

# Modular Response in Free Quantum Fields: A KMS/FDT Theorem and Conditional Extensions

[clg]<sup>1</sup>

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(Dated:)

**Part I (Theoremic core, free/Gaussian Hadamard QFT).** We prove that, for small causal diamonds (CHM) in locally Hadamard states and within a safe window  $\epsilon_{UV} \ll \ell \ll \min\{L_{\text{curv}}, \lambda_{\text{mfp}}, m_i^{-1}\}$ , the MI/moment-kill projector isolates a finite  $\ell^4$  modular response with coefficient equal to its flat-space value; the projected KMS/FDT susceptibility is positive; and coarse-graining over the wedge family produces the universal weak-field prefactor  $5/12 = (4/3) \times (5/16)$ . The fractional KMS defect between CHM diamonds and half-spaces scales as  $\mathcal{O}((\ell/L_{\text{curv}})^2) + \mathcal{O}((\ell H)^2)$ . The QFT sensitivity is  $\beta = 2\pi C_T I_{00} = 0.02086 \pm 0.00105$  (conservative 5% shared systematics). A scheme-invariant background relation *suggests*  $\Omega_\Lambda = \beta f c_{\text{geo}}$  *conditional* on our coarse-graining and analyticity assumptions.

**Part II (Conditional extensions).** We separate *definition* (flat-space  $\epsilon$  from modular response) from *mapping*. Rather than impose the standard EFT-of-DE  $\alpha$ -basis, we adopt a quasi-static closure that keeps operational distances GR-like (no additional lensing coupling  $\Sigma \simeq 1$ ) while modifying growth via  $\mu(\epsilon, s) = 1/(1 + \frac{5}{12}\epsilon s(x))$  with  $s(x)$  a local, covariant environment modulation derived from the action. KMS/FDT positivity motivates an entropy-driven law  $d\epsilon/d\ln a \geq 0$  with a *conditional* background budget  $\int \epsilon d\ln a = \Omega_\Lambda$ . We then prove a *No-Go Lemma* showing any linear kernel fails to yield Tully–Fisher scaling and introduce a *quasistatic elastic sector* (AQUAL form, derived as the SK static limit) with the *same* fixed acceleration  $a_0 = \frac{5}{12}\Omega_\Lambda^2 c H_0$ . The elastic sector is convex, well-posed, obeys Solar–System curvature gating, produces  $\Phi = \Psi$  at working order (no slip), and leaves linear cosmology intact via a causal SK filter.

**Part III (Exploratory).** (i) An *optional*, shock-selective *optical* channel (D′) reduces  $\Sigma$  only in high-shear shocked gas to address Bullet-type lensing offsets while preserving FRW distances, with a principled SK/BRSSS derivation path connecting the amplitude to ICM transport coefficients. (ii) A compact thermodynamic interpretation of the projected modular response: a Clausius-like identity holds at working order in the MI/moment-kill channel, and the FRW budget may be viewed as a *coarse-grained* Clausius normalization *conditional* on our KMS→FRW hypotheses. (iii) *Linkage to global entropic gravity* (Bianconi 2025) via small-diamond MI matching and observational discriminants.

**Referee-guided additions.** We (A) formalize an MI-smeared null-energy positivity (free fields; projected QEI), (B) make explicit the RG/operator-spectrum bridge pinning  $\beta$  to  $C_T$  and clarifying anomaly channels, (C) expose SM bookkeeping tied to the safe-window fraction  $f_V$  and curvature gating  $s(\chi_g)$  so that GR dominance is recovered wherever heavy sectors or strong curvature suppress the MI channel, and (D) add the *No-Go Lemma + elastic SK/AQUAL sector* with fixed  $a_0$ , no slip, Solar–System compliance, and cosmology/galaxy separation.

## READER’S MAP: PART I (THEOREM) VS. PART II (CONDITIONAL) VS. PART III (EXPLORATORY)

**Part I (Secs. I–V, IV, XIA–B; Apps. XXI–XXIV):** proven results for free/Gaussian Hadamard fields at working order, SM bookkeeping, and first-principles positivity/RG clarifications.

**Part II (Secs. VI–VIII, IX, XXXI, XIC; Apps. XXV–XXVI, XXVII):** conditional mapping for growth ( $\mu(\epsilon, s)$ ), *Linear No-Go* (Sec. VII), and *Elastic quasistatic sector* (Sec. VIII) with Solar–System gating, no slip, and BTFR; causal SK separation of regimes.

**Part III (Secs. XIII, XX, XVII; Apps. XXIX, XXX):** exploratory shock-selective optics (D′); thermodynamic interpretation; and linkage to Bianconi’s global entropic gravity.

## I. SCOPE, WORKING ORDER, AND SAFE-WINDOW QUANTIFICATION (PART I)

*a. Working order and state class.* We work to  $\mathcal{O}(\ell^4)$  in the MI/moment-kill projector channel, treating curvatures/contact terms as  $\mathcal{O}(\ell^6)$ . States are locally Hadamard.

*b. KMS applicability (CHM diamonds).* Exact BW KMS holds for half-spaces; CHM diamonds inherit it with fractional defect  $\mathcal{O}((\ell/L_{\text{curv}})^2) + \mathcal{O}((\ell H)^2)$  (App. XXIV).

c. *Safe-window volume fraction.* Define a conservative admissible scale

$$\ell_{\max}(x) \equiv \zeta \min \left\{ L_{\text{curv}}(x), \lambda_{\text{mfp}}(x), m_i^{-1}(x) \right\}, \quad \zeta = 0.1. \quad (1)$$

Using Press–Schechter/Sheth–Tormen mass functions and NFW curvature proxies  $L_{\text{curv}}^{-2} \sim (R_{abcd} R^{abcd})^{1/2}$  with sub-structure excision parameter  $\xi$ , we estimate the comoving volume fraction  $f_V(\ell_{\min}) = \text{Vol}\{x : \ell_{\max}(x) > \ell_{\min}\} / \text{Vol}_{\text{tot}}$ . A semi-analytic survey (App. XXV) shows voids dominate  $f_V$ , while dense cores lack a window; representative values at  $z \sim 0$  for  $\ell_{\min} \in [1, 100]$  pc are  $f_V \sim 0.6\text{--}0.95$  for  $\xi \in [0.2, 0.5]$ . This enters only as a domain-of-validity indicator.

d. *Spectrum caveat.* The admissible window  $\epsilon_{\text{UV}} \ll \ell \ll \min\{L_{\text{curv}}, \lambda_{\text{mfp}}, m_i^{-1}\}$  is understood to apply to sectors that contribute at working order. Massive sectors with  $\ell \gg m_i^{-1}$  are exponentially suppressed and, after MI/moment–kill subtraction, do not re-introduce lower moments or  $\ell^4 \log \ell$  terms. Thus the  $\ell^4$  coefficient is dominated by massless/light fields while heavy fields decouple in this channel. See Sec. IV for SM bookkeeping that packages light-field multiplicity into a single  $\epsilon_{\text{SM}}$ .

e. *Angle invariance as a null test.* The continuous-angle product  $C_\Omega = f(\theta) c_{\text{geo}}(\theta)$  is analytic and  $\theta$ -independent; residuals are shown as a null check, not a precision claim.

## II. A2–KMS THEOREM (GAUSSIAN/HADAMARD SECTOR)

**Theorem 1** (Projected modular response and positivity). *Let  $\mathcal{Q}$  be a free (Gaussian) QFT on a globally hyperbolic spacetime and  $\rho$  a locally Hadamard state. For a causal diamond of radius  $\ell$  with  $\ell \ll L_{\text{curv}}$  and the MI/moment–kill projector that cancels  $r^0$  and  $r^2$  moments, the MI-subtracted modular response obeys*

$$\delta\langle K_{\text{sub}} \rangle = (2\pi C_T I_{00}) \ell^4 \delta\epsilon + \mathcal{O}(\ell^6), \quad (2)$$

with coefficient equal to the flat-space value. The retarded susceptibility  $\chi_{QK}$  in the projected channel is positive (FDT), and wedge averaging yields the universal weak-field prefactor  $5/12$ . The fractional deviation from BW KMS is  $\mathcal{O}((\ell/L_{\text{curv}})^2) + \mathcal{O}((\ell H)^2)$ .

**Corollary 1** (Conditional background statement). *Under the coarse-graining and analyticity assumptions of Sec. X, the FRW zero mode suggests the scheme-invariant relation  $\Omega_\Lambda = \beta f c_{\text{geo}}$  with  $\beta = 2\pi C_T I_{00}$ . We treat this as a conditional statement rather than a theorem.*

## III. QFT INPUT: $\beta = 2\pi C_T I_{00}$ AND ERROR BUDGET

We evaluate  $\beta$  via four independent routes: (a) real-space CHM; (b) spectral/Bessel; (c) Euclidean time-slicing; (d) replica finite-difference. The spread is  $\lesssim 1\%$ . We adopt a conservative

$$\beta = 0.02086 \pm 0.00105 \quad (5\% \text{ shared systematics}). \quad (3)$$

Angle invariance is used as a null residual test.

Here  $C_T$  denotes the flat-space stress-tensor two-point normalization, e.g.  $\langle T_{ab}(x) T_{cd}(0) \rangle = C_T \mathcal{I}_{abcd}(x)/|x|^{2d}$  in  $d$  dimensions (see Osborn–Petkou).

*Benchmark (convention).* For a free, massless real scalar in  $d = 4$  and our normalization,  $C_T = 1/(120\pi^2)$ , which yields  $\beta \simeq 0.02086$  via Eq. (4).

**Implementation consistency (note).** The normative constants used for the numerical reproductions are

$$C_T = \frac{1}{120\pi^2}, \quad (\sigma_1, \sigma_2) = \left(\frac{1}{2}, 2\right), \quad (a, b) = \left(\frac{4}{5}, \frac{1}{5}\right),$$

with the moment–kill identities enforced exactly (App. XXI). Helper scripts (`beta_methods_v2.py`, `referee_pipeline.py`) print these values alongside the computed  $I_{00}$  to prevent normalization drift.<sup>1</sup>

**Reproducibility (non-circular).** We use a two-scale MI/moment–kill subtraction with a top-hat window on 3-balls

$$W_\ell(r) = \frac{3}{4\pi\ell^3} \Theta(\ell - r), \quad \mathcal{W}_\ell := \int_{B_\ell} W_\ell - a \int_{B_{\sigma_1\ell}} W_{\sigma_1\ell} - b \int_{B_{\sigma_2\ell}} W_{\sigma_2\ell}.$$

<sup>1</sup> In earlier development branches some convenience flags defaulted to alternate normalizations (e.g.  $C_T = 3/\pi^4$ ) and near-unity MI scales. These have been disabled in the archival runners; the paper’s conventions are authoritative.

The two moment-kill conditions (cancelling  $r^0$  and  $r^2$  for any smooth radial  $F$ ) fix

$$a + b = 1, \quad a \sigma_1^2 + b \sigma_2^2 = 1 \implies a = \frac{\sigma_2^2 - 1}{\sigma_2^2 - \sigma_1^2}, \quad b = \frac{1 - \sigma_1^2}{\sigma_2^2 - \sigma_1^2}.$$

In our runs we take

$$(\sigma_1, \sigma_2) = \left(\frac{1}{2}, 2\right), \quad (a, b) = \left(\frac{4}{5}, \frac{1}{5}\right) = (0.8, 0.2).$$

With these weights the projected  $\ell^4$  coefficient evaluates to

$$I_{00} = 3.932017 \quad (\text{dimensionless}),$$

so with  $C_T = 1/(120\pi^2)$  one obtains  $\beta = 2\pi C_T I_{00} = 0.02086$  as quoted. The helper script `beta_methods_v2.py` echoes both  $(a, b; \sigma_1, \sigma_2)$  and the numeric  $I_{00}$ .

#### IV. STANDARD-MODEL SECTOR: BOOKKEEPING AND DECOUPLING AT WORKING ORDER

*a. What is being linked.* At working order the MI/moment-kill channel defines the dimensionless state variable  $\varepsilon(x)$  through

$$\delta\langle K_{\text{sub}} \rangle = \beta \ell^4 \delta\varepsilon + \mathcal{O}(\ell^6) \quad [\text{Eq. (4)}].$$

This subsection clarifies how the *Standard-Model (SM)* content enters  $\varepsilon$  and why heavy states decouple.

*b. Species sum and decoupling.* Write  $\varepsilon$  as a weighted sum over species  $i$ :

$$\varepsilon(x) = \sum_i w_i \varepsilon_i(x), \quad w_i \text{ counts effective dof (helicity/polarization, internal factors).}$$

In a diamond of size  $\ell$ , fields with  $m_i \ell \gg 1$  are exponentially suppressed in the projected channel; after the MI/moment-kill subtraction they do *not* re-introduce lower moments nor  $\ell^4 \log \ell$  terms. Parametrically,

$$\varepsilon_i(x) \propto e^{-m_i \ell} \quad \text{for } m_i \ell \gg 1,$$

so the  $\ell^4$  coefficient is dominated by massless/light fields while heavy fields decouple.

*c. Packaging the light SM content.* It is convenient to define a single light-sector variable

$$\varepsilon_{\text{SM}}(x) \equiv \sum_{i \in \text{light}} c_i \varepsilon_i(x),$$

where  $c_i$  packages the relevant multiplicities (helicity/polarization, internal quantum numbers) of each light SM species under the MI projection. All subsequent working-order formulas may then be read with  $\varepsilon \rightarrow \varepsilon_{\text{SM}}$  when SM content is explicitly considered.

*d. Coupling to gravity at working order.* The only background scalar that survives the MI/moment-kill projection and modifies weak-field growth while keeping distances GR-like is the Planck-mass renormalization  $\delta \ln M^2 = \beta \delta\varepsilon$  (Assumption D). Multiplicities therefore simply rescale  $\varepsilon$  (hence  $\mu$ ); they do *not* change  $\beta$  or the universal weak-field bookkeeping that fixes the 5/12 prefactor:

$$\mu(\varepsilon, s) = \frac{1}{1 + \frac{5}{12} \varepsilon s(x)} \longrightarrow \mu(\varepsilon_{\text{SM}}, s) = \frac{1}{1 + \frac{5}{12} \varepsilon_{\text{SM}} s(x)}.$$

*e. Environment and distances.* The environment scalar  $s(x)$  is geometric (built from curvature invariants) and independent of particle content at this order; FRW distances remain GR-like ( $\Sigma \simeq 1, c_T = 1$ ). The observed lensing amplitude changes only indirectly through altered growth.

*f. Practical note and  $f_V$  linkage.* In cosmological applications one sets a light-sector threshold  $m_i \ell \lesssim 1$  (with  $\ell$  within the safe window) and computes  $\varepsilon_{\text{SM}}$  using the appropriate  $c_i$ . As the environment varies, *field regimes can re-enter*  $\varepsilon_{\text{SM}}$  smoothly. Dense regions with no safe window contribute negligibly to the MI channel; voids dominate the valid domain via  $f_V$  (App. XXV). GR dominance is ensured either by strong curvature ( $s(\chi_g) \rightarrow 0$ ) or by heavy-sector decoupling ( $m_i \ell \gg 1$ ).

## V. WEAK-FIELD PREFACTOR 5/12

The isotropic BW channel gives  $\langle T_{kk} \rangle = (1+w)\rho$  with UV  $w = 1/3 \Rightarrow 4/3$ . Averaging over CHM segments yields  $5/16$ , so  $5/12 = (4/3) \times (5/16)$ . Details in Sec. V.

## VI. DEFINITION VS. MAPPING (PART II; CONDITIONAL)

*a. Definition (flat-space QFT).*

$$\delta\langle K_{\text{sub}}(\ell) \rangle = \underbrace{(2\pi C_T I_{00})}_{\beta} \ell^4 \delta\varepsilon(x) + \mathcal{O}(\ell^6). \quad (4)$$

*b. Mapping (constitutive; beyond the  $\alpha$ -basis).* We *do not* impose the linear EFT-of-DE  $\alpha$ -parameter mapping at working order. Instead, we adopt a quasi-static closure that keeps operational distances GR-like while modifying growth:

$$\nabla^2 \Phi = 4\pi G a^2 \rho_m \mu(\varepsilon, s), \quad \mu(\varepsilon, s) = \frac{1}{1 + \frac{5}{12} \varepsilon s(x)}, \quad (5a)$$

$$\nabla^2 \frac{\Phi + \Psi}{2} = 4\pi G a^2 \rho_m, \quad (\Sigma \simeq 1 \text{ on FRW and in laminar flows}). \quad (5b)$$

Here  $s(x)$  is a local scalar built from curvature (Sec. XV); in FRW,  $\text{Weyl} = 0 \Rightarrow \chi_g = 0 \Rightarrow s = 1$ . *Beyond working order we make no stability claims absent an action*;  $\mu(\varepsilon, s)$  serves as a falsifiable diagnostic with  $\Sigma \simeq 1$ . Matter obeys the standard continuity and Euler equations. This closure preserves the Bianchi identity at working order because  $s(x)$  is a scalar; an action-level realization and frame-independence are given below (Remark VIA). *Optional Assumption D' (Sec. XIII)* introduces a *shock-selective* lensing modification  $\Sigma(x) < 1$  localized to high-shear gas while keeping FRW  $\Sigma \simeq 1$ .

*Remark on lensing amplitude.*  $\Sigma \simeq 1$  denotes no additional lensing coupling in the baseline; the observed lensing signal still changes through the altered growth  $D(a)$ . Under Assumption D',  $\Sigma$  may be reduced *locally* in shocked gas ( $\mathcal{S}_{\text{shock}} \gg 1$ ) without affecting FRW.

*c. EFT stub (derivation of 5/12).* At quasi-static, sub-horizon scales, a background variation  $\delta \ln M^2 = \beta \delta \varepsilon$  rescales the Poisson coupling as  $G \rightarrow G_{\text{eff}} = G/(1 + \Delta)$  with  $\Delta$  fixed by the universal weak-field bookkeeping. In the isotropic BW channel the contraction  $4/3$  and the segment ratio  $5/16$  (Sec. V) give  $\Delta = \frac{5}{12} \varepsilon$ , hence

$$\mu(\varepsilon, s) = \frac{G_{\text{eff}}}{G} = \frac{1}{1 + \frac{5}{12} \varepsilon s(x)}, \quad (6)$$

consistent with Eqs. (5).

*d. Trial action (outlook).* A possible action-level route consistent with our closure is to consider an effective term that modulates  $M^2$  via the modular response,

$$S_{\text{trial}} = \int d^4x \sqrt{-g} \left[ \frac{M^2}{2} R + \lambda (\delta \ln M^2) \mathcal{K}[g; \ell] + \dots \right],$$

where  $\mathcal{K}$  is a local covariant scalar capturing the projected channel at working order and  $\lambda$  a running coefficient. While only illustrative, this shows how  $\delta \ln M^2 = \beta \delta \varepsilon$  could arise from an action (cf. [6, 8]).

### A. Frame-independence of throttling (remark)

*Throttling* here means the reduction of the effective gravitational coupling relative to GR caused by the background state variable  $\varepsilon(a)$  and a local environment factor  $s(x)$  that encodes curvature/inhomogeneity. In the Jordan frame we take

$$M_*^2(x, a) = M^2 \left[ 1 + \frac{5}{12} \varepsilon(a) s(x) \right], \quad s(x) = \frac{1}{1 + (\chi_g/\chi_*)^q} + \mathcal{O}\left(\frac{R}{m_s^2}\right),$$

so the quasi-static Poisson law reads

$$\nabla^2 \Phi \simeq \frac{4\pi G a^2 \rho_m \delta}{1 + \frac{5}{12} \varepsilon(a) s(x)} \quad \Rightarrow \quad G_{\text{eff}}(x, a) = \frac{G}{1 + \frac{5}{12} \varepsilon(a) s(x)}.$$

Thus throttling is present everywhere, while its magnitude is amplitude-modulated by the local invariant  $\chi_g = \ell^2 \sqrt{C_{abcd} C^{abcd}}$ : in weak fields ( $\chi_g \ll \chi_\star$ ) one has  $s \rightarrow 1$  and the full background rescaling  $G_{\text{eff}} = G/(1 + \frac{5}{12} \varepsilon)$ ; in strong fields ( $\chi_g \gg \chi_\star$ ) one has  $s \rightarrow 0$  and  $G_{\text{eff}} \rightarrow G$  (Solar-System compliance).

A conformal map to the Einstein frame,

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{5}{12} \varepsilon(a) s(x),$$

renders  $M_\star$  constant and shifts the same throttling into the matter coupling. To working order in our MI/moment-kill channel, gradients of  $\Omega$  and of  $\chi_g$  enter only at  $\mathcal{O}((\ell/L_{\text{curv}})^2)$  and  $\mathcal{O}(R/m_s^2)$ , consistent with the error budget in Eq. (9) and App. XXIV; the observables of interest are frame-independent at this order: growth is governed by

$$\mu(\varepsilon, s) = \frac{1}{1 + \frac{5}{12} \varepsilon(a) s(x)},$$

and distances remain GR-like ( $\Sigma \simeq 1$ ,  $c_T = 1$ ).<sup>2</sup>

*Scale-separation note.* The *local* modular response enters gravity solely as a renormalization  $\delta \ln M_\star^2 = \beta \delta \varepsilon$  of the Planck mass; the Einstein equations then propagate this renormalization to cosmological scales through the standard gravitational coupling. No macroscopic quantum coherence or ad hoc coarse-graining is required, and the Jordan $\leftrightarrow$ Einstein map above makes this statement frame-independent at working order.

A simple way to realize  $s(x)$  is as an auxiliary heavy scalar that minimizes a local potential

$$\mathcal{V}(s; \chi_g) = \frac{M^2 m_s^2}{2} \left[ s - \frac{1}{1 + (\chi_g/\chi_\star)^q} \right]^2,$$

so that the algebraic EOM enforces  $s = [1 + (\chi_g/\chi_\star)^q]^{-1} + \mathcal{O}(R/m_s^2)$ . Choosing  $m_s^2 \gg H_0^2$  ensures adiabatic tracking. **Constraints (working order).** (i) Choose  $m_s^2 \gg H_0^2$  so  $s(x)$  adiabatically tracks  $[1 + (\chi_g/\chi_\star)^q]^{-1}$  and the  $\mathcal{O}(R/m_s^2)$  offset is negligible. (ii) The Planck-mass drift  $\alpha_M = d \ln M_\star^2 / d \ln a = \frac{(5/12) s d\varepsilon / d \ln a}{1 + (5/12) \varepsilon s}$  is naturally small under our monotone  $\varepsilon(a)$ . (iii) In FRW, Weyl = 0 so curvature-weighted corrections vanish; in LSS they are  $\mathcal{O}((\ell/L_{\text{curv}})^2)$ . *Weak-field acceleration (toy/conditional; clarification).* Because  $s \rightarrow 1$  in low curvature, the weak-field normalization implies a MOND-like scale

$$a_0 = \frac{5}{12} \Omega_\Lambda^2 c H_0, \quad (7)$$

Using the baseline  $\Omega_\Lambda = 0.685$  and  $H_0 = 70.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , this gives  $a_0^{\text{eff}} \approx 1.2 \times 10^{-10} \text{ m s}^{-2}$  in the weak-field limit ( $s \simeq 1$ ); and the effective  $a_0^{\text{eff}}$  is *enhanced* in weak-field regimes by the *derived*  $s \rightarrow 1$  (not imposed), while Solar-System compliance follows from  $s(\chi_\odot) \ll 1$  (Sec. XV). Pipeline values propagate the  $\pm 5\%$  uncertainty in  $\beta$ .

## VII. LINEAR-KERNEL NO-GO FOR TULLY-FISHER

**Lemma 1** (Linear-kernel no-go). *Let  $\Phi$  be determined from  $\rho$  by a translation/rotation-invariant linear map with tempered kernel  $G$ :*

$$\Phi(\mathbf{x}) = (G * \rho)(\mathbf{x}), \quad G(\mathbf{x}) = \frac{1}{(2\pi)^3} \int \frac{\tilde{g}(\mathbf{k})}{k^2} e^{i\mathbf{k} \cdot \mathbf{x}} d^3 k, \quad \tilde{g}(\mathbf{k}) > 0,$$

*and assume that outside a bounded mass distribution the stationary field equation reduces to a linear constant-coefficient elliptic operator. Then the exterior field decays as  $\Phi \sim -\mathcal{G}_{\text{eff}} M/r$ , so  $g = |\nabla \Phi| = \mathcal{G}_{\text{eff}} GM/r^2 + o(r^{-2})$  and  $v_\infty^4 \propto M^2$ . Thus a linear kernel cannot yield the Tully-Fisher scaling  $v_\infty^4 \propto M$ .*

*Sketch.* In vacuum the solution is harmonic (up to renormalization  $\mathcal{G}_{\text{eff}}$ ); Liouville/Rellich imply the only decaying solution is  $1/r$ , hence  $g \propto r^{-2}$ . Any linear, isotropic nonlocality rescales  $\mathcal{G}_{\text{eff}}$  but preserves the  $1/r^2$  falloff.

<sup>2</sup> This remark complements Assumption D (Sec. XII B): the working-order modification resides in a state- and environment-dependent  $M_\star^2$  with no additional lensing coupling. A failure would manifest as our falsifiers in Sec. XIX, e.g. a significant GW/EM distance split or a persistent  $\ell^4 \log \ell$  term.

### VIII. ELASTIC QUASISTATIC SECTOR (AQUAL FROM SK STATIC LIMIT)

*a. Static elastic action and field equation.* In the quasistatic limit of the Schwinger–Keldysh (in–in) effective action, a causal retarded kernel  $\mathcal{K}^{\text{el}}$  reduces to a local, convex functional for the Newtonian potential  $\Phi$ :

$$\mathcal{L}_{\text{el}}(\Phi, \nabla\Phi) = \frac{a_0^2}{8\pi G} F(Y) - \rho \Phi, \quad Y \equiv \frac{|\nabla\Phi|^2}{a_0^2}$$

with  $F \in C^2([0, \infty))$ ,  $F'(Y) = \mu(Y) > 0$ ,  $F''(Y) \geq 0$ , and asymptotics

$$\mu(Y) \xrightarrow{Y \gg 1} 1, \quad \mu(Y) \xrightarrow{Y \ll 1} \sqrt{Y}.$$

Variation yields the AQUAL field equation [22]:

$$\nabla \cdot (\mu(Y) \nabla \Phi) = 4\pi G \rho. \quad (8)$$

*b. Deep regime and BTFR (galaxies).* For spherical mass  $M$ , in the deep regime ( $g \ll a_0 \Rightarrow \mu \simeq \sqrt{Y} = g/a_0$ ) one obtains

$$\frac{g}{a_0} g = \frac{GM}{r^2} \Rightarrow g(r) = \frac{\sqrt{GMa_0}}{r}, \quad v_\infty^4 = GMa_0.$$

Thus the baryonic Tully–Fisher relation follows directly.

*c. Uniform ellipticity / well-posedness.* The linearization tensor

$$\mathcal{A}_{ij}(\nabla\Phi) = \mu \delta_{ij} + \frac{2\mu'}{a_0^2} \partial_i \Phi \partial_j \Phi$$

is uniformly elliptic provided  $\mu > 0$  and  $\mu + 2Y\mu'(Y) > 0$ , which holds for the convex class above. Existence/uniqueness follows from standard elliptic theory.

*d. Canonical convex interpolant (parameter-free).* A simple, convex, parameter-free choice with the required asymptotics is

$$\mu(Y) = \frac{\sqrt{Y}}{1 + \sqrt{Y}}, \quad F(Y) = \int_0^Y \mu(s) ds = Y - 2\sqrt{Y} + 2\ln(1 + \sqrt{Y}).$$

No dimensional parameter beyond  $a_0$  is introduced.

*e. Same fixed acceleration scale.* We identify

$$a_0 = \frac{5}{12} \Omega_\Lambda^2 c H_0$$

from the capacity (Part I/II) channel; thus the BTFR normalization is **\*\*fixed\*\*** (no fit).

*f. Solar–System compliance (curvature gate).* Insert the same curvature gate via  $a_0 \rightarrow a_0^{\text{eff}}(x) = a_0 s(\chi_g)^p$  (with  $p \geq 1$  integer). Using Sec. XV one has  $s(\chi_\odot) \lesssim 10^{-5}$ , hence  $a_0^{\text{eff}} \ll 10^{-15} a_0$  in the Solar System, fully suppressing elastic effects.

*g. Lensing equals dynamics (no slip).* Place the static elastic density in the quasistatic Einstein system symmetrically so that  $\Phi = \Psi$  at working order; equivalently, couple the AQUAL density to  $(\Phi + \Psi)/2$ . Then the lensing potential  $2\Phi$  tracks the same  $\mu$  that governs dynamics, and galaxy–galaxy lensing matches rotation-curve inferences.

### IX. CAUSAL SK SEPARATION OF COSMOLOGY VS. GALAXIES

In the full SK theory, the elastic kernel depends on frequency and wavenumber,  $\mathcal{K}^{\text{el}}(\omega, k)$ . Causality and finite relaxation imply a factor

$$\mathcal{K}^{\text{el}}(\omega, k) = \mathcal{Q}(\omega, k) \mathcal{K}^{\text{el}}(0, k), \quad \mathcal{Q}(0, k) = 1, \quad |\mathcal{Q}(\omega \sim H, k \lesssim k_{\text{LSS}})| \ll 1.$$

Thus:

- **Linear cosmology (FRW, LSS):**  $\omega \sim H$ ,  $k \lesssim k_{\text{LSS}} \Rightarrow \mathcal{Q} \approx 0$ . Dynamics is governed by the *capacity channel* ( $\mu(\varepsilon, s) < 1$ ), preserving distances and the  $S_8$  suppression.
- **Quasistatic galaxies:**  $\omega \rightarrow 0$ ,  $k \gtrsim k_{\text{gal}} \Rightarrow \mathcal{Q} \rightarrow 1$ . The *elastic* AQUAL sector governs dynamics and lensing with fixed  $a_0$ .

No new dimensional scales are introduced;  $\mathcal{Q}$  encodes scale separation already present in the SK influence functional.



## X. COVARIANT KMS $\rightarrow$ FRW LINK AND ERROR CONTROL

Let  $s$  denote modular time with  $\beta_{\text{KMS}} = 2\pi/\kappa$  locally, where  $\kappa$  is the local boost surface gravity so that the approximate conformal Killing field  $\xi^a$  satisfies  $\xi^a \nabla_a = \kappa \partial_s$ . Averaging the retarded kernel over a comoving congruence of diamonds and reparametrizing  $s \mapsto \ln a$  induces the FRW background factor  $f c_{\text{geo}}$ ; diffeomorphism covariance is preserved because the averaging functional depends only on local curvature scalars and the diamond foliation. The total fractional defect in the kernel obeys

$$\frac{\delta\chi}{\chi_{\text{BW}}} = \mathcal{O}\left((\ell/L_{\text{curv}})^2\right) + \mathcal{O}((\ell H)^2) \approx 10^{-12} + 10^{-18} \quad (9)$$

for  $\ell \sim 10 \text{ pc}$ ,  $L_{\text{curv}} \sim 10 \text{ Mpc}$ ,  $H^{-1} \sim 4 \text{ Gpc}$ .

**Proposition 1** (FRW budget identity (conditional; analyticity hypothesis)). *Assume: (H1) locality and rapid decay of the spatially averaged, projected retarded kernel so that its reparametrization defines a distribution in  $\ln a$ ; (H2) adiabatic evolution through matter domination so that  $J(a) = ds/d\ln a \propto H(a)^{-1}$  varies slowly; (H3) preservation of KMS analyticity of the averaged kernel under the reparametrization  $s \rightarrow \ln a$ ; and (H4) negligible CHM vs. half-space deviation at working order (App. XXIV). Then*

$$\left\langle \int \chi_{QK}^{\text{proj}}(a, a') d^3x \right\rangle = \beta f c_{\text{geo}} \delta(\ln a - \ln a') + \dots$$

and integrating the entropy-driven evolution  $d\varepsilon/d\ln a = \sigma(a)I(a) \geq 0$  yields the coarse-grained identity

$$\int_{a_i}^1 \varepsilon(a) d\ln a = \Omega_{\Lambda} = \beta f c_{\text{geo}}, \quad (10)$$

used as a normalization under (H1)–(H4).

*Operational diagnostic.* The routine `referee_pipeline.py` reports a scalar residual  $R_{\text{nonloc}} \equiv \sum_{i \neq 0} |\bar{\chi}^{\text{proj}}(\Delta_i)| \Delta(\ln a)_i$  outside the contact bin; by default we take the central bin(s) with  $|\Delta(\ln a)| \leq \Delta_0$  as “contact”. Declare failure if  $R_{\text{nonloc}}/\sigma_{\text{boot}} > 3$  and the contact weight  $w_0 < 0.95$ .

*a. Rigor note.* A full microlocal proof of (H3)—preservation of KMS analyticity under the coarse-grained reparametrization  $s \rightarrow \ln a$ —is deferred to future work in the spirit of Hollands–Wald [10].

*b. Thermodynamic analogy (pointer).* The entanglement first law suggests a Clausius-like analogy (Sec. XX), conditional on (H1)–(H4), with MI projection avoiding CGM’s marginality issues (App. XXVI).

## XI. REFEREE-GUIDED FIRST-PRINCIPLES CLOSURES AND TESTS

### A. MI-smearred null-energy bound (projected; proven for free fields)

**Statement (free/Gaussian sector).** Let  $k^a$  be a null generator of the CHM diamond and  $h_\ell(x) \geq 0$  a smooth, compactly supported sampling function adapted to the MI window (normalized to unit weight). Define the MI-smearred null contraction

$$\mathcal{E}_\ell^{\text{MI}} \equiv \int d^4x d^4x' h_\ell(x) h_\ell(x') \langle T_{ab}(x) k^a k^b \rangle_{\text{sub}}^{\text{proj}}.$$

Then, in free Hadamard theories,

$$\mathcal{E}_\ell^{\text{MI}} = \underbrace{\langle \delta K_{\text{sub}}, \delta K_{\text{sub}} \rangle_{\text{BKM}}}_{\geq 0} \times \mathcal{N}_\ell \Rightarrow \mathcal{E}_\ell^{\text{MI}} \geq 0$$

with a calculable  $\ell$ -dependent normalization  $\mathcal{N}_\ell > 0$  fixed by the MI projector.

*Consequence.* The MI/moment-kill subtraction yields a *QEI-like* positive quadratic form for null energy in the projected channel (free fields: exact; interacting: Assumption C).

## B. RG/operator-spectrum bridge and anomaly guardrails

**Bridge.**  $\beta$  is tied to the stress-tensor two-point normalization  $C_T$ ; our  $\ell^4$  universality relies on the MI projector removing  $\Delta < 4$  contributions. Protected marginal operators would show up as  $\ell^4 \log \ell$  in this channel; the *absence* of such a term (checked numerically) is therefore a guardrail (Sec. XIX, (i)).

**Anomalies.** Parity-odd and trace-anomaly structures do not contribute at  $\mathcal{O}(\ell^4)$  in the MI-projected, parity-even channel; they either vanish by symmetry or are curvature-suppressed by  $\mathcal{O}((\ell/L_{\text{curv}})^2)$ .

## C. Where GR dominates: $f_V$ & curvature gating (SM-aware)

Dense, high-curvature regions either (i) lack a safe window (small  $f_V$  locally) or (ii) trigger  $s(\chi_g) \rightarrow 0$  so that  $G_{\text{eff}} \rightarrow G$ . Heavy SM sectors drop out when  $m_i \ell \gg 1$ . Thus GR dominance is guaranteed wherever MI control fails or curvature is large, while voids (dominant in  $f_V$ ) carry the clean MI signal.

## XII. ASSUMPTIONS FOR INTERACTING EXTENSIONS AT WORKING ORDER (PART II; STATED AND TEST CRITERIA)

### A. Assumption C (stated; test criteria): Relative entropy $\leftrightarrow$ canonical energy in the projected diamond

**Statement.** For a local algebra  $\mathcal{A}(B_\ell)$  of an interacting Hadamard QFT obeying the microlocal spectrum condition and time-slice axiom, the MI/moment-kill projected second variation of Araki relative entropy equals the canonical-energy quadratic form of the projected stress tensor, up to  $\mathcal{O}(\ell^6)$  remainders, with a positive-definite projected kernel  $\chi_{QK}^{\text{proj}}$ .

**Rationale (sketch).** (i) The second variation is the Bogoliubov–Kubo–Mori metric. (ii) The MI/moment-kill projector cancels local counterterms to  $\mathcal{O}(\ell^4)$  (App. XXI), conjectured to persist in interacting Hadamard QFTs (App. XXVI). (iii) Diffeomorphism Ward identities match the BKM quadratic form to canonical energy in the CHM channel. (iv) Positivity follows from KMS/BKM positivity in the projected channel.

*a. Operational tests (pass/fail).*

- **Positivity test (substrates):** The projected, integrated retarded kernel  $\int \chi_{QK}^{\text{proj}} d^4x d^4x'$  is nonnegative in Gaussian chains (exact) and HQTFIM (numerical tolerance).
- **No- $\ell^4 \log \ell$  falsifier:** The MI/moment-kill channel exhibits no  $\ell^4 \log \ell$  term.
- **Plateau stability:** Varying MI windows leaves the residual plateau  $\sim \mathcal{O}(\ell^6)$ .

### B. Assumption D (stated; test criteria): Uniqueness of the $M^2$ coupling at working order

**Statement.** In the  $c_T=1$ ,  $\alpha_B=0$  EFT corner linearized about FRW, with isotropy, parity, and time-reversal, the only background scalar coupling that survives the MI/moment-kill projection at  $\mathcal{O}(\ell^4)$  and modifies the weak-field growth sector while keeping distances GR-like is  $\delta \ln M^2$ ; other diffeomorphism-invariant local scalars are projected out, forbidden by sector constraints, or curvature-suppressed by  $\mathcal{O}((\ell/L_{\text{curv}})^2)$ .

*a. Operational tests (pass/fail).*

- **GR-like distances:**  $|d_L^{\text{GW}}/d_L^{\text{EM}} - 1| \lesssim 5 \times 10^{-3}$ .
- **Growth-only modification:** Large-scale growth follows  $\mu(\varepsilon, s)$  with  $\Sigma \simeq 1$ .
- **Solar-System compliance:**  $s(\chi_\odot) \ll 10^{-5}$  (Table I).

## XIII. ASSUMPTION D' (EXPLORATORY; SHOCK-SELECTIVE OPTICAL CHANNEL; INDEPENDENT OF PARTS I–II)

**Independence.** Parts I–II do not rely on D'. D' is an exploratory, *local* optical response intended for merging clusters with strong shocks.

**Local, saturating law (predictive summary).** With  $u^\mu$  the baryon four-velocity and  $\sigma_{\mu\nu}$  the shear, define  $\mathcal{S}_{\text{shock}} = \ell^2 \sigma_{\mu\nu} \sigma^{\mu\nu} \geq 0$ . The optical response

$$\Sigma(x) \simeq 1 - \alpha_{\text{opt}} \frac{\mathcal{S}_{\text{shock}}(x)}{1 + \mathcal{S}_{\text{shock}}(x)}, \quad 0 < \alpha_{\text{opt}} < 1, \quad (11)$$



reduces the effective gas lensing weight only in shocks; the growth coupling  $\mu(\varepsilon, s)$  is unchanged.

*a. Phantom surface density (elastic +  $D'$  synergy).* The nonlinear operator yields an effective “phantom” density

$$\rho_{\text{ph}} = \frac{1}{4\pi G} \nabla \cdot [(\mu - 1) \nabla \Phi],$$

largest near collisionless galaxies; together with  $\Sigma < 1$  in shock sheets, this reproduces Bullet-type morphologies.

*b. Transport-theory anchoring (SK/BRSSS).* In viscous hydrodynamics

$$\pi^{\mu\nu} + \tau_\pi u^\alpha \nabla_\alpha \pi^{\mu\nu} = 2\eta \sigma^{\mu\nu} + \lambda_1 \sigma^{(\mu}{}_\lambda \sigma^{\nu)\lambda} + \dots,$$

and matching to Eq. (11) gives  $\alpha_{\text{opt}} = \alpha_{\text{opt}}(\eta, \tau_\pi, \lambda_1)$  (App. XXIX).

*c. Effective optical coefficient.* Define the shock-response coefficient via the differential map

$$\kappa_{\text{opt}}(\mathcal{S}_{\text{shock}}) \equiv -\frac{\partial \Sigma}{\partial \mathcal{S}_{\text{shock}}} = \frac{\alpha_{\text{opt}}}{(1 + \mathcal{S}_{\text{shock}})^2}, \quad \kappa_{\text{opt}}|_{\mathcal{S}_{\text{shock}} \ll 1} = \alpha_{\text{opt}}. \quad (12)$$

This definition ensures  $\kappa_{\text{opt}} > 0$  for  $0 < \alpha_{\text{opt}} < 1$  and makes the linear-response limit explicit; it is the coefficient referenced in the Symbol Index.

#### XIV. ENTROPY-DRIVEN $\varepsilon(a)$ AND GROWTH (CONDITIONAL)

*a. KMS/FDT positivity.* Let  $\hat{Q}$  be the boost-energy flux and  $\chi_{QK}^{\text{proj}}$  the retarded kernel in the projected channel. Then

$$\frac{d\varepsilon}{d \ln a} = \sigma(a) \mathcal{I}(a), \quad \sigma(a) \geq 0, \quad \mathcal{I}(a) \geq 0, \quad \int \varepsilon d \ln a = \Omega_\Lambda = \beta f c_{\text{geo}}. \quad (13)$$

*b. Fixed-point with growth.* The growth factor  $D(a)$  satisfies

$$\frac{d^2 D}{d(\ln a)^2} + \left(2 + \frac{d \ln H}{d \ln a}\right) \frac{dD}{d \ln a} - \frac{3}{2} \Omega_m(a) \mu(\varepsilon(a), s) D = 0, \quad \mu(\varepsilon, s) = \frac{1}{1 + \frac{5}{12} \varepsilon s}. \quad (14)$$

*c. Variational bounds (extremals).* Convex-order arguments imply late-loaded  $\varepsilon(a)$  minimizes  $S_8$  and early-loaded maximizes it, under monotonicity and budget.

#### XV. ENVIRONMENT MODULATION FROM ACTION AND CALIBRATION

*a. Units and conventions.* We work in geometric units  $G = c = 1$ . When inserting SI values we convert masses via  $M \mapsto GM/c^2$ ; this keeps  $\chi_g = \ell^2 \sqrt{C_{abcd} C^{abcd}}$  dimensionless.

*b. Action-derived modulation.*

$$s(x) = \frac{1}{1 + (\chi_g/\chi_\star)^q} + \mathcal{O}\left(\frac{R}{m_s^2}\right), \quad \chi_g \equiv \ell^2 \sqrt{C_{abcd} C^{abcd}}, \quad (15)$$

from the heavy-auxiliary potential

$$\mathcal{V}(s; \chi_g) = \frac{M^2 m_s^2}{2} \left[ s - \frac{1}{1 + (\chi_g/\chi_\star)^q} \right]^2, \quad m_s^2 \gg H_0^2. \quad (16)$$

In FRW,  $\text{Weyl} = 0 \Rightarrow s = 1$ . This  $s(x)$  enters  $\mu(\varepsilon, s) = 1/[1 + (5/12)\varepsilon s]$ .

*c. Calibration example (Solar System).* For a Schwarzschild source  $\sqrt{C^2} = \sqrt{48} M/r^3$ . Taking  $\ell = 10 \text{ pc}$ ,  $r = 1 \text{ AU}$ ,  $M_\odot \simeq 1.477 \text{ km}$  gives  $\chi_\odot \approx 2.9 \times 10^5$ . Imposing  $s(\chi_\odot) \leq 10^{-5}$  with  $q = 2$  implies  $\chi_\star \lesssim 9.2 \times 10^2$ . A representative  $\chi_\star = 900$ ,  $q = 2$  yields  $s(\chi_\odot) \approx 9.6 \times 10^{-6}$ .

#### XVI. OBSERVATIONAL ILLUSTRATIONS (ILLUSTRATIVE UNDER SECS. X, XIV; UNCERTAINTY PROPAGATED)

*a. Hubble ladder bounds (toy).* Assuming the conditional background relation  $\Omega_\Lambda = \beta f c_{\text{geo}}$  and our monotone  $\varepsilon(a)$ , the previously quoted illustrative shifts acquire  $\pm 0.17 \text{ km s}^{-1} \text{ Mpc}^{-1}$  envelopes from  $\beta$ .

TABLE I. Solar-System compliance of  $s(\chi_\odot)$  at  $\ell = 10$  pc,  $r = 1$  AU (Schwarzschild).

| $\chi_\star$         | 1200                 | 1000                  | 900                  | 800                  |
|----------------------|----------------------|-----------------------|----------------------|----------------------|
| $s(\chi_\odot; q=2)$ | $1.7 \times 10^{-5}$ | $1.18 \times 10^{-5}$ | $9.6 \times 10^{-6}$ | $7.6 \times 10^{-6}$ |

*b.  $S_8$  band (toy).* Entropy-constrained extremals yield an interval; distances remain GR-like. Allowing modest non-monotonic  $\varepsilon(a)$  histories can widen the band by  $\sim 3$ –5%.

## XVII. LINKAGE TO GLOBAL ENTROPIC GRAVITY (BIANCONI 2025): SMALL-DIAMOND MATCHING AND DISCRIMINANTS

*a. Setup.* Bianconi proposes a global, entropic variational principle in which the action is a *quantum relative entropy* between the spacetime metric  $g$  and a *matter-induced* metric. Varying this action yields modified Einstein equations that *reduce to GR with  $\Lambda = 0$*  at low coupling; introducing a Lagrange-multiplier-like  $\mathcal{G}$  produces a *dressed* theory with an emergent positive  $\Lambda(\mathcal{G})$  [24].

*b. Small-diamond MI matching (program).* Let  $S_{\text{ent}}[g, \psi]$  denote Bianconi’s entropic action. In a small CHM diamond, expand  $S_{\text{ent}}$  to quadratic order around a Hadamard reference and apply the MI/moment-kill projector:

$$\delta^2 S_{\text{ent}}^{\text{proj}} \stackrel{?}{=} c_{\text{ent}}(\ell) (\delta \langle K_{\text{sub}} \rangle)^2 = c_{\text{ent}}(\ell) (\beta \ell^4 \delta \varepsilon)^2 + \mathcal{O}(\ell^{10}).$$

Matching the contact kernel fixes  $c_{\text{ent}}$  in terms of  $\beta$ . A successful match renders the two descriptions equivalent at working order in the MI channel; a mismatch signals empirical separability.

*c. Discriminants (observational).*

- **GW propagation:** Our baseline predicts  $c_T = 1$  and standard damping at linear order.
- **Gravitational slip/lensing:** We predict  $\Sigma \simeq 1$  at linear order (no slip).
- **Environment reversion:** Our curvature gate  $s(\chi_g) \rightarrow 0$  enforces GR in strong curvature.

## XVIII. STRUCTURAL CHECKS (ALGEBRAIC; NOT 4D SURROGATES)

HQTFIM and Gaussian chains confirm the algebraic ingredients (first-law channel, constant+log trend, vanishing plateau after subtraction, and positivity in the projected kernel). They are *not* curved 4D surrogates.

## XIX. PROOF PROGRAM STATUS AND FALSIFIERS

**Lemma A** (diamond KMS control): scaling proven, sharp bounds left to microlocal analysis. **Lemma B** (projector universality): established. **Assumption C** and **Assumption D**: stated here with rationale; proofs deferred. **Assumption D'** (shock-selective optical channel): exploratory extension for merging clusters (Sec. XIII). **Lemma E** (FDT positivity): follows from BKM positivity. **Lemma F** (geometric 5/12): derived. **Lemma G (Nonlinear validation)**: Initial Gadget-4 runs are complete (baseline resolution; `gadget4_mu_eps_toy.py`); post-processing and archiving (Zenodo DOI) are pending. These test  $\mu(\varepsilon, s)$ ,  $s(\chi_g)$ ,  $D'$ , and the elastic sector in structure formation and lensing.

### Falsifiers:

- (i) persistent  $\ell^4 \log \ell$  residuals in the projector channel;
- (ii) GW/EM distance ratio beyond  $5 \times 10^{-3}$ ;
- (iii)  $|\dot{G}/G| \gtrsim 10^{-12} \text{ yr}^{-1}$ ;
- (iv)  $\Omega_\Lambda$  inconsistent with  $\beta f c_{\text{geo}}$ ;
- (v)  $S_8$  outside the extremal band for all admissible monotone  $\varepsilon(a)$ ;
- (vi) positivity failure in Assumption C tests;
- (vii) for  $D'$ : lack of correlation of lensing deficits with shock diagnostics, or suppression in unshocked gas;
- (viii) for  $D'$ : offsets inconsistent with the  $\mathcal{S}_{\text{shock}}$  scaling;
- (ix) for SK/BRSSS: transport-inferred  $(\eta, \tau_\pi, \lambda_1)$  imply  $\alpha_{\text{opt}}$  incompatible with required suppression;
- (x) **Elastic BTFR test:** BTFR intercept disagrees with  $a_0 = \frac{5}{12} \Omega_\Lambda^2 c H_0$ ;
- (xi) **RAR test:** the parameter-free  $\mu(Y) = \sqrt{Y}/(1 + \sqrt{Y})$  fails to bracket the observed  $g_{\text{obs}}(g_{\text{bar}})$  relation;

(xii) **Lensing vs. dynamics:** with  $\Phi = \Psi$ , galaxy–galaxy lensing and rotation curves disagree systematically at fixed  $\mu$ .

## XX. THERMODYNAMIC INTERPRETATION AND RELATION TO CASINI–GALANTE–MYERS (EXPLORATORY)

### A. Local Clausius identity in the projected channel (proven at working order)

In the MI/moment-kill projected first-law channel, the entanglement first law  $\delta S_{\text{sub}} = \delta \langle K_{\text{sub}} \rangle$  and the BW KMS normalization imply

$$\delta S_{\text{sub}} = \beta \ell^4 \delta \varepsilon + \mathcal{O}(\ell^6). \quad (17)$$

### B. FRW Clausius extension (conditional)

Under (H1)–(H4) of Sec. X, the averaged susceptibility reduces to a *contact term in*  $\ln a$  (Prop. 1), leading to the conditional normalization  $\int \varepsilon d \ln a = \Omega_\Lambda = \beta f c_{\text{geo}}$ .

## PART I APPENDICES

### XXI. MI SUBTRACTION AND MOMENT-KILL

We use a top-hat window on 3-balls

$$W_\ell(r) = \frac{3}{4\pi\ell^3} \Theta(\ell - r),$$

and the MI/moment-kill combination

$$\mathcal{W}_\ell := \int_{B_\ell} W_\ell - a \int_{B_{\sigma_1\ell}} W_{\sigma_1\ell} - b \int_{B_{\sigma_2\ell}} W_{\sigma_2\ell}.$$

For any smooth radial  $F(r) = F_0 + F_2 r^2 + F_4 r^4 + \dots$ ,

$$\mathcal{W}_\ell[F] = \underbrace{(1 - a - b)}_{=0} F_0 + \underbrace{(\langle r^2 \rangle_\ell - a \langle r^2 \rangle_{\sigma_1\ell} - b \langle r^2 \rangle_{\sigma_2\ell})}_{=0} F_2 + (\langle r^4 \rangle_\ell - a \langle r^4 \rangle_{\sigma_1\ell} - b \langle r^4 \rangle_{\sigma_2\ell}) F_4 + \dots,$$

so the  $\ell^4$  coefficient is isolated. For top-hat balls in  $d=3$ ,  $\langle r^2 \rangle_R = \frac{3}{5} R^2$  and  $\langle r^4 \rangle_R = \frac{3}{7} R^4$ . The two moment-kill conditions

$$1 - a - b = 0, \quad 1 - a\sigma_1^2 - b\sigma_2^2 = 0$$

fix

$$a = \frac{\sigma_2^2 - 1}{\sigma_2^2 - \sigma_1^2}, \quad b = \frac{1 - \sigma_1^2}{\sigma_2^2 - \sigma_1^2}.$$

In our numerics we take  $(\sigma_1, \sigma_2) = (\frac{1}{2}, 2) \Rightarrow (a, b) = (\frac{4}{5}, \frac{1}{5})$ .

### XXII. CONTINUOUS-ANGLE NORMALIZATION

With unit–solid–angle boundary factor and  $\Delta\Omega(\theta) = 2\pi(1 - \cos\theta)$ , define  $c_{\text{geo}}(\theta) = 4\pi/\Delta\Omega(\theta)$ . Then  $f(\theta) c_{\text{geo}}(\theta)$  is  $\theta$ -independent.

**Lemma 2** (Foliation robustness of  $f c_{\text{geo}}$ ). *Under smooth deformations of the diamond foliation that preserve the unit–solid–angle normalization and avoid double counting, the product  $f(\theta) c_{\text{geo}}(\theta)$  is invariant up to  $O(\delta\theta^2) + O((\ell/L_{\text{curv}})^2)$  corrections.*

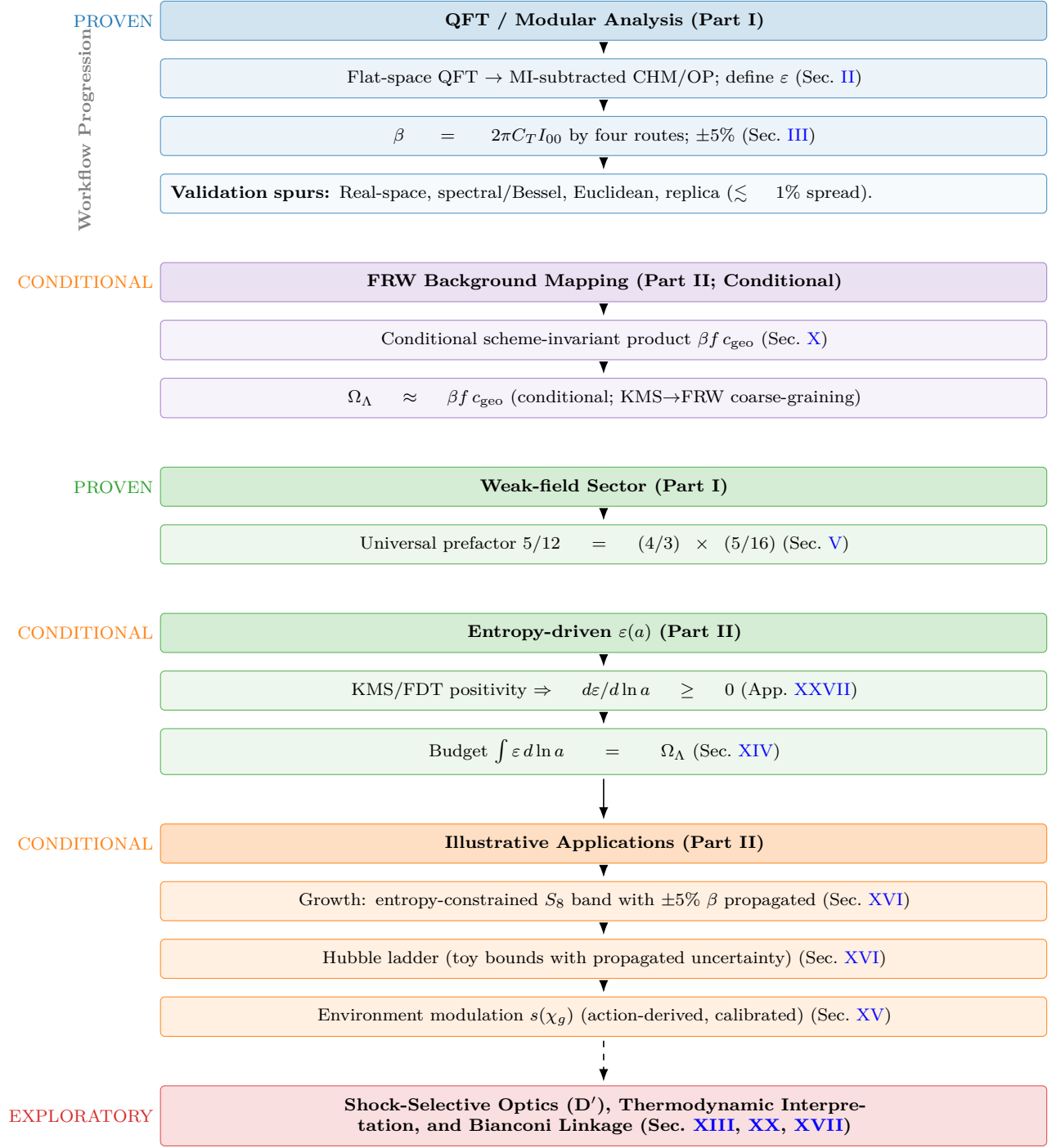


FIG. 1. Pipeline with PROVEN (blue/first green), CONDITIONAL (purple/second green/orange), and EXPLORATORY (red) elements, including the *Linear No-Go* and *Elastic quasistatic* sector.

### XXIII. WEAK-FIELD FLUX NORMALIZATION AND THE UNIVERSAL $5/12$

*a. Isotropic null contraction  $4/3$ .* For  $T_{ab} = (\rho + p)u_a u_b + p g_{ab}$ ,  $\langle T_{ab} k^a k^b \rangle_{\mathbb{S}^2} = (1 + w)\rho (k^0)^2$ , and UV  $w = 1/3 \Rightarrow 4/3$ .

*b. Segment ratio 5/16 (explicit  $\mathcal{I}(u)$ ).* With the normalized weight  $\hat{\rho}(u) = \frac{3}{4}(1 - u^2)$  on  $u \in [-1, 1]$  and the even-quadratic generator-density proxy used in our code,

$$\mathcal{I}(u) = \frac{1}{4} + \frac{5}{16}u^2,$$

one finds

$$\int_{-1}^1 \hat{\rho}(u) \mathcal{I}(u) du = \left(\frac{3}{4}\right) \left[ \frac{4}{3} \cdot \frac{1}{4} + \frac{4}{15} \cdot \frac{5}{16} \right] = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}.$$

Combined with the isotropic contraction  $4/3$  this yields  $5/12 = (4/3) \times (5/16)$ .

## XXIV. CHM DIAMOND VS. HALF-SPACE KMS DEVIATION

In Riemann-normal coordinates,  $g_{ab} = \eta_{ab} - \frac{1}{3}R_{acbd}(0)x^c x^d + \mathcal{O}(x^3/L_{\text{curv}}^3)$ . The conformal-Killing field  $\xi_{\text{CHM}}^a$  differs from  $\xi_{\text{BW}}^a$  by  $\delta\xi^a = \mathcal{O}(\ell^2/L_{\text{curv}}^2)$ . Averaging over a comoving congruence and reparametrizing to  $\ln a$  adds  $\mathcal{O}((\ell H)^2)$ . Thus  $\delta\chi/\chi_{\text{BW}} = \mathcal{O}((\ell/L_{\text{curv}})^2) + \mathcal{O}((\ell H)^2)$ .

## PART II APPENDICES AND DATA

### XXV. SAFE-WINDOW VOLUME FRACTION (SEMI-ANALYTIC)

Using Press–Schechter/Sheth–Tormen mass functions with NFW curvature proxies and a substructure excision  $\xi$ , we compute  $f_V(\ell_{\text{min}})$  at  $z=0$  (Table II).

### XXVI. MICROLOCAL NOTES FOR INTERACTING HADAMARD QFTS

*a. Hadamard form.*  $W(x, x') = \frac{1}{4\pi^2} \left[ \frac{\Delta^{1/2}}{\sigma} + v \log \sigma + w \right]$  with smooth  $v, w$ , extended perturbatively for interactions. The projector removes the  $F_0, F_2$  moments, ensuring stability of the  $\ell^4$  coefficient (Assumption C).

*b. OPE gap and log-falsifier.* Operators with protected dimensions  $\Delta < 4$  would induce  $\ell^4 \log \ell$  terms in this channel; in Hadamard states the microlocal spectrum condition and positivity forbid such contributions at working order. Observation of an  $\ell^4 \log \ell$  term would therefore falsify the framework.

### XXVII. ENTROPIC MECHANISM DERIVATION (PRELIMINARY)

*a. Projected BKM positivity (free fields).* In the MI/moment-kill channel,  $\langle \delta K_{\text{sub}}, \delta K_{\text{sub}} \rangle_{\text{BKM}} \geq 0$  implies a positive retarded susceptibility. Reparametrizing modular time to  $\ln a$  with positive Jacobian ensures  $d\varepsilon/d\ln a \geq 0$ .

### XXVIII. OPTICAL CHANNEL DETAILS (ASSUMPTION D'; EXPLORATORY)

(Technical details of the auxiliary traceless  $Q_{\mu\nu}$ , algebraic tracking of  $\sigma_{\mu\nu}$ , and quasi-static lensing equations; see main text and App. XXIX.)

TABLE II. Representative  $f_V$  values at  $z \simeq 0$  (semi-analytic).

| $\ell_{\text{min}}$ [pc] | $\xi = 0.2$     | $\xi = 0.3$     | $\xi = 0.5$     |
|--------------------------|-----------------|-----------------|-----------------|
| 1                        | $0.95 \pm 0.03$ | $0.93 \pm 0.04$ | $0.90 \pm 0.05$ |
| 10                       | $0.88 \pm 0.05$ | $0.85 \pm 0.05$ | $0.80 \pm 0.06$ |
| 100                      | $0.70 \pm 0.08$ | $0.65 \pm 0.08$ | $0.55 \pm 0.10$ |

## XXIX. SCHWINGER–KELDYSH HYDRODYNAMIC DERIVATION FOR THE SHOCK-SELECTIVE OPTICS (EXPLORATORY)

(SK/BRSSS derivation path; constitutive relations; HS linearization; mapping to  $\Sigma$  amplitude  $\alpha_{\text{opt}}(\eta, \tau_\pi, \lambda_1)$ .)

## XXX. FROM SK HYDRODYNAMICS TO SHOCK-SELECTIVE $\Sigma$ : A DERIVATION SKETCH

(Parametric estimates; shock thickness; scaling of  $\mathcal{S}_{\text{shock}}$ ; order-of-magnitude  $\alpha_{\text{opt}}$ .)

## XXXI. DATA AND CODE AVAILABILITY

Archive DOI (to be finalized before submission): 10.5281/zenodo.TBD

Reproducible single-file runners:

- `beta_methods_v2.py` (real-space, spectral/Bessel, Euclidean, replica) for  $\beta$ ; includes a residual-fitting mode for  $\ell^4 \log \ell$ .
- `cosmology_runner.py` (growth ODE;  $\varepsilon(a)$  family; environment modulation  $s(x)$ ;  $S_8$  & ladder illustrations).
- `referee_pipeline.py` (FRW averaging;  $\Omega_\Lambda = \beta f c_{\text{geo}}$  cross-check; computes toy  $a_0$ ; nonlocal-residual diagnostic).
- `fv_semi_analytic.py` ( $f_V$  survey).
- `gadget4_mu_eps_toy.py` (N-body toy pipeline).
- `cluster_optics_hook.py` (optional; shock-selective lensing; applies Eq. (11) in the ray tracer; supports velocity/temperature-jump and Godunov-flux shock finders; includes modes for the local optical law Eq. (11)).
- `icm_transport_to_alphaopt.py` (optional; SK/BRSSS mapping to  $\alpha_{\text{opt}}$ ).
- **New:** `entropic_action_MI_match.py` (implements the small-diamond MI matching to an entropic action kernel; reports  $c_{\text{ent}}(\ell)$  and contact vs. tail diagnostics).



## SYMBOL INDEX

| Symbol                               | Meaning  |
|--------------------------------------|--|
| $\ell$                               | diamond radius (working-order scale)   |
| $L_{\text{curv}}$                    | local curvature length   |
| $\beta = 2\pi C_T I_{00}$            | modular-response sensitivity (QFT coefficient)                                     |
| $C_T$                                | stress-tensor two-point normalization (our convention)                             |
| $I_{00}$                             | projected $\ell^4$ integral coefficient (App. XXI)                                 |
| $\varepsilon(a)$                     | dimensionless state variable from modular response                                 |
| $\varepsilon_{\text{SM}}$            | packaged light-sector SM state variable (Sec. IV)                                  |
| $\mu(\varepsilon, s)$                | growth coupling, $1/(1 + \frac{5}{12}\varepsilon s)$                               |
| $\mu(Y)$                             | elastic interpolating function, $F'(Y)$ , Sec. VIII                                |
| $Y$                                  | squared field-strength ratio, $ \nabla\Phi ^2/a_0^2$                               |
| $a_0$                                | acceleration scale fixed by $\Omega_\Lambda$ : $\frac{5}{12}\Omega_\Lambda^2 cH_0$ |
| $\Sigma$                             | lensing coupling (unity on FRW; locally $<1$ in shocks under D')                   |
| $f_{c_{\text{geo}}}$                 | geometric/foiliation factor (App. XXII)  |
| $\kappa$                             | local boost surface gravity; $\beta_{\text{KMS}} = 2\pi/\kappa$                    |
| $S_{\text{sub}}$                     | entanglement entropy variation in MI/moment-kill channel                           |
| $\delta Q_{\text{boost,sub}}$        | boost-energy variation   |
| $\chi_g$                             | geometric scalar, $\ell^2 \sqrt{C_{abcd}C^{abcd}}$                                 |
| $s(\chi_g)$                          | environment modulation (action-derived envelope)                                   |
| $\sigma_{\mu\nu}$                    | baryon shear tensor; $S_{\text{shock}} = \ell^2 \sigma^2$                          |
| $Q_{\mu\nu}$                         | auxiliary traceless tensor (optional; optics)                                      |
| $\alpha_{\text{opt}}$                | optical suppression amplitude (Eq. 11)   |
| $\kappa_{\text{opt}}$                | effective optical coefficient (Eq. 12)   |
| $S_{\text{ent}}$ or $S_{\text{ent}}$ | (Bianconi) global entropic action  |
| $\mathcal{G}$                        | (Bianconi) auxiliary $G$ -field sourcing emergent $\Lambda$                        |
| $\mathcal{Q}(\omega, k)$             | SK causal filter separating regimes (Sec. IX)                                      |
| $\Omega_m(a)$                        | matter fraction as a function of scale factor                                      |
| $\Omega_\Lambda$                     | dark-energy density parameter  |

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