PID Design

Lab 4: ECE 147A

Student : Corey Gravelle
Perm Number: 8094211
E-mail: cgravelle@umail.ucsb.edu

11/16/2009

Department of Electrical and Computer Engineering, UCSB

1. Abstract

The purpose of this lab is to study the properties of the proportional integral and derivative (PID) controller, and to implement it with a cart to discover any possible simulation-to-actuality discrepancies. I modeled a proportional, integrating, and differentiating path in parallel. After simulating with SIMULINK and MATLAB, I connected our models to a physical cart and found the high-frequency noise and gain was effected by the differentiating path. After tuning the functions correctly, I found the PID controller was an excellent method of controlling the cart.

I will then control the response of a cart even while it is connected to another one via a spring. One should compare the differences, if any, between theoretical simulations and actuality and draw conclusions on why these differences are made. By using two different transfer functions, one taking the second mass into account and one not, one may find the robustness of a response is not entirely dependent on the given plant.

2. Introduction

Separately, the proportional, integrating, and differentiating controllers each have their flaws. However, together, they may balance each other out to result in a near ideal controller. By simulating before applying, I will be able to predict the results of each path. However, there are always other factors involved such as path noise, solid-state variances, inertia, and mechanical problems to deal with, so it is necessary to implement with a real-life mechanical application.

One may assume the following approximations in modeling the system shown in Figure 1... F is the force supplied based on an input voltage V:

$$F = \frac{K_m K_g}{R_m r} V - \frac{K_m^2 K_g^2}{R_m r^2} \dot{x}_1$$

Given this and the accleation state equations as follows:

$$\begin{aligned} F - k(x_1 - x_2) &= m_1 \ddot{x}_1 \\ k(x_1 - x_2) &= m_2 \ddot{x}_2. \end{aligned}$$

One may find the Fourier transform and combine the equations to arrive at:

$$\begin{aligned} X_2(s)(m_2 s^2 + k) &= k X_1(s) \\ X_1(s)(m_1 s^2 + \frac{K_m^2 K_g^2}{R_m r^2} s + k) &= \frac{K_m K_g}{R_m r} V(s) + k X_2(s) \end{aligned}$$

Plugging in actual values for Km, Rm, Kg, r, m1 and m2, one will arrive at the final transfer functions:

$$
\begin{aligned}
\frac{X_1(s)}{V(s)} &= 2.97\frac{s^2+61.2}{s^4+13.24s^3+127.15s^2+810.37s} \\
\frac{X_2(s)}{X_1(s)} &= \frac{61.2}{s^2+61.2},
\end{aligned}
$$

Using these transfer functions as a model, one should be able to find desirable controllers to manage both positions with relative ease.

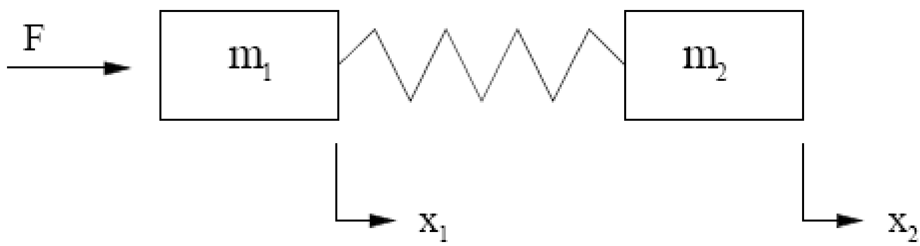


Figure 1: Free body diagram of two carts m1 and m2 connected by spring

3. Procedure

1.Design a closed-loop controller for the transfer function X1(s) / V(s).

(a) Force all poles as far left as possible, picking a K and using the root-locus plot.

(b) Plot the 0.1 magnitude step response for the closed loop system when C(s) is the following: K/2 , 3K/4 ,K, 5K/4 , and 3K/2. What gain has the fastest settling time, and where are the closed loop poles and zeros for each controller?

(c) Theoretically, for what values of C (as given as a fraction of K above) is the controller stable?

(d) For which of the above controllers does the output of the controller exceed the range 5 volts (or, *saturate*) for a step input with the above mentioned magnitude? For what range of gains would you expect saturation? How would you expect saturation to change the step response?

2. Repeat the above for X2(s) / V (s) .

4.**Results and Discussion**

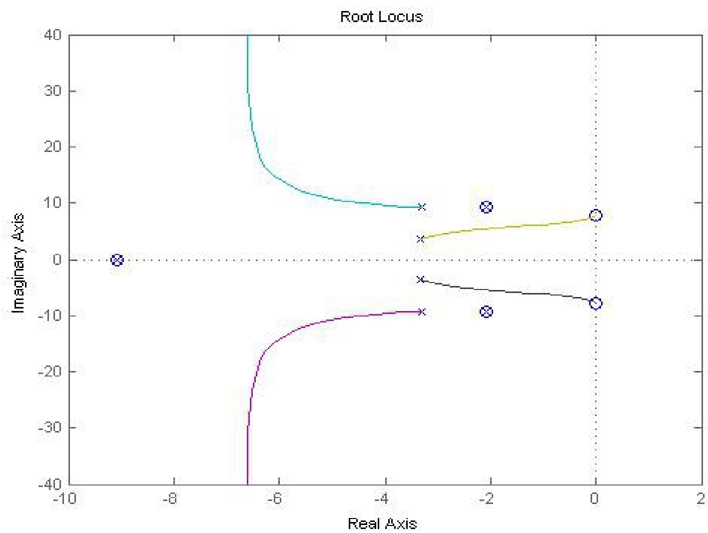


Figure 2: Root Locus Plot to find ideal K

II. *Set TR= 0 and TD= 0 to obtain a constant gain controller. As you increase the value of K, what happens to rise time, settling time, overshoot, steady-state error and stability? Select convincing plots to support your claim. How do the simulated and actual step responses vary?Select a constant gain controller which gives the system step response a small overshoot.*

To find the ideal response poles, it was necessary to line them up exactly. I found the ideal value using the overall plant X2(s) / V(s), and found this K to be 13.1.

As one increases the value of K, the rise time decreases and the settling time decreases as well (see figure 1). A proportional controller of gain K in negative feedback may be modeled by (X-Y)*K = Y; therefore, the transfer function Y/X is K/(1+K). To obtain a perfect steady-state response, the gain must be infinite. Therefore, as one increases the gain K, the steady-state error decreases as well. There is no stability issue, because for any bounded input you get a proportional output. That is, the output does not depend on s, so there is no time or frequency dependency. By the same argument, there are no rise-time issues.

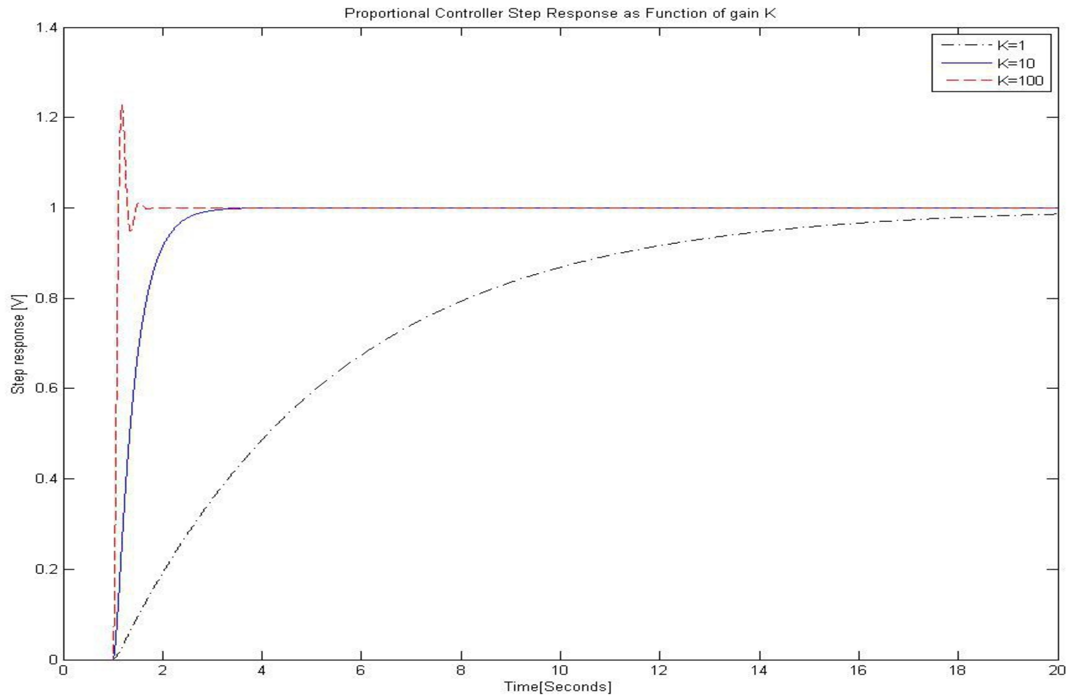
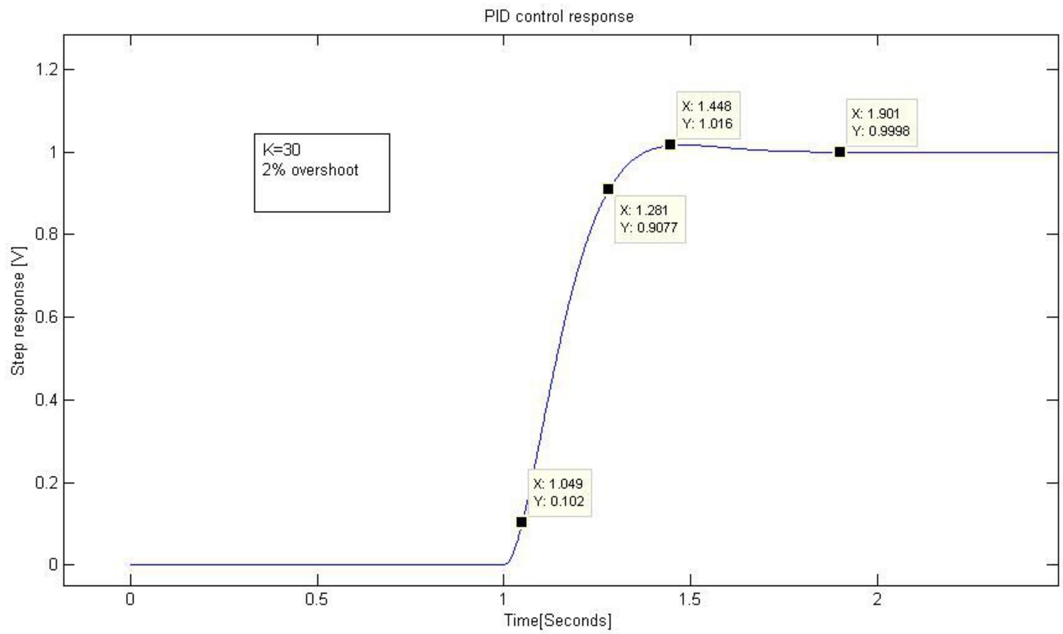


Figure 3: Comparing outputs with varying gains K

III. *If you increase TI, what happens to rise time, settling time, overshoot, and stability?*

Increasing TI results in a lengthening in the settling time but eliminates the steady-state error (see **figure 4**). Also, with a TI value of 0.01 or below, the output becomes unstable (**figure 5**) because the gain K/TI is too great. That is, if the gain is less than or equal to 0.06, it oscillates indefinitely. This is because the pole lies directly on the imaginary axis and is neither converging nor unstable (see **figure 6**). This may be found by using the root locus plot (**figure 7**).



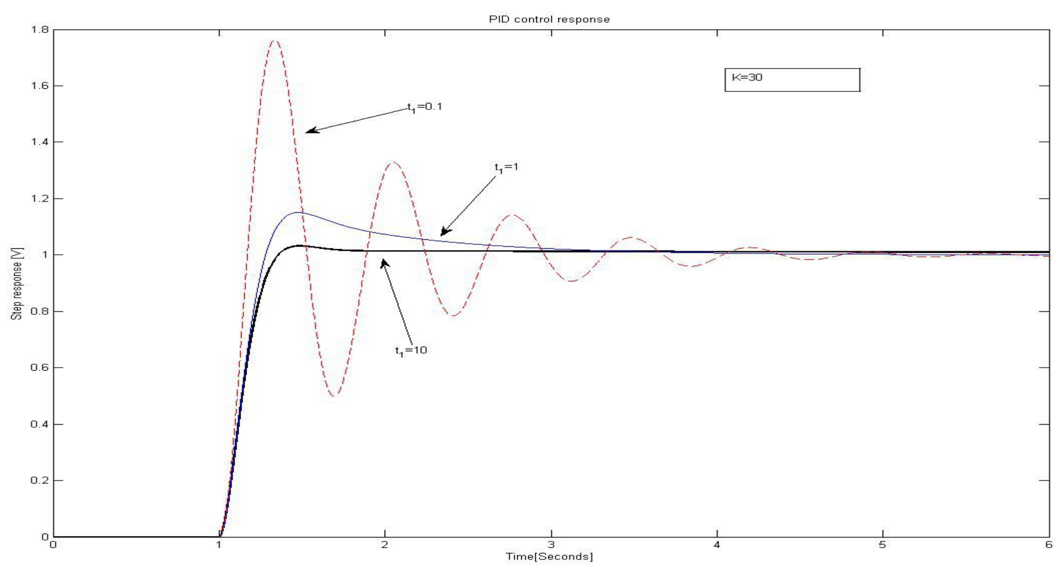


Figure 4: PID control response with varying TI

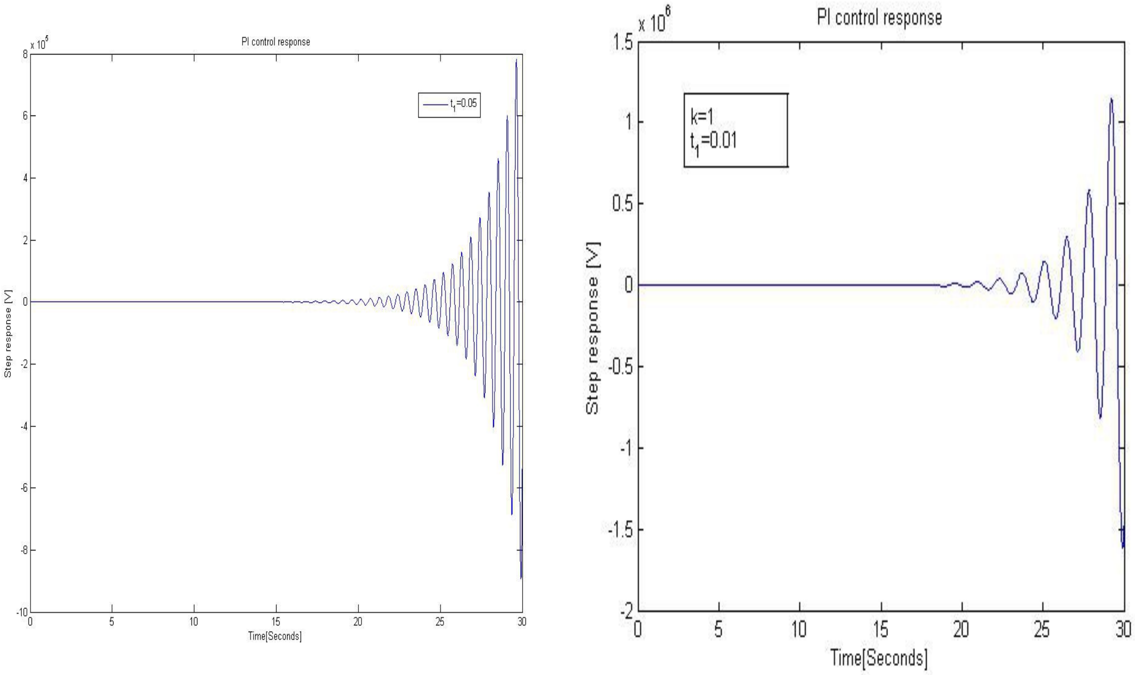


Figure 5: Unstable function for the gain K/TI at 0.05 and 0.01, respectively.

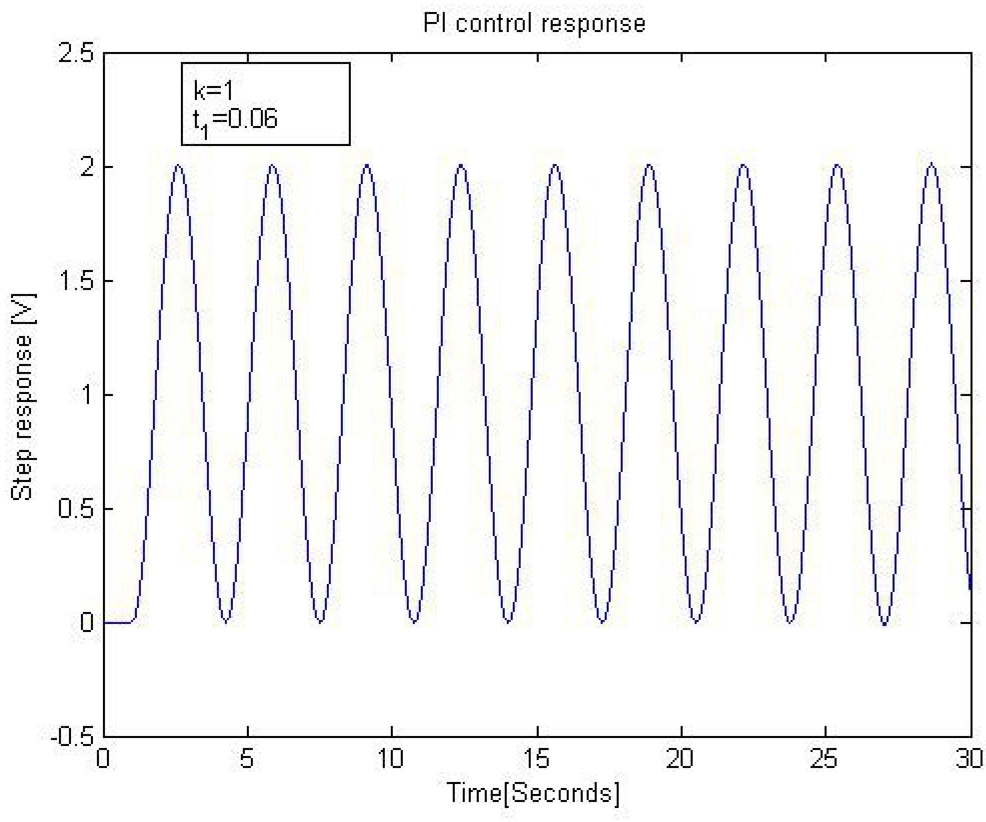


Figure 6: Oscillating response of controller as gain K/TI = 0.06.

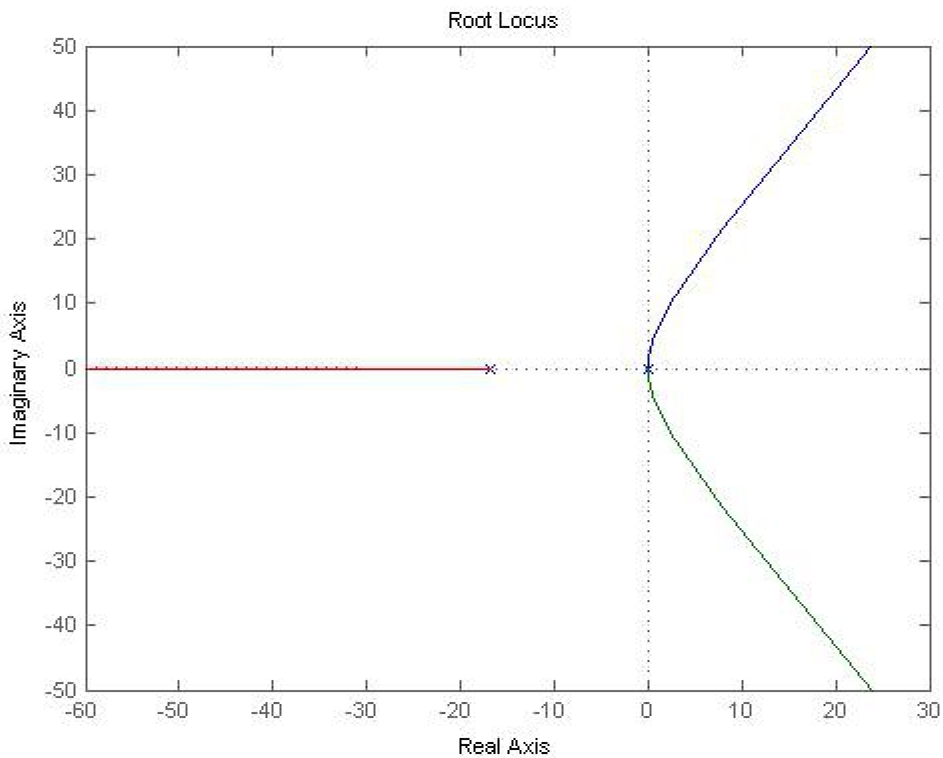


Figure 7: Root Locus plot to find gain limit of the PI controller

IV) *What happens if you select a small gain but TR is very large?*

With a small gain K and a large TR, the transfer function has only a fraction of its output effected by the integrator path. The step response has a small portion effected by the integrator and acts largely like a proportional response with a slightly quicker rise-time.

V) *Select a constant gain controller which has significant overshoot. Slowly increase TD. What happens to rise time, settling time, overshoot, steady-state error and stability?*

Increasing TD to a controller with a significant overshoot decreases the rise time a great deal. The settling time also decreases. Depending on what TD one picks, the overshoot may decrease or increase. If it is close to the desired value (TD = 0.1 in **figure 8**) then it has a smaller overshoot than the original. However, if it is increased too much, too much of the controller is dependent on the slope of the step and the overshoot increases. Also, if TD is increased further, it would lead to instability because measurement noise is high-frequency and the derivative path will amplify this.

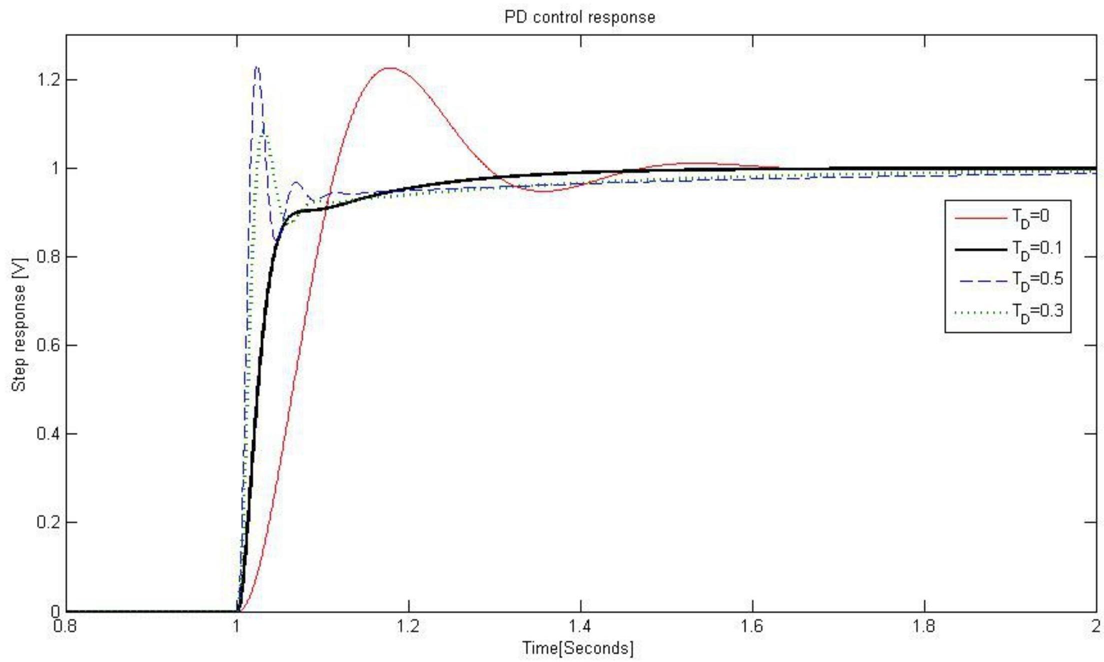


Figure 8: PD controller response with varying TD

VI) *Slowly increase TD until you hear the motor cart start to vibrate. Vibrations such as these are often caused by high gain and/or a noisy signal. In this case we have both. Can the vibration occur if you do not have derivative control?*

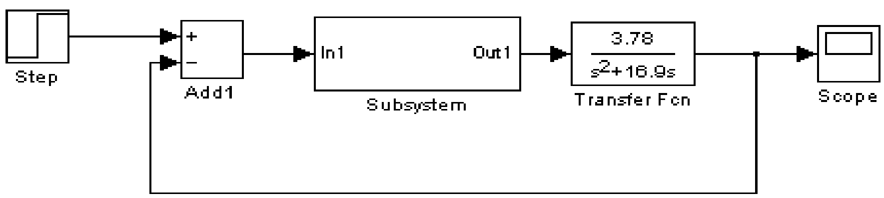
Without derivative control, the vibration would likely not occur, as the derivative control amplifies the high-frequency aspect of the noisy signal and high gain.

VII) *What is the physical significance of tau? What happens if you change tau?*

Tau is the bandwidth that the derivative is responding to; if you decrease this, the frequency wd of the signals that the derivative is acting on will decrease as well.

VII) *What plant information is required to tune a PID controller? Is it mandatory to have a model of the plant?*

You do not necessarily need to know the exact transfer function of the PID controller, though it is typically necessary to know only its response to step response data. Process reaction curves may be generated from experimental data with simulations, and from this one may find necessary gain and coefficient values.



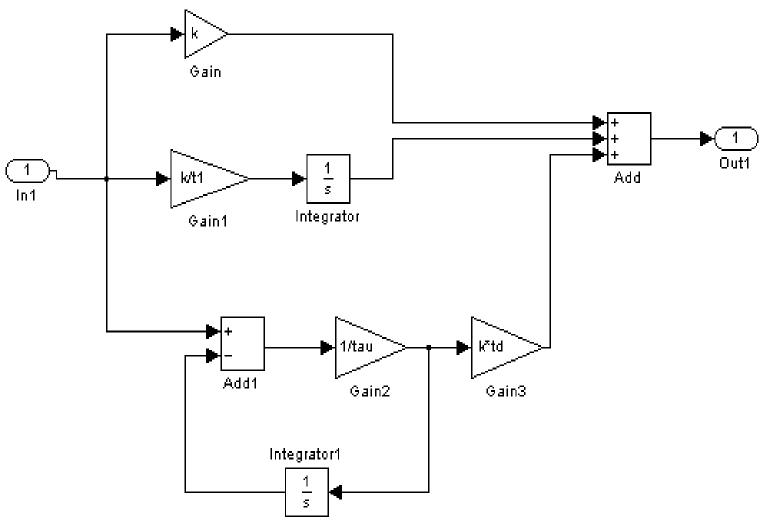


Figure 9: SIMULINK models for step-gain subsystem and overall system with cart simulation.

Conclusion

By using a PID controller, one may customize the response of an output and tune it to desired conditions. The proportional term essentially mimics the original signal through the plant, with a certain gain. However, there is a steady-state error which should be accounted for. The integrating path accounts for this error and may be tuned to decrease ringing and overshoot. The differentiating path essentially speeds up the response of the plant and counterbalances the integrator path's overshoot.