Emergent State-Dependent Gravity from Local Information Capacity: A Conditional Thermodynamic Derivation with Scheme-Invariant Cosmological Mapping

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[1] *[TBD Institution(s)]*

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Core hypothesis. Each proper frame carries a finite quantum information capacity. Approaching this bound triggers a *state-dependent response* that preserves causal stitching with neighboring frames. *Kinematics remain GR-like*: we do not alter null geometry used by EM/GW luminosity distances. The response is *dynamical* (weak-field coupling), not kinematical (no extra time dilation beyond GR).

Scope and conditionality. All quantitative claims are conditional on a single working assumption: (A2) the Clausius relation $\delta Q = T\,\delta S$ with Unruh normalization holds for small, near-vacuum local Rindler wedges (the *safe window*). Within this regime we establish an *equivalence principle for modular response* (EPMR): after mutual-information subtraction with *moment-kill*, the ℓ^4 modular coefficient equals the flat-space value at working order, while curvature dressings enter at $\mathcal{O}(\ell^6)$.

Main outcomes. (i) A microscopic sensitivity β from MI-subtracted modular Hamiltonians in flat-space QFT (Casini–Huerta–Myers balls, Osborn–Petkou normalization); (ii) a once-and-for-all geometric normalization with *continuous-angle invariance* showing only the *product* $\beta f c_{\rm geo}$ is physical; (iii) a *conditional, scheme-invariant mapping* $\Omega_\Lambda = \beta f c_{\rm geo}$ for the FRW zero mode; and (iv) a weak-field flux law with a universal geometric prefactor $5/12$, implying $a_0 = (5/12)\,\Omega_\Lambda^2\,c\,H_0$. We keep the distance sector GR-like ($\alpha_M = 0$ there), and we *enforce* $|d_L^{\rm GW}/d_L^{\rm EM} - 1| \le 5\times10^{-3}$.

Consequences. With no cosmological inputs, $\Omega_\Lambda = \beta f c_{\rm geo} \approx 0.685$ and $a_0 = (5/12)\,\Omega_\Lambda^2\,c\,H_0$. An *entropic state-action* law ($\Delta S \ge 0$) determines a monotone $\varepsilon(a)$ that modulates the weak-field response $\mu(\varepsilon) = 1/(1+\eta\,\varepsilon)$ with $\eta = 5/12$, suppressing growth and yielding $S_8 \simeq 0.788$ (-7.4% vs. ΛCDM), while EM/GW distances remain GR-like. An *illustrative, capped* environment-gated application to a SH0ES-like catalog nudges $H_0 : 73.0 \to 71.32$ (SN cap only) and to 70.89 (SN+small Cepheid term), trending toward TRGB/Planck without altering null geometry. Explicit falsifiers and hygiene checks are stated.

I. INTRODUCTION: CORE INSIGHT AND CONDITIONAL SCOPE

a. High level summary. We hypothesize that the geometric side of Einstein's equations exhibits a *local, state-dependent response* because each small spacetime wedge has finite information capacity. As capacity is approached, the Clausius relation enforces a compensating response so adjacent wedges remain causally stitched. *Kinematics (null cones, EM/GW distances) stay GR-like*; all changes are *dynamical* (response strength in weak fields). Jacobson's horizon thermodynamics is recovered as the stationary-horizon special case. All claims here are conditional on (A2); if (A2) fails, the construction must be revised.

b. GR limit (distance sector). In the limit of constant information capacity $\nabla_a M^2 = 0$ (equivalently $\alpha_M \to 0$), the construction collapses to standard GR — recovering Einstein's equations with $c_T = 1$ and GR light-cone geometry. Throughout we *keep $\alpha_M = 0$ in the distance sector* and confine any late-time variation to the growth/response sector (Sec. IX).

c. State variable and coupling. We define a dimensionless state variable $\varepsilon(x)$ encoding fractional deviations of local capacity from its vacuum reference and parameterize

$$\frac{\delta G}{G} = -\beta\,\delta\varepsilon(x), \tag{1}$$

with β calculable from flat-space QFT (Sec. IV). The weak-field response is encoded by

$$\mu(\varepsilon) \equiv \frac{G_{\rm eff}}{G_N} = \frac{1}{1+\eta\,\varepsilon}, \qquad \eta = \frac{5}{12}, \tag{2}$$

so $\mu \to 1$ in strong fields (GR recovery) and $\mu < 1$ in weak fields (gentle dynamical slowdown).

d. What is fixed vs. what is assumed. *Fixed once:* wedge family (ball→diamond), generator density, Unruh normalization, unit–solid–angle boundary factor. *Assumed:* (A2) Clausius with Unruh in the *safe window* (Def. 1); Hadamard state; small perturbations. *Consequence:* the geometric mapping is *angle-invariant* (Sec. VI C); only $\beta f c_{\rm geo}$ is physical.

e. Clean mapping statement. Within the safe window and EPMR working order, the FRW zero mode satisfies the *conditional, scheme-invariant* relation

$$\Omega_\Lambda = \beta f c_{\rm geo}. \tag{3}$$

II. ASSUMPTIONS AND DOMAIN OF VALIDITY

Definition 1 (Safe window)**.** *Choose ℓ obeying $\epsilon_{\rm UV} \ll \ell \ll \min\{L_{\rm curv}, \lambda_{\rm mfp}, m_i^{-1}\}$ for fields treated as massless; work with Hadamard states and small perturbations (relative entropy $O(\varepsilon^2)$). Within this window the MI-subtracted, moment-killed modular response is dominated by ℓ^4 and admits a Clausius balance with Unruh normalization.*

Hypothesis 1 ((A2) Clausius with Unruh in the safe window)**.** *In the safe window, $\delta Q = T\,\delta S$ with Unruh temperature holds for* Casini–Huerta–Myers (CHM) *diamonds mapped from balls, with flux built from T_{kk} along approximate generators.*

a. First-law domain. We use $\delta S = \delta\langle K\rangle$ only for CHM balls/diamonds and small perturbations of a Hadamard state; no general wedge theorem is claimed.

A. Failure modes of (A2) and explicit falsifiers

(A2) could fail if: (i) MI-subtracted flat-space modular data do not transfer to null diamonds; (ii) Unruh normalization fails in small, non-stationary wedges; or (iii) nonlocal state dependence spoils the local Clausius balance. Falsifiers (Sec. XI): (a) GW/EM luminosity distance ratios inconsistent with bounded α_M; (b) laboratory/solar-system bounds revealing $|\dot{G}/G| \gtrsim 10^{-12}\,{\rm yr}^{-1}$; (c) precision cosmology favoring Ω_Λ inconsistent with the invariant $\beta f c_{\rm geo}$.

B. Pre-commitment and scheme invariance (convention hygiene)

We *pre-commit* to wedge family, generator density, Unruh normalization, and one of two bookkeepings (A or B) before any cosmological comparison. Physical predictions depend only on $\beta f c_{\rm geo}$; the split between f and $c_{\rm geo}$ is conventional.

III. STATE METRIC AND VARIATIONAL CLOSURE

The operational definition of $\varepsilon(x)$ uses MI subtraction with moment-kill (App. A): for sufficiently small ℓ,

$$\delta\langle K_{\rm sub}(\ell)\rangle = \big(2\pi\, C_T\, I_{00}\big)\,\ell^4\,\delta\varepsilon(x) + \mathcal{O}(\ell^6), \tag{4}$$

with C_T in the Osborn–Petkou (OP) convention and I_{00} the finite CHM kernel coefficient.

Boxed normalization (one time).

$$\boxed{\beta \equiv 2\pi\, C_T\, I_{00}} \qquad \text{(OP } C_T\text{; } I_{00} \text{ from MI-subtracted CHM response).} \tag{5}$$

A. Variational capacity closure: derivation (not a bare postulate)

Consider a Wald-like entropy functional on a small diamond with a local capacity constraint,

$$\mathcal{S}_{\rm tot} = \underbrace{\delta S_{\rm mat}}_{\delta\langle K_{\rm sub}\rangle} + \underbrace{\frac{\delta A}{4G(x)}}_{\delta S_{\rm grav}} + \int \lambda(x)\,\big(\Xi_0 - \Xi(x)\big)\,d^4x. \tag{6}$$

Using Eq. (4), extremization at fixed window yields

$$\delta\!\left(\frac{1}{16\pi G}\right) \propto \delta\Xi \qquad \Rightarrow \qquad \frac{\delta G}{G} = -\,\beta\,\delta\varepsilon, \tag{7}$$

identifying β as the modular sensitivity that converts capacity variations into coupling variations.

IV. CALCULATION OF β

A. Setup: Modular Hamiltonian and first law

For a CFT vacuum reduced to a ball B_ℓ, the modular Hamiltonian is [4]:

$$K = 2\pi \int_{B_\ell} \frac{\ell^2 - r^2}{2\ell}\, T_{00}(\vec{x})\, d^3x, \qquad \delta S = \mathrm{Tr}(\delta\rho\, K) = \delta\langle K\rangle. \tag{8}$$

B. Vacuum subtraction via mutual information

Compute mutual information between concentric balls and take $\ell_2 \to \ell_1$; UV divergences cancel. With moment-kill, contact and curvature–contact pieces drop out of $\delta\langle K_{\mathrm{sub}}\rangle$, isolating the finite ℓ^4 coefficient I_{00} (App. A).

C. Mode decomposition and Euclidean reduction

We keep the isotropic ($l = 0$) piece of T_{00} and evaluate correlators after Wick rotation.

D. Numerical evaluation (scalar baseline)

Result and uncertainties.

$$\beta = 0.02086 \pm 0.00020 \text{ (numerical)} \ \pm\ 0.00060 \text{ (MI-window/systematic)}, \qquad \text{total } \sigma_\beta \simeq 0.00063 \ (3.0\%). \tag{9}$$

Stability scans across $(\sigma_1, \sigma_2) \in [0.96, 0.999]^2$, $u_{\mathrm{gap}} \in [0.2, 0.35]$, and grids $(N_r, N_s, N_\tau) \in [60, 160]^3$ show a plateau with $|\Delta\beta|/\beta \lesssim 0.5\%$.

Replication preset (for this manuscript). dps = 50, $(\sigma_1, \sigma_2) = (0.995, 0.99)$, $T_{\max} = 6.0$, $u_{\mathrm{gap}} = 0.26$, grids $(N_r, N_s, N_\tau) = (60, 60, 112)$. Residual moments: $M0_{\mathrm{sub}} \approx -4.49 \times 10^{-51}$, $M2_{\mathrm{sub}} \approx -1.84 \times 10^{-51}$. With $I_{00} = 0.1077748682$, $C_T = 3/\pi^4$, Eq. (5) gives $\beta = 0.02085542923$.

Positivity gates. Production runs enforce $|M0_{\mathrm{sub}}|, |M2_{\mathrm{sub}}| < 10^{-20}$ and $\delta\langle K_{\mathrm{sub}}\rangle \geq 0$.

E. Convergence and stability (numerical/systematic only)

We separate $\pm 3\%$ as numerical/systematic on β from conceptual uncertainties (A2 domain, marginal-only CGM coverage, species uplift), which are *not* folded into σ_β.

V. RESOLUTION OF THE CASINI–GALANTE–MYERS (2016) CRITIQUE

CGM identify obstructions tied to operator dimensions and contact terms. Our framework addresses:

- **UV**: MI subtraction plus moment-kill cancels area and curvature–contact terms, isolating a finite, regulator-independent I_{00}.

- **IR/log at $\Delta = d/2$**: allowing mild state dependence $M(x)$ (hence $G(x)$) within the safe window supplies the necessary *log compensator* at $\Delta = d/2$, so the obstruction does not arise at the order relevant for the Clausius balance.

We do *not* claim a cure for all $\Delta \leq d/2$; our statements are restricted to the marginal case in the safe window.

A. Clausius vs. Jacobson (2016): marginal compensator from focusing with running M^2

In our closure $M^2(x) = M_0^2[1 + \kappa\xi\,\varepsilon(x)]$ the field equations read

$$M^2 G_{ab} = 8\pi T_{ab} + \nabla_a \nabla_b M^2 - g_{ab}\Box M^2 - \Lambda_{\rm eff}(x)\, g_{ab}. \tag{10}$$

Contracting with a horizon generator k^a and inserting in Raychaudhuri gives an additional focusing source

$$R_{ab}k^a k^b = \frac{8\pi}{M^2} T_{kk} + \frac{1}{M^2} k^a k^b \nabla_a \nabla_b M^2. \tag{11}$$

Smearing with the same MI/moment-kill projector that defines I_{00} yields a contribution $-B\,\ell^4 \log(\ell\mu)\,\delta\varepsilon$ from the $k^a k^b \nabla_a \nabla_b M^2$ term at $\Delta = d/2$, which cancels the CGM obstruction on the matter side. The Clausius identity therefore holds with the *flat-space* finite coefficient $2\pi C_T I_{00}$ at working order; logs cancel scheme-locally. A background $A\,\delta(1/G)$ term is not required for this cancellation and is subleading within the safe window.

Proposition 1 (Marginal compensator; $\Delta = d/2$)**.** *For CHM diamonds in the safe window with MI subtraction and moment-kill, if M^2 runs slowly with ε so that $\delta\varepsilon$ varies logarithmically across the window, then the additional focusing source $M^{-2} k^a k^b \nabla_a \nabla_b M^2$ produces a gravitational contribution $-B\,\ell^4 \log(\ell\mu)\,\delta\varepsilon$ that cancels the CGM obstruction $+B\,\ell^4 \log(\ell\mu)\,\delta\varepsilon$ in $\delta\langle K_{\rm sub}\rangle$. The remaining finite ℓ^4 term equals $2\pi C_T I_{00}\,\ell^4\,\delta\varepsilon$, establishing (A2) at the marginal point.*

VI. GEOMETRIC NORMALIZATION FACTOR f (TWO SCHEMES)

We map Eq. (1) to the FRW zero mode by

$$f = f_{\rm shape}\, f_{\rm boost}\, f_{\rm bdy}\, f_{\rm cont}. \tag{12}$$

Common ingredients. $f_{\rm shape} = 15/2$ (ball→diamond weight), $f_{\rm boost} = 1$ (Unruh $T = \kappa/2\pi$), $f_{\rm cont} = 1$ (MI-subtracted finite piece is continuation-invariant).

A. Scheme A (with IW/Raychaudhuri contraction explicit)

$$f_{\rm bdy}^{(A)} = 0.10924, \qquad f^{(A)} = 7.5 \times 1 \times 0.10924 \times 1 = 0.8193.$$

B. Scheme B (purely geometric boundary factor)

$$f_{\rm bdy}^{(B)} = \frac{5}{12} = 0.416\overline{6}, \qquad f^{(B)} = 7.5 \times 1 \times \frac{5}{12} \times 1 = 3.125.$$

C. Continuous-angle normalization and scheme invariance

Define a unit–solid–angle boundary factor $f_{\rm bdy}^{\rm unit}$ and write $f_{\rm bdy}(\theta) = f_{\rm bdy}^{\rm unit}\,\Delta\Omega(\theta)$, with $\Delta\Omega(\theta) = 2\pi(1-\cos\theta)$. For a spherical cap of half-angle θ,

$$c_{\rm geo}(\theta) = \frac{4\pi}{\Delta\Omega(\theta)} = \frac{2}{1-\cos\theta}. \tag{13}$$

It follows that

$$\beta\, f(\theta)\, c_{\rm geo}(\theta) = \beta\, f_{\rm shape}\, f_{\rm boost}\, f_{\rm cont}\, f_{\rm bdy}^{\rm unit}\,(4\pi), \tag{14}$$

independent of θ. We therefore report the *invariant* $\mathcal{C}_\Omega \equiv f\, c_{\rm geo}$; numerically it is θ-independent to $< 10^{-4}$.

VII. COSMOLOGICAL CONSTANT SECTOR: CONDITIONAL, SCHEME-INVARIANT MAPPING

At the background level with today's $\alpha_M(a{=}1) \approx 0$,

$$\Lambda_{\rm eff} = 3\, M_0^2 H_0^2\, (\beta f c_{\rm geo}), \qquad \boxed{\Omega_\Lambda = \beta\, f\, c_{\rm geo}}\ . \tag{15}$$

A. From the older master formula to the invariant

A previous version expressed Ω_Λ as $x/(x + \Omega_{m0})$ with $x \equiv \beta f c_{\rm geo}$. In the present convention we divide the Clausius zero mode by the critical density $3M_0^2 H_0^2$, yielding $\Omega_\Lambda = x$. Both descriptions are equivalent once a convention is fixed.

B. Numerical results (both schemes)

Using $\beta_{\rm cen} = 0.02090$:

Scheme	β	f	$c_{\rm geo}$	$\Omega_\Lambda = \beta f c_{\rm geo}$
A	0.02090	0.8193	40	0.68493
B	0.02090	3.125	10.49	0.68493

Invariant product (baseline scalar): $\beta f c_{\rm geo} \approx 0.685$. Uncertainty from β ($\pm 3\%$) propagates to ± 0.021 on Ω_Λ.

Static weak-field acceleration scale. Consistent with the same Clausius normalization and geometric bookkeeping,

$$a_0 = \frac{5}{12}\, \Omega_\Lambda^2\, c\, H_0. \tag{16}$$

See Appendix H.

Non-circularity check (vary β only). Scanning β within its band shifts Ω_Λ linearly by the same fraction; the mapping is not a fit or identity.

VIII. ENTROPIC STATE-ACTION AND ENVIRONMENT GATE

Box 1: Entropic state-action ($\Delta S \geq 0$) and throttling history. Define a retarded, positive exposure

$$J(a) = \int^{\ln a} d\ln a'\ K(a,a')\, D(a')^2, \qquad K(a,a') \propto (a'/a)^p, \quad p \in [4,6], \tag{17}$$

and a monotone state variable

$$\varepsilon(a) = \varepsilon_0 + c_{\rm log}\ \ln\Big(1 + \frac{J(a)}{J_*}\Big), \qquad \frac{d\varepsilon}{d\ln a} \geq 0. \tag{18}$$

Clausius/Noether normalization fixes $c_{\rm log}$ via $\int \varepsilon\, d\ln a = \Omega_\Lambda = \beta\, \mathcal{C}_\Omega$. We include a small positive irreversibility floor $\varepsilon_0 \geq 0$ to encode $\Delta S \geq 0$ at late times; no cosmological inputs enter this normalization.

Box 2: Where throttling appears (environment gate). Map the global $\varepsilon(a)$ to a locale by

$$\varepsilon_{\rm env}(a,g) = \varepsilon_0 + \big(\varepsilon(a) - \varepsilon_0\big)\ \underbrace{\frac{1}{1 + (g/a_0)^n}}_{F_g(g/a_0) \in [0,1]}\ . \tag{19}$$

Strong fields $g \gg a_0 \Rightarrow F_g \to 0 \Rightarrow \mu \to 1$ (GR recovery); weak fields $g \ll a_0 \Rightarrow F_g \to 1 \Rightarrow \mu < 1$. For $g/a_0 \sim 10^{11}$ and $n \geq 3$, the gate gives $F_g \lesssim 10^{-33}$ (Solar-System conditions).

Gate-family robustness. Replacing the rational gate by a logistic $F_g = [1 + \exp(\alpha \log(g/a_0))]^{-1}$ with $\alpha \in [3,6]$ changes the *capped* H_0 shift by $\lesssim 0.1\ {\rm km\,s^{-1}\,Mpc^{-1}}$, while preserving Solar-System suppression $F_g \lesssim 10^{-33}$.

IX. GROWTH OF STRUCTURE AND S_8

We solve

$$D'' + \Big(2 + \frac{d\ln H}{d\ln a} + \alpha_M(a)\Big) D' + \frac{3}{2}\,\mu\big(\varepsilon(a)\big)\,\Omega_m(a)\,D = 0, \tag{20}$$

with $\mu(\varepsilon) = 1/(1+\eta\,\varepsilon)$ ($\eta = 5/12$). We keep $\alpha_M = 0$ in the *distance sector* and may allow a small $\alpha_M \propto \varepsilon$ in the *growth sector* only; in the calculations reported here we use $\kappa = 2$ and $\xi = 2.5$ in the growth calculations.

Using the entropic $\varepsilon(a)$ above and no re-tuning of Ω_Λ, we obtain

$$S_8 \simeq 0.788 \quad (-7.4\% \text{ vs. } \Lambda\text{CDM}), \tag{21}$$

robust to kernel powers $p \in \{4, 5, 6\}$ at the $< 10^{-3}$ level.

X. ILLUSTRATIVE HUBBLE-LADDER ENVIRONMENT CORRECTION (CAPPED)

Using the same $\varepsilon_{\rm env}(a, g)$ and a *sign-definite, first-principles* mapping for standardized SN/Cepheid residuals ("Theory+"), we confine source-side adjustments to observed host-systematic scales (caps ≤ 0.05 mag for SNe and ≤ 0.03 mag for same-host Cepheids). On an SH0ES-like catalog this nudges

$$H_0: \quad 73.0 \;\rightarrow\; 71.32 \text{ (SN cap only)}, \qquad \rightarrow\; 70.89 \text{ (SN cap + small Cepheid term)}, \tag{22}$$

without altering EM distances. These values are *illustrative, capped bounds*, not fitted predictions; environment-trend falsifiers (residual vs. host g/a_0; same-host Cepheid limits) are stated in Sec. XI.

XI. PREDICTIONS, PARAMETER TRANSLATIONS, AND FALSIFIABILITY

1. **GW/EM luminosity-distance ratio.** For a running Planck mass,

$$\frac{d_L^{\rm GW}(z)}{d_L^{\rm EM}(z)} = \exp\left[\frac{1}{2}\int_0^z \frac{\alpha_M(z')}{1+z'}\,dz'\right], \tag{23}$$

   frame invariant; depends only on the integrated α_M [6]. We enforce $|d_L^{\rm GW}/d_L^{\rm EM} - 1| \leq 5 \times 10^{-3}$.

2. **Mapping $\dot G/G$ to α_M.** $\alpha_M \equiv d\ln M^2/d\ln a = -(\dot G/G)/H$. At $z = 0$, $\alpha_M(0) = -(\dot G/G)_0/H_0$.

3. **What it does *not* mimic.** With $\alpha_T = \alpha_B = 0$, linear slip remains GR-like and the model does not by itself fit strong-lensing clusters; transition regimes require the full anisotropic kernel (future work).

XII. CONSISTENCY: BIANCHI IDENTITY AND FRW

Starting from Eq. (25), the contracted Bianchi identity and $\nabla_\mu T^{\mu\nu} = 0$ imply

$$\boxed{\nabla_b \Lambda_{\rm eff} = \tfrac{1}{2}\,R\,\nabla_b M^2}\,. \tag{24}$$

In FRW with $\alpha_M(a{=}1) \approx 0$, this is automatically satisfied at the present epoch (App. F).

XIII. UNCERTAINTY BUDGET (SUMMARY)

Source	Impact on H_0	Impact on S_8
β (numerical/systematic $\pm 3\%$)	n/a	$\ll 10^{-3}$ via normalization
Kernel power $p \in [4, 6]$	n/a	$< 10^{-3}$
GW/EM bound input	n/a	enforces $\lvert d_L^{\rm GW}/d_L^{\rm EM} - 1\rvert \leq 5 \times 10^{-3}$
Host proxy $\pm 50\%$	$\lesssim 0.2$ km s^{-1} Mpc^{-1} (uncapped only)	n/a

XIV. CONCEPTUAL PLACEMENT AND GR LIMIT

At background/linear order:

$$M^2(x)\,G_{ab} = 8\pi\,T_{ab} + \nabla_a\nabla_b M^2 - g_{ab}\,\Box M^2 - \Lambda_{\rm eff}(x)\,g_{ab}. \tag{25}$$

This is the standard $F(\phi)R$ (Jordan) structure in the $c_T = 1$, no-braiding corner ($\alpha_T = 0, \alpha_B = 0$); the sole background function is α_M [12]. Our constitutive closure fixes M^2 as a *functional* of Ξ. If $\nabla_a M^2 = 0$ ($\alpha_M \to 0$), Eq. (25) reduces to Einstein's equation with constant M and (if present) a constant zero mode. Under $\tilde{g}_{ab} = (M^2/M_0^2)g_{ab}$, frame-invariant signatures remain (notably $d_L^{\rm GW}/d_L^{\rm EM}$).

XV. CONCLUSION

Finite information capacity drives a *state-dependent response*. Each proper frame has a maximum entanglement load; as this threshold is approached, the response preserves causal stitching while keeping null geometry GR-like. Combining modular-Hamiltonian calculations (CHM/OP), MI subtraction, and a state-dependent $G(x)$, we obtain a *conditional, scheme-invariant* mapping $\Omega_\Lambda = \beta f c_{\rm geo}$ and a weak-field relation $a_0 = (5/12)\,\Omega_\Lambda^2 c H_0$. An entropic state-action law ($\Delta S \geq 0$) determines a monotone $\varepsilon(a)$ that suppresses growth ($S_8 \simeq 0.788$). A capped, environment-gated ladder illustration nudges SH0ES downward without altering distances. The framework is falsifiable and strictly limited to the safe window; beyond that domain, it is an invitation for further work.

Appendix A: Moment-kill identities and contact-term cancellation

Choose (a, b) so that for any smooth radial $F(r) = F_0 + F_2 r^2 + \mathcal{O}(r^4)$,

$$\int_{B_\ell} W_\ell F(r)\,d^3x - a\int_{B_{\sigma_1\ell}} W_{\sigma_1\ell} F(r)\,d^3x - b\int_{B_{\sigma_2\ell}} W_{\sigma_2\ell} F(r)\,d^3x = \mathcal{O}(\ell^6), \tag{A1}$$

canceling r^0 and r^2 moments. The surviving $\mathcal{O}(\ell^4)$ piece defines I_{00}.

Appendix B: Derivation of the Constitutive Factor f

1. Ball vs diamond (shape)

$W_\ell(r) = (\ell^2 - r^2)/(2\ell)$ yields $\mathcal{J}_{\rm ball} = \frac{4\pi}{15}\ell^4$. On the diamond horizon, $|v|$ with $A(v) = 4\pi(\ell^2 - v^2)$ yields $\mathcal{J}_{\rm hor} = 2\pi\ell^4$. Thus $f_{\rm shape} = 15/2$.

2. Boost and continuation

Unruh $T = \kappa/2\pi \Rightarrow f_{\rm boost} = 1$; after MI subtraction the finite coefficient is continuation invariant, so $f_{\rm cont} = 1$.

3. Boundary vs bulk: two bookkeepings

Let $u = v/\ell \in [-1, 1]$ and $\hat\rho_{\mathcal{D}}(u) = \frac{3}{4}(1 - u^2)$ with $\int_{-1}^{1} \hat\rho\,du = 1$. The geometric segment ratio is

$$R_{\rm seg} = \frac{\int_0^1 u(1-u^2)\hat\rho\,du}{\int_0^1 (1-u^2)\hat\rho\,du} = \frac{5}{16} = 0.3125.$$

Scheme A: include an isotropic IW/Raychaudhuri normalization $C_{\rm IW}$ so $C_{\rm contr} = (4/3)\,C_{\rm IW}$, giving $f_{\rm bdy}^{(A)} \simeq 0.10924$, hence $f^{(A)} = 0.8193$.

Scheme B: retain only geometric weights, including the isotropic null contraction (4/3) but not the additional IW factor. Then $f_{\rm bdy}^{(B)} = (4/3) \times (5/16) = 5/12$ and $f^{(B)} = 3.125$.

Appendix C: Integral definition and conventions for $c_{\rm geo}$

Define

$$c_{\rm geo} \equiv \frac{\displaystyle\int_{\text{FRW patch}} (\delta Q/T)_{\rm FRW}}{\displaystyle\int_{\text{local wedge}} (\delta Q/T)_{\rm wedge}}. \tag{C1}$$

For a cap of half-angle $\theta_\star$ with $\Delta\Omega = 2\pi(1-\cos\theta_\star)$,

$$c_{\rm geo} = \frac{4\pi}{\Delta\Omega} = \frac{2}{1-\cos\theta_\star}. \tag{C2}$$

Two consistent conventions (no double counting).

- **Scheme A (minimal wedge)**: $\boxed{c_{\rm geo} = 40}$, i.e. $\Delta\Omega^{(A)}_{\rm wedge} = 4\pi/40$ ($\cos\theta_\star^{(A)} = 19/20$).
- **Scheme B (equal-flux cap)**: imposing the no-double-counting rule for $\hat{\rho}_{\mathcal{D}}$ and $f^{(B)}$ yields $\boxed{c^{(B)}_{\rm geo} \simeq 10.49}$ ($\cos\theta_\star^{(B)} \simeq 0.80934$).

Appendix D: FRW zero-mode mapping (sketch)

With $M^2(a) = M_0^2[1+\mathcal{O}(\alpha_M)]$ and today $\alpha_M \simeq 0$:

$$\Lambda_{\rm eff} = 3H_0^2\, M_0^2\, (\beta\, f\, c_{\rm geo}), \qquad \Omega_\Lambda = \beta f c_{\rm geo}. \tag{D1}$$

Appendix E: EFT-of-DE mapping (summary)

At leading order we sit in the $c_T = 1$, no-braiding corner with $\alpha_T = 0 = \alpha_B$ and only $\alpha_M(a)$ active [12].

Appendix F: Bianchi-identity derivation for Eq. (24)

Starting from Eq. (25) and using $\nabla_a G^{ab} = 0$, $\nabla_a T^{ab} = 0$, and commutators on M^2 yields $\nabla_b \Lambda_{\rm eff} = \frac{1}{2}\, R\, \nabla_b M^2$.

Appendix G: Small-wedge Clausius domain and curvature suppression (EPMR)

Lemma H.1 (First-law domain). For Hadamard states in a Riemann-normal patch and small perturbations with $S(\rho|\rho_0) = \mathcal{O}(\varepsilon^2)$, the entanglement first law $\delta S = \delta\langle K\rangle + \mathcal{O}(\varepsilon^2)$ holds for sufficiently small diamonds.

Lemma H.2 (Moment-kill + MI subtraction). With $K_{\rm sub}$ of Eq. (4) choosing (a, b) to cancel the zeroth and second radial moments, contact and curvature–contact terms up to $\mathcal{O}(\ell^2)$ cancel in $\delta\langle K_{\rm sub}\rangle$.

Proposition H.1 (Curvature suppression and EPMR). After MI subtraction and moment-kill, the leading surviving isotropic term is $\mathcal{O}(\ell^4)$ and equals the *flat-space* modular coefficient; curvature dressings enter at $\mathcal{O}(\ell^6)$ within the safe window.

Appendix H: Weak-field flux law and the universal prefactor 5/12

A. Ingredients and regime. Consider Eq. (25) with $\delta G/G = -\beta\,\delta\varepsilon$ and the zero-mode mapping $\Omega_\Lambda = \beta f c_{\rm geo}$. Work in the static, weak-field limit (Newtonian gauge, $|\Phi|/c^2 \ll 1$, $\partial_t \to 0$) and within the safe window.

B. Quasilinear flux law. The $\nabla\nabla M^2$ terms renormalize the flux of $\nabla\Phi$. Coarse-graining over the wedge family yields

$$\nabla\cdot\big[\mu(Y)\,\nabla\Phi\big] = 4\pi G\,\rho_b, \qquad Y \equiv \frac{|\nabla\Phi|}{a_0}, \tag{H1}$$

with $\mu \to 1$ for $Y \gg 1$ and $\mu \sim Y$ for $Y \ll 1$.

C. Normalization from the homogeneous zero mode. The only late-time acceleration scale is $a_H \equiv cH_0$. Matching the static-flux normalization to the homogeneous Clausius zero mode with the same boundary–segment bookkeeping yields the *universal geometric constant* 5/12, hence

$$\boxed{a_0 = \frac{5}{12}\,(\beta f c_{\rm geo})^2\, c\, H_0 = \frac{5}{12}\,\Omega_\Lambda^2\, c\, H_0}\,. \tag{H2}$$

Angle/scheme invariant by Sec. VI C.

D. Scope and caveats. Applies in the static, weak-field, safe-window regime. Transition regimes $Y \sim 1$ and strong-lensing clusters require the full anisotropic kernel (future work).

Appendix I: Species uplift and C_T in OP normalization

In OP convention [10], the modular sensitivity factorizes as $\beta = 2\pi\, C_T\, I_{00}$. Our numerical calculation determines the geometric/kinematic coefficient I_{00} (after MI subtraction and moment-kill); matter content enters only through C_T. For free fields, C_T is known analytically (scalars, fermions, vectors) in OP normalization. For mixed content and finite masses one may form an effective

$$C_T^{\rm eff}(\ell) = \sum_i \Theta(1 - \ell m_i)\, C_T^{(i)},$$

so that species with $\ell m_i \gg 1$ decouple in the late-time safe window. The invariant $\beta\,\mathcal{C}_\Omega$ and hence Ω_Λ are therefore stable within our quoted β systematics across reasonable late-time windows.

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