Beyond simple maps - Integrating space and time with Bayesian models

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October 8, 2021

Presentation Structure

- Spatial and temporal demography
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- Data sources
- Modeling strategies
- Results & visualizations
- Wrap up

Spatial Demography

Space & Time

- Future directions in spatial demography report

Space & Time data models

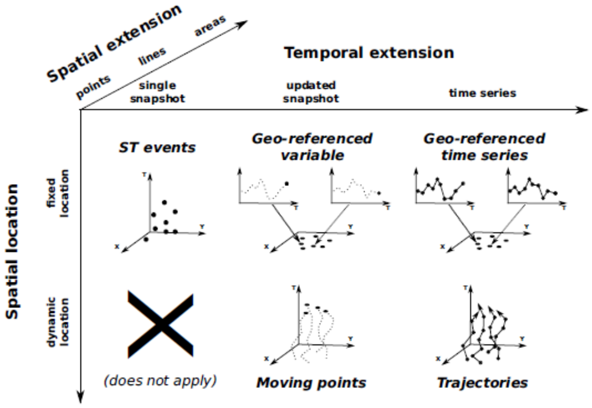

Fig. 44.1. Context for ST Clustering

- Kisilevich et al 2010

Complexities

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Data sources

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How to combine these things?

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Hierarchical Models

- I specify a Bayesian Hierarchical model for the age-standardized mortality rate
- $$Y_{ij} \sim N(\mu_{ij}, \tau_y)$$
- $$\mu_{ij} = \beta_0 + x'\beta + \gamma_j * black_i + u_j +$$
- $$\gamma_1 * \text{time} + \gamma_2 * (\text{time} * black_i) + \gamma_3 * (\text{time} * seg_i)$$
- $$\gamma_j \sim \text{CAR}(\bar{\gamma}_j, \tau_\gamma / n_j)$$
- $$u_j \sim \text{CAR}(\bar{u}_j, \tau_u / n_j)$$
- Vague Gamma priors for all the τ's
- Vague Normal priors for all the fixed effect β's and γ's

Methods - Bayesian analysis

- This type of model is commonly used in epidemiology and public health
- Various types of data likelihoods may be used
- Need to get at:

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$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

- Traditionally, we would get $p(\theta|y)$ by:
  - either figuring out what the full conditionals for all our model parameters are (hard)
  - Use some form of MCMC to arrive at the posterior marginal distributions for our parameters (time consuming)

Methods - INLA approach

- Integrated Nested Laplace Approximation - Rue, Martino & Chopin (2009)
- One of several techniques that approximate the marginal and conditional posterior densities
  - Laplace, PQL, E-M, Variational Bayes
- Assumes all random effects in the model are latent, zero-mean Gaussian random field, x with some precision matrix
  - The precision matrix depends on a small set of hyperparameters
- Attempts to construct a joint Gaussian approximation for $p(x|\theta, y)$
  - where θ is a small subset of hyper-parameters

Methods - INLA approach

- Apply these approximations to arrive at:
- $\tilde{\pi}(x_i|y) = \int \tilde{\pi}(x_i|\theta, y)\tilde{\pi}(\theta|y)d\theta$
- $\tilde{\pi}(\theta_j|y) = \int \tilde{\pi}(\theta|y)d\theta_{-j}$
- where each $\tilde{\pi}(.|.)$ is an approximated conditional density of its parameters
- Approximations to $\pi(x_i|y)$ are computed by approximating both $\pi(\theta|y)$ and $\pi(x_i|\theta, y)$ using numerical integration to integrate out the nuisance parameters.
  - This is possible if the dimension of θ is small.
- Approximations to $\tilde{\pi}(\theta|y)$ are based on the Laplace appoximation of the marginal posterior density for $\pi(x, \theta|y)$
- Their approach relies on numerical integration of the posterior of the latent field, as opposed to a pure Gaussian approximation of it

INLA in R

```
library(INLA)
```

Unstructured Model

```
mod1<-std_rate~male+black+scale(lths)+pershigdis*year
+f(year,model="iid") +f(conum, model="iid")
```

Spatially structured Model with Random Slope

```
mod2<-std_rate~male+black+scale(lths)+pershigdis*year
+f(conum, model="bym", graph="usagraph.gra") +f(year,
model="iid") +f(year, black,
model="besag",graph="usagraph.gra")
```

Discussion

- We see that, while there is a persistence of the gap in black-white mortality:
  - The mortality gap appears to be fairly consistent over time
  - In highly segregated areas, the mortality difference are decreasing
  - Suggests some evidence to support the Williams and Collins (2001) perspective
- INLA allows for rapid deployment of Bayesian statistical models with latent Gaussian random effects
  - Faster and *generally* as accurate as MCMC
  - Potentially an attractive solution for problems where large data/complex models may make MCMC less desireable

Thank you!

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