Homework #1

Problem 1

Assume a satellite in a circular 500km orbit inclined 45deg with an inertial matrix of

$$\mathbf{J}_{cg}^{b} = \begin{bmatrix} 250 & 0 & 0 \\ 0 & 300 & 0 \\ 0 & 0 & 500 \end{bmatrix} \text{ kg m}^2$$

rotating with an initial angular velocity

$$\boldsymbol{\omega}^{b}(t_0) = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^{\mathrm{T}} \text{ deg/s}$$

you can choose an arbitrary initial attitude. Design a B-dot controller that stabilizes the angular velocity in less than 6000 seconds.

A very simple dipole model for Earth's magnetic field is

$$\mathbf{B}^{n} = B_0 \left(\frac{R_e}{\|\mathbf{r}\|} \right)^3 \begin{bmatrix} \cos(\lambda) \\ 0 \\ 2\sin(\lambda) \end{bmatrix} \tag{1}$$

$$\mathbf{B}^{i} = \left(\mathbf{T}_{i}^{n}\right)^{\mathrm{T}} \mathbf{B}^{n} \tag{2}$$

where $B_0 = 3.12e - 5$ Tesla is the mean value of magnetic field at the equator, R_e is Earth's equatorial radius, λ is the latitude, and $\mathbf{T}_{i}^{n}$ is the inertial-to-NED DCM.

The North-East-Down (NED or n) frame is one in which the x-axis is along the local north direction, the y-axis along the local east direction, and the z-axis along the local down direction. If the inertial z-axis $\mathbf{i}_z$ is pointed along the earth axis (ignoring the difference between geographic and magnetic north), and if $\mathbf{r}^{i}$ is the position of the spacecraft in inertial coordinates, we have that:

$$\mathbf{n}_{z}^{i} = -\mathbf{r}^{i}/\|\mathbf{r}^{i}\| \qquad \text{(down direction)} \tag{3}$$

$$\mathbf{n}_{y}^{i} = (\mathbf{n}_{z}^{i} \times \mathbf{i}_{z}^{i})/\|\mathbf{n}_{z}^{i} \times \mathbf{i}_{z}^{i}\| \qquad \text{(east direction)} \tag{4}$$

$$\mathbf{n}_{x}^{i} = \mathbf{y}_{n}^{i} \times \mathbf{n}_{z}^{i} \qquad \text{(north direction)} \tag{5}$$

$$\mathbf{T}_{i}^{n} = \begin{bmatrix} (\mathbf{n}_{x}^{i})^{\mathrm{T}} \\ (\mathbf{n}_{y}^{i})^{\mathrm{T}} \\ (\mathbf{n}_{z}^{i})^{\mathrm{T}} \end{bmatrix} \tag{6}$$

Plot the angular velocity history vs. time.

Problem 2

Consider the following inertia matrix:

$$\mathbf{J}_{cg}^{b} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 75 & 0 \\ 0 & 0 & 50 \end{bmatrix} \text{ kg m}^2$$

numerically integrate Euler's rotational equation

$$\dot{\boldsymbol{\omega}}^b_{b/i} = -(\mathbf{J}^b_{cg})^{-1}\Big(\boldsymbol{\omega}^b_{b/i} \times (\mathbf{J}^b_{cg}\boldsymbol{\omega}^b_{b/i})\Big)$$

for 2.5 hours assuming torque-free motion. Use the following initial conditions

- $\omega(0) = [0 \;\; 0.01 \;\; 0]^T$
- $\omega(0) = [0.01 \;\; 0.0001 \;\; 0.0001]^T$
- $\omega(0) = [0.0001 \;\; 0.01 \;\; 0.0001]^T$

Explain the differences between these three cases.

Problem 3

A cylindrical satellite has inertia matrix:

$$\mathbf{J}^b_{cg} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 60 \end{bmatrix} \text{ kg m}^2$$

and initial angular velocity

$$\boldsymbol{\omega}^b_{b/i}(0) = \begin{bmatrix} 1 & 0 & 10 \end{bmatrix}^{\mathrm{T}} \text{ deg/s}$$

The initial attitude is such that the inertial z-axis $\mathbf{i}_z$ expressed in body-coordinates is given by $\mathbf{i}^b_z(0) = \mathbf{J}^b_{cg}\boldsymbol{\omega}^b_{b/i}$ and the inertial and body y-axes are aligned $\mathbf{i}_z(0) = \mathbf{b}_y(0)$.

Integrate the attitude of this torque-free system for 3000 seconds and plot the following quantities:

1. The angular velocity expressed in inertial coordinates
2. The body z-axes expressed in inertial coordinates

Problem 4

Assume a 400 km equatorial posigrade circular orbit, and assume the satellite has nominally the body x-axis pointing in the direction of motion, the body y-axis pointing to nadir, and it is spinning at orbit rate around the z-axis (i.e. this is an Earth pointing satellite). At time t = 0 the satellite position in the ECI frame is

$$\mathbf{r}^i(0) = \begin{bmatrix} 0 & -(R_e + 400km) & 0 \end{bmatrix}^{\mathrm{T}}$$

where R_e is the equatorial Earth's radius. The inertial matrix is

$$\mathbf{J}^b_{cg} = \begin{bmatrix} 90 & 0 & 0 \\ 0 & 70 & 0 \\ 0 & 0 & 60 \end{bmatrix} \text{ kg m}^2$$

assume nominal initial attitude and a slightly perturbed initial angular velocity

$$\boldsymbol{\omega}^b_{b/i}(0) = \begin{bmatrix} 0.0001 & 0.0001 & \omega_{orb\ rate} \end{bmatrix}^{\mathrm{T}} \text{ deg/s}$$

Propagate the attitude dynamics and quaternion kinematics for 5 orbits and plot the results. Introduce gravity gradient perturbations and repeat the above propagation.