarly define the cost from cooling. In the third term, we have the cost from deviating from the ideal temperature. The first and second terms in equation (86) define the cost from load shedding and generation curtailment respectively. Finally, in equation (87) we have the expected cost in the following stages, given our regulation decision in this stage.

Equation (88) enforces the constraint that some combination operating TCL units, load curtailment, and generation curtailment provide the necessary balancing. Constraint (89) defines the temperature state of each unit.

Equations (90) through (93) give the bounds on the TCL units and shedding decisions.

For $t = 2 \dots T$, the problem is

$$Q_t(\Xi_{t-1}, \omega_t) = \min_{\Xi_t} \Big\{ \sum_{k\in\mathcal{K}} \mathrm{C}^{\mathrm{TCL,H}}_{k,t} t^{\mathrm{H}}_{k,t} + \sum_{k\in\mathcal{K}} \mathrm{C}^{\mathrm{TCL,C}}_{k,t} t^{\mathrm{C}}_{k,t} + \sum_{k\in\mathcal{K}} \mathbf{C}^{\mathbf{TCL,Temp}}_{k,t} (T_{k,t} - \mathrm{T}^{\mathrm{I}}_{k,t}) \tag{94}$$

$$+ \mathrm{C}^{\mathrm{Sl,up}} s_t^{\mathrm{up}} + \mathrm{C}^{\mathrm{Sl,dn}} s_t^{\mathrm{dn}} \tag{95}$$

$$+ \mathbb{E}_{\mathbb{P}}(Q_{t+1}(\Xi_t, \omega_{t+1}))\Big\} \tag{96}$$

$$\text{s.t.} \quad -\sum_{k\in\mathcal{K}} t_{k,t}^{\mathrm{H}} - \sum_{k\in\mathcal{K}} t_{k,t}^{\mathrm{C}} + s_t^{\mathrm{up}} - s_t^{\mathrm{dn}} = \mathrm{d}_t(\omega_t) - \mathrm{w}_t(\omega_t) \tag{97}$$

$$T_{k,t} = (1-\mathrm{a}_k)T_{k,t-1} + \mathrm{a}_k \cdot \mathrm{T}_{k,t}^{\mathrm{A}} + \mathrm{h}_k \cdot t_{k,t}^{\mathrm{H}} - \mathrm{c}_k \cdot t_{k,t}^{\mathrm{C}} \qquad \forall k \in \mathcal{K} \tag{98}$$

$$0 \le t_{k,t}^{\mathrm{H}} \le \bar{\mathrm{P}}_{k,t}^{\mathrm{TCL,H}} \qquad \forall k \in \mathcal{K} \tag{99}$$

$$0 \le t_{k,t}^{\mathrm{C}} \le \bar{\mathrm{P}}_{k,t}^{\mathrm{TCL,C}} \qquad \forall k \in \mathcal{K} \tag{100}$$

$$0 \le s_t^{\mathrm{up}} \tag{101}$$

$$0 \le s_t^{\mathrm{dn}} \tag{102}$$

Complete Model

Now defining the complete model for $t = 1, \ldots, T$

$$Q_t(\Xi_{t-1}, \omega_t) = \min_{\Xi_t} \Big\{ \sum_{i\in\mathcal{I}} \mathrm{C}_{i,t}^{\mathrm{Con,up}} c_{i,t}^{\mathrm{up}} + \sum_{i\in\mathcal{I}} \mathrm{C}_{i,t}^{\mathrm{Con,dn}} c_{i,t}^{\mathrm{dn}} \tag{103}$$

$$+ \sum_{j\in\mathcal{J}} \mathrm{C}_{j,t}^{\mathrm{DL,dn}} d_{j,t}^{\mathrm{dn}} + \sum_{j\in\mathcal{J}} \mathrm{C}_{j,t}^{\mathrm{DL,ct}} d_{j,t}^{\mathrm{ct}} \tag{104}$$

$$+ \sum_{k\in\mathcal{K}} \mathrm{C}_{k,t}^{\mathrm{TCL,H}} t_{k,t}^{\mathrm{H}} + \sum_{k\in\mathcal{K}} \mathrm{C}_{k,t}^{\mathrm{TCL,C}} t_{k,t}^{\mathrm{C}} + \sum_{k\in\mathcal{K}} \mathbf{C}_{k,t}^{\mathbf{TCL,Temp}} (T_{k,t} - \mathrm{T}_{k,t}^{\mathrm{I}}) \tag{105}$$

$$+ \mathrm{C}^{\mathrm{Sl,up}} s_t^{\mathrm{up}} + \mathrm{C}^{\mathrm{Sl,dn}} s_t^{\mathrm{dn}} \tag{106}$$

$$+ \mathbb{E}_{\mathbb{P}}(Q_{t+1}(\Xi_t, \omega_{t+1}))\Big\} \tag{107}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{I}} c_{i,t}^{\mathrm{up}} - \sum_{i\in\mathcal{I}} c_{i,t}^{\mathrm{dn}} - \sum_{j\in\mathcal{J}} d_{j,t}^{\mathrm{dn}} \tag{108}$$

$$-\sum_{k\in\mathcal{K}} t_{j,t}^{\mathrm{H}} - \sum_{k\in\mathcal{K}} t_{j,t}^{\mathrm{C}} + s_t^{\mathrm{up}} - s_t^{\mathrm{dn}} = \mathrm{d}_t(\omega_t) - \mathrm{w}_t(\omega_t) \tag{109}$$

$$e_{j,t} \ge \mathrm{E}_{j,t}^{\mathrm{Req}} \qquad \forall j \in \mathcal{J} \tag{110}$$

$$d_{j,t}^{\mathrm{dn}} \le d_{j,t-1}^{\mathrm{dn}} + \mathrm{R}_{j,t}^{\mathrm{up}} \qquad \forall j \in \mathcal{J} \tag{111}$$

$$d_{j,t}^{\mathrm{dn}} \ge d_{j,t-1}^{\mathrm{dn}} - \mathrm{R}_{j,t}^{\mathrm{dn}} \qquad \forall j \in \mathcal{J} \tag{112}$$

$$e_{j,t} = e_{j,t-1} + d_{j,t}^{\mathrm{dn}} + d_{j,t}^{\mathrm{ct}} \qquad \forall j \in \mathcal{J} \tag{113}$$

$$T_{k,t} = (1-\mathrm{a}_k)T_{k,t-1} + \mathrm{a}_k \cdot \mathrm{T}_{k,t}^{\mathrm{A}} + \mathrm{h}_k \cdot t_{j,t}^{\mathrm{H}} - \mathrm{c}_k \cdot t_{k,t}^{\mathrm{C}} \qquad \forall k \in \mathcal{K} \tag{114}$$

$$0 \leq c_{i,t}^{\mathrm{up}} \leq \bar{\mathrm{P}}_{i,t}^{\mathrm{Con,up}} \qquad \forall i \in \mathcal{I} \tag{115}$$

$$0 \leq c_{i,t}^{\mathrm{dn}} \leq \bar{\mathrm{P}}_{i,t}^{\mathrm{Con,dn}} \qquad \forall i \in \mathcal{I} \tag{116}$$

$$0 \leq d_{j,t}^{\mathrm{dn}} \leq \bar{\mathrm{P}}_{j,t}^{\mathrm{DL,dn}} \qquad \forall j \in \mathcal{J} \tag{117}$$

$$0 \leq d_{j,t}^{\mathrm{ct}} \leq \bar{\mathrm{P}}_{j,t}^{\mathrm{DL,ct}} \qquad \forall j \in \mathcal{J} \tag{118}$$

$$0 \leq e_{j,t} \leq \mathrm{E}_{j}^{\mathrm{Cap}} \qquad \forall j \in \mathcal{J} \tag{119}$$

$$0 \leq t_{k,t}^{\mathrm{H}} \leq \bar{\mathrm{P}}_{k,t}^{\mathrm{TCL,H}} \qquad \forall k \in \mathcal{K} \tag{120}$$

$$0 \leq t_{k,t}^{\mathrm{C}} \leq \bar{\mathrm{P}}_{k,t}^{\mathrm{TCL,C}} \qquad \forall k \in \mathcal{K} \tag{121}$$

$$0 \leq s_{t}^{\mathrm{up}} \tag{122}$$

$$0 \leq s_{t}^{\mathrm{dn}} \tag{123}$$

With Ξ_t including the set of variables $c_{i,t}^{\mathrm{up}}$, $c_{i,t}^{\mathrm{up}}$, $d_{j,t}^{\mathrm{dn}}$, $d_{j,t}^{\mathrm{ct}}$, $e_{j,t}$, $t_{k,t}^{\mathrm{H}}$, $t_{k,t}^{\mathrm{C}}$, $T_{k,t}$, s_t^{up}, s_t^{dn}, decided at stage t ($\forall t \in \mathcal{T}$).