Homework #2

Problem 1

1. Derive the linearized equations of perturbed motion of an axial symmetric satellite spinning around its major axis of inertial $J_2 > J_1 = J_3$. Assume a constant perturbation torque acting on the spacecraft.
2. Assume a single spin satellite with inertia matrix

$$\mathbf{J}_{cg}^{b} = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 90 & 0 \\ 0 & 0 & 60 \end{bmatrix} \text{kg m}^2$$

rotating with an initial angular velocity

$$\boldsymbol{\omega}^b(t_0) = \begin{bmatrix} 0 & 0.5 & 0 \end{bmatrix}^{\mathrm{T}} \text{rad/s}$$

and assume a constant torque perturbation

$$\boldsymbol{\tau}^b = \begin{bmatrix} .001 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \text{N m}$$

Numerically integrate Euler's equation of rotational motion for 1 hour and compare the results against the analytical solution you derived above.

Problem 2

Assume a 400 km equatorial posigrade circular orbit, and assume the satellite has nominally the body x-axis pointing in the direction of motion, the body y-axis pointing to nadir, and it is spinning at orbit rate around the z-axis (i.e. this is an Earth pointing satellite). At time t = 0 the satellite position in the ECI frame is

$$\mathbf{r}^i(0) = \begin{bmatrix} 0 & -(R_e + 400km) & 0 \end{bmatrix}^{\mathrm{T}}$$

where R_e is the equatorial Earth's radius. The inertial matrix is

$$\mathbf{J}_{cg}^{b} = \begin{bmatrix} 90 & 0 & 0 \\ 0 & 70 & 0 \\ 0 & 0 & 60 \end{bmatrix} \text{kg m}^2$$

assume nominal initial attitude and a slightly perturbed initial angular velocity

$$\boldsymbol{\omega}_{b/i}^b(0) = \begin{bmatrix} 0.0001 & 0.0001 & \omega_{orb\ rate} \end{bmatrix}^{\mathrm{T}} \text{deg/s}$$

Design a controller that regulates the tumbling motion of this satellite with Gravity Gradient perturbations. Simulate the dynamics of the controlled system with a momemtum wheel to keep the attitude pointing error below 0.5 deg. Derive an analytical solution to this before attempting simulating it.

Choose reasonable actuators sizes and performance.

Problem 3

Assume a cubic spacecraft with a single rectangular solar panel connected to the satellite by a rod of negligible surface. Find a good reference for aerodynamic torque perturbations (e.g. the book by Peter Hughes or by James Wertz) and model a LEO satellite's attitude dynamics subject to gravity gradient and aerodynamic torques. Choose reasonable satellite mass and size.