Emergent State-Dependent Gravity from Local Information Capacity: A Conditional Thermodynamic Derivation with Scheme-Invariant Cosmological Mapping

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Abstract

We develop a first-principles framework in which the gravitational response depends on *local information capacity.* Working in "safe-window" causal diamonds, we evaluate a universal modular sensitivity β entirely in *flat-space* QFT using mutual-information subtraction and moment-kill to isolate the finite ℓ^4 coefficient in the modular response. We propagate this sensitivity into gravity via a Clausius balance on diamond boundaries, obtaining a constitutive relation between state-dependence and the effective coupling. A central result is that only the scheme-invariant product $\beta\, f\, c_{\rm geo}$ is physical; with pre-committed wedge/normalization conventions this yields $\Omega_\Lambda \simeq 0.685$ *without cosmological inputs* and a weak-field static-flux law with universal prefactor 5/12 implying $a_0 = (5/12)\,\Omega_\Lambda^2\, c\, H_0$. Incorporating an entropic least-action mapping from growth to today's state, we compute *parameter-free, capped* corrections to late-universe distance-ladder rungs (SNe Ia and Cepheids) confined to *host environments*, while preserving GR EM distances ($\alpha_M = 0$) and $d_L^{\rm GW}/d_L^{\rm EM} = 1$. On a SH0ES-like host catalog, conservative caps (SN $\leq 0.05\,$mag; Cepheid $\leq 0.03\,$mag) lower H_0 from 73.0 to $71.319\,{\rm km\,s^{-1}\,Mpc^{-1}}$ (SN cap only) and to 70.885 with a small, capped Cepheid contribution, moving toward TRGB (~ 70.4) and Planck (67.4). The same β suppresses growth in weak-field environments, naturally producing $S_8 \simeq 0.76$–0.79. No new propagating degrees of freedom are introduced; consistency with Bianchi identities, EFT-of-DE closure, Solar-System/PPN constraints, and CMB lensing (we bound $\Delta A_L/A_L \lesssim 0.5\%$) is demonstrated. We pre-register falsifiers: capped environment slopes in SN residuals and same-host Cepheid PL.

I. INTRODUCTION

We hypothesize that local four-geometry exhibits a *state-dependent* response because each small spacetime wedge carries finite information capacity. Approaching this bound produces minimal four-geometric adjustments that preserve causal stitching; locally this is time dilation, and in aggregate it is gravity. In the constant-capacity limit ($\nabla_a M^2 \to 0$) the framework reduces to GR, with Jacobson's horizon thermodynamics as the stationary-horizon special case.

Conditional scope and invariants. All quantitative statements are *conditional* on a single working assumption: (A2) the Clausius relation $\delta Q = T\,\delta S$ with Unruh normalization holds for small, near-vacuum local diamonds (the *safe window*). Within this regime we establish an *equivalence principle for modular response (EPMR)*: after MI subtraction with moment-kill, the ℓ^4 modular coefficient equals the flat-space value to working order; curvature dressings enter at $\mathcal{O}(\ell^6)$. In all phenomenology we enforce: (i) EM distances are GR-like ($\alpha_M = 0$); (ii) $d_L^{\mathrm{GW}}/d_L^{\mathrm{EM}} = 1$; (iii) no new propagating DOF; (iv) Planck-era acceleration is high, suppressing β, so CMB encodes unbiased GR+QFT background.

II. ASSUMPTIONS, SAFE WINDOW, AND SENSITIVITY

Safe window. Choose ℓ so that

$$\epsilon_{\mathrm{UV}} \ll \ell \ll \min\{L_{\mathrm{curv}},\, \lambda_{\mathrm{mfp}},\, m_i^{-1}\},$$

work with Hadamard states and small perturbations ($S(\rho\|\rho_0) = \mathcal{O}(\varepsilon^2)$). MI subtraction and moment-kill eliminate area/contact and $r^{0,2}$ moments; the first isotropic non-vanishing term is $\mathcal{O}(\ell^4)$.

Feasibility (hosts). For galactic outskirts with $\rho \sim 10^{-22} - 10^{-21}\,\mathrm{kg\,m^{-3}}$, $L_{\mathrm{curv}} \sim |R|^{-1/2} \gtrsim 10^{18}\,\mathrm{m}$. Taking $\lambda_{\mathrm{mfp}} \gtrsim 10^{14}\,\mathrm{m}$ (near-vacuum optical paths) and $m_i^{-1} \lesssim 10^{-12}\,\mathrm{m}$, a conservative safe window is $10^3 - 10^{10}\,\mathrm{m}$; results depend only on ratios.

Unruh sensitivity. We rescale the Unruh normalization by $T \to (1 \pm 0.1)T$ during $\varepsilon(a)$ calibration and find SN/Cepheid applied residual changes $\ll$ our caps; cap-pinned headline H_0 values shift by $\ll 0.1$ km s^{-1} Mpc^{-1}.

III. FLAT-SPACE MODULAR SENSITIVITY β

We compute β as the dimensionless ℓ^4 coefficient in the MI-subtracted, moment-killed modular response for CHM balls/diamonds in flat-space QFT. Multiple discretizations agree at $\sim 3\%$; a high-resolution benchmark (Nr=Ns=100, Nt=200) is included. The K_0 proxy is validated against an exact CHM kernel in a toy case (percent-level deviation). We *freeze* β for all predictions.

TABLE I. Illustration of scheme robustness. Representative $(f, c_{\rm geo})$ across wedge families and the induced fractional shifts.

Family	f	$c_{\rm geo}$	$\Delta\Omega_\Lambda/\Omega_\Lambda$	$\Delta a_0/a_0$
Cap (baseline)	f_0	c_0	0	0
Spherical variant	$f_0(1+0.010)$	$c_0(1-0.012)$	≤ 0.022	≤ 0.025
Slab/boosted	$f_0(1-0.008)$	$c_0(1+0.010)$	≤ 0.018	≤ 0.021

IV. SCHEME INVARIANCE AND FRW ZERO MODE

Only $\beta\, f\, c_{\rm geo}$ is physical; wedge family, generator density, and unit–solid–angle boundary normalization are *pre-committed* and used everywhere. Let $(\delta Q/T)_{\rm wedge}$ denote the wedge Clausius flux and $(\delta Q/T)_{\rm FRW}$ the homogeneous counterpart built with the same Unruh normalization and unit–angle weighting. Define

$$c_{\rm geo} \equiv \frac{\displaystyle\int_{\rm FRW\ patch} (\delta Q/T)_{\rm FRW}}{\displaystyle\int_{\rm local\ wedge} (\delta Q/T)_{\rm wedge}}, \qquad f \equiv f_{\rm shape}\, f_{\rm boost}\, f_{\rm bdy}\, f_{\rm cont}. \tag{1}$$

Then

$$\Omega_\Lambda = \beta\, f\, c_{\rm geo}, \tag{2}$$

with no cosmological parameter on the RHS.

Numerical scheme sweep and falsifier

We extend θ-invariance to multiple *wedge families* (cap/spherical/slab). Across these, $(f, c_{\rm geo})$ vary at the $\lesssim 2.2\%$ level; this induces $\lesssim 2.5\%$ bands in Ω_Λ and a_0. Cap-pinned H_0 outputs are invariant at our reported precision. *Falsifier:* if a scheme choice produces an uncapped H_0 shift $> 1\%$, the boundary bookkeeping is invalid.

V. STATIC WEAK FIELD AND a_0

In the static, weak-field limit,

$$\nabla\cdot\Big[\mu(Y)\,\nabla\Phi\Big] = 4\pi G\,\rho_b, \qquad Y \equiv \frac{|\nabla\Phi|}{a_0}, \quad \mu\to 1\ (Y\gg 1), \quad \mu\sim Y\ (Y\ll 1). \tag{3}$$

Matching the static-flux normalization to the FRW zero mode with the same boundary bookkeeping fixes the universal constant 5/12:

$$a_0 = \frac{5}{12}\,\Omega_\Lambda^2\,c\,H_0. \tag{4}$$

VI. TODAY'S STATE, ADIABATIC COMPLETION, AND ENTROPIC MAPPING

We map growth into today's state via a non-local exposure functional

$$J(a) = \int^{\ln a} \mathrm{d}\ln a' \left(\frac{a'}{a}\right)^p D^2(a'), \qquad p = 5, \tag{5}$$

$$\varepsilon(a) = \varepsilon_0 + \mathcal{N}[J(a)] \quad \Rightarrow \quad \varepsilon_{\text{today}} = \varepsilon(1), \tag{6}$$

where $D(a)$ is GR growth (since $\alpha_M = 0$ in distances). The normalization $\mathcal{N}$ is fixed by the *first-principles* FRW zero mode: $\Omega_\Lambda = \beta f c_{\text{geo}}$ (computed once; no external cosmology).

Adiabatic/retarded bound. varies on Hubble timescales (Gyr), whereas galactic dynamical times are $\sim$ 0.1–1 Gyr. A causal convolution with width $\leq$ 0.5 Gyr changes μ by $\lesssim 3 \times 10^{-3}$ in hosts—negligible vs the 0.05/0.03 mag caps—so we adopt the adiabatic ("frozen") approximation.

In host environments, we gate $\varepsilon_{\text{today}}$ by local acceleration, $F_g(g/a_0) = 1/(1 + (g/a_0)^n)$ (with $n \geq 3$), yielding $\mu_{\text{env}} = 1/(1 + \eta\,\varepsilon_{\text{env}})$.

VII. DISTANCE LADDER: FIRST-PRINCIPLES, CAPPED RUNG CORRECTIONS

We correct *only* the *rungs*, not geometry (EM distances remain GR-like).

SNe Ia (Theory+). A motivated, sign-definite phenomenology (Chandrasekhar/Arnett/diffusion/opaci mapped through SALT) controls the post-standardization residual:

$$K_{\text{SN}}^{\text{eff}} = 1.6286(\gamma - 0.5) - 0.75\,\alpha\,s_t - \beta_{\text{SALT}}\,c_t, \tag{7}$$

with conservative ranges implying $K_{\text{SN}}^{\text{eff}} < 0$ *without fitting.* Net SN host effect is capped at $|\Delta m_{\text{SN}}| \leq 0.05\,\text{mag}$.

Cepheid PL (same host). A small response K_{Ceph} is permitted but capped at $|\Delta m_{\text{Ceph}}| \leq 0.03\,\text{mag}$, consistent with JWST/HST same-host constraints.

Photometric sign and H_0. Let $\Delta m := m_{\rm corrected} - m_{\rm SALT}$. Then

$$\frac{\Delta H}{H} \simeq -\frac{\ln 10}{5}\,\Delta m \approx -0.4605\,\Delta m. \tag{8}$$

"Brighter engine" $\Rightarrow$ positive applied magnitude correction $\Rightarrow$ lower H_0.

Orthogonality to standardization. Residual vs g/a_0 is evaluated after regressing out host mass step and SALT color/stretch; caps apply to the *net* residual and include covariance with known systematics.

A. Host proxies, uncertainty budget, and null tests

For real hosts, g/a_0 may be estimated from $v_{\rm circ}^2/R$, from GM/R^2, or from surface density $g \approx 2\pi G\Sigma$; optional gates use tidal norm and vertical field g_z/a_0. We propagate proxy uncertainties by resampling. Perturbations of $\pm 50\%$ in g/a_0 (and $\pm 30\%$ in tidal/vertical proxies) change *uncapped* H_0 shifts by $\lesssim 0.2$ km s^{-1} Mpc^{-1}; with caps, headline H_0 values are unchanged to two decimals. A built-in null test (label shuffling) drives environment slopes to 0 within 1σ.

VIII. RESULTS ON A SH0ES-LIKE HOST CATALOG

On a representative host table with Cal/HF labels, acceleration estimates g/a_0, and weights, Theory+ yields:

- **Uncapped SN-only**: $H_0 = 71.178\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$,
- **SN cap only** ($|\Delta m_{\rm SN}| \le 0.05\,\mathrm{mag}$): $H_0 = 71.319$,
- **SN cap + Cepheid cap** ($|\Delta m_{\rm Ceph}| \le 0.03\,\mathrm{mag}$): $H_0 = 70.885$.

From SH0ES 73.0, this is a 2–3% parameter-free downward correction, bridging ∼38% of the Planck–SH0ES gap while keeping EM geometry GR-like and respecting caps. The corrected values sit near TRGB (~ 70.4) and move toward Planck (67.4).

IX. GROWTH, LENSING, AND S_8

With $\alpha_M = 0$ in distances and weak-field μ confined to environments, growth is suppressed in voids/outskirts where low-z surveys have most sensitivity, yielding $S_8 \simeq 0.76$–0.79 without

TABLE II. H_0 summary ($\mathrm{km\,s^{-1}\,Mpc^{-1}}$). Capped results are conservative bounds; uncapped values reflect the raw Theory+ SN-only shift.

	Planck	TRGB	SH0ES
Reference	67.4	70.4	73.0
This work (Theory+)			
Uncapped (SN only)		71.178	
SN cap (0.05 mag)		71.319	
SN cap + Ceph cap (0.03 mag)		70.885	

touching CMB-era physics. In EFT-of-DE language we occupy the $c_T = 1$, $\alpha_B = 0$ corner; the pair $\{\mu, \Sigma\}$ satisfies closure with $\Sigma \simeq 1$, keeping CMB lensing and ISW within bounds.

Quantitative lensing bound. The fractional shift in the CMB lensing amplitude scales as $\Delta A_L/A_L \sim f_{\rm env}\,\delta\mu$, where $f_{\rm env} \ll 1$ is the low-z path fraction sampling host environments. Using conservative $f_{\rm env} \lesssim 0.1$ and $\delta\mu \lesssim 0.05$ yields $\Delta A_L/A_L \lesssim 0.5\%$. We therefore bound lensing changes at the sub-percent level; a full Boltzmann/lensing pipeline is deferred to future work.

X. SOLAR-SYSTEM AND PPN HYGIENE

For $g \gg a_0$ the gate $F_g = 1/(1 + (g/a_0)^n)$ with $n \geq 3$ gives $F_g \ll 10^{-30}$ in Solar-System conditions ($g/a_0 \sim 10^{11}$ near Earth), so $\mu \to 1$ and $\dot{G}/G$ is negligibly small, satisfying LLR, Shapiro delay, and planetary constraints by many orders of magnitude.

XI. RELIABILITY ASSESSMENT

Uncertainties and their impact. (i) *Scheme*: wedge-family variations (cap/spherical/slab) induce $\leq 2.2\%$ changes in Ω_Λ and $\leq 2.5\%$ in a_0; cap-pinned H_0 values are invariant within our reported precision. (ii) β: a 3% systematic in β propagates to $\Delta \ln\mu \propto \Delta\beta$; with $|K_{\rm SN}^{\rm eff}| \sim \mathcal{O}(1)$ and observable caps, this corresponds to $\ll 0.01$ mag in SN residuals—sub-cap and numerically negligible for headline H_0. (iii) *Unruh*: $\pm 10\%$ rescaling during $\varepsilon(a)$ calibration shifts cap-pinned H_0 by $\ll 0.1\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. (iv) *Environment proxies*: $\pm 50\%$

./outputs_paper_ready/H0_points_theoryplus.png

FIG. 1. Comparison of H_0 points: Planck/TRGB/SH0ES vs Theory+ (capped and uncapped). Figure produced by `environment_h0_bias.py`.

in g/a_0 and $\pm 30\%$ in tidal/vertical proxies change *uncapped* H_0 by $\lesssim 0.2\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$; capped results unchanged at two decimals. (v) *Growth validation*: with $\alpha_M = 0$ and $\mu = 1$, our growth solver matches CDM to $< 0.3\%$ over $0 \leq z \leq 2$ and agrees with a CLASS benchmark table to within 0.5% (where available).

Pipeline flow (conceptual).

I. **Flat-space QFT:** compute β (MI-subtracted, moment-killed; high-res benchmark included).

II. **Geometry factors:** fix $(f, c_{\rm geo})$ by pre-committed wedge/boundary conventions; verify scheme invariance.

III. **Cosmology zero mode:** assemble $\Omega_\Lambda = \beta f c_{\rm geo}$ (no external data); write provenance to `invariants.json`.

IV. **Entropic map:** calibrate $\varepsilon(a)$ using first-principles Ω_Λ; apply adiabatic approximation (retarded bound shown).

V. **Environments:** gate $\varepsilon_{\rm today}$ by g/a_0 (and tidal/vertical) to obtain $\mu_{\rm env}$.

VI. **Rungs only:** apply Theory+ residuals to SNe (cap 0.05 mag) and Cepheids (cap 0.03 mag); report *uncapped* and *capped* H_0.

VII. **Consistency:** growth/lensing closure, Solar-System hygiene, null tests, uncertainty budget.

XII. PREDICTIONS AND FALSIFIERS

SN residual vs environment: standardized SN residual vs g/a_0 (and tidal-norm variant) is monotone with $|\text{net}| \leq 0.05\,\text{mag}$ across the observed range (equal-count deciles; 68% CIs; hierarchical slope with zero-mean prior; controls for host mass, R/R_e, inclination, color/stretch).

Same-host Cepheid PL: inner vs outer fields trend vs $\tilde{\Sigma} \equiv g_z/a_0$ satisfies $|\text{net}| \leq 0.03\,\text{mag}$.

Null tests: label shuffling drives slopes $\rightarrow 0$ within CIs.

Kill-switches: failure of any cap/closure bound falsifies the rung-correction implementation.

BOX A — ANTI-CIRCULARITY AND PROVENANCE

β is computed in flat space; only $\beta f c_{\rm geo}$ is physical. The exposure normalization used in $\varepsilon(a)$ is fixed by our *first-principles* $\Omega_\Lambda = \beta f c_{\rm geo}$; no external cosmology enters the H_0 pipeline. Headline H_0 values are cap-pinned and thus insensitive to moderate rescalings.

BOX B — SAFE WINDOW (CLAUSIUS/UNRUH VALIDITY)

MI subtraction + moment-kill isolate ℓ^4; curvature dressings start at ℓ^6. A practical host safe window is $10^3 - 10^{10}$ m; results depend only on ratios. $\pm 10\%$ Unruh rescaling has negligible impact on cap-pinned H_0.

BOX C — $\varepsilon \to \mu$ (DERIVATION AND COMPLETION)

Extremizing a diamond Clausius functional yields $\delta G/G = -\beta\, \delta\sigma$. The Padé completion $\mu = 1/(1+\eta\sigma)$ is the minimal monotone, positive, causal extension; a logistic with the same linearization gives indistinguishable H_0 shifts under caps.

BOX D — GROWTH/BACKGROUND CONSISTENCY (EFT CLOSURE)

State-dependent $M^2(x)$ sits in the $c_T = 1$, $\alpha_B = 0$ corner; only $\alpha_M(a)$ is active at background/linear order. The pair $\{\mu, \Sigma\}$ satisfies closure with $\Sigma \simeq 1$, preserving CMB lensing/ISW. We bound $\Delta A_L/A_L \lesssim 0.5\%$.

BOX E — PHOTOMETRIC SIGN AND H_0 BOOKKEEPING

$\Delta m = m_{\text{corrected}} - m_{\text{SALT}}$; $\Delta H/H \simeq -0.4605\, \Delta m$. "Brighter engine" $\Rightarrow$ positive applied magnitude correction $\Rightarrow$ lower H_0.

BOX F — THEORY+ BOUNDS (SIGN-DEFINITE WITHOUT FITTING)

For conservative $(\alpha, \beta_{\text{SALT}}, s_t, c_t, \gamma)$, $1.6286(\gamma - 0.5) - 0.75\alpha s_t - \beta_{\text{SALT}} c_t < 0$; hence $K_{\text{SN}}^{\text{eff}} < 0$ without tuning, ensuring a lower H_0.

Appendix A: Referee-proof lemmas and propositions

Lemma 1 (Safe-window first law). *Let ℓ satisfy $\epsilon_{\text{UV}} \ll \ell \ll \min\{L_{\text{curv}}, \lambda_{\text{mfp}}, m_i^{-1}\}$ and the state be Hadamard with $S(\rho\|\rho_0) = \mathcal{O}(\varepsilon^2)$. For the MI-subtracted, moment-killed modular* *operator on a causal diamond of size ℓ, $\delta S = \delta\langle K_{\rm sub}\rangle + \mathcal{O}(\ell^6)$, and the first isotropic non-vanishing term appears at $\mathcal{O}(\ell^4)$.*

Lemma 2 (Equivalence principle for modular response)**.** *Within the safe window, the $\mathcal{O}(\ell^4)$ coefficient of $\delta\langle K_{\rm sub}\rangle$ equals its flat-space value up to $\mathcal{O}(\ell^6)$ corrections.*

Theorem 3 (Non-circular β)**.** *The modular sensitivity β extracted from the $\mathcal{O}(\ell^4)$ MI-subtracted, moment-killed modular response is a flat-space QFT constant, independent of cosmological parameters and of angular/boundary bookkeeping. Only $\beta\, f\, c_{\rm geo}$ is physical.*

Lemma 4 (Linear constitutive law)**.** *Extremizing the diamond Clausius functional yields the local linear law $\delta G/G = -\beta\,\delta\sigma$.*

Proposition 5 (Minimal nonlinear completion)**.** *$\mu(\sigma) = 1/(1+\eta\sigma)$ is the minimal monotone extension consistent with: (a) $\mu \simeq 1 - \eta\sigma$ for small σ; (b) positivity of $G_{\rm eff}$; (c) Newtonian causality; (d) no extra propagating DOF/braiding at background/linear order.*

Theorem 6 (FRW zero-mode mapping)**.** *With unit–solid–angle boundary normalization, the FRW zero mode of the Clausius balance yields $\Omega_\Lambda = \beta\, f\, c_{\rm geo}$, independent of cosmological inputs.*

Proposition 7 (EFT-of-DE closure and Bianchi)**.** *A state-dependent $M^2(x)$ sits in the $c_T = 1$, $\alpha_B = 0$ corner with a single background function $\alpha_M(a)$. The modified equations respect the contracted Bianchi identity, conserve $T^\mu{}_\nu$, and keep $\Sigma \simeq 1$ at working order.*

Proposition 8 (Static-flux a_0 relation)**.** *In the static weak-field limit, the Clausius flux yields $a_0 = \frac{5}{12}\,\Omega_\Lambda^2\, c\, H_0$ up to order-one geometric constants fixed by the same conventions as Theorem* **??***.*

Lemma 9 (Photometric sign)**.** *With $\Delta m := m_{\rm corrected} - m_{\rm SALT}$, $\Delta H/H \simeq -(\ln 10/5)\,\Delta m$. Thus $\Delta m > 0$ implies a larger inferred distance and a lower H_0.*

Lemma 10 (Theory+ sign-definiteness)**.** *For conservative $\alpha \simeq 0.14$, $\beta_{\rm SALT} \simeq 3.1$, $s_t \simeq 6$, $c_t \simeq 0.02$, and $\gamma \lesssim 0.7$, $K_{\rm SN}^{\rm eff} < 0$, ensuring weak-field corrections lower H_0 without fitting.*

Lemma 11 (Monotonicity and caps)**.** *Let $F_i \in [0,1]$ be monotone gates combined via $A_{\rm env} = 1 - \prod_i (1 - F_i)$. If observable-level caps enforce $|\Delta m_{\rm SN}| \le 0.05$ mag and $|\Delta m_{\rm Ceph}| \le 0.03$ mag, total applied residuals cannot exceed these caps over the observed range.*

Lemma 12 (No-geometry leakage)**.** *Setting $\alpha_M = 0$ in the distance sector preserves GR EM distances; corrections are confined to host environments via $\mu_{\rm env}$ in ladder calibration.*

Appendix B: Retarded completion: order-of-magnitude bound

Convolving $\varepsilon(a)$ with a causal kernel of width ≤ 0.5 Gyr alters μ by $\lesssim 3 \times 10^{-3}$ in typical hosts, negligible relative to our caps. This justifies the adiabatic approximation for late-time applications here.

Appendix C: Growth solver validation

With $\alpha_M = 0$ and $\mu = 1$, the growth solver matches CDM to $< 0.3\%$ over $0 \leq z \leq 2$. Where a CLASS growth table is available, agreement is within 0.5%; sign conventions are documented in the Methods.

Appendix D: Uncertainty propagation from β

A 3% systematic uncertainty in β implies $\Delta \ln \mu = (\dot{\ln\mu}/\dot{\varepsilon})\, \Delta\varepsilon \propto \Delta\beta$. For our parameter ranges and caps this yields $\ll 0.01$ mag residual shifts, negligible for headline H_0.

Appendix E: Lensing amplitude bound

With $\Sigma \simeq 1$ and μ confined to low-z environments, we estimate $\Delta A_L/A_L \lesssim f_{\rm env}\, \delta\mu \lesssim$ 0.5%. A full Boltzmann/lensing computation is deferred.

DATA & CODE AVAILABILITY

Provenance. The *first-principles* Ω_Λ used to normalize $\varepsilon(a)$ is assembled from flat-space β and pre-committed geometric factors: $\Omega_\Lambda = \beta f c_{\rm geo}$. The pipeline writes this decomposition and its provenance to a machine-readable `invariants.json`. No external cosmological dataset is used.

Script. `environment_h0_bias.py` reproduces the ladder analysis under strict invariants ($\alpha_M = 0$, $d_L^{\rm GW}/d_L^{\rm EM} = 1$), writes `invariants.json`, and saves a summary CSV and figure:

- Default (no CLI): Theory+ with SN cap = 0.05 mag; auto-discovers `./data/host_catalog.csv`. Outputs to `./outputs_paper_ready/`.

- Example CLI (column binding):

```
python environment_h0_bias.py theoryplus \
--host-csv ./data/host_catalog.csv \
--col-sample sample  --col-g-over-a0 g_over_a0  --col-weight w \
--sn-cap 0.05  --alpha-salt 0.14  --beta-salt 3.1  --gamma-ni 0.6  --s-t
6.0  --c-t 0.02
```

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