Homework #3

Problem 1

James Webb Space Telescope (JWST) has six reaction wheels with moment of maximum inertia $J_w = 0.1295$ kg m^2 and maximum wheel speed of 6000 rpm . The columns of matrix $\boldsymbol{X}^b_{rw}$ below are the orientation of each wheel in body coordinates

$$\boldsymbol{X}^b_{rw} = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2}\cos 30^o & \frac{\sqrt{3}}{2}\cos 30^o & 0 & -\frac{\sqrt{3}}{2}\cos 30^o & -\frac{\sqrt{3}}{2}\cos 30^o \\ \cos 30^o & \frac{1}{2}\cos 30^o & -\frac{1}{2}\cos 30^o & -\cos 30^o & -\frac{1}{2}\cos 30^o & \frac{1}{2}\cos 30^o \\ \sin 30^o & \sin 30^o & \sin 30^o & \sin 30^o & \sin 30^o & \sin 30^o \end{bmatrix}$$

JWST's inertial matrix $\boldsymbol{J}^b_{cm}$ is given by

$$\boldsymbol{J}^b_{cm} = \begin{bmatrix} 67946 & -83 & 11129 \\ -83 & 90061 & 103 \\ 11129 & 103 & 45821 \end{bmatrix} \text{ kg m}^2 \tag{1}$$

At time $t = 0$, JWST has zero angular velocity $\boldsymbol{\omega}_{b/i}$ and inertial-to-body attitude given by $\boldsymbol{q}^b_i(0) = [0 \ \ 0 \ \ 0 \ \ 1]^{\mathrm{T}}$, where the quaternion has scalar part LAST.

Do the following:

1. Calculate the final attitude of JWST when executing a 900 second slew with nominal angular velocity of $\boldsymbol{\omega}^b_{b/i} = \begin{bmatrix} 0 & 0 & 1/300 \end{bmatrix}^{\mathrm{T}}$ deg/s.
2. Design a controller that
   - calculates the torque needed to perform the above attitude slew (i.e. it controls the angular velocity and attitude histories to match the two values you calculated for $0 \leq t \leq 900$ s)
   - calculates the torque needed to hold the final attitude for all times $t > 900$ s
3. Simulate reaction wheels and their steering law such that
   - RW command: calculates the commanded wheels angular acceleration needed to deliver the control torque
   - Reaction Wheels: models the dynamics of the wheel, that it, it integrates the angular acceleration command to obtain the wheels angular velocity, angular momentum, and torque
4. Simulate the JWST kinematic/dynamics controlled by the wheels torque
5. The top level of your code must contain 3 separate functions/blocks/subsystems (you can add a fourth one with initial conditions if you so wish)
   (a) Flight Software: this contains three flight software components
      i. Calculation of the nominal attitude (a.k.a. maneuver/pointing logic)
      ii. Attitude controller
      iii. RW command
   (b) Flight Hardware: this contains one actuator

        i. 6-wheel RW system

    (c) Environment: this contains

        i. Attitude dynamics
        ii. Attitude kinematics

6. Run your code for 1800 seconds
7. Include a snapshot of all your code with your homework (do not attache the source files)

Produce the following plots:

1. Angular velocity of the satellite in body coordinates vs. time (the actual value, not the nominal value)
2. Inertial-to-body attitude quaternion components vs. time (the actual value, not the nominal value)
3. The angular velocity of each of the six wheels vs. time
4. The torque generated by the wheels in body coordinates vs. time

Problem 2

Code up a controller and steering law, assuming

- The inertias, orbit, and CMGs of the International Space Station
- Holding LVLH attitude (notice that this is not an equilibrium point as the ISS inertia is not diagonal in body coordinates)
- Gravity gradient perturbation