Emergent State-Dependent Gravity from Local Information Capacity: A Conditional Thermodynamic Derivation with Scheme-Invariant Cosmological Mapping

[clg]¹

¹[TBD Institution(s)]
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Core hypothesis. Each proper frame carries a finite quantum information capacity. Approaching this bound triggers a *state-dependent response* that preserves causal stitching with neighboring frames. *Kinematics remain GR-like*: we do not alter null geometry used by EM/GW luminosity distances. The response is *dynamical* (weak-field coupling), not kinematical (no extra time dilation beyond GR).

Scope and conditionality. All quantitative claims are conditional on a single working assumption: (A2) the Clausius relation $\delta Q = T \, \delta S$ with Unruh normalization holds for small, near-vacuum local Rindler wedges (the safe window). Within this regime we establish an equivalence principle for modular response (EPMR): after mutual-information subtraction with moment-kill, the ℓ^4 modular coefficient equals the flat-space value at working order, while curvature dressings enter at $\mathcal{O}(\ell^6)$. See Theorem 1 for the working-order statement and error control.

Main outcomes. (i) A microscopic sensitivity β from MI-subtracted modular Hamiltonians in flat-space QFT (Casini–Huerta–Myers balls, Osborn–Petkou normalization); (ii) a once-and-for-all geometric normalization with continuous-angle invariance showing only the product $\beta f c_{\rm geo}$ is physical; (iii) a conditional, scheme-invariant mapping $\Omega_{\Lambda} = \beta f c_{\rm geo}$ for the FRW zero mode; and (iv) a weak-field flux law with a universal geometric prefactor 5/12, implying $a_0 = (5/12) \Omega_{\Lambda}^2 c H_0$. We keep the distance sector GR-like ($\alpha_M = 0$ there), and we enforce $|d_L^{\rm GW}/d_L^{\rm EM} - 1| \leq 5 \times 10^{-3}$.

Consequences. With no cosmological inputs, $\Omega_{\Lambda} = \beta f c_{\rm geo} \approx 0.685$ and $a_0 = (5/12) \, \Omega_{\Lambda}^2 \, c \, H_0$. An entropic state-action law $(\Delta S \geq 0)$ determines a monotone $\varepsilon(a)$ that modulates the weak-field response $\mu(\varepsilon) = 1/(1+\eta\,\varepsilon)$ with $\eta = 5/12$, suppressing growth and yielding $S_8 \simeq 0.788$ (-7.4% vs. ΛCDM), while EM/GW distances remain GR-like. An illustrative, capped environment-gated application to a SH0ES-like catalog nudges $H_0: 73.0 \rightarrow 71.32$ (SN cap only) and to 70.89 (SN+small Cepheid term), trending toward TRGB/Planck without altering null geometry. Explicit falsifiers and hygiene checks are stated.

I. INTRODUCTION: CORE INSIGHT AND CONDITIONAL SCOPE

- a. High level summary. We hypothesize that the geometric side of Einstein's equations exhibits a local, state-dependent response because each small spacetime wedge has finite information capacity. As capacity is approached, the Clausius relation enforces a compensating response so adjacent wedges remain causally stitched. Kinematics (null cones, EM/GW distances) stay GR-like; all changes are dynamical (response strength in weak fields). Jacobson's horizon thermodynamics is recovered as the stationary-horizon special case. All claims here are conditional on (A2); if (A2) fails, the construction must be revised. Our working-order result is stated as Theorem 1 (App. I).
- b. GR limit (distance sector). In the limit of constant information capacity $\nabla_a M^2 = 0$ (equivalently $\alpha_M \to 0$), the construction collapses to standard GR recovering Einstein's equations with $c_T = 1$ and GR light-cone geometry. Throughout we keep $\alpha_M = 0$ in the distance sector and confine any late-time variation to the growth/response sector (Sec. X).
- c. State variable and coupling. We define a dimensionless state variable $\varepsilon(x)$ encoding fractional deviations of local capacity from its vacuum reference and parameterize

$$\frac{\delta G}{G} = -\beta \, \delta \varepsilon(x),\tag{1}$$

with β calculable from flat-space QFT (Sec. IV). The weak-field response is encoded by

$$\mu(\varepsilon) \equiv \frac{G_{\text{eff}}}{G_N} = \frac{1}{1 + \eta \, \varepsilon}, \qquad \eta = \frac{5}{12},$$
 (2)

so $\mu \to 1$ in strong fields (GR recovery) and $\mu < 1$ in weak fields (gentle dynamical slowdown). Why $n = \frac{5}{10}$. The coefficient follows from the same unit-solid-angle Noether normalization when $\mu = \frac{5}{10}$.

Why $\eta = \frac{5}{12}$. The coefficient follows from the same unit-solid-angle Noether normalization used in the FRW zero mode. Coarse-graining the $\nabla \nabla M^2$ terms over the CHM wedge family yields a quasilinear flux law with a universal boundary-segment ratio; the isotropic null contraction contributes (4/3) and the segment geometry contributes (5/16), giving (4/3) × (5/16) = 5/12. Appendix J shows the identical factor fixing the static acceleration scale $a_0 = (5/12)\Omega_{\Lambda}^2 cH_0$; using the same bookkeeping in the weak-field $\mu(\varepsilon)$ guarantees scheme/angle invariance.

- d. What is fixed vs. what is assumed. Fixed once: wedge family (ball \rightarrow diamond), generator density, Unruh normalization, unit–solid–angle boundary factor. Assumed: (A2) Clausius with Unruh in the safe window (Def. 1); Hadamard state; small perturbations. Consequence: the geometric mapping is angle-invariant (Sec. VII C); only $\beta f c_{\text{geo}}$ is physical.
- e. Clean mapping statement. Within the safe window and EPMR working order, the FRW zero mode satisfies the conditional, scheme-invariant relation

$$\Omega_{\Lambda} = \beta f c_{\text{geo}}. \tag{3}$$

Physical intuition (causal stitching). A small diamond acts like a local "channel" for stress—energy: its modular dynamics fixes how much information the region can throughput per unit affine time. When the channel nears capacity, the response does not alter null kinematics (so EM/GW distances remain GR-like); instead it adjusts the dynamical coupling so adjacent diamonds exchange flux without tearing causal links. This is the content of our "causal stitching" language: the effective $G_{\rm eff}$ shifts just enough to preserve flux continuity while keeping the light cone intact. In strong fields the capacity is ample and GR is recovered; in weak fields capacity is tighter and a gentle dynamical slowdown appears.

II. ASSUMPTIONS AND DOMAIN OF VALIDITY

Definition 1 (Safe window). Choose ℓ obeying $\epsilon_{\rm UV} \ll \ell \ll \min\{L_{\rm curv}, \lambda_{\rm mfp}, m_i^{-1}\}$ for fields treated as massless; work with Hadamard states and small perturbations (relative entropy $O(\varepsilon^2)$). Within this window the MI-subtracted, moment-killed modular response is dominated by ℓ^4 and admits a Clausius balance with Unruh normalization.

Hypothesis 1 ((A2) Clausius with Unruh in the safe window). In the safe window, $\delta Q = T \, \delta S$ with Unruh temperature holds for Casini-Huerta-Myers (CHM) diamonds mapped from balls, with flux built from T_{kk} along approximate generators.

- a. Working-order theorem. Assuming Lemmas H.1–H.2 and Proposition H.1 (App. I), the small-diamond Clausius identity holds to $\mathcal{O}(\ell^4)$ with $\mathcal{O}(\ell^6)$ corrections; cf. Theorem 1. The marginal $\Delta = d/2$ compensator is summarized in Lemma 1.
- b. First-law domain. We use $\delta S = \delta \langle K \rangle$ only for CHM balls/diamonds and small perturbations of a Hadamard state; no general wedge theorem is claimed.

A. Failure modes of (A2) and explicit falsifiers

(A2) could fail if: (i) MI-subtracted flat-space modular data do not transfer to null diamonds; (ii) Unruh normalization fails in small, non-stationary wedges; or (iii) nonlocal state dependence spoils the local Clausius balance. Falsifiers (Sec. XII): (a) GW/EM luminosity distance ratios inconsistent with bounded α_M ; (b) laboratory/solar-system bounds revealing $|\dot{G}/G| \gtrsim 10^{-12} \, \mathrm{yr}^{-1}$; (c) precision cosmology favoring Ω_{Λ} inconsistent with the invariant $\beta f c_{\mathrm{geo}}$.

B. Pre-commitment and scheme invariance (convention hygiene)

We pre-commit to wedge family, generator density, Unruh normalization, and one of two bookkeepings (A or B) before any cosmological comparison. Physical predictions depend only on $\beta f c_{\text{geo}}$; the split between f and c_{geo} is conventional.

III. STATE METRIC AND VARIATIONAL CLOSURE

The operational definition of $\varepsilon(x)$ uses MI subtraction with moment-kill (App. C): for sufficiently small ℓ ,

$$\delta \langle K_{\text{sub}}(\ell) \rangle = (2\pi C_T I_{00}) \ell^4 \delta \varepsilon(x) + \mathcal{O}(\ell^6), \tag{4}$$

with C_T in the Osborn-Petkou (OP) convention and I_{00} the finite CHM kernel coefficient. Boxed normalization (one time).

$$\beta \equiv 2\pi C_T I_{00}$$
 (OP C_T ; I_{00} from MI-subtracted CHM response). (5)

A. Variational capacity closure: derivation (not a bare postulate)

Consider a Wald-like entropy functional on a small diamond with a local capacity constraint,

$$S_{\text{tot}} = \underbrace{\delta S_{\text{mat}}}_{\delta \langle K_{\text{sub}} \rangle} + \underbrace{\frac{\delta A}{4G(x)}}_{\delta S_{\text{grav}}} + \int \lambda(x) \left(\Xi_0 - \Xi(x)\right) d^4 x. \tag{6}$$

Using Eq. (4), extremization at fixed window yields

$$\delta\left(\frac{1}{16\pi G}\right) \propto \delta\Xi \qquad \Rightarrow \qquad \frac{\delta G}{G} = -\beta \,\delta\varepsilon,$$
 (7)

identifying β as the modular sensitivity that converts capacity variations into coupling variations.

IV. CALCULATION OF β

A. Setup: Modular Hamiltonian and first law

For a CFT vacuum reduced to a ball B_{ℓ} , the modular Hamiltonian is [4]:

$$K = 2\pi \int_{B_{\ell}} \frac{\ell^2 - r^2}{2\ell} T_{00}(\vec{x}) d^3x, \qquad \delta S = \text{Tr}(\delta \rho K) = \delta \langle K \rangle.$$
 (8)

B. Vacuum subtraction via mutual information

Compute mutual information between concentric balls and take $\ell_2 \to \ell_1$; UV divergences cancel. With moment-kill, contact and curvature–contact pieces drop out of $\delta \langle K_{\text{sub}} \rangle$, isolating the finite ℓ^4 coefficient I_{00} (App. C).

C. Mode decomposition and Euclidean reduction

We keep the isotropic (l = 0) piece of T_{00} and evaluate correlators after Wick rotation.

D. Numerical evaluation (scalar baseline)

Result and uncertainties.

$$\beta = 0.02086 \pm 0.00020$$
 (numerical) ± 0.00060 (MI-window/systematic), total $\sigma_{\beta} \simeq 0.00063$ (3.0%). (9)

Stability scans across $(\sigma_1, \sigma_2) \in [0.96, 0.999]^2$, $u_{\text{gap}} \in [0.2, 0.35]$, and grids $(N_r, N_s, N_\tau) \in [60, 160]^3$ show a plateau with $|\Delta \beta|/\beta \lesssim 0.5\%$.

Replication preset (for this manuscript). dps = 50, $(\sigma_1, \sigma_2) = (0.995, 0.99)$, $T_{\text{max}} = 6.0$, $u_{\text{gap}} = 0.26$, grids $(N_r, N_s, N_\tau) = (60, 60, 112)$. Residual moments: $M0_{\text{sub}} \approx -4.49 \times 10^{-51}$, $M2_{\text{sub}} \approx -1.84 \times 10^{-51}$. With $I_{00} = 0.1077748682$, $C_T = 3/\pi^4$, Eq. (5) gives $\beta = 0.02085542923$.

Positivity gates. Production runs enforce $|M0_{\rm sub}|, |M2_{\rm sub}| < 10^{-20}$ and $\delta \langle K_{\rm sub} \rangle \geq 0$.

E. Convergence and stability (numerical/systematic only)

We separate $\pm 3\%$ as numerical/systematic on β from conceptual uncertainties (A2 domain, marginal-only CGM coverage, species uplift), which are *not* folded into σ_{β} .

F. Independent QFT routes to β and robustness

To test that β is not an artifact of a single discretization, we implemented four independent determinations that share only the OP/CHM convention and the MI–subtracted first–law setup:

- (a) Real-space CHM kernel + MI subtraction (baseline). Direct quadrature of the CHM ball modular kernel in real space with mutual-information subtraction and moment-kill to remove r^0 and r^2 moments, isolating the finite ℓ^4 coefficient I_{00} (App. C).
- (b) Momentum-space spectral/Fourier-Bessel route. Evaluate the isotropic ($\ell = 0$) piece via a spectral representation for $\langle T_{00}T_{00}\rangle$ and integrate against the (Bessel) transform of the CHM weight; implement MI subtraction in k-space.
- (c) **Euclidean correlator time-slicing.** Wick rotate to τ , compute the τ -sliced correlation with independent quadrature and reconstruct the modular response; this provides an orthogonal check on the time dimension and on the handling of the Euclidean gap parameter.
- (d) Replica-geometry finite-difference check. A small- δn finite difference of replica entropies confirms contact-term cancellation and reproduces the finite I_{00} within numerical error.

Each route was scanned over MI windows $(\sigma_1, \sigma_2) \in [0.96, 0.999]^2$, Euclidean gaps $u_{\rm gap} \in [0.2, 0.35]$, and grids $(N_r, N_s, N_\tau) \in [60, 160]^3$. The method-to-method spread of β is $\leq 1\%$, and the total numerical/systematic uncertainty quoted in Eq. (5) remains $\simeq 3\%$ when including MI-window and discretization effects. Reporting the scheme-invariant combination βC_{Ω} further reduces apparent variation, since C_{Ω} is fixed by the unit-solid-angle normalization and is angle-invariant to $< 10^{-4}$ (Sec. VII C). A compact robustness summary is given in Table I.

TABLE I. Robustness of β across independent QFT routes and scans. Entries show the fractional deviation relative to the baseline real–space CHM result; ranges reflect MI–window and grid scans. The *invariant* product βC_{Ω} exhibits sub–percent dispersion.

Route	$\Delta eta/eta$	$\Delta(\beta C_{\Omega})/(\beta C_{\Omega})$
Real–space $CHM + MI$ (baseline)	0 (by definition)	0
Momentum-space spectral (Bessel)	$\lesssim 1\%$	$\lesssim 0.5\%$
Euclidean correlator time–slicing	$\lesssim 1\%$	$\lesssim 0.5\%$
Replica—geometry finite—difference	$\lesssim 1\%$	$\lesssim 0.5\%$

V. MICROPHYSICAL SUBSTRATE VALIDATIONS (HQTFIM AND GAUSSIAN CHAINS)

To test the structural assumptions used throughout our continuum calculation—namely (i) the entanglement first law in the linear window, (ii) a constant+log dependence on region size ℓ for the MI–subtracted modular response, and (iii) a near–zero residual "plateau" after subtracting $[1, \log \ell]$ —we implemented two independent microscopic testbeds:

- (a) an interacting transverse–field Ising chain (HQTFIM) solved by exact diagonalization, and
- (b) a free–fermion (Gaussian) chain, where the modular kernel on a block is known exactly from the correlation matrix. Both systems are *independent* of the continuum integrals that determine β , and therefore provide external checks of the assumptions entering the safe–window Clausius balance.

Key results (numbers are from the reproducible runs shipped with this manuscript).

- **HQTFIM** (spin chain): first-law RMS($\delta S \delta \langle K \rangle$) = 2.18 × 10⁻⁵; residual plateau mean $\simeq -4.34 \times 10^{-19}$ with standard error $\simeq 3.27 \times 10^{-5}$; clean [1, log ℓ] trend for $\delta \langle K \rangle (\ell)$. Quick validations: (i) δg -scan is linear with $R^2 \simeq 0.984$; (ii) boundary swap (PBC \leftrightarrow OBC) leaves the plateau unchanged within error; (iii) block-range and size scans show only mild drifts (no finite-size pathology).
- Gaussian (free fermion) chain: the discrete first-law holds exactly in our implementation (RMS= 0) via $\delta S = \text{Tr}[(\delta C) h_0] = \delta \langle K \rangle$, where $h_0 = \log[(I C_0)C_0^{-1}]$ on the block; the fitted slope versus $\log \ell$ is $a_1 = +1.119$ and the residual plateau mean is consistent with zero with standard error ~ 0.10 over $\ell = 20...100$.

These tests are not a computation of the cosmological β ; they are *structural validations* showing that MI–subtracted modular response in concrete microphysics exhibits the same (first–law, constant+log, plateau) features assumed in the OP/CHM flat–space calculation.

TABLE II. Substrate validation metrics (see App. A for definitions). "Plateau" refers to the mean residual after subtracting $a_0 + a_1 \log \ell$ from $\delta(K)(\ell)$. HQTFIM errors reflect finite-size (L = 10-12) and linear-response truncation; quick-validate scans (dg, PBC/OBC, size) show no finite-size pathology at our precision.

Model	Settings	First-law RMS	Plateau mean \pm SE	Notes
HQTFIM	$L=1012,\ell\in[2,6]$	2.18×10^{-5}	$(-4.34 \pm 32.7) \times 10^{-6}$	δg -linear, PBC/OBC PASS
Gaussian fermion	$L = 200, \mathrm{PBC}, \ell \in [20,100]$	0	$\approx 0 \pm 9.75 \times 10^{-2}$	exact first law, log-trend

VI. RESOLUTION OF THE CASINI-GALANTE-MYERS (2016) CRITIQUE

CGM identify obstructions tied to operator dimensions and contact terms. Our framework addresses:

- UV: MI subtraction plus moment-kill cancels area and curvature–contact terms, isolating a finite, regulator-independent I_{00} .
- IR/log at $\Delta = d/2$: allowing mild state dependence M(x) (hence G(x)) within the safe window supplies the necessary log compensator at $\Delta = d/2$, so the obstruction does not arise at the order relevant for the Clausius balance.

We do not claim a cure for all $\Delta \leq d/2$; our statements are restricted to the marginal case in the safe window.

A. Clausius vs. Jacobson (2016): marginal compensator from focusing with running M^2

In our closure $M^2(x) = M_0^2[1 + \kappa \xi \varepsilon(x)]$ the field equations read

$$M^2 G_{ab} = 8\pi T_{ab} + \nabla_a \nabla_b M^2 - g_{ab} \square M^2 - \Lambda_{\text{eff}}(x) g_{ab}. \tag{10}$$

Contracting with a horizon generator k^a and inserting in Raychaudhuri gives an additional focusing source

$$R_{ab}k^{a}k^{b} = \frac{8\pi}{M^{2}}T_{kk} + \frac{1}{M^{2}}k^{a}k^{b}\nabla_{a}\nabla_{b}M^{2}.$$
 (11)

Smearing with the same MI/moment-kill projector that defines I_{00} yields a contribution $-B \ell^4 \log(\ell \mu) \delta \varepsilon$ from the $k^a k^b \nabla_a \nabla_b M^2$ term at $\Delta = d/2$, which cancels the CGM obstruction on the matter side. The Clausius identity therefore holds with the *flat-space* finite coefficient $2\pi C_T I_{00}$ at working order; logs cancel scheme-locally. A background $A \delta(1/G)$ term is not required for this cancellation and is subleading within the safe window.

Proposition 1 (Marginal compensator; $\Delta = d/2$). For CHM diamonds in the safe window with MI subtraction and moment-kill, if M^2 runs slowly with ε so that $\delta \varepsilon$ varies logarithmically across the window, then the additional focusing source $M^{-2}k^ak^b\nabla_a\nabla_bM^2$ produces a gravitational contribution $-B\ell^4\log(\ell\mu)\delta\varepsilon$ that cancels the CGM obstruction $+B\ell^4\log(\ell\mu)\delta\varepsilon$ in $\delta(K_{\rm sub})$. The remaining finite ℓ^4 term equals $2\pi C_T I_{00}\ell^4\delta\varepsilon$, establishing (A2) at the marginal point.

VII. GEOMETRIC NORMALIZATION FACTOR f (TWO SCHEMES)

We map Eq. (1) to the FRW zero mode by

$$f = f_{\text{shape}} f_{\text{boost}} f_{\text{bdv}} f_{\text{cont}}. \tag{12}$$

Common ingredients. $f_{\text{shape}} = 15/2$ (ball \rightarrow diamond weight), $f_{\text{boost}} = 1$ (Unruh $T = \kappa/2\pi$), $f_{\text{cont}} = 1$ (MI-subtracted finite piece is continuation-invariant).

A. Scheme A (with IW/Raychaudhuri contraction explicit)

$$f_{\text{bdy}}^{(A)} = 0.10924, \qquad f^{(A)} = 7.5 \times 1 \times 0.10924 \times 1 = 0.8193.$$

B. Scheme B (purely geometric boundary factor)

$$f_{\text{bdy}}^{(B)} = \frac{5}{12} = 0.416\overline{6}, \qquad f^{(B)} = 7.5 \times 1 \times \frac{5}{12} \times 1 = 3.125.$$

C. Continuous-angle normalization and scheme invariance

Define a unit-solid-angle boundary factor $f_{\text{bdy}}^{\text{unit}}$ and write $f_{\text{bdy}}(\theta) = f_{\text{bdy}}^{\text{unit}} \Delta\Omega(\theta)$, with $\Delta\Omega(\theta) = 2\pi(1 - \cos\theta)$. For a spherical cap of half-angle θ ,

$$c_{\text{geo}}(\theta) = \frac{4\pi}{\Delta\Omega(\theta)} = \frac{2}{1 - \cos\theta}.$$
 (13)

It follows that

$$\beta f(\theta) c_{\text{geo}}(\theta) = \beta f_{\text{shape}} f_{\text{boost}} f_{\text{cont}} f_{\text{bdy}}^{\text{unit}} (4\pi), \tag{14}$$

independent of θ . We therefore report the invariant $\mathcal{C}_{\Omega} \equiv f c_{\text{geo}}$; numerically it is θ -independent to $< 10^{-4}$.

VIII. COSMOLOGICAL CONSTANT SECTOR: CONDITIONAL, SCHEME-INVARIANT MAPPING

Roadmap. This section isolates the FRW zero mode. All angle/scheme dependence is carried by the single invariant $\mathcal{C}_{\Omega} = f c_{\text{geo}}$; hence only the product $\beta \mathcal{C}_{\Omega}$ matters. No cosmological input enters.

At the background level with today's $\alpha_M(a=1) \approx 0$,

$$\Lambda_{\text{eff}} = 3 M_0^2 H_0^2 \left(\beta f c_{\text{geo}} \right), \qquad \boxed{\Omega_{\Lambda} = \beta f c_{\text{geo}}} . \tag{15}$$

A. From the older master formula to the invariant

A previous version expressed Ω_{Λ} as $x/(x + \Omega_{m0})$ with $x \equiv \beta f c_{\text{geo}}$. In the present convention we divide the Clausius zero mode by the critical density $3M_0^2H_0^2$, yielding $\Omega_{\Lambda} = x$. Both descriptions are equivalent once a convention is fixed.

B. Numerical results (both schemes)

Using $\beta_{\text{cen}} = 0.02090$:

Scheme	β	f	$c_{ m geo}$	$\Omega_{\Lambda} = \beta f c_{\rm geo}$
A	0.02090	0.8193	40	0.68493
В	0.02090	3.125	10.49	0.68493

Invariant product (baseline scalar): $\beta f c_{\text{geo}} \approx 0.685$. Uncertainty from β (±3%) propagates to ±0.021 on Ω_{Λ} . Static weak-field acceleration scale. Consistent with the same Clausius normalization and geometric bookkeeping,

$$a_0 = \frac{5}{12} \,\Omega_{\Lambda}^2 \, c \, H_0. \tag{16}$$

See Appendix J.

Non-circularity check (vary β only). Scanning β within its band shifts Ω_{Λ} linearly by the same fraction; the mapping is not a fit or identity.

IX. ENTROPIC STATE-ACTION AND ENVIRONMENT GATE

What this section does. Here we specify the minimal, sign-definite evolution law for the state variable $\varepsilon(a)$ consistent with $\Delta S \ge 0$ (entropic state–action), and explain how it gates into environments through $F_g(g/a_0)$. No new cosmological parameters are introduced; c_{\log} is fixed by the Clausius/Noether integral.

Box 1: Entropic state-action ($\Delta S \geq 0$) and throttling history. Define a retarded, positive exposure

$$J(a) = \int_{-1}^{\ln a} d\ln a' \ K(a, a') \ D(a')^2, \qquad K(a, a') \propto (a'/a)^p, \quad p \in [4, 6], \tag{17}$$

and a monotone state variable

$$\varepsilon(a) = \varepsilon_0 + c_{\log} \ln\left(1 + \frac{J(a)}{J_*}\right), \qquad \frac{d\varepsilon}{d\ln a} \ge 0.$$
 (18)

Clausius/Noether normalization fixes c_{\log} via $\int \varepsilon d \ln a = \Omega_{\Lambda} = \beta C_{\Omega}$. We include a small positive irreversibility floor $\varepsilon_0 \geq 0$ to encode $\Delta S \geq 0$ at late times; no cosmological inputs enter this normalization.

Box 2: Where throttling appears (environment gate). Map the global $\varepsilon(a)$ to a locale by

$$\varepsilon_{\text{env}}(a,g) = \varepsilon_0 + \left(\varepsilon(a) - \varepsilon_0\right) \underbrace{\frac{1}{1 + (g/a_0)^n}}_{F_a(g/a_0) \in [0,1]}.$$
(19)

Strong fields $g \gg a_0 \Rightarrow F_g \to 0 \Rightarrow \mu \to 1$ (GR recovery); weak fields $g \ll a_0 \Rightarrow F_g \to 1 \Rightarrow \mu < 1$. For $g/a_0 \sim 10^{11}$ and $n \geq 3$, the gate gives $F_g \lesssim 10^{-33}$ (Solar-System conditions).

Gate-family robustness. Replacing the rational gate by a logistic $F_g = [1 + \exp(\alpha \log(g/a_0))]^{-1}$ with $\alpha \in [3, 6]$ changes the capped H_0 shift by $\lesssim 0.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$, while preserving Solar-System suppression $F_g \lesssim 10^{-33}$. Why the rational gate. We adopt $F_g = 1/[1 + (g/a_0)^n]$ because (i) it is the unique C^1 rational family with the correct asymptotes $F_g \to 1$ ($g \ll a_0$) and $F_g \to 0$ ($g \gg a_0$), (ii) it gives analytic Solar-System suppression $F_g \lesssim 10^{-33}$ for $g/a_0 \sim 10^{11}$ with $n \geq 3$, and (iii) it mirrors common quasilinear closures in weak-field modified Poisson equations. Our logistic check shows the H_0 bounds are insensitive to this choice within the caps.

X. GROWTH OF STRUCTURE AND S_8

We solve

$$D'' + \left(2 + \frac{d\ln H}{d\ln a} + \alpha_M(a)\right)D' + \frac{3}{2}\mu(\varepsilon(a))\Omega_m(a)D = 0,$$
(20)

with $\mu(\varepsilon) = 1/(1 + \eta \varepsilon)$ ($\eta = 5/12$). We keep $\alpha_M = 0$ in the distance sector and may allow a small $\alpha_M \propto \varepsilon$ in the growth sector only; in the calculations reported here we use $\kappa = 2$ and $\xi = 2.5$ in the growth calculations.

Using the entropic $\varepsilon(a)$ above and no re-tuning of Ω_{Λ} , we obtain

$$S_8 \simeq 0.788 \quad (-7.4\% \text{ vs. } \Lambda \text{CDM}),$$
 (21)

robust to kernel powers $p \in \{4, 5, 6\}$ at the $< 10^{-3}$ level.

A. Survey-facing predictions and falsifiers

- RSD / growth: With $\varepsilon(a)$ fixed by Clausius normalization and $\mu = 1/(1 + \eta \varepsilon)$, we predict a redshift–dependent suppression $f\sigma_8(z)$ lower than Λ CDM by 5–9% over $z \in [0,1]$; the shape is monotone with mild curvature and no free z–parameters. Euclid/Roman measurements at $\pm 2\%$ precision decisively test this.
- Weak lensing: Two-point shear spectra are altered at the \lesssim few-percent level in amplitude but preserve $\Sigma \simeq 1$ at the sub-percent level in our working runner (distance sector GR-like).
- SN residuals vs host field: Standardized residuals exhibit a sign-definite trend with host g/a_0 ; the slope is bounded by the SN cap (0.05 mag) and should persist with same-host controls.
- Same-host Cepheids: Residual trends must be ≤ 0.03 mag; a larger systematic would falsify our capped illustration.
- **GW/EM distances:** We enforce $|d_L^{\text{GW}}/d_L^{\text{EM}} 1| \le 5 \times 10^{-3}$; any larger deviation over $0 < z \lesssim 1$ falsifies the working closure.

XI. ILLUSTRATIVE HUBBLE-LADDER ENVIRONMENT CORRECTION (CAPPED)

Using the same $\varepsilon_{\text{env}}(a, g)$ and a *sign-definite*, *first-principles* mapping for standardized SN/Cepheid residuals ("Theory+"), we confine source-side adjustments to observed host-systematic scales (caps ≤ 0.05 mag for SNe and ≤ 0.03 mag for same-host Cepheids). On an SH0ES-like catalog this nudges

$$H_0: 73.0 \rightarrow 71.32 \text{ (SN cap only)}, \rightarrow 70.89 \text{ (SN cap + small Cepheid term)},$$
 (22)

without altering EM distances. These values are illustrative, capped bounds, not fitted predictions; environment-trend falsifiers (residual vs. host g/a_0 ; same-host Cepheid limits) are stated in Sec. XII.

A. Uncapped vs. capped Hubble-ladder runs

In addition to the conservative, capped illustration (Sec. XI), we also run the identical environment–gated mapping without caps. The uncapped run demonstrates that the downward shift in H_0 is not an artifact of the caps; caps merely serve as a conservative systematic control.

Configuration	$H_0 [{\rm km s^{-1} Mpc^{-1}}]$
Baseline (catalog value)	73.0
Uncapped, SN-only (Theory+)	71.178
Capped, SN-only (0.05 mag)	71.319
Capped, SN + small Cepheid term $(0.05/0.03 \text{ mag})$	70.885

Two points follow. First, the direction and order of magnitude of the shift are already present in the uncapped run (SN-only: $73.0 \rightarrow 71.178$), showing that the sign-definite environment response is sufficient. Second, the caps tighten susceptibility to outliers and known catalog systematics; the capped SN+Cepheid combination (70.885) provides a conservative bound. As in the rest of the paper, EM distances remain GR-like; no cosmological parameters are fitted.

XII. PREDICTIONS, PARAMETER TRANSLATIONS, AND FALSIFIABILITY

1. **GW/EM luminosity-distance ratio.** For a running Planck mass,

$$\frac{d_L^{\text{GW}}(z)}{d_L^{\text{EM}}(z)} = \exp\left[\frac{1}{2} \int_0^z \frac{\alpha_M(z')}{1+z'} dz'\right],\tag{23}$$

frame invariant; depends only on the integrated α_M [6]. We enforce $|d_L^{\rm GW}/d_L^{\rm EM}-1| \leq 5 \times 10^{-3}$.

- 2. Mapping \dot{G}/G to α_M . $\alpha_M \equiv d \ln M^2/d \ln a = -(\dot{G}/G)/H$. At z = 0, $\alpha_M(0) = -(\dot{G}/G)_0/H_0$.
- 3. What it does not mimic. With $\alpha_T = \alpha_B = 0$, linear slip remains GR-like and the model does not by itself fit strong-lensing clusters; transition regimes require the full anisotropic kernel (future work).

XIII. CONSISTENCY: BIANCHI IDENTITY AND FRW

Starting from Eq. (25), the contracted Bianchi identity and $\nabla_{\mu}T^{\mu\nu}=0$ imply

$$\nabla_b \Lambda_{\text{eff}} = \frac{1}{2} R \nabla_b M^2 \quad . \tag{24}$$

In FRW with $\alpha_M(a=1) \approx 0$, this is automatically satisfied at the present epoch (App. H).

XIV. UNCERTAINTY BUDGET (SUMMARY)

Source	Impact on H_0	Impact on S_8
β (numerical/systematic $\pm 3\%$)	n/a	$\ll 10^{-3}$ via normalization
Kernel power $p \in [4, 6]$	n/a	$< 10^{-3}$
GW/EM bound input	n/a	enforces $ d_L^{\text{GW}}/d_L^{\text{EM}} - 1 \le 5 \times 10^{-3}$
Host proxy $\pm 50\%$	$\lesssim 0.2 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ (uncapped only)}$	n/a

XV. CONCEPTUAL PLACEMENT AND GR LIMIT

At background/linear order:

$$M^{2}(x) G_{ab} = 8\pi T_{ab} + \nabla_{a} \nabla_{b} M^{2} - g_{ab} \square M^{2} - \Lambda_{\text{eff}}(x) g_{ab}.$$
 (25)

This is the standard $F(\phi)R$ (Jordan) structure in the $c_T = 1$, no-braiding corner ($\alpha_T = 0$, $\alpha_B = 0$); the sole background function is α_M [12]. Our constitutive closure fixes M^2 as a functional of Ξ . If $\nabla_a M^2 = 0$ ($\alpha_M \to 0$), Eq. (25) reduces to Einstein's equation with constant M and (if present) a constant zero mode. Under $\tilde{g}_{ab} = (M^2/M_0^2)g_{ab}$, frame-invariant signatures remain (notably $d_L^{\text{GW}}/d_L^{\text{EM}}$).

XVI. DATA AND CODE AVAILABILITY

All figures and numbers quoted for the substrate checks can be reproduced with two single-file runners included in the repository:

- 1. hqtfim_capacity_probe.py (spin chain). Default run produces first_law_check.png, dK_vs_logl.png, residual_after_miki and summary.json. Passing --quick-validate additionally writes quick_dg_scan.csv/png, quick_size_scan.csv/png, quick_pbc_compare.json, quick_block_compare.json, and validation_report.txt.
- 2. gaussian_capacity_probe.py (Gaussian chain). Default run produces first_law_check.png, dK_vs_logl.png, residual_after_subtraction.png, and summary.json.

These scripts have no cosmological inputs and are intended for rapid referee validation of the structural assumptions used in the continuum calculation.

XVII. CONCLUSION

Finite information capacity drives a state-dependent response. Each proper frame has a maximum entanglement load; as this threshold is approached, the response preserves causal stitching while keeping null geometry GR-like. Combining modular-Hamiltonian calculations (CHM/OP), MI subtraction, and a state-dependent G(x), we obtain a conditional, scheme-invariant mapping $\Omega_{\Lambda} = \beta f c_{\text{geo}}$ and a weak-field relation $a_0 = (5/12) \Omega_{\Lambda}^2 c H_0$. An entropic state-action law $(\Delta S \geq 0)$ determines a monotone $\varepsilon(a)$ that suppresses growth $(S_8 \simeq 0.788)$. A capped, environment-gated ladder illustration nudges SH0ES downward without altering distances. The framework is falsifiable and strictly limited to the safe window; beyond that domain, it is an invitation for further work.

Appendix A: Substrate validation protocol (definitions and quick checks)

First-law RMS. For a set of block sizes $\{\ell_i\}$,

$$\mathrm{RMS} \equiv \sqrt{\frac{1}{N} \sum_{i} \left(\delta S(\ell_i) - \delta \langle K \rangle(\ell_i) \right)^2}.$$

Plateau statistic. Fit $\delta \langle K \rangle(\ell) = a_0 + a_1 \log \ell$ on the chosen window; define $r(\ell) \equiv \delta \langle K \rangle(\ell) - (a_0 + a_1 \log \ell)$. Report the sample mean \bar{r} and its standard error $SE = \sigma_r / \sqrt{N}$.

Quick validations. (i) δ -scan: vary the deformation amplitude (e.g. $\delta g \in \{0.001, 0.002, 0.005\}$ in HQTFIM); in the linear domain, RMS scales $\propto \delta$ and \bar{r} stays consistent with 0 within SE. (ii) Boundary swap: PBC \leftrightarrow OBC should leave \bar{r} unchanged within SE. (iii) Block-range stability: small changes of $[\ell_{\min}, \ell_{\max}]$ change a_1 only mildly. (iv) Size scan: increasing L reduces RMS/SE slightly; large drifts would flag finite-size effects.

Appendix B: Gaussian-chain formulas used in Sec. V

For a 1D free-fermion chain with single-particle Hamiltonian $H = U \operatorname{diag}(\varepsilon_k) U^{\dagger}$ and Fermi projector $P = U \Theta(-H) U^{\dagger}$, the correlation matrix is C = P. For a spatial block A with restriction C_A , the block modular kernel is

$$h_0 = \log[(I - C_A)C_A^{-1}],$$

and the entanglement first law gives

$$\delta S_A = \operatorname{Tr}_A[(\delta C_A) h_0] = \delta \langle K_A \rangle,$$

so the first–law RMS vanishes up to numerical roundoff. The observed constant+log dependence of $\delta \langle K_A \rangle(\ell)$ and the near–zero residual after subtracting $a_0 + a_1 \log \ell$ provide an analytic benchmark for the substrate validations.

Appendix C: Moment-kill identities and contact-term cancellation

Choose (a, b) so that for any smooth radial $F(r) = F_0 + F_2 r^2 + \mathcal{O}(r^4)$,

$$\int_{B_{\ell}} W_{\ell} F(r) d^3 x - a \int_{B_{\sigma_1 \ell}} W_{\sigma_1 \ell} F(r) d^3 x - b \int_{B_{\sigma_2 \ell}} W_{\sigma_2 \ell} F(r) d^3 x = \mathcal{O}(\ell^6), \tag{C1}$$

canceling r^0 and r^2 moments. The surviving $\mathcal{O}(\ell^4)$ piece defines I_{00} .

Appendix D: Derivation of the Constitutive Factor f

1. Ball vs diamond (shape)

 $W_{\ell}(r) = (\ell^2 - r^2)/(2\ell)$ yields $\mathcal{J}_{\text{ball}} = \frac{4\pi}{15}\ell^4$. On the diamond horizon, |v| with $A(v) = 4\pi(\ell^2 - v^2)$ yields $\mathcal{J}_{\text{hor}} = 2\pi\ell^4$. Thus $f_{\text{shape}} = 15/2$.

2. Boost and continuation

Unruh $T = \kappa/2\pi \Rightarrow f_{\text{boost}} = 1$; after MI subtraction the finite coefficient is continuation invariant, so $f_{\text{cont}} = 1$.

3. Boundary vs bulk: two bookkeepings

Let $u = v/\ell \in [-1, 1]$ and $\hat{\rho}_{\mathcal{D}}(u) = \frac{3}{4}(1 - u^2)$ with $\int_{-1}^{1} \hat{\rho} du = 1$. The geometric segment ratio is

$$R_{\text{seg}} = \frac{\int_0^1 u(1-u^2)\hat{\rho} \, du}{\int_0^1 (1-u^2)\hat{\rho} \, du} = \frac{5}{16} = 0.3125.$$

Scheme A: include an isotropic IW/Raychaudhuri normalization $C_{\rm IW}$ so $C_{\rm contr} = (4/3) C_{\rm IW}$, giving $f_{\rm bdy}^{(A)} \simeq 0.10924$, hence $f^{(A)} = 0.8193$.

Scheme B: retain only geometric weights, including the isotropic null contraction (4/3) but not the additional IW factor. Then $f_{\text{bdy}}^{(B)} = (4/3) \times (5/16) = 5/12$ and $f^{(B)} = 3.125$.

Appendix E: Integral definition and conventions for c_{geo}

Define

$$c_{\text{geo}} \equiv \frac{\int_{\text{FRW patch}} (\delta Q/T)_{\text{FRW}}}{\int_{\text{local wedge}} (\delta Q/T)_{\text{wedge}}}.$$
 (E1)

For a cap of half-angle θ_{\star} with $\Delta\Omega = 2\pi(1-\cos\theta_{\star})$,

$$c_{\text{geo}} = \frac{4\pi}{\Delta\Omega} = \frac{2}{1 - \cos\theta_{\star}}.$$
 (E2)

Two consistent conventions (no double counting).

- Scheme A (minimal wedge): $c_{\text{geo}} = 40$, i.e. $\Delta\Omega_{\text{wedge}}^{(A)} = 4\pi/40 \ (\cos\theta_{\star}^{(A)} = 19/20)$.
- Scheme B (equal-flux cap): imposing the no-double-counting rule for $\hat{\rho}_{\mathcal{D}}$ and $f^{(B)}$ yields $c_{\text{geo}}^{(B)} \simeq 10.49$ (cos $\theta_{\star}^{(B)} \simeq 0.80934$).

Appendix F: FRW zero-mode mapping (sketch)

With $M^2(a) = M_0^2[1 + \mathcal{O}(\alpha_M)]$ and today $\alpha_M \simeq 0$:

$$\Lambda_{\text{eff}} = 3H_0^2 M_0^2 (\beta f c_{\text{geo}}), \qquad \Omega_{\Lambda} = \beta f c_{\text{geo}}. \tag{F1}$$

Appendix G: EFT-of-DE mapping (summary)

At leading order we sit in the $c_T = 1$, no-braiding corner with $\alpha_T = 0 = \alpha_B$ and only $\alpha_M(a)$ active [12].

Appendix H: Bianchi-identity derivation for Eq. (24)

Starting from Eq. (25) and using $\nabla_a G^{ab} = 0$, $\nabla_a T^{ab} = 0$, and commutators on M^2 yields $\nabla_b \Lambda_{\text{eff}} = \frac{1}{2} R \nabla_b M^2$.

Appendix I: Small-wedge Clausius domain and curvature suppression (EPMR)

Lemma H.1 (First-law domain). For Hadamard states in a Riemann-normal patch and small perturbations with $S(\rho|\rho_0) = \mathcal{O}(\varepsilon^2)$, the entanglement first law $\delta S = \delta \langle K \rangle + \mathcal{O}(\varepsilon^2)$ holds for sufficiently small diamonds.

Lemma H.2 (Moment-kill + MI subtraction). With K_{sub} of Eq. (4) choosing (a, b) to cancel the zeroth and second radial moments, contact and curvature–contact terms up to $\mathcal{O}(\ell^2)$ cancel in $\delta \langle K_{\text{sub}} \rangle$.

Proposition H.1 (Curvature suppression and EPMR). After MI subtraction and moment-kill, the leading surviving isotropic term is $\mathcal{O}(\ell^4)$ and equals the *flat-space* modular coefficient; curvature dressings enter at $\mathcal{O}(\ell^6)$ within the safe window.

Theorem 1 (Working-order small-diamond Clausius/Unruh). Let the state be Hadamard and consider CHM diamonds of linear size ℓ inside the safe window of Def. 1. With mutual-information subtraction and moment-kill as in App. C, the modular first law $\delta S = \delta \langle K_{\rm sub} \rangle$ and the Clausius identity with Unruh normalization hold to working order:

$$\delta \langle K_{\text{sub}} \rangle = (2\pi C_T I_{00}) \ell^4 \delta \varepsilon + \mathcal{O}(\ell^6), \qquad \frac{\delta Q}{T} = \delta S + \mathcal{O}(\ell^6),$$

so that the finite ℓ^4 coefficient equals its flat-space value and curvature dressings start at $\mathcal{O}(\ell^6)$. Proof sketch. Lemma H.1 gives the first-law domain; Lemma H.2 removes the r^0, r^2 moments and any curvature-contact pieces; Proposition H.1 then enforces the $\mathcal{O}(\ell^6)$ onset of curvature. At the marginal point $\Delta = d/2$, the logarithmic obstruction is cancelled by the slow running of M^2 (Lemma 1/Prop. 1), leaving the flat ℓ^4 finite coefficient at working order.

Lemma 1 (Marginal compensator ($\Delta = d/2$)). Within the safe window, if $M^2(x)$ runs slowly with ε so that $\delta \varepsilon$ varies logarithmically across the window, the additional focusing source $M^{-2}k^ak^b\nabla_a\nabla_bM^2$ contributes a term that cancels the $\ell^4 \log(\ell\mu) \delta \varepsilon$ obstruction in $\delta \langle K_{\rm sub} \rangle$. See Proposition 1 for the detailed continuum derivation.

Appendix J: Weak-field flux law and the universal prefactor 5/12

A. Ingredients and regime. Consider Eq. (25) with $\delta G/G = -\beta \delta \varepsilon$ and the zero-mode mapping $\Omega_{\Lambda} = \beta f c_{\text{geo}}$. Work in the static, weak-field limit (Newtonian gauge, $|\Phi|/c^2 \ll 1$, $\partial_t \to 0$) and within the safe window.

B. Quasilinear flux law. The $\nabla \nabla M^2$ terms renormalize the flux of $\nabla \Phi$. Coarse-graining over the wedge family yields

$$\nabla \cdot \left[\mu(Y) \, \nabla \Phi \right] = 4\pi G \, \rho_b, \qquad Y \equiv \frac{|\nabla \Phi|}{a_0}, \tag{J1}$$

with $\mu \to 1$ for $Y \gg 1$ and $\mu \sim Y$ for $Y \ll 1$.

C. Normalization from the homogeneous zero mode. The only late-time acceleration scale is $a_H \equiv cH_0$. Matching the static-flux normalization to the homogeneous Clausius zero mode with the same boundary–segment bookkeeping yields the universal geometric constant 5/12, hence

$$a_0 = \frac{5}{12} (\beta f c_{\text{geo}})^2 c H_0 = \frac{5}{12} \Omega_{\Lambda}^2 c H_0$$
 (J2)

Angle/scheme invariant by Sec. VII C.

D. Scope and caveats. Applies in the static, weak-field, safe-window regime. Transition regimes $Y \sim 1$ and strong-lensing clusters require the full anisotropic kernel (future work).

Appendix K: Species uplift and C_T in OP normalization

In OP convention [10], the modular sensitivity factorizes as $\beta = 2\pi C_T I_{00}$. Our numerical calculation determines the geometric/kinematic coefficient I_{00} (after MI subtraction and moment-kill); matter content enters only through C_T . For free fields, C_T is known analytically (scalars, fermions, vectors) in OP normalization. For mixed content and finite masses one may form an effective

$$C_T^{\text{eff}}(\ell) = \sum_i \Theta(1 - \ell m_i) C_T^{(i)},$$

so that species with $\ell m_i \gg 1$ decouple in the late-time safe window. The invariant βC_{Ω} and hence Ω_{Λ} are therefore stable within our quoted β systematics across reasonable late-time windows. For the late-time safe window relevant here, massive species with $\ell m_i \gg 1$ are exponentially suppressed in C_T^{eff} ; scanning realistic mixtures shifts βC_{Ω} at the sub-percent level, well below our quoted numerical/systematic on β .

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