$\operatorname{Midterm}$ - ASE 389P.8 Satellite Control Systems

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1 Part 1

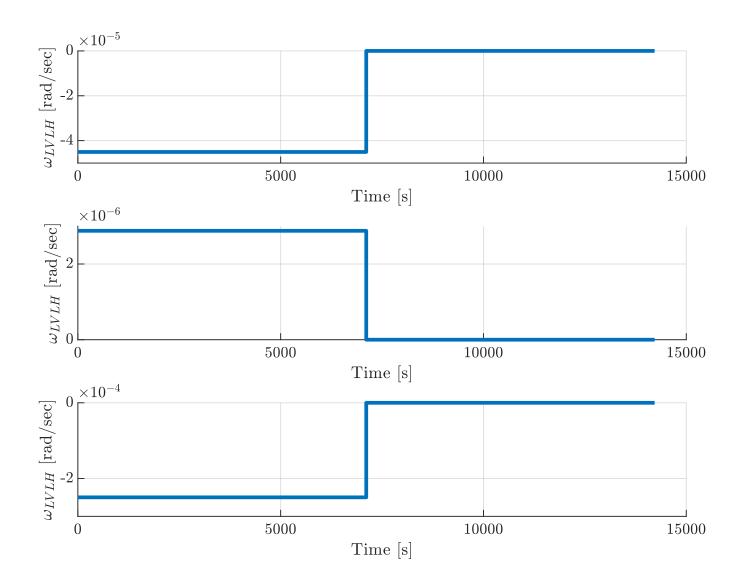


Figure 1: The nominal LVLH to body angular rates. We begin with a constant rate maneuver, followed by a LVLH hold.

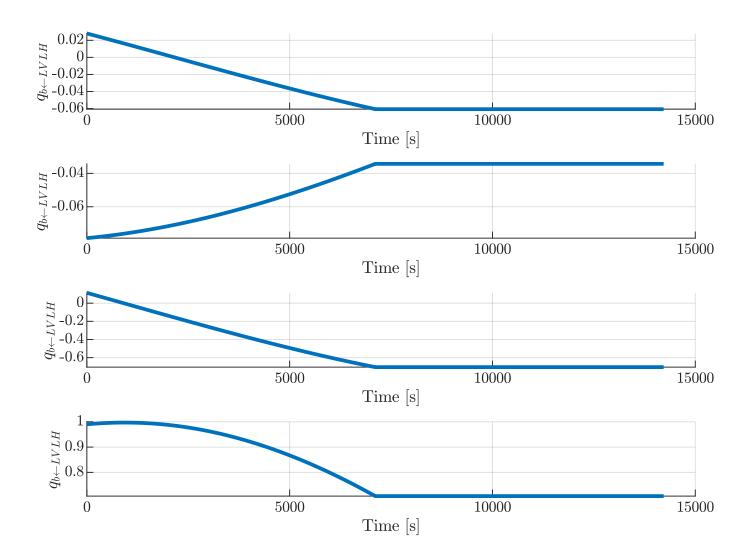


Figure 2: The nominal LVLH to body quaternion. Here we see that the quaternion is commanded to be constant at the end of the maneuver.

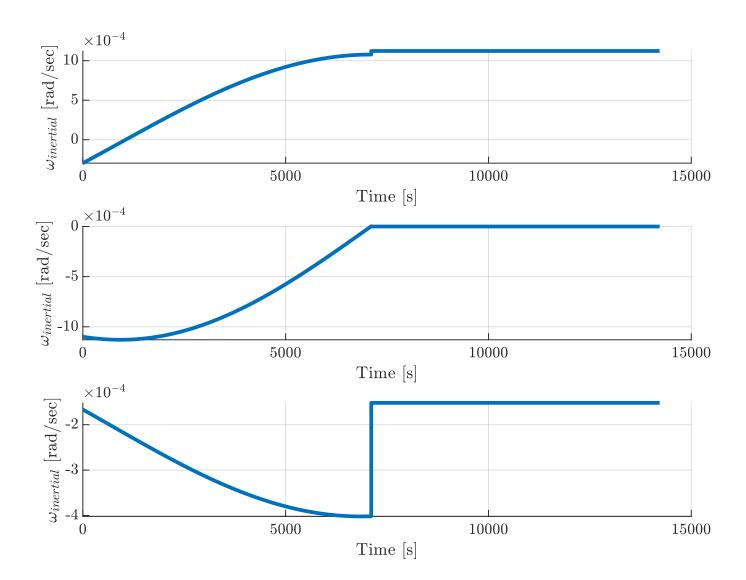


Figure 3: The nominal inertial to body angular rates. These are not zero after the maneuver because the LVLH frame is rotating.

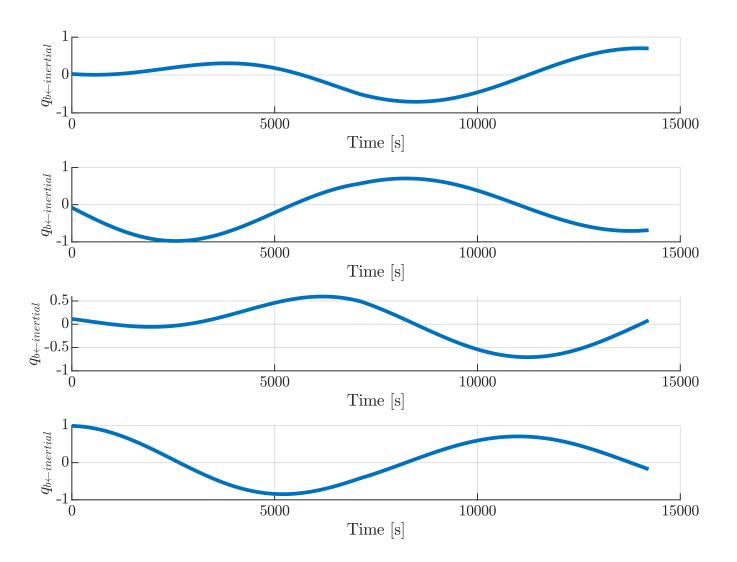


Figure 4: The nominal inertial to body quaternion. This is not constant after the maneuver because the LVLH frame is rotating.

2 Part 2

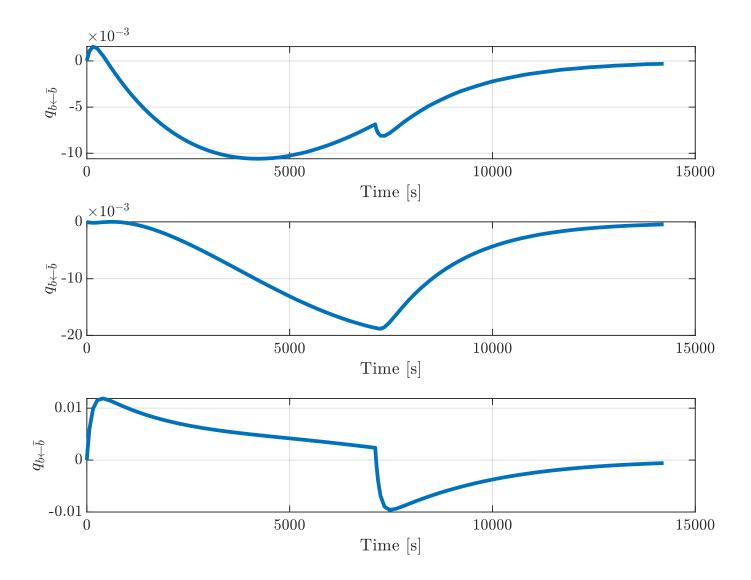


Figure 5: The attitude control error as represented by the vector components of the error quaternion. The commanded body rate profile is discontinuous at the beginning and end of the maneuver, resulting in overshoot and poor tracking performance after those moments.

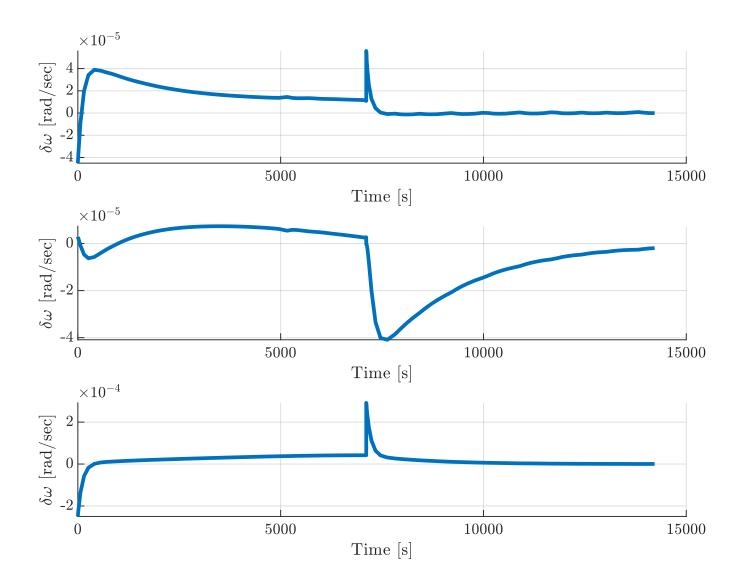


Figure 6: The attitude rate error. The commanded body rate profile is discontinuous at the beginning and end of the maneuver, resulting in overshoot and poor performance after those moments.

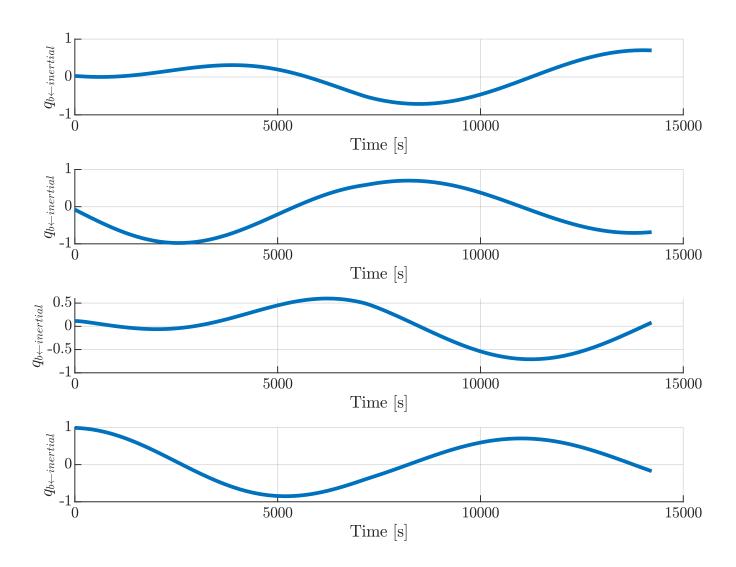


Figure 7: The true inertial to body quaternion.

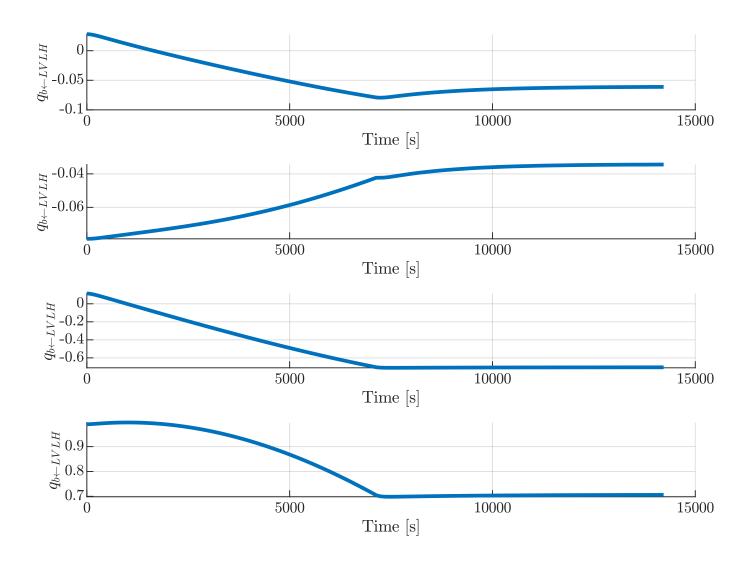


Figure 8: The true LVLH to body quaternion.

3 Part 3

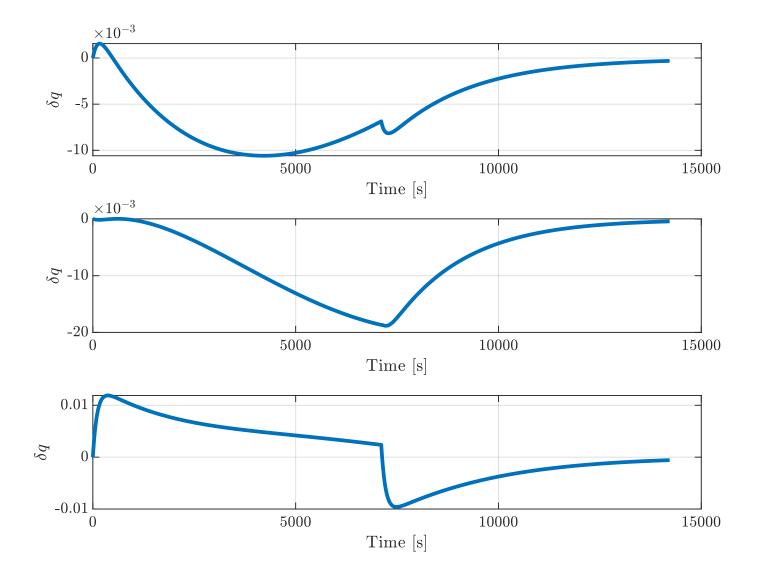


Figure 9: The attitude control error as represented by the vector components of the error quaternion. The commanded body rate profile is discontinuous at the beginning and end of the maneuver, resulting in overshoot and poor performance after those moments.

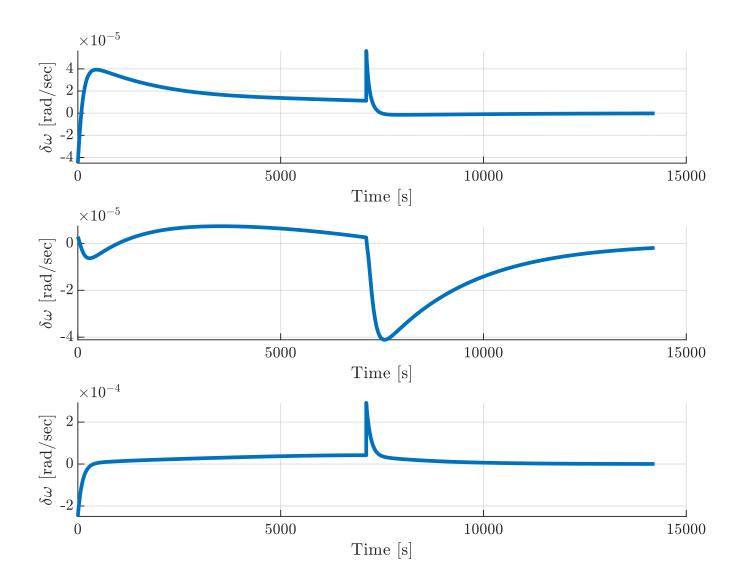


Figure 10: The attitude rate error. The commanded body rate profile is discontinuous at the beginning and end of the maneuver, resulting in overshoot and poor performance after those moments.

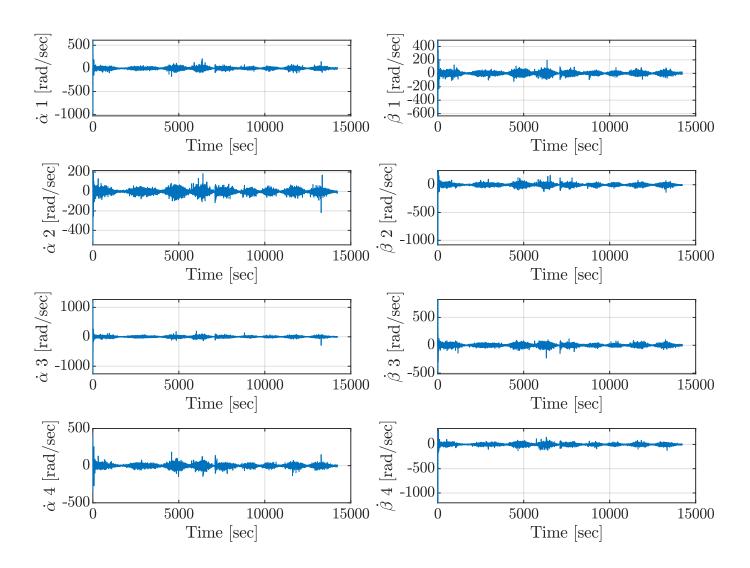


Figure 11: The gimbal rates for each CMG. These rates are extremely high and physically unrealizable. This occurs because our KD controller gains are very high and the system contains no logic to saturate the commanded gimbal rate.

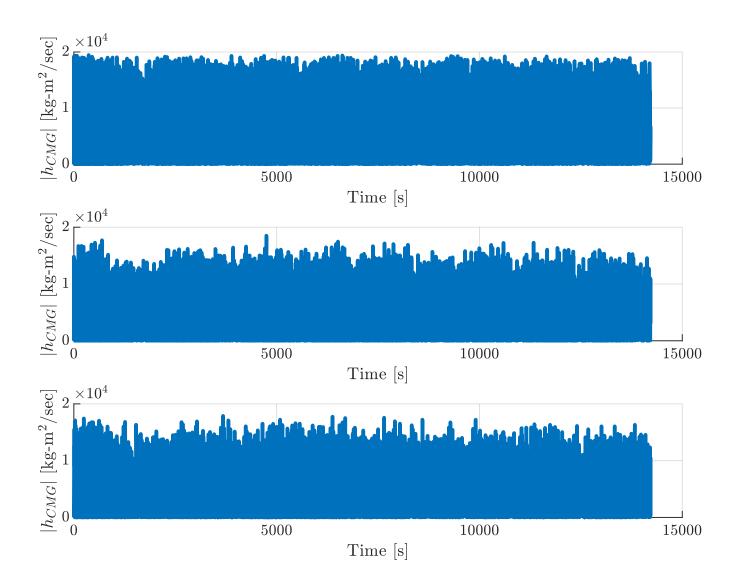


Figure 12: The magnitude of each element of the angular momentum vector for the CMG array. The rate of change of this vector is extremely high due to the physically unrealizable gimbal rates.

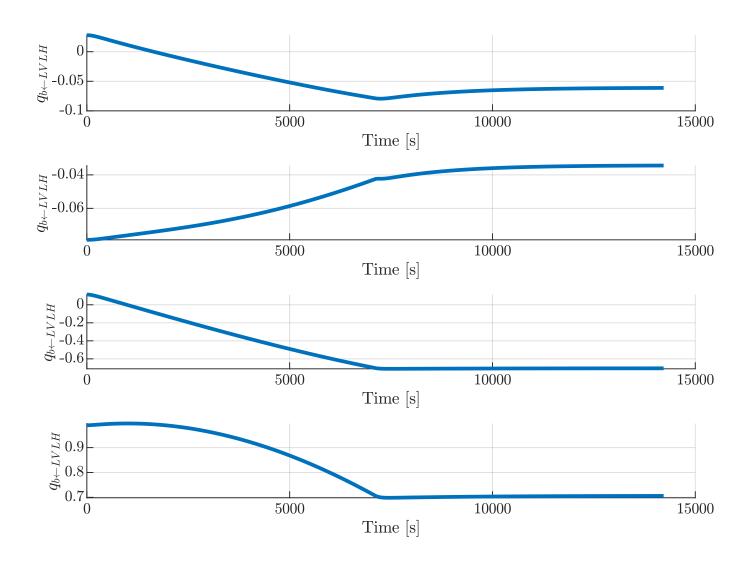


Figure 13: The true LVLH to body attitude quaternion.

4 Code

```
%% Setup
clear
close all
clc
addpath("..\util\")
%% Problem Initalization
% Gravity
mu = 398600; %km3/s2
% Orbital elements
a = 6371 + 400;
e = 0;
i = 0;
Ohm = 0;
w = 0;
theta = 0;
[r inertial 0, v inertial 0] = OE2State(a, e, i, Ohm, w, theta);
% Find orbit rate
n = sqrt(mu/a^3);
% Find LVLH rotation rate
w LVLH wrt inertial in LVLH = [0, -n, 0]';
% Initial rotation between LVLH and inertial
x LVLH inertial 0 = v inertial 0/norm(v inertial 0);
z_LVLH_inertial_0 = -r_inertial_0/norm(r_inertial_0);
y_LVLH_inertial_0 = cross(z_LVLH_inertial_0,x_LVLH_inertial_0);
T_inertial2LVLH_0 = [x_LVLH_inertial_0'; y_LVLH_inertial_0'; z_LVLH_inertial 0'];
q inertial2LVLH 0 = DCM2Quat(T inertial2LVLH 0);
% Rotation formulations
J = [24181836 \ 3783405 \ 3898808
    3783405 37621803 -1171849
    3898808 -1171849 51576634];
w_b_LVLH_0 = [0 \ 0 \ 0]'; % Initial LVLH rotation rate, rad/sec
q_LVLH2body_0 = [0.028, -0.0788, 0.1141, 0.9899]'; % Initial attitude quaternion
%q LVLH2body f = q LVLH2body 0;
q LVLH2body f = [-0.0607, -0.0343, -0.7045, 0.7062]'; % Attitude quaternion at end of the \checkmark
manuever
% Initial pose and rate in inertial
q inertial2body 0 = QuatProduct(q LVLH2body 0,q inertial2LVLH 0);
w body wrt inertial 0 = QuatTransform(q LVLH2body 0,w LVLH wrt inertial in LVLH) + ✔
w b LVLH 0;
% Final simulation time
Tf = 2*7110:
```

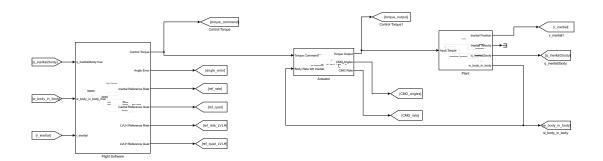
```
% Tf = 100;
% Tf = 10000;
% Final manuever time
Tf man = 7110;
% CMG momentum
h0 = 4881;
% Maximum CMG rates
rate max = Inf*(pi/180); % Rad/sec
%% Design the maneuver in the LVLH frame
% Change in quaternion
dq LVLH = QuatProduct(q LVLH2body f,QuatInv(q LVLH2body 0));
% Euler axis and angle change
[dtheta LVLH, dn LVLH] = Quat2AxisAngle(dq LVLH);
% Find angular rate in rad/sec
w b LVLH man = dtheta LVLH/Tf man*dn LVLH;
%% Nonlinear controller design
kp nonlin = 500;
kd nonlin = 500000;
%% Main
use CMG = false;
out_no_CMG = sim("simulink\midterm sim.slx");
use CMG = true;
out_w_CMG = sim("simulink\midterm_sim.slx");
%% Extract information for Part 1
tout_no_CMG = out_no_CMG.tout;
q_LVLH2body_ref = squeeze(out_no_CMG.ref_quat_LVLH);
w body LVLH ref = squeeze(out no CMG.ref rate LVLH);
q_inertial2body_ref = squeeze(out_no_CMG.ref_quat);
w_body_inertial_ref_no_CMG = squeeze(out_no_CMG.ref_rate);
%% Extract Informatin for Part 2
err quat no CMG = squeeze(out no CMG.error quat);
q inertial2body no CMG = squeeze(out no CMG.quat);
w body inertial no CMG = squeeze(out no CMG.w);
q_inertial2LVLH_no_CMG = squeeze(out_no_CMG.q_inertial2LVLH);
```

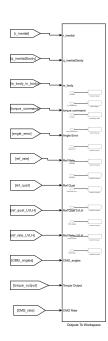
```
% Find quaternion from body to LVLH
q_LVLH2body_no_CMG = zeros(size(q_inertial2LVLH_no_CMG));
for ii = 1:length(tout no CMG)
    q LVLH2body no CMG(:,ii) = QuatProduct(q inertial2body no CMG(:,ii),QuatInv✔
(q inertial2LVLH no CMG(:,ii)));
end
%% Extract Information for Part 3
tout_w_CMG = out_w_CMG.tout;
w_body_inertial_w_CMG = squeeze(out_w_CMG.w);
q inertial2LVLH w CMG = squeeze(out w CMG.q inertial2LVLH);
q_inertial2body_w_CMG = squeeze(out_w_CMG.quat);
CMG_rates = squeeze(out_w_CMG.CMG_rates);
err quat w CMG = squeeze(out w CMG.error quat);
w body inertial ref w CMG = squeeze(out w CMG.ref rate);
CMG h = squeeze(out w CMG.CMG h);
% Find quaternion from body to LVLH
q LVLH2body w CMG = zeros(size(q inertial2LVLH no CMG));
for ii = 1:length(tout w CMG)
    q LVLH2body w CMG(:,ii) = QuatProduct(q inertial2body w CMG(:,ii),QuatInv ✓
(q inertial2LVLH w CMG(:,ii)));
end
%% Plotting Part 1
figure
for ii = 1:3
   subplot(3,1,ii)
   hold on
   plot(tout_no_CMG, w_body LVLH ref(ii,:),'LineWidth',2)
   xlabel('Time [s]','Interpreter','latex')
   ylabel('$\omega_{LVLH}$ [rad/sec]','Interpreter','latex')
    grid on
end
saveas(gcf,"latex/figs/P1Q1.pdf")
for ii = 1:4
   subplot(4,1,ii)
   hold on
    plot(tout no CMG, q LVLH2body ref(ii,:),'LineWidth',2)
   xlabel('Time [s]','Interpreter','latex')
    ylabel("$q {b \leftarrow LVLH}$","Interpreter","latex")
end
saveas(gcf,"latex/figs/P1Q2.pdf")
```

```
figure
for ii = 1:3
    subplot(3,1,ii)
    hold on
    plot(tout no CMG, w body inertial ref no CMG(ii,:), 'LineWidth',2)
    xlabel('Time [s]','Interpreter','latex')
    ylabel('$\omega {inertial}$ [rad/sec]','Interpreter','latex')
    grid on
end
saveas(gcf,"latex/figs/P1Q3.pdf")
figure
for ii = 1:4
    subplot(4,1,ii)
    hold on
    plot(tout no CMG, q inertial2body ref(ii,:),'LineWidth',2)
    xlabel('Time [s]')
    ylabel("$q {b \leftarrow inertial}$",'Interpreter','latex')
    grid on
end
saveas(gcf,"latex/figs/P1Q4.pdf")
%% Plotting Part 2
figure
for ii = 1:3
   subplot(3,1,ii)
    plot(tout_no_CMG, err_quat_no_CMG(ii,:),'LineWidth',2)
    xlabel('Time [s]','Interpreter','latex')
    ylabel("$q {b \leftarrow \bar{b}}$","Interpreter","latex")
    grid on
end
saveas(gcf,"latex/figs/P2Q1.pdf")
figure
for ii = 1:3
    subplot(3,1,ii)
    hold on
   plot(tout_no_CMG, w_body_inertial_ref_no_CMG(ii,:) - w_body_inertial_no_CMG ✓
(ii,:),'LineWidth',2)
    xlabel('Time [s]', "Interpreter", "latex")
    ylabel('$\delta \omega$ [rad/sec]',"Interpreter","latex")
    grid on
end
saveas(gcf,"latex/figs/P2Q2.pdf")
figure
for ii = 1:4
    subplot(4,1,ii)
    plot(tout no CMG, q inertial2body no CMG(ii,:),'LineWidth',2)
```

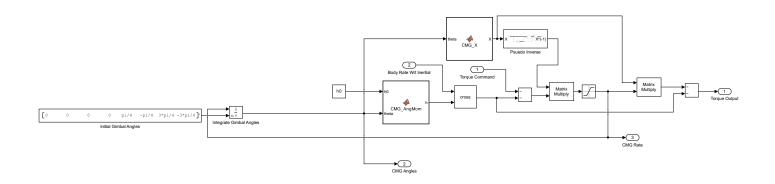
```
xlabel('Time [s]','Interpreter','latex')
    ylabel("$q {b \leftarrow inertial}$","Interpreter","latex")
    grid on
end
saveas(gcf,"latex/figs/P2Q3.pdf")
figure
for ii = 1:4
   subplot(4,1,ii)
    hold on
    plot(tout_no_CMG, q_LVLH2body_no_CMG(ii,:),"LineWidth",2)
    xlabel('Time [s]', "Interpreter", "latex")
    ylabel("$q {b \leftarrow LVLH}$","Interpreter","latex")
    grid on
end
saveas(gcf,"latex/figs/P2Q4.pdf")
%% Plotting Part 3
figure
for ii = 1:3
    subplot(3,1,ii)
    plot(tout_w_CMG, err_quat_w_CMG(ii,:),"LineWidth",2)
    xlabel('Time [s]',"Interpreter","latex")
    ylabel("$\delta q$","Interpreter","latex")
    grid on
end
saveas(gcf,"latex/figs/P3Q1.pdf")
figure
for ii = 1:3
    subplot(3,1,ii)
    hold on
    plot(tout_w_CMG, w_body_inertial_ref_w_CMG(ii,:) - w_body_inertial_w_CMG(ii,:),"
LineWidth",2)
    xlabel('Time [s]', "Interpreter", "latex")
    ylabel('$\delta \omega$ [rad/sec]',"Interpreter","latex")
    grid on
saveas(gcf,"latex/figs/P3Q2.pdf")
figure
for ii = 1:4
   subplot(4,2,2*ii-1)
    plot(tout w CMG,CMG rates(ii,:))
    xlabel('Time [sec]', "Interpreter", "latex")
    ylabel(strcat("$\dot{\alpha}$ ",num2str(ii)," [rad/sec]"),"Interpreter",'latex')
    grid on
    subplot(4,2,2*ii)
    plot(tout w CMG, CMG rates(ii+4,:))
```

```
xlabel('Time [sec]', "Interpreter", "latex")
    ylabel(strcat("$\dot{\beta}$ ",num2str(ii)," [rad/sec]"),"Interpreter",'latex')
    grid on
end
saveas(gcf,"latex/figs/P3Q3.pdf")
figure
for ii = 1:3
    subplot(3,1,ii)
    hold on
    plot(tout_w_CMG, abs(CMG_h(ii,:)),"LineWidth",2)
    xlabel('Time [s]', "Interpreter", "latex")
    ylabel('\$|h_{CMG}|)\$ [kg-m\text{textsuperscript}\{2\}/sec]',"Interpreter","latex")
    grid on
end
saveas(gcf,"latex/figs/P3Q4.pdf")
figure
for ii = 1:4
    subplot(4,1,ii)
    hold on
    plot(tout_w_CMG, q_LVLH2body_w_CMG(ii,:),"LineWidth",2)
    xlabel('Time [s]',"Interpreter","latex")
    ylabel("$q {b \leftarrow LVLH}$","Interpreter","latex")
    grid on
end
saveas(gcf,"latex/figs/P3Q5.pdf")
```



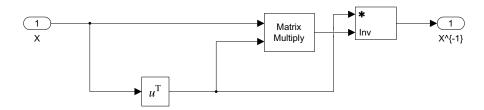


Actuator

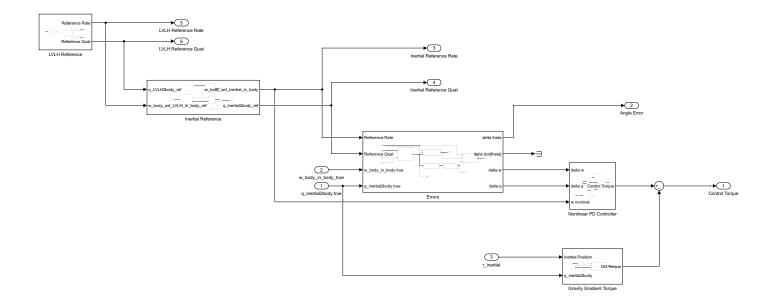


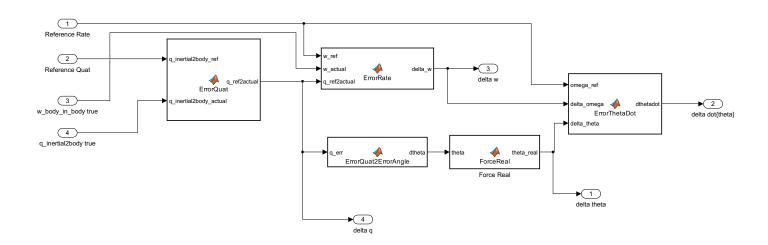
```
function h = CMG_AngMom(h0, theta)
% Produces the angular momentum vector for the CMGs
% Extract alpha and betas
alphas = theta(1:4);
betas = theta(5:8);

h = zeros(3,1);
for ii = 1:4
    h = h + h0*[sin(alphas(ii)); cos(alphas(ii))*cos(betas(ii)); cos(alphas(ii))*sin(betas(ii))];
end
```



Flight Software





function theta_real = ForceReal(theta)
theta_real = real(theta);

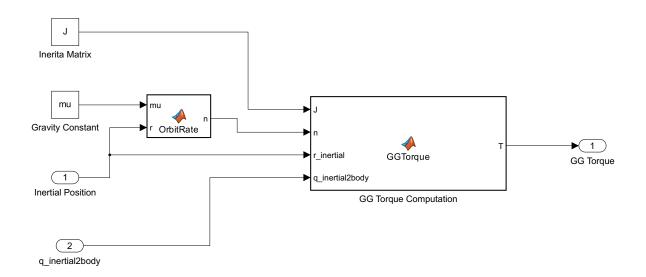
```
function q_ref2actual = ErrorQuat(q_inertial2body_ref, q_inertial2body_actual)
q_ref2actual = QuatProduct(q_inertial2body_actual,QuatInv(q_inertial2body_ref));
% Force normalization
q_ref2actual = q_ref2actual/norm(q_ref2actual);
```

```
function delta_w = ErrorRate(w_ref, w_actual, q_ref2actual)
delta_w = w_actual - QuatTransform(q_ref2actual,w_ref);
```

```
function dtheta = ErrorQuat2ErrorAngle(q_err)
% Quaternion components
q v = real(q_err(1:3));
q_s = real(q_err(4));

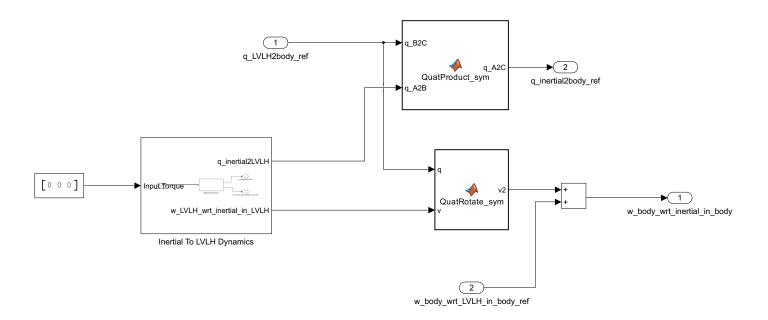
dtheta = [atan2(q_v(1),q_s);
    atan2(q_v(2),q_s);
    atan2(q_v(3),q_s)];
```

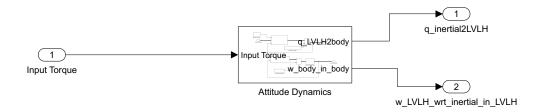
function dthetadot = ErrorThetaDot(omega_ref, delta_omega, delta_theta)
dthetadot = delta_omega - cross(omega_ref,delta_theta);

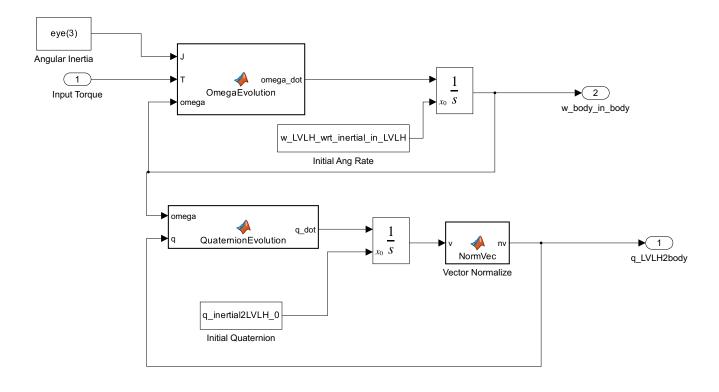


```
function T = GGTorque(J, n, r_inertial, q_inertial2body)
% Down in body frame
down_inertial = -r_inertial/norm(r_inertial);
down_body = QuatTransform(q_inertial2body,down_inertial);
% Gravity gradient torque
T = 3*n^2*cross(down_body,J*down_body);
```

```
function n = OrbitRate(mu,r)
n = sqrt(mu/norm(r)^3);
```







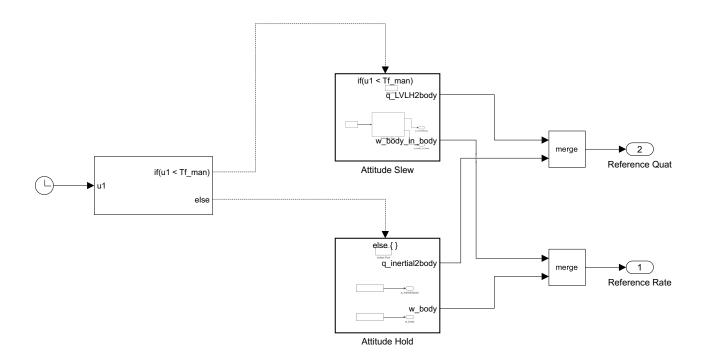
function q_dot = QuaternionEvolution(omega, q)
intermed = [omega; 0];
q_dot = 0.5*QuatProduct(intermed,q);

function omega_dot = OmegaEvolution(J, T, omega) omega_dot = $J \setminus (T - cross(omega, J*omega))$;

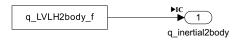
function nv = NormVec(v)
nv = v/norm(v);

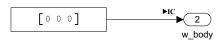
function q_A2C = QuatProduct_sym(q_B2C,q_A2B)
q_A2C = QuatProduct(q_B2C,q_A2B);

function v2 = QuatRotate_sym(q,v)
v2 = QuatTransform(q,v);

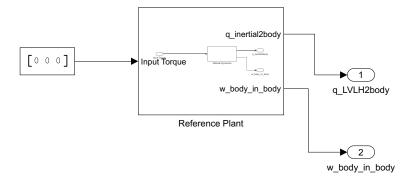


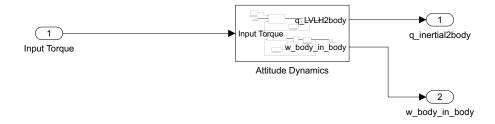
else { }
Action Port

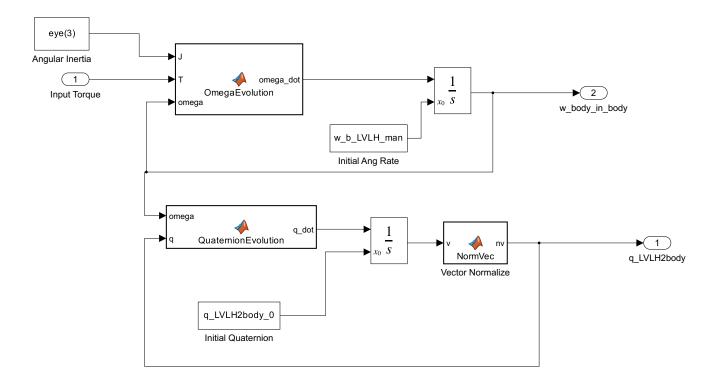




(u1 < Tf_man Action Port



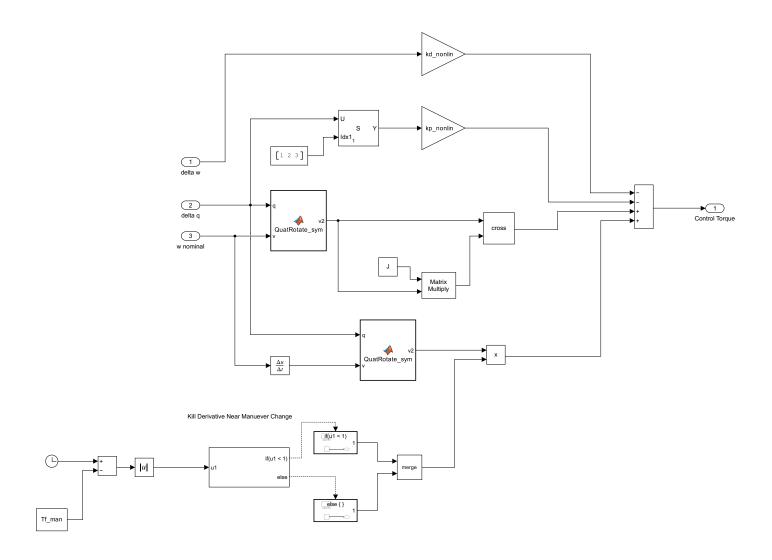


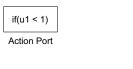


```
function q_dot = QuaternionEvolution(omega, q)
intermed = [omega; 0];
q_dot = 0.5*QuatProduct(intermed,q);
```

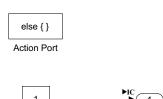
function omega_dot = OmegaEvolution(J, T, omega) omega_dot = $J \setminus (T - cross(omega, J*omega))$;

function nv = NormVec(v)
nv = v/norm(v);



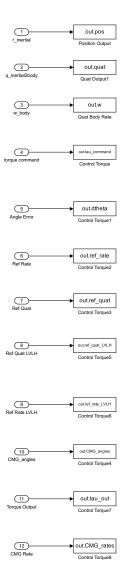




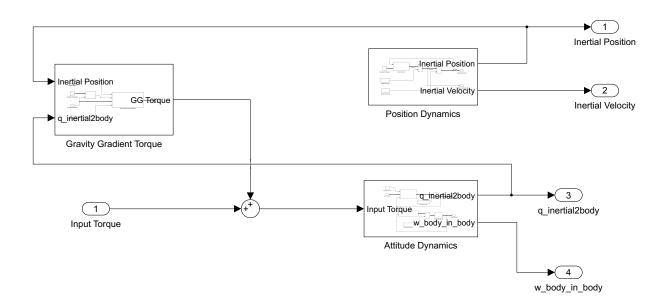


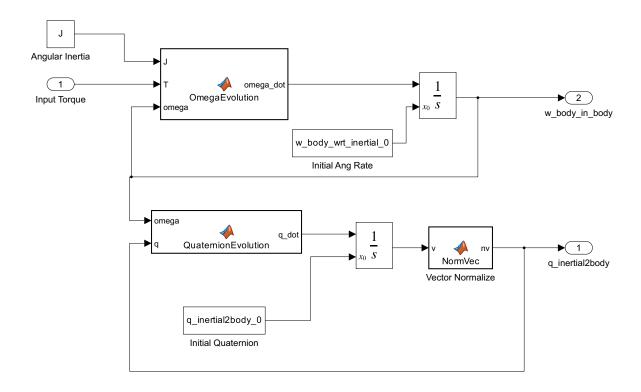
function v2 = QuatRotate_sym(q,v)
v2 = QuatTransform(q,v);

function v2 = QuatRotate_sym(q,v)
v2 = QuatTransform(q,v);



Plant

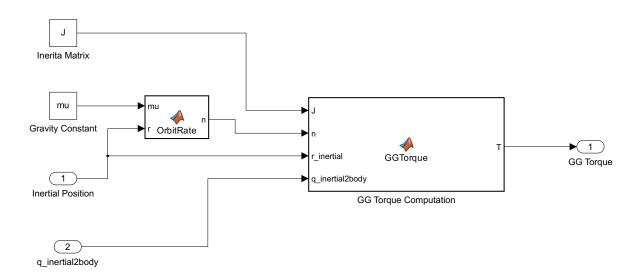




```
function q_dot = QuaternionEvolution(omega, q)
intermed = [omega; 0];
q_dot = 0.5*QuatProduct(intermed,q);
```

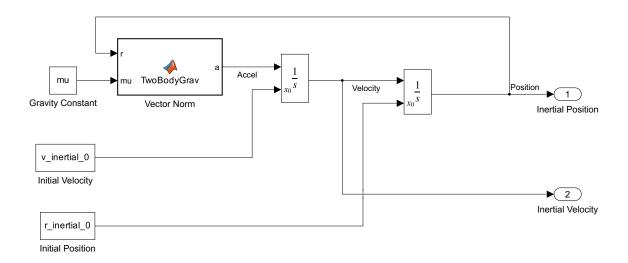
function omega_dot = OmegaEvolution(J, T, omega) omega_dot = $J \setminus (T - cross(omega, J*omega))$;

function nv = NormVec(v)
nv = v/norm(v);



```
function T = GGTorque(J, n, r_inertial, q_inertial2body)
% Down in body frame
down inertial = -r_inertial/norm(r_inertial);
down_body = QuatTransform(q_inertial2body,down_inertial);
% Gravity gradient torque
T = 3*n^2*cross(down_body,J*down_body);
```

```
function n = OrbitRate(mu,r)
n = sqrt(mu/norm(r)^3);
```



```
function a = TwoBodyGrav(r,mu)
nr = norm(r);
a = -mu*r/(nr^3);
```