Final Exam

Problem 1 (100 Points)

A spacecraft is in a 45 degree of inclination, 400km of altitude, circular orbit of your choice. The spacecraft is holding an inertially fixed attitude and you can assume the true inertial-to-body attitude is exactly constant:

$$\begin{aligned} \boldsymbol{q}_i^b(t) &= [0\ \ 0\ \ 0\ \ 1]^{\mathrm{T}} \qquad \forall t \\ \boldsymbol{\omega}_{b/i}^b(t) &= [0\ \ 0\ \ 0]^{\mathrm{T}} \qquad \forall t \end{aligned}$$

The spacecraft is equipped with a gyro and a magnetometer. The gyro has an angular random walk of 0.15 deg/$\sqrt{\text{hour}}$ and a bias instability of 0.3 deg/hour. Model the magnetometer error as a zero mean, white sequence with standard deviation of $2.5 \cdot 10^{-7}$ Tesla per axis. Run both sensors at 1 Hz.

Design a Kalman filter to estimate the attitude and gyro bias, assume an initial uncertainty of 1 degree per axis and an initial gyro bias uncertainty of 1 deg/hour. Assume the two sensors are perfectly aligned. Randomize the true initial gyro bias and the initial estimated quaternion and show the evolution of the estimation error and of the predicted estimation error covariance for a duration of 8 orbits, do a single simulation run, not a Monte Carlo. Create six plots, one per filter state. In each plot co-plot the estimation error and the square-root of the corresponding covariance matrix diagonal component. Plot both the negative and positive square root value (total of three lines per plot). You can feed the true position of the spacecraft to the Kalman filter.

You can use the simple dipole model for Earth's magnetic field from HW1, that is:

$$\boldsymbol{B}^n = B_0 \left(\frac{R_e}{\|\boldsymbol{r}\|} \right)^3 \begin{bmatrix} \cos(\lambda) \\ 0 \\ 2\sin(\lambda) \end{bmatrix} \tag{1}$$

$$\boldsymbol{B}^i = (\boldsymbol{T}_i^n)^{\mathrm{T}}\, \boldsymbol{B}^n \tag{2}$$

where $B_0 = 3.12 \cdot 10^{-5}$ Tesla is the mean value of magnetic field at the equator, R_e is Earth's equatorial radius, λ is the latitude, and $\boldsymbol{T}_i^n$ is the inertial-to-NED DCM.

The North-East-Down (NED or n) frame is one in which the x-axis is along the local north direction, the y-axis along the local east direction, and the z-axis along the local down direction. If the inertial z-axis $\boldsymbol{i}_z$ is pointed along the earth axis (you can ignore the difference between geographic and magnetic north), and if $\boldsymbol{r}^i$ is the position of the spacecraft in inertial coordinates, we have that:

$$\boldsymbol{n}_z^i = -\boldsymbol{r}^i / \|\boldsymbol{r}^i\| \qquad \text{(down direction)} \tag{3}$$

$$\boldsymbol{n}_y^i = (\boldsymbol{n}_z^i \times \boldsymbol{i}_z^i) / \|\boldsymbol{n}_z^i \times \boldsymbol{i}_z^i\| \qquad \text{(east direction)} \tag{4}$$

$$\boldsymbol{n}_x^i = \boldsymbol{y}_n^i \times \boldsymbol{n}_z^i \qquad \text{(north direction)} \tag{5}$$

$$\boldsymbol{T}_i^n = \begin{bmatrix} (\boldsymbol{n}_x^i)^{\mathrm{T}} \\ (\boldsymbol{n}_y^i)^{\mathrm{T}} \\ (\boldsymbol{n}_z^i)^{\mathrm{T}} \end{bmatrix} \tag{6}$$