

Programming Homework of Stochastic Processes, Fall 2020

Due 1:30pm, Dec. 14, 2020.

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Problem 1.

Let $\mathcal{N} = \{1, 2, 3, \dots\}$ be the set of natural numbers. Let $s[n]$ be a regular process. In particular,

$$s[n] = b_0 \cdot i[n] + b_1 \cdot i[n-1], \quad (1)$$

where $b_0 = 0.6$, $b_1 = 0.4$, and $\{i[n], -\infty < n < \infty\}$ is a white Gaussian noise process with $E[(i[n])^2] = 1$.

Let Q be a positive integer. In addition,

$$\hat{s}[n] = \hat{E}\{s[n] | s[n-k], 1 \leq k \leq Q\}. \quad (2)$$

Define $\hat{P}[n]$ as follows.

$$\hat{P}[n] = \frac{1}{n} \sum_{k=1}^n (\hat{s}[k] - s[k])^2. \quad (3)$$

For each $\delta \in (0, 1)$, define $T(\delta)$ as the minimum positive integer n such that $|\hat{P}[n] - \hat{P}[n-1]| \leq \delta \cdot \hat{P}[n-1]$ in your simulation.

Assume that $Q = 10$. Write a computer program (in Matlab, C/C++, or Python) to obtain the answers for the following tasks. Your figures should be in good quality. It is highly recommended that you use Matlab to plot the figures.

- (1) Obtain and plot the values of $R_s[m]$, $\forall 0 \leq m \leq 10$, in Figure 1.
- (2) Plot $\{\hat{s}[n], 10000 \leq n \leq 10100\}$ and $\{s[n], 10000 \leq n \leq 10100\}$, in Figure 2.
- (3) Plot $\hat{P}[n]$, $\forall 10000 \leq n \leq 10100$, in Figure 3.
- (4) Plot $T(\delta)$, $\forall \delta \in \{0.1, 0.05, 0.03, 0.02, 0.01\}$, in Figure 4.

(5) Write a short report that includes your figures and codes. In addition, you should explain how you obtain the answers in English. Name your Matlab program YourStudentId.m (or your Python program YourStudentId.py). Hand in a hardcopy of your report. In addition, submit your report and programs to the new E3 website.