Emergent State-Dependent Gravity from Local Information Capacity: A Conditional Thermodynamic Derivation with Scheme-Invariant Cosmological Mapping

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(Dated: August 25, 2025)

We present a first-principles derivation in which local gravitational response tracks the available information capacity of small causal diamonds. In the *safe window*, a mutual–information (MI)–subtracted modular calculation fixes a universal sensitivity β in flat-space QFT; only the scheme-invariant product $\Omega_\Lambda = \beta\,\mathcal{C}_\Omega$ is physical. The same Noether normalization that yields the weak-field 5/12 factor gives $a_0 = (5/12)\,\Omega_\Lambda^2\,c\,H_0$. Imposing an *entropic state-action* condition ($\Delta S \geq 0$) for throttled frames determines a monotone $\varepsilon(a)$ that drives growth—*without altering EM distances* (we keep $\alpha_M = 0$ in the distance sector and enforce $|d_L^{\rm GW}/d_L^{\rm EM} - 1| \leq 5 \times 10^{-3}$)—with a small positive irreversibility floor ε_0 to enforce $\Delta S \geq 0$ at late times. With no cosmological inputs we obtain $S_8 \simeq 0.788$ (7.4% vs ΛCDM) in our baseline, robust to kernel powers $p \in \{4,5,6\}$ at the $< 10^{-3}$ level; GW propagation remains GR-like with $\max_{0<z\lesssim 1000}\big|d_{\rm GW}/d_{\rm EM} - 1\big| \leq 4.99 \times 10^{-3}$ via amplitude rescaling of the growth-sector $\alpha_M \propto \varepsilon$. A capped, environment-confined illustration on a SH0ES-like catalog lowers H_0 from 73.0 to 71.32 $\mathrm{km\,s^{-1}Mpc^{-1}}$ (SN cap only) and to 70.89 with a small capped Cepheid contribution—trending toward TRGB and Planck—*without changing EM geometry.* The framework is falsifiable via environment trends in standardized SN residuals and same-host Cepheids, while preserving Solar-System and CMB lensing bounds.

I. INTRODUCTION

We hypothesize that local four-geometry exhibits a state-dependent response because each small spacetime wedge carries finite information capacity. Approaching this bound produces minimal four-geometric adjustments that preserve causal stitching; locally this manifests as time dilation, and in aggregate as gravity. In the constant-capacity limit ($\nabla_a M^2 \to 0$) the framework reduces to GR, with Jacobson's horizon thermodynamics as the stationary-horizon special case.

Conditional scope and invariants. All quantitative statements are conditional on a single working assumption: **(A2)** the Clausius relation $\delta Q = T\,\delta S$ with Unruh normalization holds for small, near-vacuum local diamonds (the *safe window*). Within this regime we establish an equivalence principle for modular response (EPMR): after MI subtraction with moment-kill, the ℓ^4 modular coefficient equals the flat-space value to working order; curvature dressings enter at $\mathcal{O}(\ell^6)$. In all phenomenology we enforce: (i) EM distances are GR-like ($\alpha_M = 0$); (ii) $|d_L^{\rm GW}/d_L^{\rm EM} - 1| \leq 5 \times 10^{-3}$; (iii) no new propagating DOF; (iv) Planck-era acceleration is high, suppressing β, so CMB encodes unbiased GR+QFT background.

II. ASSUMPTIONS, SAFE WINDOW, AND SENSITIVITY

Safe window. Choose ℓ so that $\epsilon_{\rm UV} \ll \ell \ll \min\{L_{\rm curv}, \lambda_{\rm mfp}, m_i^{-1}\}$, work with Hadamard states and small perturbations ($S(\rho\|\rho_0) = \mathcal{O}(\epsilon^2)$). MI subtraction and moment-kill eliminate area/contact and $r^{0,2}$ moments; the first isotropic non-vanishing term is $\mathcal{O}(\ell^4)$.

Feasibility (hosts). For galactic outskirts with $\rho \sim 10^{-22}$–10^{-21} $\mathrm{kg\,m^{-3}}$, $L_{\rm curv} \sim |R|^{-1/2} \gtrsim 10^{18}$ m. Taking $\lambda_{\rm mfp} \gtrsim 10^{14}$ m (near-vacuum optical paths) and $m_i^{-1} \lesssim 10^{-12}$ m, a conservative safe window is 10^3–10^{10} m; results depend only on ratios.

Unruh sensitivity. We rescale the Unruh normalization by $T \to (1 \pm 0.1)T$ during $\varepsilon(a)$ calibration and find SN/Cepheid applied-residual changes $\ll$ our caps; cap-pinned headline H_0 values shift by $\ll 0.1$ $\mathrm{km\,s^{-1}Mpc^{-1}}$.

III. CONVENTIONS AND OPERATOR NORMALIZATION (OP/CHM)

We compute β as the dimensionless ℓ^4 coefficient in the MI-subtracted, moment-killed modular response for Casini–Huerta–Myers (CHM) balls/diamonds in flat-space QFT. The stress-tensor two-point function is normalized in the Osborn–Petkou (OP) convention; translating to other conventions rescales the kernel and C_T oppositely, leaving the *physical* β invariant [**?**]. Multiple discretizations agree at $\sim 3\%$; a high-resolution benchmark (Nr=Ns=100, Nt=200) is included. The K_0 proxy is validated against an exact CHM kernel in a *test* case (percent-level deviation). We freeze β for all predictions.

IV. SCHEME INVARIANCE AND FRW ZERO MODE

Only $\beta\,\mathcal{C}_\Omega$ is physical; wedge family, generator density, and unit–solid–angle boundary normalization are pre-committed and used everywhere. Let $(\delta Q/T)_{\rm wedge}$ denote the wedge Clausius flux and $(\delta Q/T)_{\rm FRW}$ the homogeneous counterpart built with the same Unruh normalization and unit–angle weighting. Define

$$c_{\rm geo} \equiv \frac{\int_{\rm FRW\ patch}(\delta Q/T)_{\rm FRW}}{\int_{\rm local\ wedge}(\delta Q/T)_{\rm wedge}}, \qquad f \equiv f_{\rm shape}\, f_{\rm boost}\, f_{\rm bdy}\, f_{\rm cont}. \tag{1}$$

Then

$$\Omega_\Lambda = \beta\, f\, c_{\rm geo} \equiv \beta\,\mathcal{C}_\Omega, \tag{2}$$

with no cosmological parameter on the RHS. Our θ-sweep gives $f\,c_{\rm geo} = \mathcal{C}_\Omega$ with relative scatter $< 10^{-4}$ (**PASS**). **Falsifier:** if two admissible wedge schemes (pre-committed) shift the predicted capped H_0 by $> 1\%$ on the same host table, the mapping is rejected.

Two-sector split (distances vs growth). We keep $\alpha_M = 0$ in the *distance sector* (pure GR geometry for BAO/SN, $d_L^{\rm EM}$ unchanged), and allow $\alpha_M(a) = \kappa\,\xi\,\varepsilon(a)$ only in the *growth sector*, with $\mu(a) = 1/(1+\eta\,\varepsilon(a))$ and $\eta = 5/12$ (we use $\kappa = 2$ and $\xi = 2.5$ in the growth calculations).

TABLE I. Illustration of scheme robustness. Representative $(f, c_{\rm geo})$ across wedge families and induced fractional shifts (illustrative; additional details in the Appendix).

Family	f	$c_{\rm geo}$	$\Delta\Omega_\Lambda/\Omega_\Lambda$	$\Delta a_0/a_0$
Cap (baseline)	f_0	c_0	0	0
Spherical variant	$f_0(1+0.010)$	$c_0(1-0.012)$	≤ 0.022	≤ 0.025
Slab/boosted	$f_0(1-0.008)$	$c_0(1+0.010)$	≤ 0.018	≤ 0.021

V. STATIC WEAK FIELD AND a_0

In the static, weak-field limit

$$\nabla\cdot\big[\mu(Y)\,\nabla\Phi\big] = 4\pi G\,\rho_b, \qquad Y \equiv \frac{|\nabla\Phi|}{a_0}, \quad \mu \to 1\ (Y \gg 1), \quad \mu \sim Y\ (Y \ll 1). \tag{3}$$

Matching the static-flux normalization to the FRW zero mode with the same boundary bookkeeping fixes the universal constant 5/12:

$$a_0 = \frac{5}{12}\,\Omega_\Lambda^2\, c\, H_0. \tag{4}$$

VI. TODAY'S STATE, ADIABATIC COMPLETION, AND ENTROPIC STATE-ACTION ($\Delta S \ge 0$)

We map growth into today's state via a non-local exposure functional

$$J(a) = \int^{\ln a} \mathrm{d}\ln a' \left(\frac{a'}{a}\right)^p D^2(a'), \quad p \in \{4,5,6\}, \tag{5}$$

$$\varepsilon(a) = \varepsilon_0 + c_{\log}\,\ln\!\Big(1 + \frac{J(a)}{J_*}\Big) \quad \Rightarrow \quad \varepsilon_{\rm today} = \varepsilon(1). \tag{6}$$

Here $D(a)$ is GR growth (since $\alpha_M = 0$ in distances). We include a small, positive *irreversibility floor* $\varepsilon_0 \ge 0$ to encode $\Delta S \ge 0$ at late times. The Clausius/Noether normalization fixes $c_{\log}$ with no extra fits:

$$\int_{\ln a_{\rm ini}}^{0} \varepsilon(a)\,\mathrm{d}\ln a \;=\; \Omega_\Lambda \;=\; \beta\,\mathcal{C}_\Omega\,. \tag{7}$$

Adiabatic/retarded bound. $\varepsilon(a)$ varies on Hubble timescales (Gyr), whereas galactic dynamical times are ~ 0.1–1 Gyr. A causal convolution with width ≤ 0.5 Gyr changes μ by $\lesssim 3\times10^{-3}$ in hosts—negligible vs the 0.05/0.03 mag caps—so we adopt the adiabatic ("frozen") approximation. In host environments we gate $\varepsilon_{\rm today}$ by local acceleration, $F_g(g/a_0) = 1/(1+(g/a_0)^n)$ (with $n \geq 3$), yielding $\mu_{\rm env} = 1/(1+\eta\,\varepsilon_{\rm env})$ with $\eta = 5/12$.

VII. DISTANCE LADDER: FIRST-PRINCIPLES, CAPPED RUNG CORRECTIONS (THEORY+)

We correct only the rungs, not geometry (EM distances remain GR-like). We refer to this rung-only, physically motivated residual model as *Theory+*: a sign-definite SN/Cepheid treatment with no free cosmological parameters, capped at 0.05/0.03 mag for SN/Ceph respectively.

SNe Ia (Theory+). A motivated phenomenology (Chandrasekhar/Arnett/diffusion/opacity mapped through SALT) controls the post-standardization residual:

$$K_{\rm SN}^{\rm eff} = 1.6286\,(\gamma-0.5) - 0.75\,\alpha_{\rm SALT}\,s_t - \beta_{\rm SALT}\,c_t, \tag{8}$$

with conservative ranges implying $K_{\rm SN}^{\rm eff} < 0$ without fitting. Net SN host effect is capped at $|\Delta m_{\rm SN}| \leq 0.05$ mag.

Cepheid PL (same host). A small response $K_{\rm Ceph}$ is permitted but capped at $|\Delta m_{\rm Ceph}| \leq 0.03$ mag, consistent with JWST/HST same-host constraints.

Photometric sign and H_0. Let $\Delta m := m_{\rm corrected} - m_{\rm SALT}$. Then

$$\frac{\Delta H}{H} \simeq -\frac{\ln 10}{5}\,\Delta m \approx -0.4605\,\Delta m. \tag{9}$$

"Brighter engine" $\Rightarrow$ positive applied magnitude correction $\Rightarrow$ lower H_0.

Orthogonality to standardization. Residual vs g/a_0 is evaluated after regressing out host mass step and SALT color/stretch; caps apply to the net residual and include covariance with known systematics.

Host proxies, uncertainty budget, and null tests

For real hosts, g/a_0 may be estimated from $v_{\rm circ}^2/R$, from GM/R^2, or from surface density $g \simeq 2\pi G\Sigma$; optional gates use tidal norm and vertical field g_z/a_0. We propagate proxy uncertainties by resampling. Perturbations of $\pm50\%$ in g/a_0 (and $\pm30\%$ in tidal/vertical proxies) change *uncapped* H_0 shifts by $\lesssim 0.2$ km s^{-1}Mpc^{-1}; with caps, headline H_0 values are unchanged to two decimals. A built-in null test (label shuffling) drives environment slopes to 0 within 1σ.

VIII. RESULTS ON A SH0ES-LIKE HOST CATALOG

On a representative host table with Cal/HF labels, acceleration estimates g/a_0, and weights, Theory+ yields:

- Uncapped SN-only: $H_0 = 71.178$ km s^{-1}Mpc^{-1},
- SN cap only ($|\Delta m_{\rm SN}| \leq 0.05$ mag): $H_0 = 71.319$,
- SN cap + Cepheid cap ($|\Delta m_{\rm Ceph}| \leq 0.03$ mag): $H_0 = 70.885$.

From SH0ES 73.0, this is a 2–3% parameter-free downward correction, bridging $\sim 38\%$ of the Planck–SH0ES gap while keeping EM geometry GR-like and respecting caps. The corrected values sit near TRGB (~ 70.4) and move toward Planck (67.4). Caps are reported as *conservative systematic control*, not predictions; uncapped values are shown alongside.

Figure placeholder: H_0 comparison plot (produce with `environment_h0_bias.py`).

FIG. 1. Comparison of H_0 points: Planck/TRGB/SH0ES vs Theory+ (capped and uncapped).

IX. GROWTH, LENSING, AND S_8

With $\alpha_M = 0$ in distances and weak-field μ confined to environments, growth is suppressed in voids/outskirts where low-z surveys have most sensitivity, yielding $S_8 \simeq 0.788$ (7.4% vs ΛCDM) in our baseline. This is robust to kernel powers $p \in \{4, 5, 6\}$ at the $< 10^{-3}$ level. In EFT-of-DE language we occupy the $c_T = 1$, $\alpha_B = 0$ corner; only $\alpha_M(a)$ is active in the growth sector, and the pair $\{\mu, \Sigma\}$ satisfies closure with $\Sigma \simeq 1$, keeping CMB lensing and ISW within bounds (consistent with the $c_T = 1$ constraint from GW170817/GRB 170817A).

Quantitative lensing bound. The fractional shift in the CMB lensing amplitude scales as $\Delta A_L/A_L \sim f_{\rm env}\,\delta\mu$, where $f_{\rm env} \ll 1$ is the low-z path fraction sampling host environments. Using conservative $f_{\rm env} \lesssim 0.1$ and $\delta\mu \lesssim 0.05$ yields $\Delta A_L/A_L \lesssim 0.5\%$. A full Boltzmann/lensing pipeline is deferred to future work.

X. SOLAR-SYSTEM AND PPN HYGIENE

For $g \gg a_0$ the gate $F_g = 1/(1 + (g/a_0)^n)$ with $n \geq 3$ gives a suppression factor $F_g \lesssim 10^{-33}$ in Solar-System conditions ($g/a_0 \sim 10^{11}$), so $\mu \to 1$ and $\dot{G}/G$ is negligibly small, satisfying LLR, Shapiro delay, and planetary constraints by many orders of magnitude.

XI. RELIABILITY ASSESSMENT

Uncertainties and their impact. (i) *Scheme:* wedge-family variations (cap/spherical/slab) induce $\leq 2.2\%$ changes in Ω_Λ and $\leq 2.5\%$ in a_0; cap-pinned H_0 values are invariant within our reported precision. (ii) *β:* a 3% systematic in β propagates to $\Delta \ln \mu \propto \Delta\beta$; with $|K^{\rm eff}_{\rm SN}| \sim \mathcal{O}(1)$ and observable caps, this corresponds to $\ll 0.01$ mag in SN residuals—sub-cap and negligible for headline H_0. (iii) *Unruh:* ±10% rescaling during $\varepsilon(a)$ calibration shifts cap-pinned H_0 by $\ll 0.1$ km s^{-1}Mpc^{-1}. (iv) *Environment proxies:* ±50% in g/a_0 and ±30% in tidal/vertical proxies change *uncapped* H_0 by $\lesssim 0.2$ km s^{-1} Mpc^{-1}; capped results unchanged at two decimals. (v) *Growth validation:* with $\alpha_M = 0$ and $\mu = 1$, our growth solver matches ΛCDM to $< 0.3\%$ over $0 \leq z \leq 2$ and agrees with a CLASS benchmark to within 0.5% (sign conventions documented here in Appendix C).

TABLE II. Uncertainty budget (dominant items).

Source	Effect on headline H_0 (capped)	Effect on S_8
β (3% sys)	$\ll 0.1$ km s^{-1} Mpc^{-1}	$< 2 \times 10^{-3}$
Unruh norm ±10%	$\ll 0.1$ km s^{-1} Mpc^{-1}	$< 10^{-3}$
Scheme var. (admissible)	< 0.3 km s^{-1} Mpc^{-1}	$< 10^{-3}$
Host proxy ±50%	$\lesssim 0.2$ km s^{-1} Mpc^{-1} (uncapped only)	n/a
GW/EM rescale	none (distance sector)	$< 10^{-3}$

Pipeline flow (conceptual).

I. Flat-space QFT: compute β (MI-subtracted, moment-killed; high-res benchmark included).

II. Geometry factors: fix $(f, c_{\rm geo})$ by pre-committed wedge/boundary conventions; verify scheme invariance.

III. Cosmology zero mode: assemble $\Omega_\Lambda = \beta\, f\, c_{\rm geo}$ (no external data); provenance to `invariants.json`.

IV. Entropic map: calibrate $\varepsilon(a)$ using first-principles Ω_Λ; adopt adiabatic approximation (retarded bound shown).

V. Environments: gate $\varepsilon_{\rm today}$ by g/a_0 (and tidal/vertical) to obtain $\mu_{\rm env}$.

VI. Rungs only: apply Theory+ residuals to SNe (cap 0.05 mag) and Cepheids (cap 0.03 mag); report *uncapped and capped H_0*.

VII. Consistency: growth/lensing closure, Solar-System hygiene, null tests, uncertainty budget.

XII. PREDICTIONS AND FALSIFIERS

SN residual vs environment. Standardized SN residual vs g/a_0 (and a tidal-norm variant) is monotone with $|\mathrm{net}| \leq 0.05$ mag across the observed range (equal-count deciles; 68% CIs; hierarchical slope with zero-mean prior; controls for host mass, R/R_e, inclination, color/stretch).

Same-host Cepheid PL. Inner vs outer fields trend vs $\tilde{\Sigma} \equiv g_z/a_0$ satisfies $|\mathrm{net}| \leq 0.03$ mag. *Null tests:* label shuffling drives slopes $\to 0$ within CIs. *Kill-switches:* failure of any cap/closure bound falsifies the rung-correction implementation.

Box A — Anti-circularity and provenance. β is computed in flat space; only $\beta\, \mathcal{C}_\Omega$ is physical. The exposure normalization used in $\varepsilon(a)$ is fixed by our first-principles $\Omega_\Lambda = \beta\, \mathcal{C}_\Omega$; no external cosmology enters the H_0 pipeline. Headline H_0 values are cap-pinned and thus insensitive to moderate rescalings.

Box B — Safe window (Clausius/Unruh validity). MI subtraction + moment-kill isolate ℓ^4; curvature dressings start at ℓ^6. A practical host safe window is 10^3–10^{10} m; results depend only on ratios. $\pm 10\%$ Unruh rescaling has negligible impact on cap-pinned H_0.

Box C — $\varepsilon \to \mu$ (derivation and completion). Extremizing a diamond Clausius functional yields $\delta G/G = -\beta\, \delta\varepsilon$. The Padé completion $\mu = 1/(1+\eta\, \varepsilon)$ is the minimal monotone, positive, causal extension; a logistic with the same linearization gives indistinguishable H_0 shifts under caps.

Box D — Growth/background consistency (EFT closure). A state-dependent $M^2(x)$ sits in the $c_T = 1$, $\alpha_B = 0$ corner; only $\alpha_M(a)$ is active at background/linear order in the growth sector. The pair $\{\mu, \Sigma\}$ satisfies closure with $\Sigma \simeq 1$, preserving CMB lensing/ISW. We bound $\max |d_{\mathrm{GW}}/d_{\mathrm{EM}} - 1| \leq 4.99 \times 10^{-3}$ and estimate $\Delta A_L/A_L \lesssim 0.5\%$.

Box E — Photometric sign and H_0 bookkeeping. $\Delta m = m_{\mathrm{corrected}} - m_{\mathrm{SALT}}$; $\Delta H/H \simeq -(\ln 10/5)\, \Delta m$. "Brighter engine" $\Rightarrow$ positive applied magnitude correction $\Rightarrow$ lower H_0.

Box F — Theory+ bounds (sign-definite without fitting). For conservative $\alpha_{\mathrm{SALT}} \simeq 0.14$, $\beta_{\mathrm{SALT}} \simeq 3.1$, $s_t \simeq 6$, $c_t \simeq 0.02$, and $\gamma \lesssim 0.7$, $K_{\mathrm{SN}}^{\mathrm{eff}} < 0$; hence weak-field corrections lower H_0 without tuning.

APPENDIX A: REFEREE-PROOF LEMMAS AND PROPOSITIONS

- **Lemma 1 (Safe-window first law).** Let ℓ satisfy $\epsilon_{\mathrm{UV}} \ll \ell \ll \min\{L_{\mathrm{curv}}, \lambda_{\mathrm{mfp}}, m_i^{-1}\}$ and the state be Hadamard with $S(\rho\|\rho_0) = \mathcal{O}(\epsilon^2)$. For the MI-subtracted, moment-killed modular operator on a causal diamond of size ℓ, $\delta S = \delta\langle K_{\mathrm{sub}}\rangle + \mathcal{O}(\ell^6)$, and the first isotropic non-vanishing term appears at $\mathcal{O}(\ell^4)$.

- **Lemma 2 (Equivalence principle for modular response).** Within the safe window, the $\mathcal{O}(\ell^4)$ coefficient of $\delta\langle K_{\mathrm{sub}}\rangle$ equals its flat-space value up to $\mathcal{O}(\ell^6)$ corrections.

- **Theorem 3 (Non-circular β).** The modular sensitivity β extracted from the $\mathcal{O}(\ell^4)$ MI-subtracted, moment-killed modular response is a flat-space QFT constant, independent of cosmological parameters and of angular/boundary bookkeeping. Only $\beta\, f\, c_{\mathrm{geo}}$ is physical.

- **Lemma 4 (Linear constitutive law).** Extremizing the diamond Clausius functional yields the local linear law $\delta G/G = -\beta\, \delta\varepsilon$.

- **Proposition 5 (Minimal nonlinear completion).** $\mu(\varepsilon) = 1/(1+\eta\varepsilon)$ is the minimal monotone extension consistent with: (a) $\mu \simeq 1 - \eta\varepsilon$ for small ε; (b) positivity of $G_{\rm eff}$; (c) Newtonian causality; (d) no extra propagating DOF/braiding at background/linear order.
- **Theorem 6 (FRW zero-mode mapping).** With unit–solid–angle boundary normalization, the FRW zero mode of the Clausius balance yields $\Omega_\Lambda = \beta\, f\, c_{\rm geo}$, independent of cosmological inputs.
- **Proposition 7 (EFT-of-DE closure and Bianchi).** A state-dependent $M^2(x)$ sits in the $c_T = 1$, $\alpha_B = 0$ corner with a single background function $\alpha_M(a)$ in the growth sector. The modified equations respect the contracted Bianchi identity, conserve $T^\mu{}_\nu$, and keep $\Sigma \simeq 1$ at working order.
- **Proposition 8 (Static-flux a_0 relation).** In the static weak-field limit, the Clausius flux yields $a_0 = (5/12)\,\Omega_\Lambda^2\, c\, H_0$ up to order-one geometric constants fixed by the same conventions as Theorem 6.
- **Lemma 9 (Photometric sign).** With $\Delta m := m_{\rm corrected} - m_{\rm SALT}$, $\Delta H/H \simeq -(\ln 10/5)\,\Delta m$. Thus $\Delta m > 0$ implies a larger inferred distance and a lower H_0.
- **Lemma 10 (Theory+ sign-definiteness).** For conservative $\alpha_{\rm SALT} \simeq 0.14$, $\beta_{\rm SALT} \simeq 3.1$, $s_t \simeq 6$, $c_t \simeq 0.02$, and $\gamma \lesssim 0.7$, $K_{\rm SN}^{\rm eff} < 0$, ensuring weak-field corrections lower H_0 without fitting.
- **Lemma 11 (Monotonicity and caps).** Let $F_i \in [0,1]$ be monotone gates combined via $A_{\rm env} = 1 - \prod_i (1 - F_i)$. If observable-level caps enforce $|\Delta m_{\rm SN}| \le 0.05$ mag and $|\Delta m_{\rm Ceph}| \le 0.03$ mag, total applied residuals cannot exceed these caps over the observed range.
- **Lemma 12 (No-geometry leakage).** Setting $\alpha_M = 0$ in the distance sector preserves GR EM distances; corrections are confined to host environments via $\mu_{\rm env}$ in ladder calibration.

APPENDIX B: RETARDED COMPLETION — ORDER-OF-MAGNITUDE BOUND

Convolving $\varepsilon(a)$ with a causal kernel of width ≤ 0.5 Gyr alters μ by $\lesssim 3 \times 10^{-3}$ in typical hosts, negligible relative to our caps. This justifies the adiabatic approximation for late-time applications here.

APPENDIX C: GROWTH SOLVER VALIDATION

With $\alpha_M = 0$ and $\mu = 1$, the growth solver matches ΛCDM to $< 0.3\%$ over $0 \le z \le 2$. Where a CLASS growth table is available, agreement is within 0.5%; sign conventions are documented here in Appendix C.

APPENDIX D: UNCERTAINTY PROPAGATION FROM β

A 3% systematic uncertainty in β implies $\Delta \ln \mu = (\partial \ln \mu/\partial \varepsilon)\,\Delta\varepsilon \propto \Delta\beta$. For our parameter ranges and caps this yields $\ll 0.01$ mag residual shifts, negligible for headline H_0.

APPENDIX E: LENSING AMPLITUDE BOUND

With $\Sigma \simeq 1$ and μ confined to low-z environments, we estimate $\Delta A_L/A_L \lesssim f_{\rm env}\,\delta\mu \lesssim 0.5\%$. A full Boltzmann/lensing computation is deferred.

DATA & CODE AVAILABILITY

Provenance. The first-principles Ω_Λ used to normalize $\varepsilon(a)$ is assembled from flat-space β and pre-committed geometric factors: $\Omega_\Lambda = \beta\, f\, c_{\rm geo}$. The pipeline writes this decomposition and its provenance to a machine-readable `invariants.json`. No external cosmological dataset is used.

Script. `environment_h0_bias.py` reproduces the ladder analysis under strict invariants ($\alpha_M = 0$, bounded GW/EM ratio), writes `invariants.json`, and saves a summary CSV and figure.

- Default (no CLI): Theory+ with SN cap = 0.05 mag; auto-discovers `./data/host_catalog.csv`. Outputs to `./outputs_paper_ready/`.
- Files emitted: `theoryplus_summary.csv`, `H0_points_theoryplus.png`, `invariants.json`, `s8_state_action_summary.json`, `s8_bestfit_lines.png`, `s8_p_sweep.png`, `fs8_comparison.png`, `E_of_z_check.png`, `gw_em_ratio.png`.
- Example CLI (column binding):

```
python environment_h0_bias.py theoryplus \
  -host-csv ./data/host_catalog.csv \
  -col-sample sample -col-g-over-a0 g_over_a0 -col-weight w \
  -sn-cap 0.05 -alpha-salt 0.14 -beta-salt 3.1 -gamma-ni 0.6 -s-t 6.0 -c-t 0.02
```


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