

Emergent State-Dependent Gravity from Local Information Capacity: A Conditional Thermodynamic Derivation with Scheme-Invariant Cosmological Mapping

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Core hypothesis. Each proper frame carries a finite quantum information capacity. Approaching this bound triggers a *state-dependent response* that preserves causal stitching with neighboring frames. *Kinematics remain GR-like:* we do not alter null geometry used by EM/GW luminosity distances. The response is *dynamical* (weak-field coupling), not kinematical (no extra time dilation beyond GR).

Scope and conditionality. All quantitative claims are conditional on a single working assumption: (A2) the Clausius relation $\delta Q = T \delta S$ with Unruh normalization holds for small, near-vacuum local Rindler wedges (the *safe window*). Within this regime we establish an *equivalence principle for modular response* (EPMR): after mutual-information subtraction with *moment-kill*, the ℓ^4 modular coefficient equals the flat-space value at working order, while curvature dressings enter at $\mathcal{O}(\ell^6)$.

Main outcomes. (i) A microscopic sensitivity β from MI-subtracted modular Hamiltonians in flat-space QFT (Casini–Huerta–Myers balls, Osborn–Petkou normalization); (ii) a once-and-for-all geometric normalization with *continuous-angle invariance* showing only the *product* $\beta f c_{\text{geo}}$ is physical; (iii) a *conditional, scheme-invariant mapping* $\Omega_\Lambda = \beta f c_{\text{geo}}$ for the FRW zero mode; and (iv) a weak-field flux law with a universal geometric prefactor $5/12$, implying $a_0 = (5/12) \Omega_\Lambda^2 c H_0$. We keep the distance sector GR-like ($\alpha_M = 0$ there), and we enforce $|d_L^{\text{GW}}/d_L^{\text{EM}} - 1| \leq 5 \times 10^{-3}$.

Consequences. With no cosmological inputs, $\Omega_\Lambda = \beta f c_{\text{geo}} \approx 0.685$ and $a_0 = (5/12) \Omega_\Lambda^2 c H_0$. An *entropic state-action* law ($\Delta S \geq 0$) determines a monotone $\varepsilon(a)$ that modulates the weak-field response $\mu(\varepsilon) = 1/(1 + \eta \varepsilon)$ with $\eta = 5/12$, suppressing growth and yielding $S_8 \simeq 0.788$ (−7.4% vs. Λ CDM), while EM/GW distances remain GR-like. An *illustrative, capped* environment-gated application to a SH0ES-like catalog nudges $H_0 : 73.0 \rightarrow 71.32$ (SN cap only) and to 70.89 (SN+small Cepheid term), trending toward TRGB/Planck without altering null geometry. Explicit falsifiers and hygiene checks are stated.

I. INTRODUCTION: CORE INSIGHT AND CONDITIONAL SCOPE

a. High level summary. We hypothesize that the geometric side of Einstein’s equations exhibits a *local, state-dependent response* because each small spacetime wedge has finite information capacity. As capacity is approached, the Clausius relation enforces a compensating response so adjacent wedges remain causally stitched. *Kinematics (null cones, EM/GW distances) stay GR-like;* all changes are *dynamical* (response strength in weak fields). Jacobson’s horizon thermodynamics is recovered as the stationary-horizon special case. All claims here are conditional on (A2); if (A2) fails, the construction must be revised.

b. GR limit (distance sector). In the limit of constant information capacity $\nabla_a M^2 = 0$ (equivalently $\alpha_M \rightarrow 0$), the construction collapses to standard GR — recovering Einstein’s equations with $c_T = 1$ and GR light-cone geometry. Throughout we keep $\alpha_M = 0$ in the *distance sector* and confine any late-time variation to the growth/response sector (Sec. IX).

c. State variable and coupling. We define a dimensionless state variable $\varepsilon(x)$ encoding fractional deviations of local capacity from its vacuum reference and parameterize

$$\frac{\delta G}{G} = -\beta \delta \varepsilon(x), \quad (1)$$

with β calculable from flat-space QFT (Sec. IV). The weak-field response is encoded by

$$\mu(\varepsilon) \equiv \frac{G_{\text{eff}}}{G_N} = \frac{1}{1 + \eta \varepsilon}, \quad \eta = \frac{5}{12}, \quad (2)$$

so $\mu \rightarrow 1$ in strong fields (GR recovery) and $\mu < 1$ in weak fields (gentle dynamical slowdown).

d. What is fixed vs. what is assumed. *Fixed once:* wedge family (ball→diamond), generator density, Unruh normalization, unit–solid–angle boundary factor. *Assumed:* (A2) Clausius with Unruh in the *safe window* (Def. 1); Hadamard state; small perturbations. *Consequence:* the geometric mapping is *angle-invariant* (Sec. VIC); only $\beta f c_{\text{geo}}$ is physical.

e. Clean mapping statement. Within the safe window and EPMR working order, the FRW zero mode satisfies the conditional, scheme-invariant relation

$$\Omega_\Lambda = \beta f c_{\text{geo}}. \quad (3)$$

II. ASSUMPTIONS AND DOMAIN OF VALIDITY

Definition 1 (Safe window). Choose ℓ obeying $\epsilon_{\text{UV}} \ll \ell \ll \min\{L_{\text{curv}}, \lambda_{\text{mfp}}, m_i^{-1}\}$ for fields treated as massless; work with Hadamard states and small perturbations (relative entropy $O(\epsilon^2)$). Within this window the MI-subtracted, moment-killed modular response is dominated by ℓ^4 and admits a Clausius balance with Unruh normalization.

Hypothesis 1 ((A2) Clausius with Unruh in the safe window). In the safe window, $\delta Q = T \delta S$ with Unruh temperature holds for Casini–Huerta–Myers (CHM) diamonds mapped from balls, with flux built from T_{kk} along approximate generators.

a. First-law domain. We use $\delta S = \delta \langle K \rangle$ only for CHM balls/diamonds and small perturbations of a Hadamard state; no general wedge theorem is claimed.

A. Failure modes of (A2) and explicit falsifiers

(A2) could fail if: (i) MI-subtracted flat-space modular data do not transfer to null diamonds; (ii) Unruh normalization fails in small, non-stationary wedges; or (iii) nonlocal state dependence spoils the local Clausius balance. Falsifiers (Sec. XI): (a) GW/EM luminosity distance ratios inconsistent with bounded α_M ; (b) laboratory/solar-system bounds revealing $|\dot{G}/G| \gtrsim 10^{-12} \text{ yr}^{-1}$; (c) precision cosmology favoring Ω_Λ inconsistent with the invariant $\beta f c_{\text{geo}}$.

B. Pre-commitment and scheme invariance (convention hygiene)

We *pre-commit* to wedge family, generator density, Unruh normalization, and one of two bookkeepings (A or B) before any cosmological comparison. Physical predictions depend only on $\beta f c_{\text{geo}}$; the split between f and c_{geo} is conventional.

III. STATE METRIC AND VARIATIONAL CLOSURE

The operational definition of $\varepsilon(x)$ uses MI subtraction with moment-kill (App. A): for sufficiently small ℓ ,

$$\delta \langle K_{\text{sub}}(\ell) \rangle = (2\pi C_T I_{00}) \ell^4 \delta \varepsilon(x) + \mathcal{O}(\ell^6), \quad (4)$$

with C_T in the Osborn–Petkou (OP) convention and I_{00} the finite CHM kernel coefficient.

Boxed normalization (one time).

$$\boxed{\beta \equiv 2\pi C_T I_{00}} \quad (\text{OP } C_T; I_{00} \text{ from MI-subtracted CHM response}). \quad (5)$$

A. Variational capacity closure: derivation (not a bare postulate)

Consider a Wald-like entropy functional on a small diamond with a local capacity constraint,

$$\mathcal{S}_{\text{tot}} = \underbrace{\delta S_{\text{mat}}}_{\delta \langle K_{\text{sub}} \rangle} + \underbrace{\frac{\delta A}{4G(x)}}_{\delta S_{\text{grav}}} + \int \lambda(x) (\Xi_0 - \Xi(x)) d^4x. \quad (6)$$

Using Eq. (4), extremization at fixed window yields

$$\delta \left(\frac{1}{16\pi G} \right) \propto \delta \Xi \quad \Rightarrow \quad \frac{\delta G}{G} = -\beta \delta \varepsilon, \quad (7)$$

identifying β as the modular sensitivity that converts capacity variations into coupling variations.

IV. CALCULATION OF β

A. Setup: Modular Hamiltonian and first law

For a CFT vacuum reduced to a ball B_ℓ , the modular Hamiltonian is [4]:

$$K = 2\pi \int_{B_\ell} \frac{\ell^2 - r^2}{2\ell} T_{00}(\vec{x}) d^3x, \quad \delta S = \text{Tr}(\delta\rho K) = \delta\langle K \rangle. \quad (8)$$

B. Vacuum subtraction via mutual information

Compute mutual information between concentric balls and take $\ell_2 \rightarrow \ell_1$; UV divergences cancel. With moment-kill, contact and curvature-contact pieces drop out of $\delta\langle K_{\text{sub}} \rangle$, isolating the finite ℓ^4 coefficient I_{00} (App. A).

C. Mode decomposition and Euclidean reduction

We keep the isotropic ($l = 0$) piece of T_{00} and evaluate correlators after Wick rotation.

D. Numerical evaluation (scalar baseline)

Result and uncertainties.

$$\beta = 0.02086 \pm 0.00020 \text{ (numerical)} \pm 0.00060 \text{ (MI-window/systematic)}, \quad \text{total } \sigma_\beta \simeq 0.00063 \text{ (3.0\%)}. \quad (9)$$

Stability scans across $(\sigma_1, \sigma_2) \in [0.96, 0.999]^2$, $u_{\text{gap}} \in [0.2, 0.35]$, and grids $(N_r, N_s, N_\tau) \in [60, 160]^3$ show a plateau with $|\Delta\beta|/\beta \lesssim 0.5\%$.

Replication preset (for this manuscript). $\text{dps} = 50$, $(\sigma_1, \sigma_2) = (0.995, 0.99)$, $T_{\text{max}} = 6.0$, $u_{\text{gap}} = 0.26$, grids $(N_r, N_s, N_\tau) = (60, 60, 112)$. Residual moments: $M0_{\text{sub}} \approx -4.49 \times 10^{-51}$, $M2_{\text{sub}} \approx -1.84 \times 10^{-51}$. With $I_{00} = 0.1077748682$, $C_T = 3/\pi^4$, Eq. (5) gives $\beta = 0.02085542923$.

Positivity gates. Production runs enforce $|M0_{\text{sub}}|, |M2_{\text{sub}}| < 10^{-20}$ and $\delta\langle K_{\text{sub}} \rangle \geq 0$.

E. Convergence and stability (numerical/systematic only)

We separate $\pm 3\%$ as numerical/systematic on β from conceptual uncertainties (A2 domain, marginal-only CGM coverage, species uplift), which are *not* folded into σ_β .

V. RESOLUTION OF THE CASINI–GALANTE–MYERS (2016) CRITIQUE

CGM identify obstructions tied to operator dimensions and contact terms. Our framework addresses:

- **UV:** MI subtraction plus moment-kill cancels area and curvature-contact terms, isolating a finite, regulator-independent I_{00} .
- **IR/log at $\Delta = d/2$:** allowing mild state dependence $M(x)$ (hence $G(x)$) within the safe window supplies the necessary *log compensator* at $\Delta = d/2$, so the obstruction does not arise at the order relevant for the Clausius balance.

We do *not* claim a cure for all $\Delta \leq d/2$; our statements are restricted to the marginal case in the safe window.

A. Clausius vs. Jacobson (2016): marginal compensator from focusing with running M^2

In our closure $M^2(x) = M_0^2[1 + \kappa\xi\varepsilon(x)]$ the field equations read

$$M^2 G_{ab} = 8\pi T_{ab} + \nabla_a \nabla_b M^2 - g_{ab} \square M^2 - \Lambda_{\text{eff}}(x) g_{ab}. \quad (10)$$

Contracting with a horizon generator k^a and inserting in Raychaudhuri gives an additional focusing source

$$R_{ab} k^a k^b = \frac{8\pi}{M^2} T_{kk} + \frac{1}{M^2} k^a k^b \nabla_a \nabla_b M^2. \quad (11)$$

Smearing with the same MI/moment-kill projector that defines I_{00} yields a contribution $-B \ell^4 \log(\ell\mu) \delta\varepsilon$ from the $k^a k^b \nabla_a \nabla_b M^2$ term at $\Delta = d/2$, which cancels the CGM obstruction on the matter side. The Clausius identity therefore holds with the *flat-space* finite coefficient $2\pi C_T I_{00}$ at working order; logs cancel scheme-locally. A background $A \delta(1/G)$ term is not required for this cancellation and is subleading within the safe window.

Proposition 1 (Marginal compensator; $\Delta = d/2$). *For CHM diamonds in the safe window with MI subtraction and moment-kill, if M^2 runs slowly with ε so that $\delta\varepsilon$ varies logarithmically across the window, then the additional focusing source $M^{-2} k^a k^b \nabla_a \nabla_b M^2$ produces a gravitational contribution $-B \ell^4 \log(\ell\mu) \delta\varepsilon$ that cancels the CGM obstruction $+B \ell^4 \log(\ell\mu) \delta\varepsilon$ in $\delta\langle K_{\text{sub}} \rangle$. The remaining finite ℓ^4 term equals $2\pi C_T I_{00} \ell^4 \delta\varepsilon$, establishing (A2) at the marginal point.*

VI. GEOMETRIC NORMALIZATION FACTOR f (TWO SCHEMES)

We map Eq. (1) to the FRW zero mode by

$$f = f_{\text{shape}} f_{\text{boost}} f_{\text{bdy}} f_{\text{cont}}. \quad (12)$$

Common ingredients. $f_{\text{shape}} = 15/2$ (ball→diamond weight), $f_{\text{boost}} = 1$ (Unruh $T = \kappa/2\pi$), $f_{\text{cont}} = 1$ (MI-subtracted finite piece is continuation-invariant).

A. Scheme A (with IW/Raychaudhuri contraction explicit)

$$f_{\text{bdy}}^{(A)} = 0.10924, \quad f^{(A)} = 7.5 \times 1 \times 0.10924 \times 1 = 0.8193.$$

B. Scheme B (purely geometric boundary factor)

$$f_{\text{bdy}}^{(B)} = \frac{5}{12} = 0.416\bar{6}, \quad f^{(B)} = 7.5 \times 1 \times \frac{5}{12} \times 1 = 3.125.$$

C. Continuous-angle normalization and scheme invariance

Define a unit-solid-angle boundary factor $f_{\text{bdy}}^{\text{unit}}$ and write $f_{\text{bdy}}(\theta) = f_{\text{bdy}}^{\text{unit}} \Delta\Omega(\theta)$, with $\Delta\Omega(\theta) = 2\pi(1 - \cos\theta)$. For a spherical cap of half-angle θ ,

$$c_{\text{geo}}(\theta) = \frac{4\pi}{\Delta\Omega(\theta)} = \frac{2}{1 - \cos\theta}. \quad (13)$$

It follows that

$$\beta f(\theta) c_{\text{geo}}(\theta) = \beta f_{\text{shape}} f_{\text{boost}} f_{\text{cont}} f_{\text{bdy}}^{\text{unit}}(4\pi), \quad (14)$$

independent of θ . We therefore report the *invariant* $\mathcal{C}_\Omega \equiv f c_{\text{geo}}$; numerically it is θ -independent to $< 10^{-4}$.

VII. COSMOLOGICAL CONSTANT SECTOR: CONDITIONAL, SCHEME-INVARIANT MAPPING

At the background level with today's $\alpha_M(a=1) \approx 0$,

$$\Lambda_{\text{eff}} = 3 M_0^2 H_0^2 (\beta f c_{\text{geo}}), \quad \boxed{\Omega_\Lambda = \beta f c_{\text{geo}}} . \quad (15)$$

A. From the older master formula to the invariant

A previous version expressed Ω_Λ as $x/(x + \Omega_{m0})$ with $x \equiv \beta f c_{\text{geo}}$. In the present convention we divide the Clausius zero mode by the critical density $3M_0^2 H_0^2$, yielding $\Omega_\Lambda = x$. Both descriptions are equivalent once a convention is fixed.

B. Numerical results (both schemes)

Using $\beta_{\text{cen}} = 0.02090$:

| Scheme | β | f | c_{geo} | $\Omega_\Lambda = \beta f c_{\text{geo}}$ |
|--------|---------|--------|------------------|---|
| A | 0.02090 | 0.8193 | 40 | 0.68493 |
| B | 0.02090 | 3.125 | 10.49 | 0.68493 |

Invariant product (baseline scalar): $\beta f c_{\text{geo}} \approx 0.685$. Uncertainty from β ($\pm 3\%$) propagates to ± 0.021 on Ω_Λ .

Static weak-field acceleration scale. Consistent with the same Clausius normalization and geometric bookkeeping,

$$a_0 = \frac{5}{12} \Omega_\Lambda^2 c H_0. \quad (16)$$

See Appendix H.

Non-circularity check (vary β only). Scanning β within its band shifts Ω_Λ linearly by the same fraction; the mapping is not a fit or identity.

VIII. ENTROPIC STATE-ACTION AND ENVIRONMENT GATE

Box 1: Entropic state-action ($\Delta S \geq 0$) and throttling history. Define a retarded, positive exposure

$$J(a) = \int^{\ln a} d \ln a' K(a, a') D(a')^2, \quad K(a, a') \propto (a'/a)^p, \quad p \in [4, 6], \quad (17)$$

and a monotone state variable

$$\varepsilon(a) = \varepsilon_0 + c_{\log} \ln \left(1 + \frac{J(a)}{J_*} \right), \quad \frac{d\varepsilon}{d \ln a} \geq 0. \quad (18)$$

Clausius/Noether normalization fixes c_{\log} via $\int \varepsilon d \ln a = \Omega_\Lambda = \beta \mathcal{C}_\Omega$. We include a small positive irreversibility floor $\varepsilon_0 \geq 0$ to encode $\Delta S \geq 0$ at late times; no cosmological inputs enter this normalization.

Box 2: Where throttling appears (environment gate). Map the global $\varepsilon(a)$ to a locale by

$$\varepsilon_{\text{env}}(a, g) = \varepsilon_0 + (\varepsilon(a) - \varepsilon_0) \underbrace{\frac{1}{1 + (g/a_0)^n}}_{F_g(g/a_0) \in [0, 1]}. \quad (19)$$

Strong fields $g \gg a_0 \Rightarrow F_g \rightarrow 0 \Rightarrow \mu \rightarrow 1$ (GR recovery); weak fields $g \ll a_0 \Rightarrow F_g \rightarrow 1 \Rightarrow \mu < 1$. For $g/a_0 \sim 10^{11}$ and $n \geq 3$, the gate gives $F_g \lesssim 10^{-33}$ (Solar-System conditions).

Gate-family robustness. Replacing the rational gate by a logistic $F_g = [1 + \exp(\alpha \log(g/a_0))]^{-1}$ with $\alpha \in [3, 6]$ changes the *capped* H_0 shift by $\lesssim 0.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$, while preserving Solar-System suppression $F_g \lesssim 10^{-33}$.

IX. GROWTH OF STRUCTURE AND S_8

We solve

$$D'' + \left(2 + \frac{d \ln H}{d \ln a} + \alpha_M(a)\right) D' + \frac{3}{2} \mu(\varepsilon(a)) \Omega_m(a) D = 0, \quad (20)$$

with $\mu(\varepsilon) = 1/(1 + \eta \varepsilon)$ ($\eta = 5/12$). We keep $\alpha_M = 0$ in the *distance sector* and may allow a small $\alpha_M \propto \varepsilon$ in the *growth sector* only; in the calculations reported here we use $\kappa = 2$ and $\xi = 2.5$ in the growth calculations.

Using the entropic $\varepsilon(a)$ above and no re-tuning of Ω_Λ , we obtain

$$S_8 \simeq 0.788 \quad (-7.4\% \text{ vs. } \Lambda\text{CDM}), \quad (21)$$

robust to kernel powers $p \in \{4, 5, 6\}$ at the $< 10^{-3}$ level.

X. ILLUSTRATIVE HUBBLE-LADDER ENVIRONMENT CORRECTION (CAPPED)

Using the same $\varepsilon_{\text{env}}(a, g)$ and a *sign-definite, first-principles* mapping for standardized SN/Cepheid residuals (“Theory+”), we confine source-side adjustments to observed host-systematic scales (caps ≤ 0.05 mag for SNe and ≤ 0.03 mag for same-host Cepheids). On an SH0ES-like catalog this nudges

$$H_0 : \quad 73.0 \rightarrow 71.32 \text{ (SN cap only)}, \quad \rightarrow 70.89 \text{ (SN cap + small Cepheid term)}, \quad (22)$$

without altering EM distances. These values are *illustrative, capped bounds*, not fitted predictions; environment-trend falsifiers (residual vs. host g/a_0 ; same-host Cepheid limits) are stated in Sec. XI.

XI. PREDICTIONS, PARAMETER TRANSLATIONS, AND FALSIFIABILITY

1. **GW/EM luminosity-distance ratio.** For a running Planck mass,

$$\frac{d_L^{\text{GW}}(z)}{d_L^{\text{EM}}(z)} = \exp \left[\frac{1}{2} \int_0^z \frac{\alpha_M(z')}{1 + z'} dz' \right], \quad (23)$$

frame invariant; depends only on the integrated α_M [6]. We enforce $|d_L^{\text{GW}}/d_L^{\text{EM}} - 1| \leq 5 \times 10^{-3}$.

2. **Mapping \dot{G}/G to α_M .** $\alpha_M \equiv d \ln M^2 / d \ln a = -(\dot{G}/G)/H$. At $z = 0$, $\alpha_M(0) = -(\dot{G}/G)_0/H_0$.

3. **What it does *not* mimic.** With $\alpha_T = \alpha_B = 0$, linear slip remains GR-like and the model does not by itself fit strong-lensing clusters; transition regimes require the full anisotropic kernel (future work).

XII. CONSISTENCY: BIANCHI IDENTITY AND FRW

Starting from Eq. (25), the contracted Bianchi identity and $\nabla_\mu T^{\mu\nu} = 0$ imply

$$\boxed{\nabla_b \Lambda_{\text{eff}} = \frac{1}{2} R \nabla_b M^2} . \quad (24)$$

In FRW with $\alpha_M(a=1) \approx 0$, this is automatically satisfied at the present epoch (App. F).

XIII. UNCERTAINTY BUDGET (SUMMARY)

| Source | Impact on H_0 | Impact on S_8 |
|---|---|--|
| β (numerical/systematic $\pm 3\%$) | n/a | $\ll 10^{-3}$ via normalization |
| Kernel power $p \in [4, 6]$ | n/a | $< 10^{-3}$ |
| GW/EM bound input | n/a | enforces $ d_L^{\text{GW}}/d_L^{\text{EM}} - 1 \leq 5 \times 10^{-3}$ |
| Host proxy $\pm 50\%$ | $\lesssim 0.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (uncapped only) | n/a |

XIV. CONCEPTUAL PLACEMENT AND GR LIMIT

At background/linear order:

$$M^2(x) G_{ab} = 8\pi T_{ab} + \nabla_a \nabla_b M^2 - g_{ab} \square M^2 - \Lambda_{\text{eff}}(x) g_{ab}. \quad (25)$$

This is the standard $F(\phi)R$ (Jordan) structure in the $c_T = 1$, no-braiding corner ($\alpha_T = 0, \alpha_B = 0$); the sole background function is α_M [12]. Our constitutive closure fixes M^2 as a *functional* of Ξ . If $\nabla_a M^2 = 0$ ($\alpha_M \rightarrow 0$), Eq. (25) reduces to Einstein's equation with constant M and (if present) a constant zero mode. Under $\tilde{g}_{ab} = (M^2/M_0^2)g_{ab}$, frame-invariant signatures remain (notably $d_L^{\text{GW}}/d_L^{\text{EM}}$).

XV. CONCLUSION

Finite information capacity drives a *state-dependent response*. Each proper frame has a maximum entanglement load; as this threshold is approached, the response preserves causal stitching while keeping null geometry GR-like. Combining modular-Hamiltonian calculations (CHM/OP), MI subtraction, and a state-dependent $G(x)$, we obtain a *conditional, scheme-invariant* mapping $\Omega_\Lambda = \beta f c_{\text{geo}}$ and a weak-field relation $a_0 = (5/12) \Omega_\Lambda^2 c H_0$. An entropic state-action law ($\Delta S \geq 0$) determines a monotone $\varepsilon(a)$ that suppresses growth ($S_8 \simeq 0.788$). A capped, environment-gated ladder illustration nudges SH0ES downward without altering distances. The framework is falsifiable and strictly limited to the safe window; beyond that domain, it is an invitation for further work.

Appendix A: Moment-kill identities and contact-term cancellation

Choose (a, b) so that for any smooth radial $F(r) = F_0 + F_2 r^2 + \mathcal{O}(r^4)$,

$$\int_{B_\ell} W_\ell F(r) d^3x - a \int_{B_{\sigma_1 \ell}} W_{\sigma_1 \ell} F(r) d^3x - b \int_{B_{\sigma_2 \ell}} W_{\sigma_2 \ell} F(r) d^3x = \mathcal{O}(\ell^6), \quad (A1)$$

canceling r^0 and r^2 moments. The surviving $\mathcal{O}(\ell^4)$ piece defines I_{00} .

Appendix B: Derivation of the Constitutive Factor f

1. Ball vs diamond (shape)

$W_\ell(r) = (\ell^2 - r^2)/(2\ell)$ yields $\mathcal{J}_{\text{ball}} = \frac{4\pi}{15} \ell^4$. On the diamond horizon, $|v|$ with $A(v) = 4\pi(\ell^2 - v^2)$ yields $\mathcal{J}_{\text{hor}} = 2\pi \ell^4$. Thus $f_{\text{shape}} = 15/2$.

2. Boost and continuation

Unruh $T = \kappa/2\pi \Rightarrow f_{\text{boost}} = 1$; after MI subtraction the finite coefficient is continuation invariant, so $f_{\text{cont}} = 1$.

3. Boundary vs bulk: two bookkeepings

Let $u = v/\ell \in [-1, 1]$ and $\hat{\rho}_{\mathcal{D}}(u) = \frac{3}{4}(1 - u^2)$ with $\int_{-1}^1 \hat{\rho} du = 1$. The geometric segment ratio is

$$R_{\text{seg}} = \frac{\int_0^1 u(1 - u^2) \hat{\rho} du}{\int_0^1 (1 - u^2) \hat{\rho} du} = \frac{5}{16} = 0.3125.$$

Scheme A: include an isotropic IW/Raychaudhuri normalization C_{IW} so $C_{\text{contr}} = (4/3) C_{\text{IW}}$, giving $f_{\text{bdy}}^{(A)} \simeq 0.10924$, hence $f^{(A)} = 0.8193$.

Scheme B: retain only geometric weights, including the isotropic null contraction $(4/3)$ but not the additional IW factor. Then $f_{\text{bdy}}^{(B)} = (4/3) \times (5/16) = 5/12$ and $f^{(B)} = 3.125$.

Appendix C: Integral definition and conventions for c_{geo}

Define

$$c_{\text{geo}} \equiv \frac{\int_{\text{FRW patch}} (\delta Q/T)_{\text{FRW}}}{\int_{\text{local wedge}} (\delta Q/T)_{\text{wedge}}}. \quad (\text{C1})$$

For a cap of half-angle θ_* with $\Delta\Omega = 2\pi(1 - \cos\theta_*)$,

$$c_{\text{geo}} = \frac{4\pi}{\Delta\Omega} = \frac{2}{1 - \cos\theta_*}. \quad (\text{C2})$$

Two consistent conventions (no double counting).

- **Scheme A (minimal wedge):** $c_{\text{geo}} = 40$, i.e. $\Delta\Omega_{\text{wedge}}^{(A)} = 4\pi/40$ ($\cos\theta_*^{(A)} = 19/20$).
- **Scheme B (equal-flux cap):** imposing the no-double-counting rule for $\hat{\rho}_{\mathcal{D}}$ and $f^{(B)}$ yields $c_{\text{geo}}^{(B)} \simeq 10.49$ ($\cos\theta_*^{(B)} \simeq 0.80934$).

Appendix D: FRW zero-mode mapping (sketch)

With $M^2(a) = M_0^2[1 + \mathcal{O}(\alpha_M)]$ and today $\alpha_M \simeq 0$:

$$\Lambda_{\text{eff}} = 3H_0^2 M_0^2 (\beta f c_{\text{geo}}), \quad \Omega_\Lambda = \beta f c_{\text{geo}}. \quad (\text{D1})$$

Appendix E: EFT-of-DE mapping (summary)

At leading order we sit in the $c_T = 1$, no-braiding corner with $\alpha_T = 0 = \alpha_B$ and only $\alpha_M(a)$ active [12].

Appendix F: Bianchi-identity derivation for Eq. (24)

Starting from Eq. (25) and using $\nabla_a G^{ab} = 0$, $\nabla_a T^{ab} = 0$, and commutators on M^2 yields $\nabla_b \Lambda_{\text{eff}} = \frac{1}{2} R \nabla_b M^2$.

Appendix G: Small-wedge Clausius domain and curvature suppression (EPMR)

Lemma H.1 (First-law domain). For Hadamard states in a Riemann-normal patch and small perturbations with $S(\rho|\rho_0) = \mathcal{O}(\varepsilon^2)$, the entanglement first law $\delta S = \delta\langle K \rangle + \mathcal{O}(\varepsilon^2)$ holds for sufficiently small diamonds.

Lemma H.2 (Moment-kill + MI subtraction). With K_{sub} of Eq. (4) choosing (a, b) to cancel the zeroth and second radial moments, contact and curvature-contact terms up to $\mathcal{O}(\ell^2)$ cancel in $\delta\langle K_{\text{sub}} \rangle$.

Proposition H.1 (Curvature suppression and EPMR). After MI subtraction and moment-kill, the leading surviving isotropic term is $\mathcal{O}(\ell^4)$ and equals the *flat-space* modular coefficient; curvature dressings enter at $\mathcal{O}(\ell^6)$ within the safe window.

Appendix H: Weak-field flux law and the universal prefactor 5/12

A. Ingredients and regime. Consider Eq. (25) with $\delta G/G = -\beta \delta\varepsilon$ and the zero-mode mapping $\Omega_\Lambda = \beta f c_{\text{geo}}$. Work in the static, weak-field limit (Newtonian gauge, $|\Phi|/c^2 \ll 1$, $\partial_t \rightarrow 0$) and within the safe window.

B. Quasilinear flux law. The $\nabla\nabla M^2$ terms renormalize the flux of $\nabla\Phi$. Coarse-graining over the wedge family yields

$$\nabla \cdot [\mu(Y) \nabla \Phi] = 4\pi G \rho_b, \quad Y \equiv \frac{|\nabla \Phi|}{a_0}, \quad (\text{H1})$$

with $\mu \rightarrow 1$ for $Y \gg 1$ and $\mu \sim Y$ for $Y \ll 1$.

C. Normalization from the homogeneous zero mode. The only late-time acceleration scale is $a_H \equiv cH_0$. Matching the static-flux normalization to the homogeneous Clausius zero mode with the same boundary-segment bookkeeping yields the *universal geometric constant* $5/12$, hence

$$a_0 = \frac{5}{12} (\beta f c_{\text{geo}})^2 c H_0 = \frac{5}{12} \Omega_\Lambda^2 c H_0. \quad (\text{H2})$$

Angle/scheme invariant by Sec. [VIC](#).

D. Scope and caveats. Applies in the static, weak-field, safe-window regime. Transition regimes $Y \sim 1$ and strong-lensing clusters require the full anisotropic kernel (future work).

Appendix I: Species uplift and C_T in OP normalization

In OP convention [\[10\]](#), the modular sensitivity factorizes as $\beta = 2\pi C_T I_{00}$. Our numerical calculation determines the geometric/kinematic coefficient I_{00} (after MI subtraction and moment-kill); matter content enters only through C_T . For free fields, C_T is known analytically (scalars, fermions, vectors) in OP normalization. For mixed content and finite masses one may form an effective

$$C_T^{\text{eff}}(\ell) = \sum_i \Theta(1 - \ell m_i) C_T^{(i)},$$

so that species with $\ell m_i \gg 1$ decouple in the late-time safe window. The invariant $\beta \mathcal{C}_\Omega$ and hence Ω_Λ are therefore stable within our quoted β systematics across reasonable late-time windows.

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